Optics for laser measurements

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Outline

Geometrical optics
- lenses
- ray tracing
- aberrations

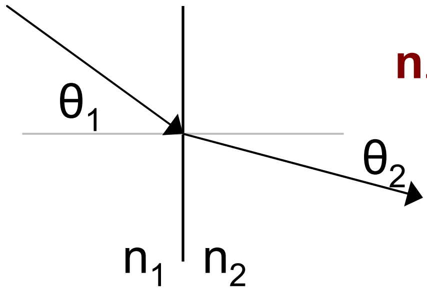
Physical optics
- diffraction and Fourier optics
- Gaussian optics
- reflections and coatings

More stuff
- reflections
- coatings
- interference
- polarization optics

Power of refracting surfaces

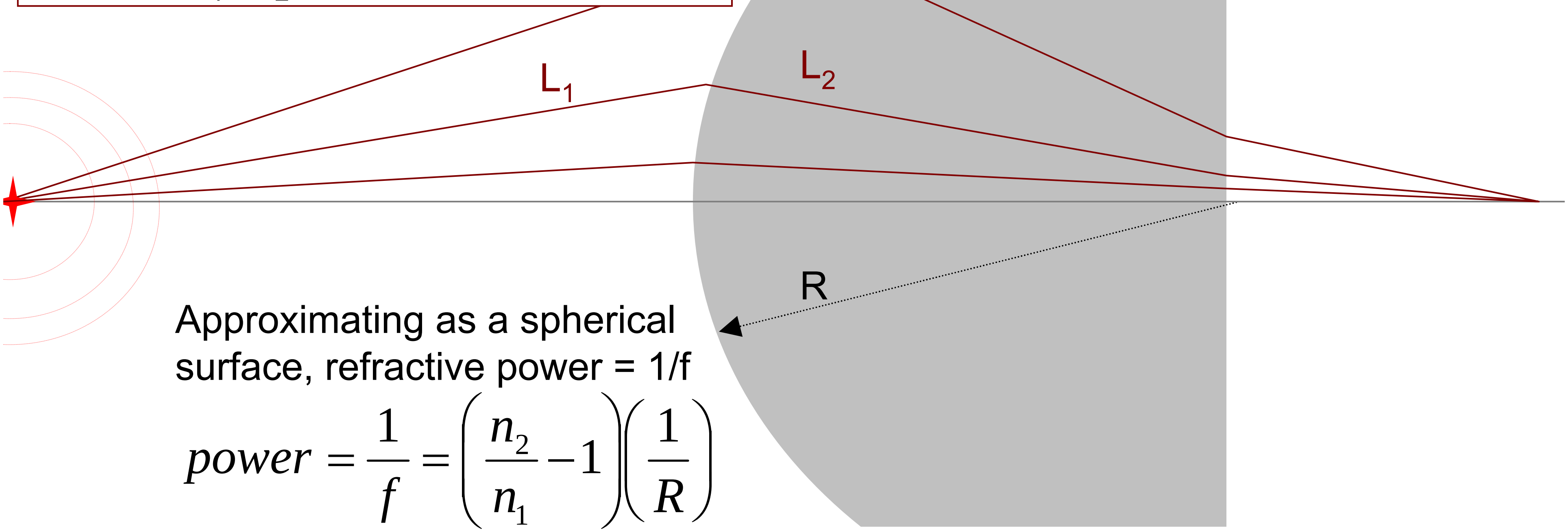
Fermat's principle of least time: $n_1L_1 + n_2L_2$ *is stationary* $(d(\Sigma nL)/dL = 0)$

This principle leads to *Snell's law*



$$\mathbf{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

and gives curved surfaces focusing power

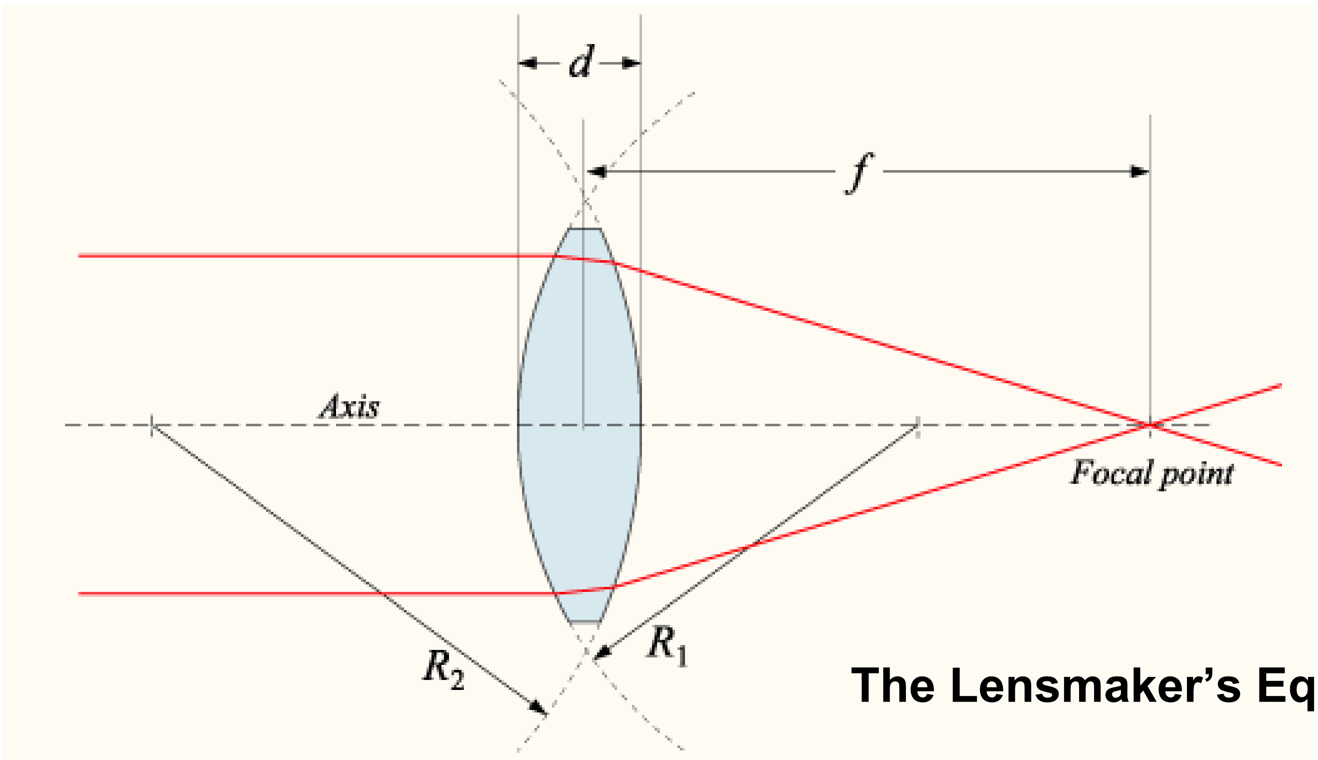


Approximating as a spherical surface, refractive power = 1/f

$$power = \frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R}\right)$$

Thick lens (and lensmaker's equation)

