Matching a close coupling asymptotic form to the "half–discrete" representation of the 2 particle free–particle test case : Psi0 = G0 Phi0 with Phi0 = Exp[–a r1 –b r2]

■ Definition of the "half–discrete" representation of G0

```
In[221]:= Clear[p, eps, a, b, K, L, ene, nmax, z, r1, r2, r1p, r2p, s, c, kmax, z, G, rho, BesH]

          a = 2;
          b = 2;

          p[eps_] := If[eps > 0, Sqrt[2 * eps], I * Sqrt[-2 * eps]]
          Kn[n_, L_] := n * Pi / L;

          Psi0[r1_, r2_, K_] :=
           Evaluate[(2 / L) * Sum[Sin[r2 * Kn[n, L]] * (Kn[n, L] / (b^2 + Kn[n, L]^2)) *
               2 * (Exp[-a * r1] - Exp[I * p[(1 / 2) * K^2 - (1 / 2) * Kn[n, L]^2] * r1]) /
                (a^2 + p[(1 / 2) * K^2 - (1 / 2) * Kn[n, L]^2]^2), {n, 1, nmax}]];
```

■ "exact asymptotic form of the wave function Psi0

```
In[224]:= Psiasymp[r1_, r2_, K_] :=
           - Exp[-I * 3 * Pi / 4] * 2 * Sqrt[2 / Pi] * Exp[I * K * rho[r1, r2]] * (1 / Sqrt[K * rho[r1, r2]]) *
            (K * c[r1, r2] / (a^2 + (K * c[r1, r2])^2)) * (K * s[r1, r2] / (b^2 + (K * s[r1, r2])^2))
```

■ singly differential cross section from the outgoing flux of the exact Psi0

Note the normalization is such that
Total xsec = Integral[difflux[Sqrt[2*energy1]] , K] d energy1

```
In[225]:= diffflux[k1_, AK_] :=
```

$$(1 / (k1 * \text{Sqrt}[AK^2 - k1^2])) * 4 * (2 / \text{Pi}) \left(\frac{k1 * \text{Sqrt}[AK^2 - k1^2]}{(a^2 + k1^2)\,(b^2 + AK^2 - k1^2)} \right)^2 ;$$

■ Test that integral against earlier result for K=2, where total xsec = 0.00654729

```
In[226]:= a = 2;
          b = 2;
          Ktot = 2;
          NIntegrate[diffflux[Sqrt[2 * ene], Ktot], {ene, 0.0001, 0.5 * Ktot^2 - .00001}]

Out[226]= 0.00654728
```

■ Matching by the following prescription amp[n] = <chi n(r1)|Psi0(r1,r2,K)>*exp[–I p[[n]] r2]*Sqrt[p[[n]]]

▪ Derive the projection by a particle in a box function on box of length L1 onto the function Sin[Kn[n,L2]*r]

```
In[227]:= Clear[L1, L2, Kn, Lmax, r];
          InputForm[Integrate[Sqrt[2 / L1] * Sin[Kn[m, L1] * r] * Sin[Kn[n, L2] * r], {r, 0, L1}]]

  Out[227]//InputForm=
        (Sqrt[L1^(-1)]*(Kn[m, L1]*
              Sin[L1*Kn[m, L1] - L1*Kn[n, L2]] +
            Kn[n, L2]*Sin[L1*Kn[m, L1] - L1*Kn[n, L2]] -
            Kn[m, L1]*Sin[L1*Kn[m, L1] + L1*Kn[n, L2]] +
            Kn[n, L2]*Sin[L1*Kn[m, L1] + L1*Kn[n, L2]]))/
          (Sqrt[2]*(Kn[m, L1] - Kn[n, L2])*(Kn[m, L1] + Kn[n, L2]))
```