A lens is transmitting optic that has refractive power

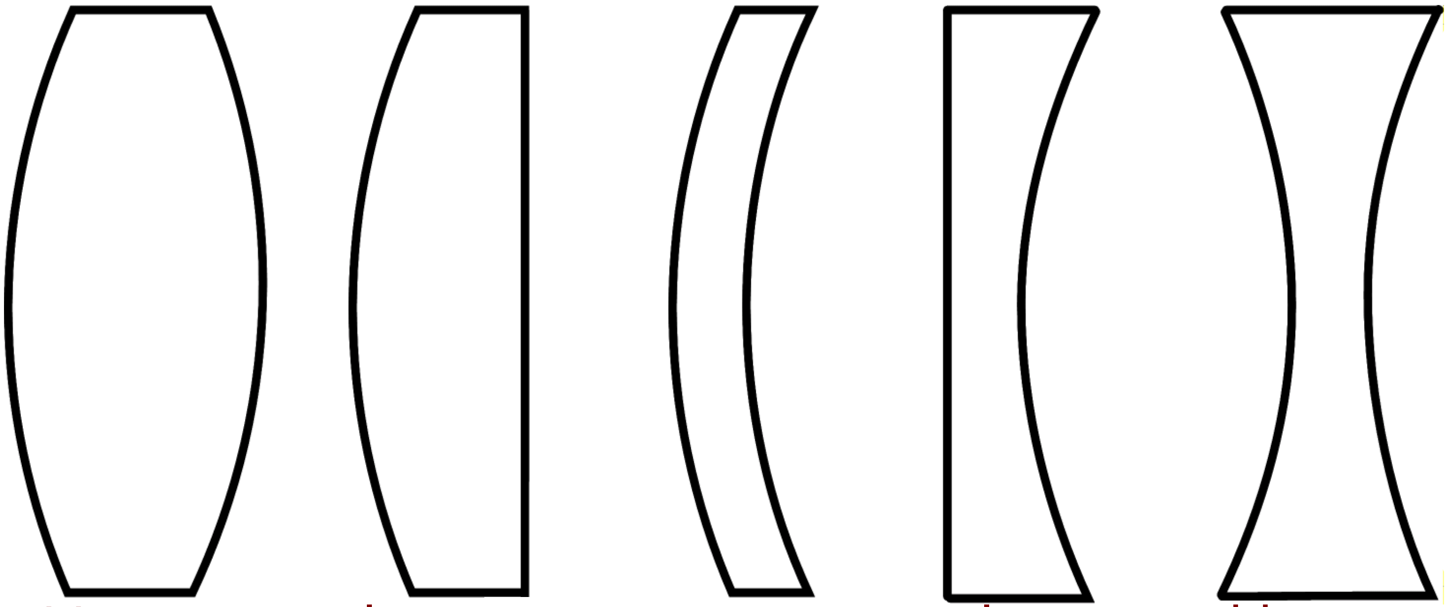


A collimated input beam focuses to focal point f

The Lensmaker's Equation

$$power = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right)$$

Types of lenses



biconvex

plano-convex

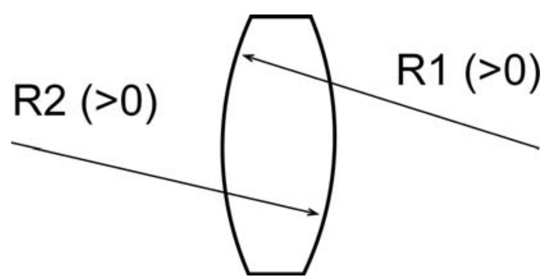
convex-concave

plano-concave

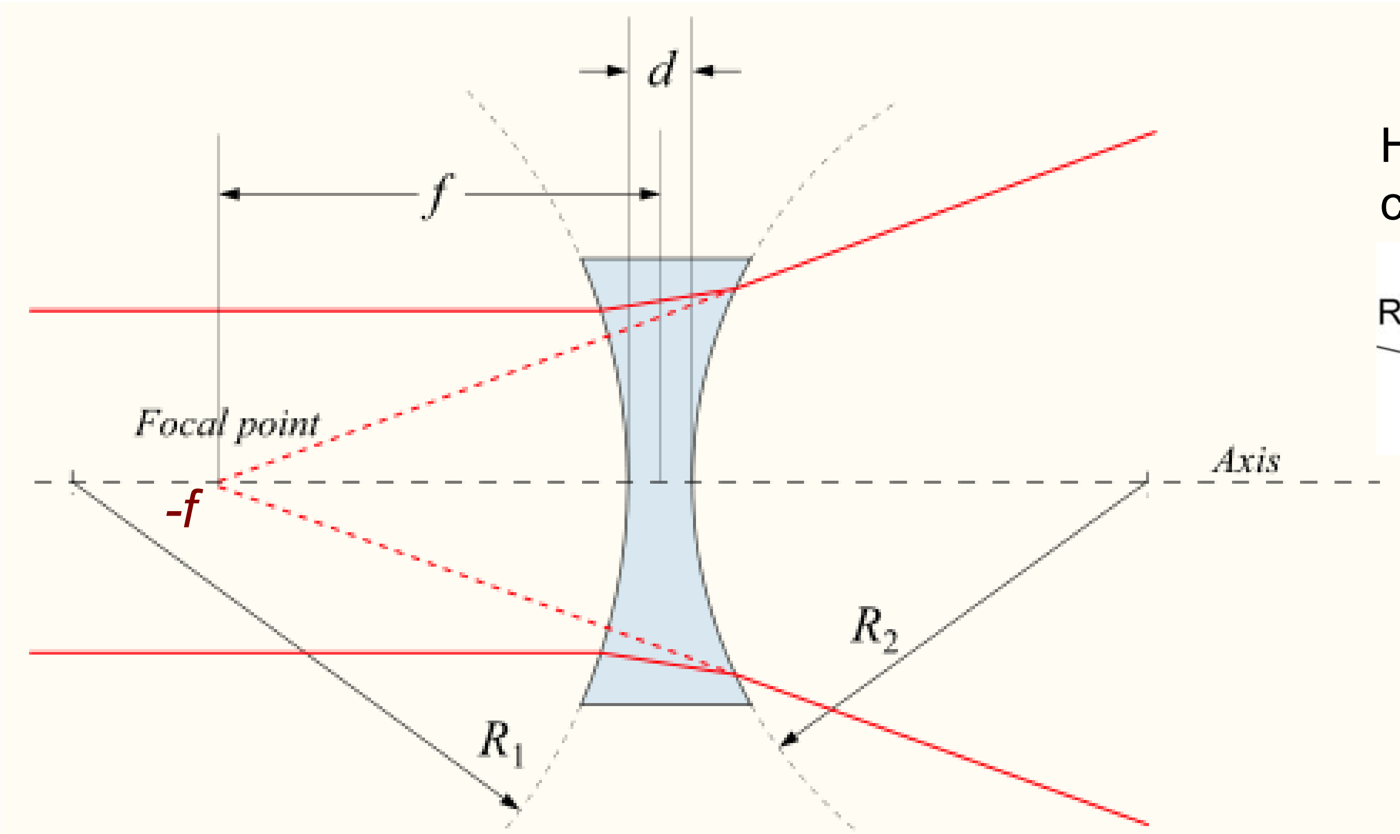
biconcave

Coddington shape factor

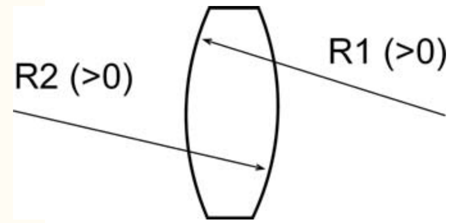
$$q = \frac{R_2 + R_1}{R_2 - R_1}$$



A lens with negative power



Here the radii of curvatures are < 0



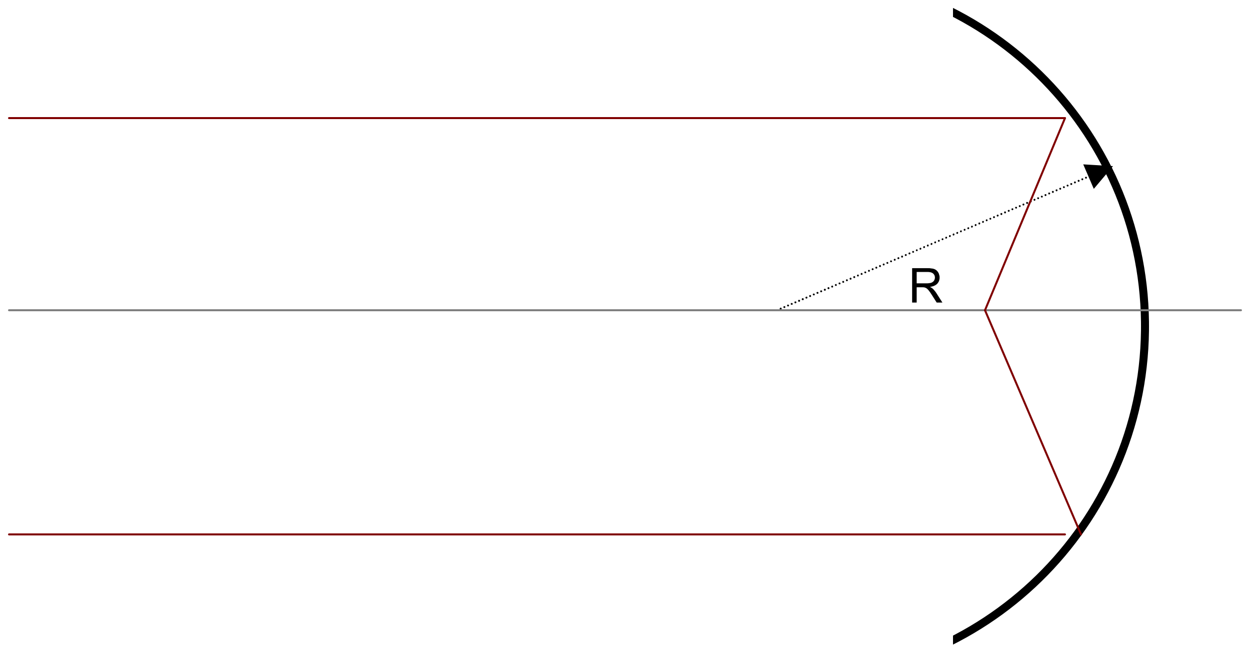
$$power = \frac{1}{f} = -(n-1)\left(\frac{1}{|R_1|} + \frac{1}{|R_2|} - \frac{(n-1)d}{nR_1R_2}\right)$$

Curved mirrors have power

For a mirror, n = -1

$$power = \frac{1}{f} = (n-1)\left(\frac{1}{R}\right)$$

so f = -R/2

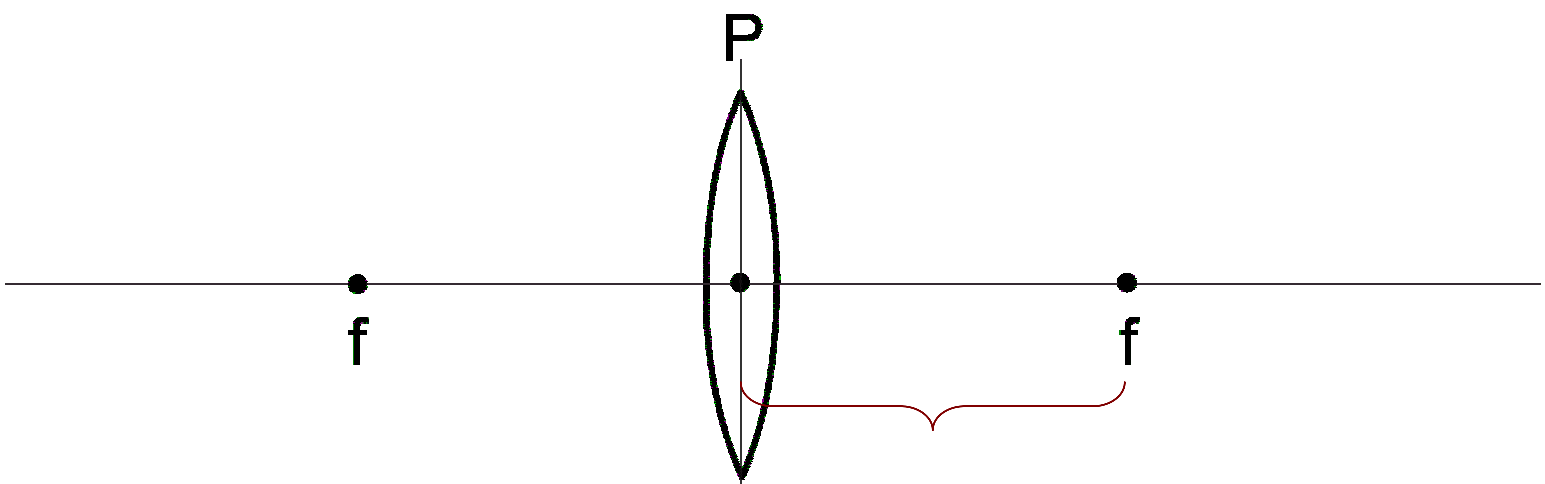


The thin lens – a useful simplification

A thin lens has radii of curvatures greater than thickness (R_1, R_2 > d)