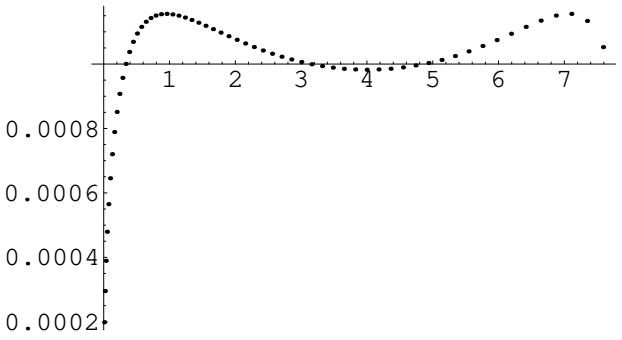
■ Define the amplitude from the projection WITHOUT the McCarthy weights which come later

```
In[228]:= Clear[p, a, b, Kn, eps, n, m, L1, L2, nmax, K];
          p[eps_] := If[eps > 0, Sqrt[2 * eps], I * Sqrt[-2 * eps]]
          a = 2;
          b = 2;

          Kn[n_, L_] := n * Pi / L;

          nmax = IntegerPart[1.2 * K * L2 / Pi];

          amp [m_, L1_, L2_, r1_] := Evaluate[Sqrt[p[(1 / 2) * K^2 - (1 / 2) * Kn[m, L1]^2]] *
             Exp[-I * p[(1 / 2) * K^2 - (1 / 2) * Kn[m, L1]^2] * r1] * (2 / L2) *
             Sum[((Sqrt[L1^(-1)] * (Kn[m, L1] * Sin[L1 * Kn[m, L1] - L1 * Kn[n, L2]] +
            Kn[n, L2] * Sin[L1 * Kn[m, L1] - L1 * Kn[n, L2]] -
            Kn[m, L1] * Sin[L1 * Kn[m, L1] + L1 * Kn[n, L2]] +
            Kn[n, L2] * Sin[L1 * Kn[m, L1] + L1 * Kn[n, L2]])) /
           (Sqrt[2] * (Kn[m, L1] - Kn[n, L2]) * (Kn[m, L1] + Kn[n, L2]))) *
               (Kn[n, L2] / (b^2 + Kn[n, L2]^2)) * 2 *
               (Exp[-a * r1] - Exp[I * p[(1 / 2) * K^2 - (1 / 2) * Kn[n, L2]^2] * r1]) /
                (a^2 + p[(1 / 2) * K^2 - (1 / 2) * Kn[n, L2]^2]^2),
              {n, 1, nmax}]];
```

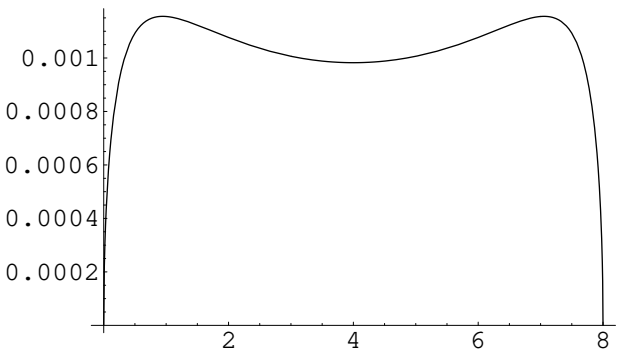
■ Now Finally do the test of using both Box sizes the SAME L1 = L2

In[230]:=
```
Clear[mopen, L1, L2];
K = 4;
L1 = 50;
L2 = 50.0001;
mopen = IntegerPart[K * L1 / Pi];
Print[" open channels = ", mopen // N]
Print[" terms in wave function =", nmax // N]

plotvec = {};
Do[{
   ampsqwt = (Abs[amp [m, L1, L2, rmatch]]^2) /  ((Kn[m + 1, L1]^2 - Kn[m - 1, L1]^2) / 4);
  plotvec = Append[plotvec, { 0.5 * Kn[m, L1]^2, ampsqwt}]
 },
 {m, 2, mopen - 1}];

plotdiscrete = ListPlot[plotvec, PlotRange -> All]

diffflux[k1_, AK_] :=

 (1 / ( k1 * Sqrt[AK^2 - k1^2])) * 4 * (2 / Pi) ( k1 * Sqrt[AK^2 - k1^2] / ((a^2 + k1^2) (b^2 + AK^2 - k1^2)) )^2 ;

pexact = Plot[diffflux[Sqrt[eps1 * 2], K], {eps1, 0, K^2 / 2}, PlotRange -> All]
Show[plotdiscrete, pexact]
```