For a thin lens in air

$$power = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

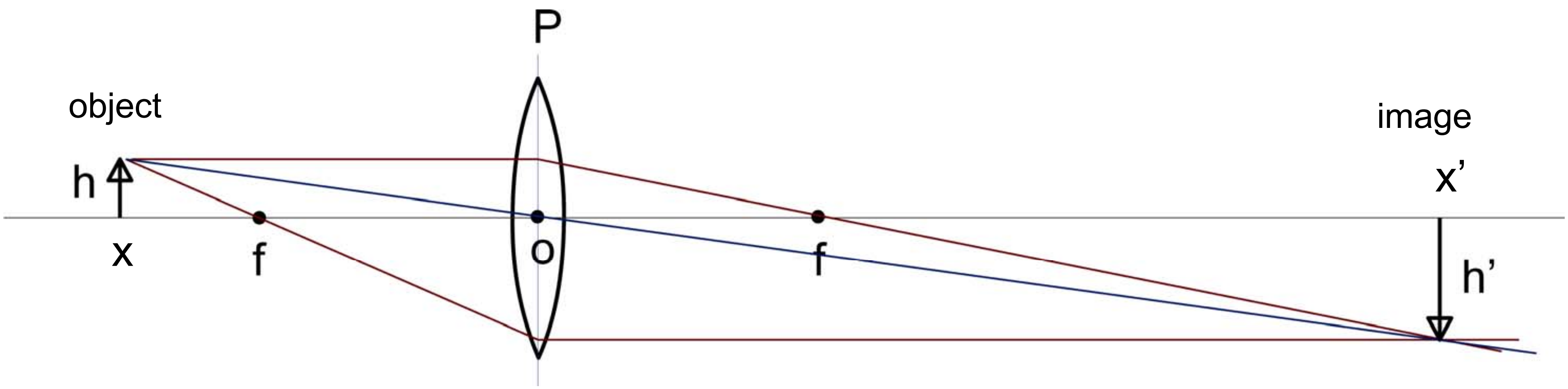


f is measured from the *principal plane* P

A thick lens may have two non-coincident principal planes

Simple ray tracing

An **object** h units high creates an **image** h’ units high



Can map the object to the image using two of three rays:

- Ray parallel to axis to principal plane, then though back focal point
- Ray through front focal point to principal plane, then parallel to axis
- Ray through nodal point

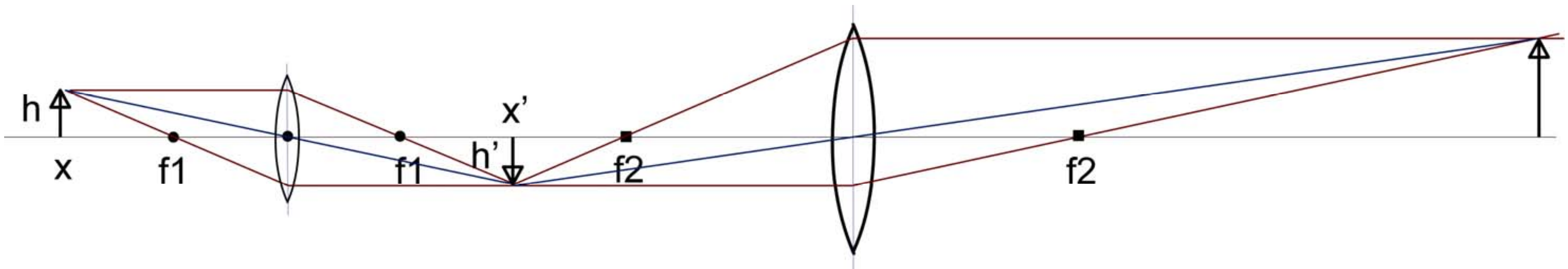
Points x and x’ are called *conjugates*

see http://www.colorado.edu/physics/phet/simulations/lens/lens.swf

Simple two-lens system

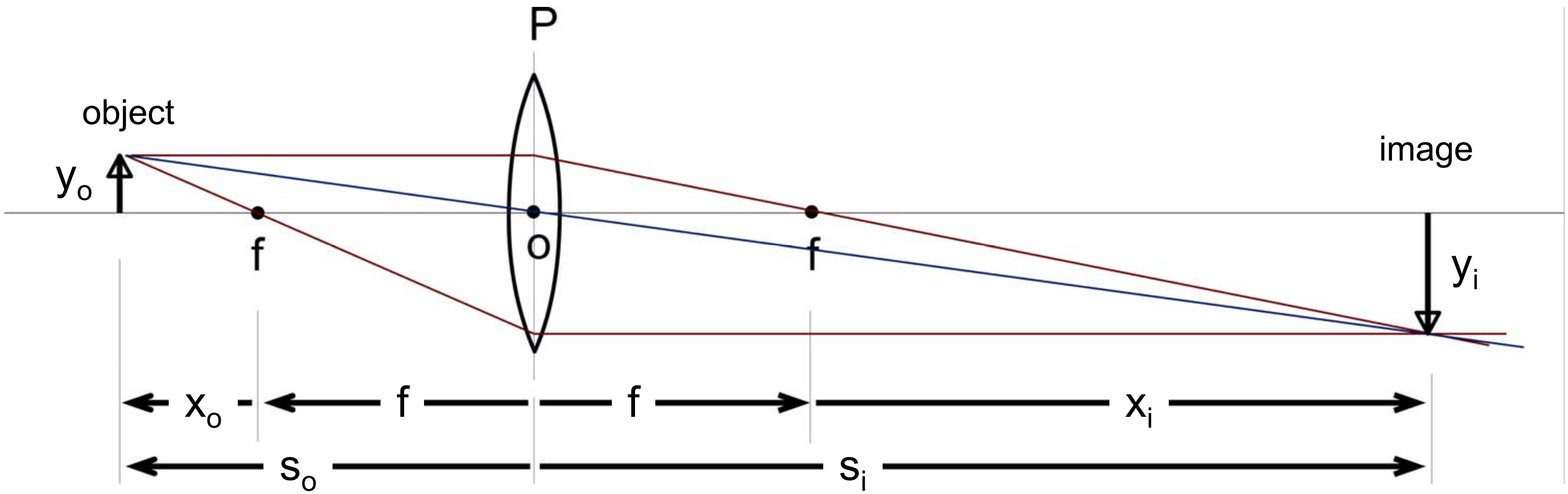
If lenses are separated by more than sum of focal lengths

- find an intermediate image
- map the intermediate image to the final image



Question…How does the lens diameter affect the image???

Imaging equations



Gaussian lens equation: $$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

Newtonian lens equation: $$x_o x_i = f^2$$

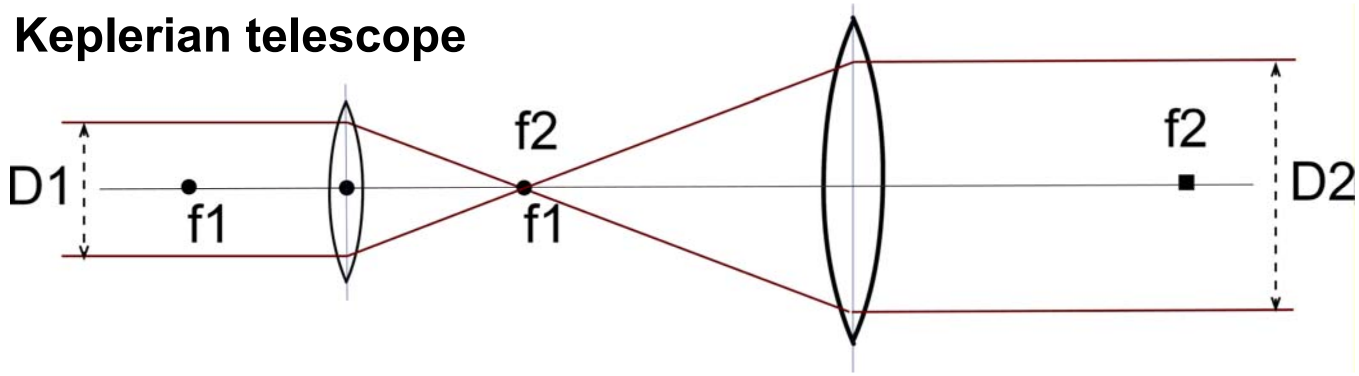
Transverse magnification

$$M = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

Beam expanders

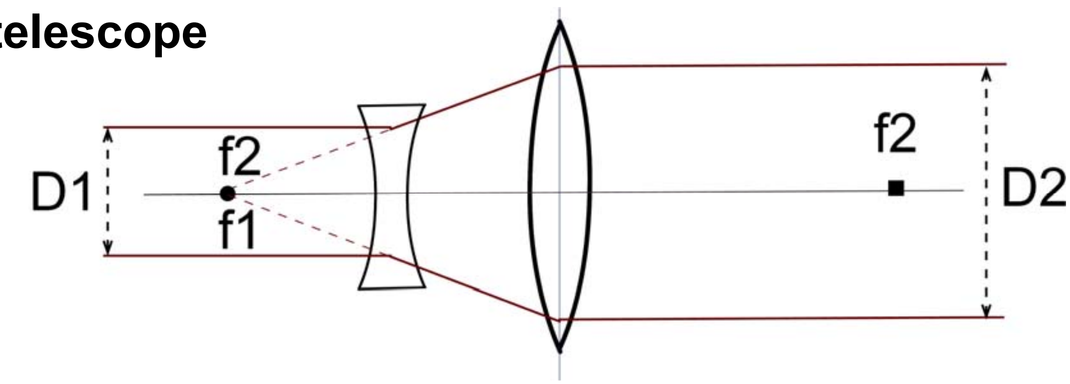
Can expand beam by separating two lenses by the sum of their focal lengths

Keplerian telescope



$$\frac{D_2}{D_1} = \frac{f_2}{f_1}$$

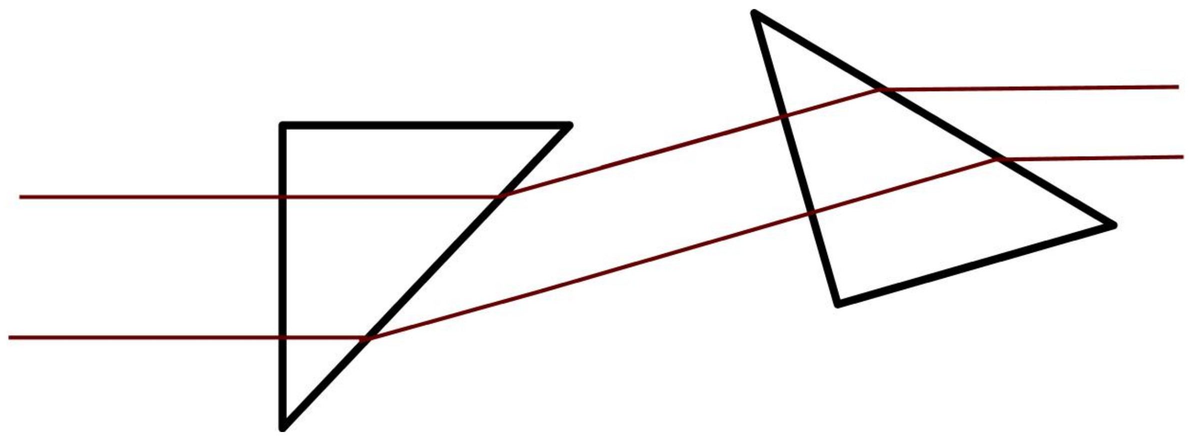
Galilean telescope



Does the lens diameter matter?
Which telescope is better for high pulse-energy lasers?

Beam shaping

Non-circular beams (e.g., laser diodes) can be “circularized” in one-dimension using cylindrical lenses or prisms

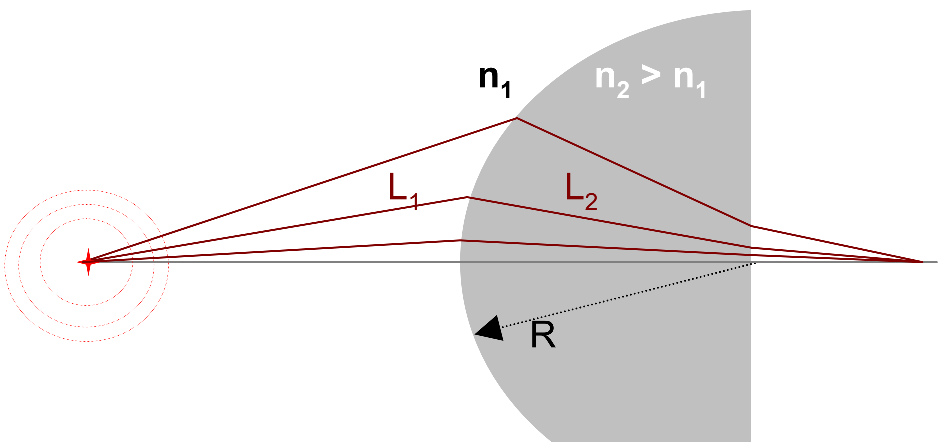


This method provides 1-d compression/expansion

To avoid the beam offset, use cylindrical lenses in a telescope

Aberrations

The perfect (or "stigmatic") imaging discussed before occurs for hyperboliods



Ray optics assumes spherical surfaces…

- *this only works in the "paraxial" (small angle, where sin x =x) limit*

- *we ignored $x^3/3! + x^5/5! - \ldots$ in the expansion of sin x*

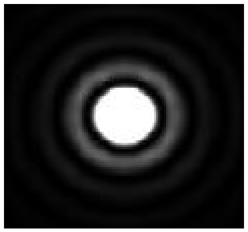
Aberrations exist when the marginal rays don't focus at the same place as the paraxial rays