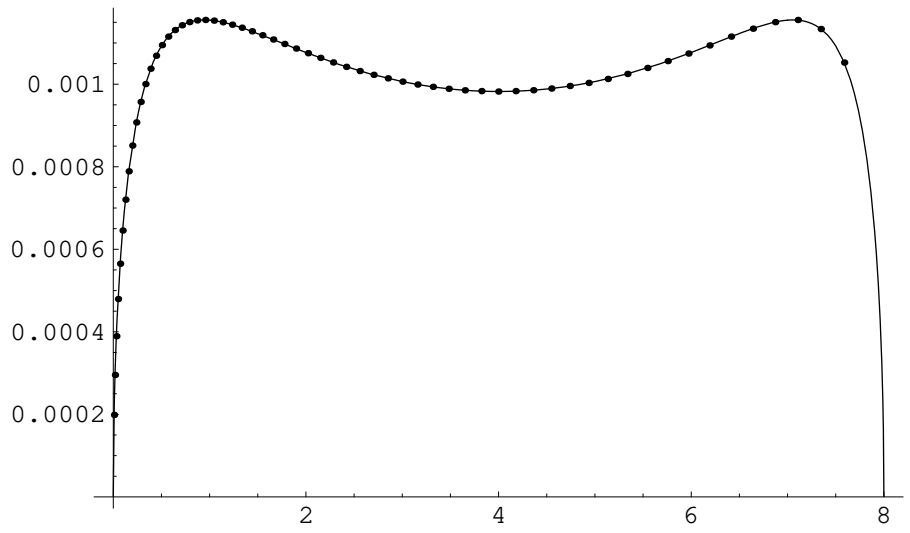
```
 open channels = 63.

 terms in wave function =76.
```