The third-order aberrations are:

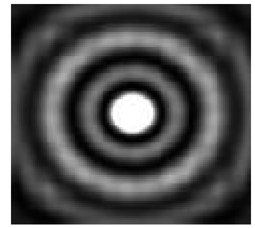
- spherical
- coma
- astigmatism
- field curvature
- distortion

increasing dependence on distance from image plane axis

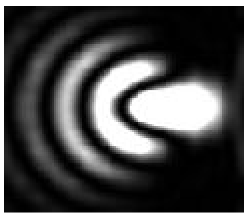
Focused spots with aberration



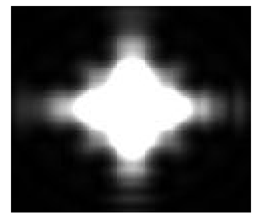
Perfect focus



Spherical aberration

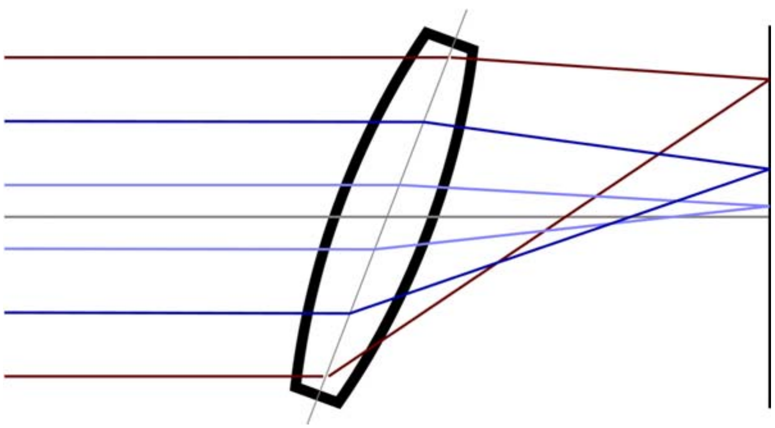


Coma

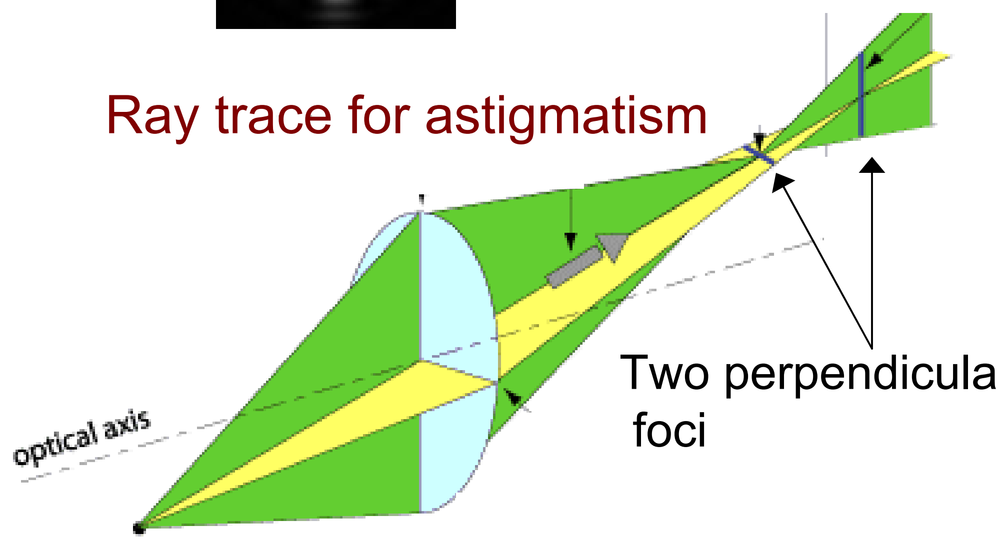


Astigmatism

Ray trace for coma



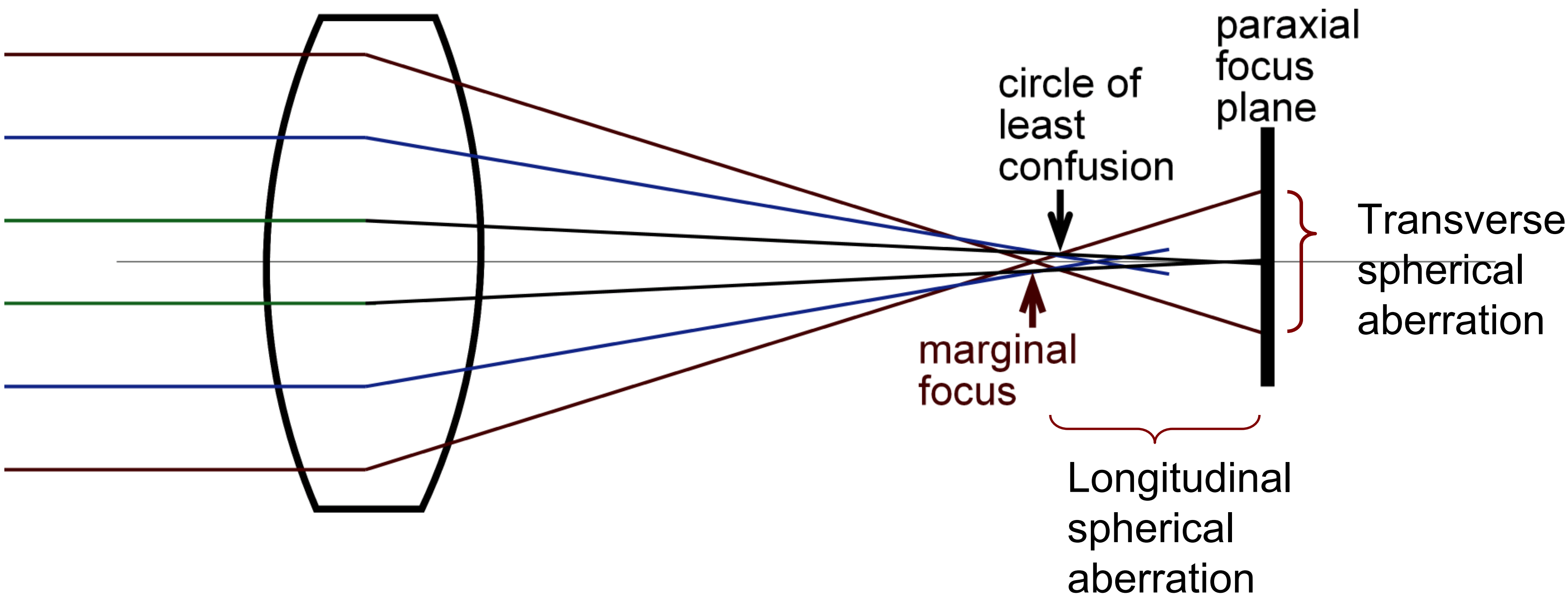
Ray trace for astigmatism



Astigmatism is common in laser diodes

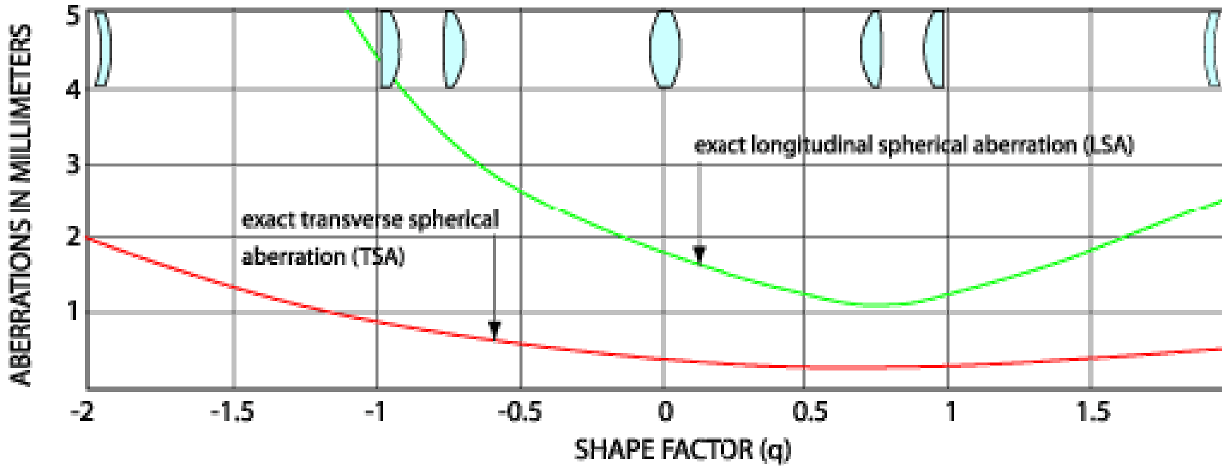
Spherical aberration

Spherical aberration causes on-axis distortion



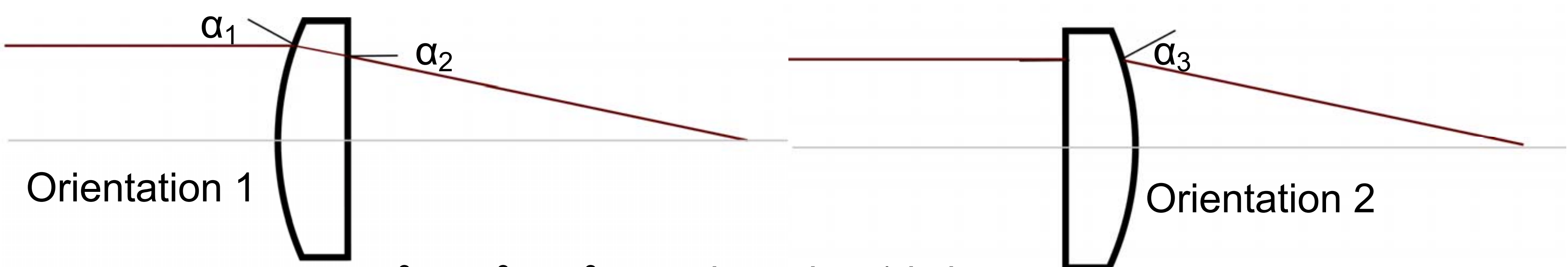
- SA causes a radially symmetric spread of the focused point
- Smallest "spot" will occur in front of paraxial focus plane

SA depends on lens shape



http://optics.mellesgriot.com/opguide/fo_3_3.htm

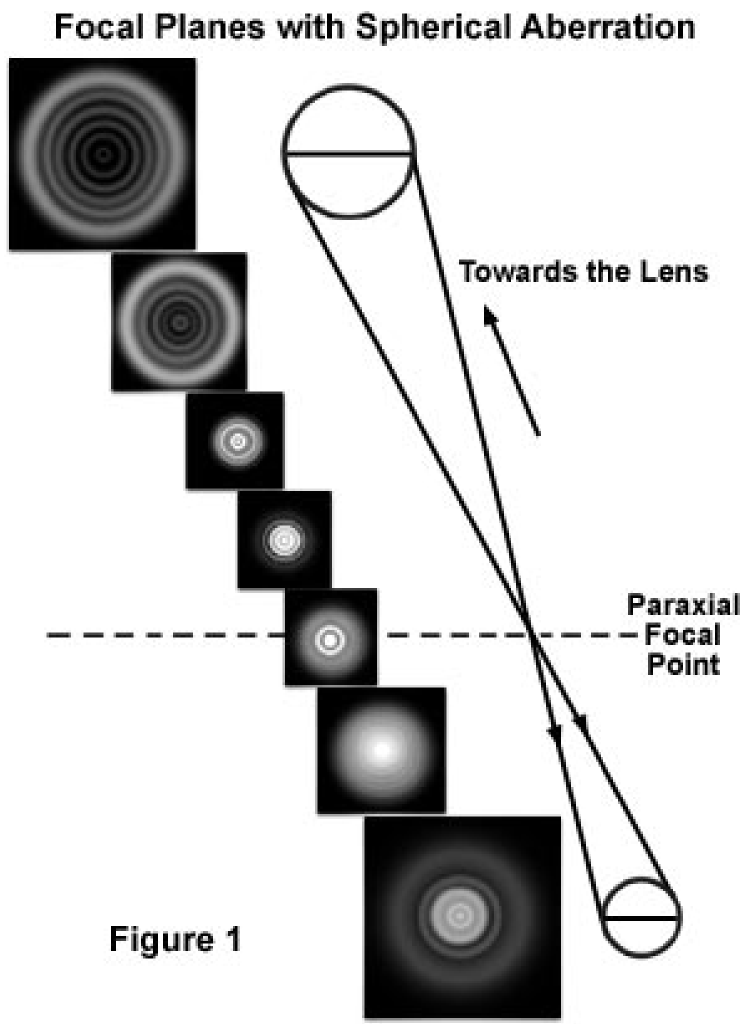
Aberration is minimized when the incident angles to the optical surface are minimized