Out[232]= - Graphics -



Out[233]= - Graphics -



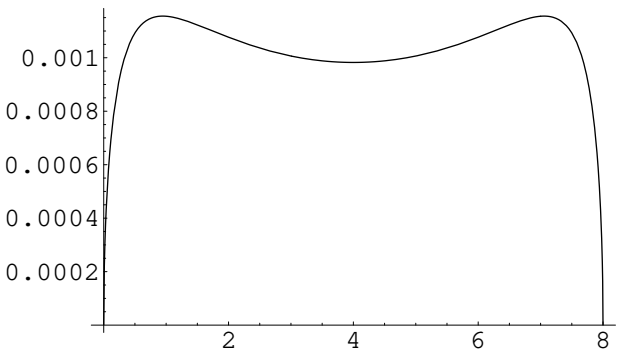
Out[234]= - Graphics -

■ Box sizes slightly DIFFERENT with L2 > L1

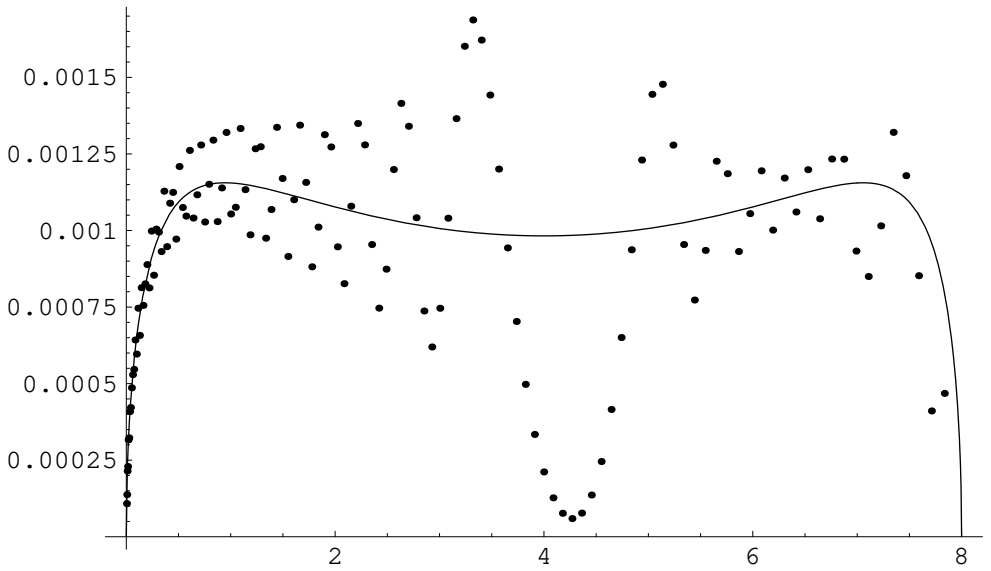
In[260]:=
```
Clear[mopen, L1, L2];
K = 4;
L1 = 100;
L2 = 105.01;
mopen = IntegerPart[K * L1 / Pi];
Print[" open channels = ", mopen // N]
Print[" terms in wave function =", nmax // N]

plotvec = {};
Do[{
   ampsqwt = (Abs[amp [m, L1, L2, rmatch]]^2) /  ((Kn[m + 1, L1]^2 - Kn[m - 1, L1]^2) / 4);
  plotvec = Append[plotvec, { 0.5 * Kn[m, L1]^2, ampsqwt}]
 },
 {m, 2, mopen - 1}];

plotdiscrete = ListPlot[plotvec, PlotRange -> All]

diffflux[k1_, AK_] :=

 (1 / ( k1 * Sqrt[AK^2 - k1^2])) * 4 * (2 / Pi) ( k1 * Sqrt[AK^2 - k1^2] / ((a^2 + k1^2) (b^2 + AK^2 - k1^2)) )^2 ;

pexact = Plot[diffflux[Sqrt[eps1 * 2], K], {eps1, 0, K^2 / 2}, PlotRange -> All]
Show[plotdiscrete, pexact]
```

```
 open channels = 127.

 terms in wave function =160.
```

Out[262]= - Graphics -



Out[263]= - Graphics -

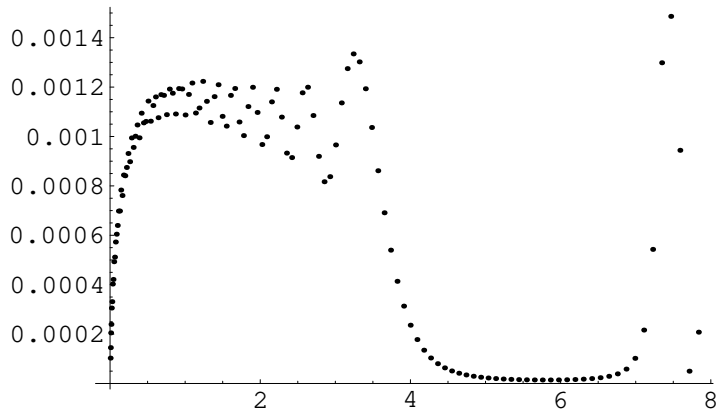


Out[264]= - Graphics -

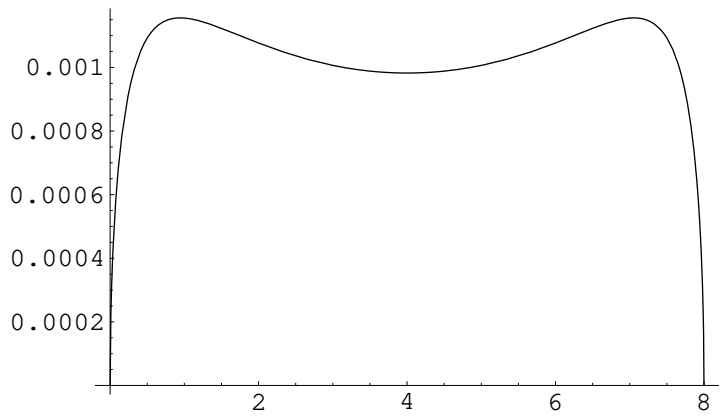
■ Box sizes DIFFERENT with L2 = twice L1

In[245]:=
```
Clear[mopen, L1, L2];
K = 4;
L1 = 100;
L2 = 200.0001;
mopen = IntegerPart[K * L1 / Pi];
Print[" open channels = ", mopen // N]
Print[" terms in wave function =", nmax // N]

plotvec = {};
Do[{
   ampsqwt = (Abs[amp [m, L1, L2, rmatch]]^2) /  ((Kn[m + 1, L1]^2 - Kn[m - 1, L1]^2) / 4);
  plotvec = Append[plotvec, { 0.5 * Kn[m, L1]^2, ampsqwt}]
 },
 {m, 2, mopen - 1}];

plotdiscrete = ListPlot[plotvec, PlotRange -> All]

diffflux[k1_, AK_] :=

 (1 / ( k1 * Sqrt[AK^2 - k1^2])) * 4 * (2 / Pi) ( k1 * Sqrt[AK^2 - k1^2] / ((a^2 + k1^2) (b^2 + AK^2 - k1^2)) )^2 ;

pexact = Plot[diffflux[Sqrt[eps1 * 2], K], {eps1, 0, K^2 / 2}, PlotRange -> All]
Show[plotdiscrete, pexact]
```

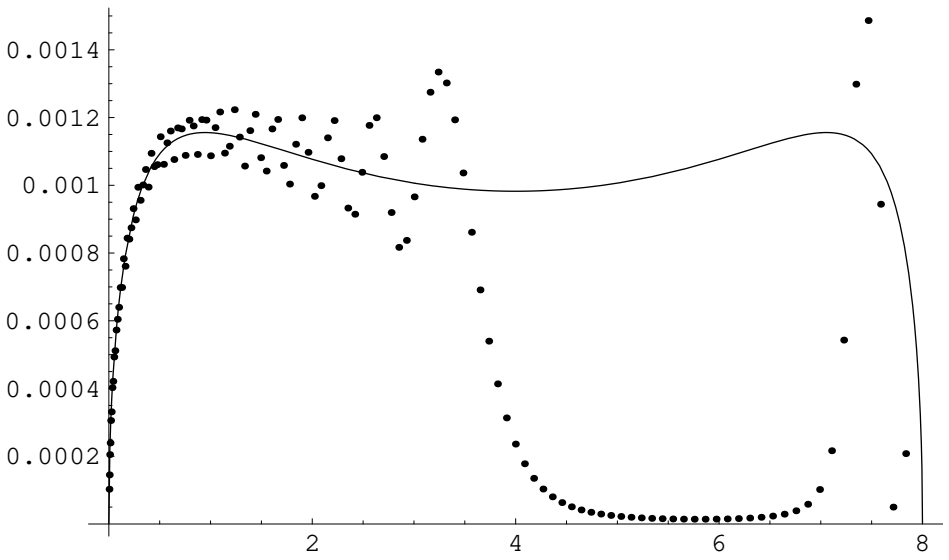
```
 open channels = 127.

 terms in wave function =305.
```



Out[247]= - Graphics -



Out[248]= **-** Graphics **-**

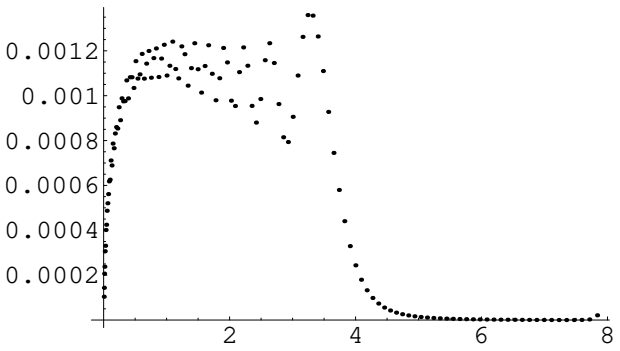


Out[249]= **-** Graphics **-**

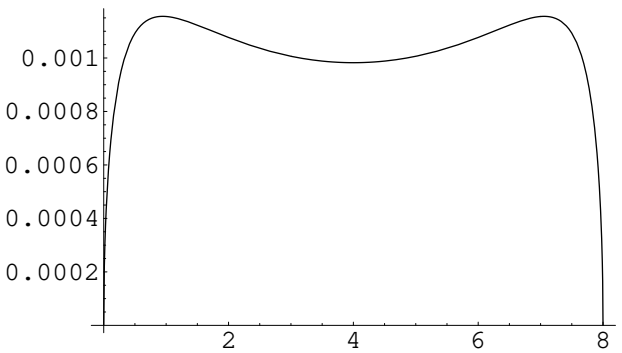
■ Box sizes with L2 = 5 L1

In[265]:=
```
Clear[mopen, L1, L2];
K = 4;
L1 = 100;
L2 = 500.0001;
mopen = IntegerPart[K * L1 / Pi];
Print[" open channels = ", mopen // N]
Print[" terms in wave function =", nmax // N]

plotvec = {};
Do[{
   ampsqwt = (Abs[amp [m, L1, L2, rmatch]]^2) /  ((Kn[m + 1, L1]^2 - Kn[m - 1, L1]^2) / 4);
  plotvec = Append[plotvec, { 0.5 * Kn[m, L1]^2, ampsqwt}]
 },
 {m, 2, mopen - 1}];

plotdiscrete = ListPlot[plotvec, PlotRange -> All]

diffflux[k1_, AK_] :=

 (1 / ( k1 * Sqrt[AK^2 - k1^2])) * 4 * (2 / Pi) ( k1 * Sqrt[AK^2 - k1^2] / ((a^2 + k1^2) (b^2 + AK^2 - k1^2)) )^2 ;

pexact = Plot[diffflux[Sqrt[eps1 * 2], K], {eps1, 0, K^2 / 2}, PlotRange -> All]
Show[plotdiscrete, pexact]
```

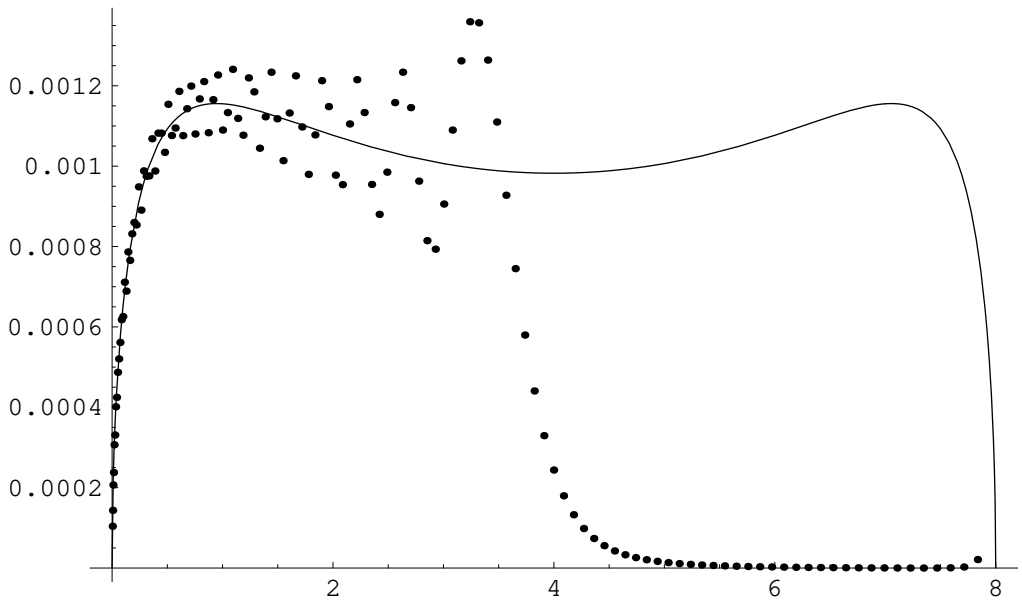
```
 open channels = 127.

 terms in wave function =763.
```



Out[267]= - Graphics -



Out[268]= - Graphics -



Out[269]= - Graphics -