$\alpha_1^3 + \alpha_2^3 < \alpha_3^3$, so orientation 1 is better

Rule of thumb: point most curved surface towards most distant conjugate

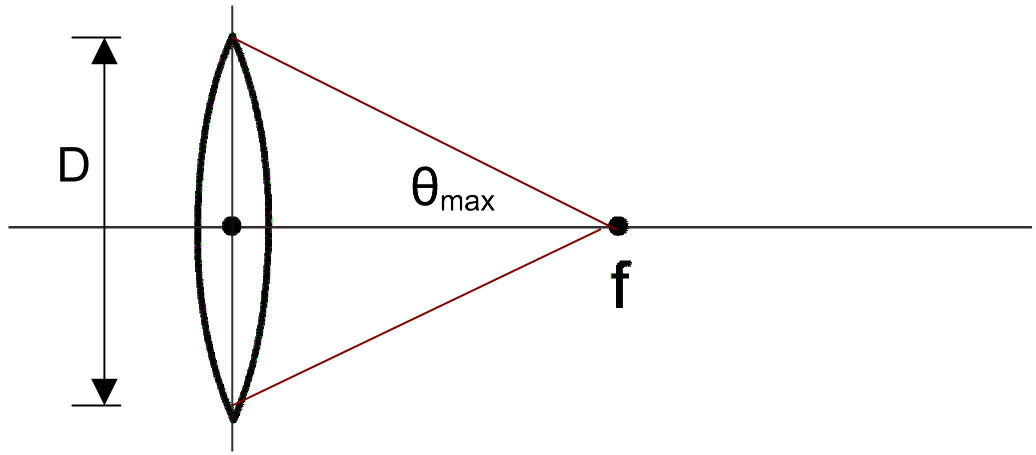
Pictures of SA though focus



Java tutorial at http://www.olympusmicro.com/primer/anatomy/aberrationhome.html

Numerical aperture & F-number

Numerical aperture

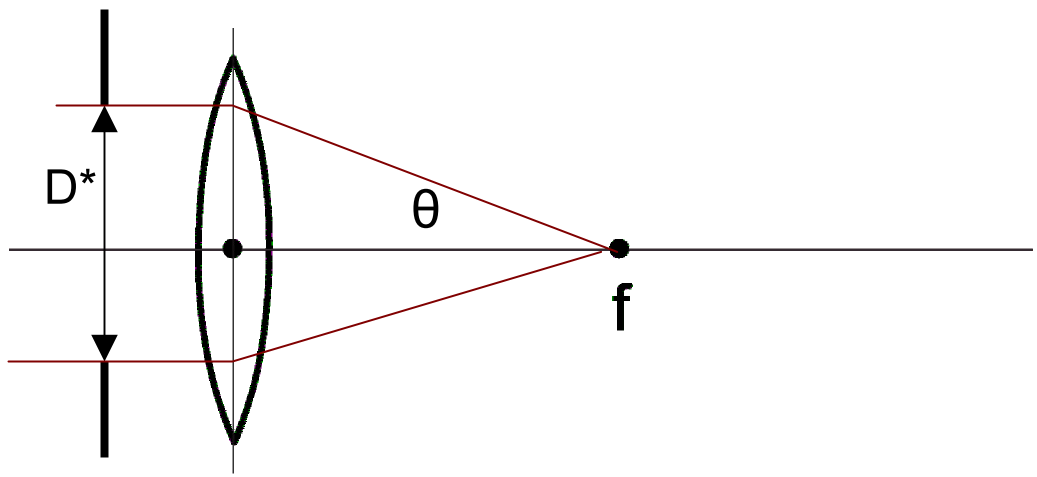


$NA = n \sin \theta_{max}$

n = 1 for lens in air

$F\# = f/D$

One can "stop down" to reduce the NA or increase the F#



$NA = n \sin \theta$

$F\# = f/D^*$

Aberration scaling

The severity of aberrations generally depends on the aperture stop diameter (D) and the off-axis distance of the image point (h')

Aberration	Dependence on D	Dependence on h'
Spherical aberration	D^3	none
Coma	D^2	h'
Astigmatism	D	h'^2

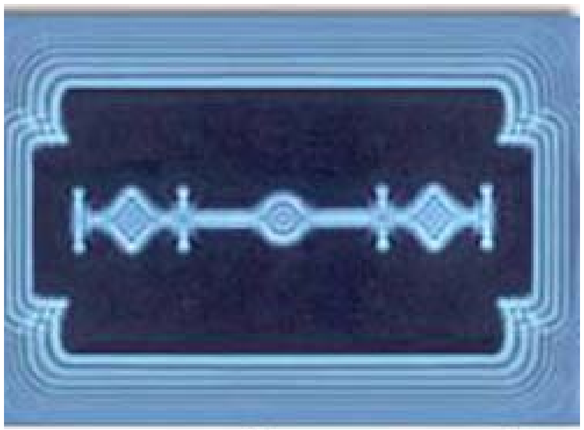
High NA or low F# systems have more SA than low NA / high F# systems

Physical optics

In geometric optics, a bundle of rays parallel to the axis will focus to a point

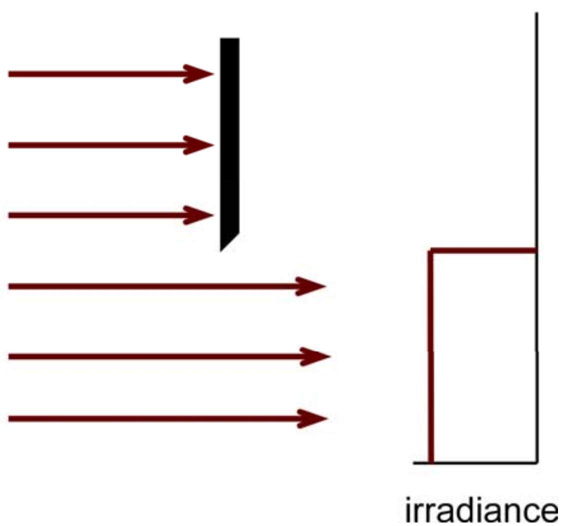
In real life, the spot size is finite even if aberrations are zero

Diffraction

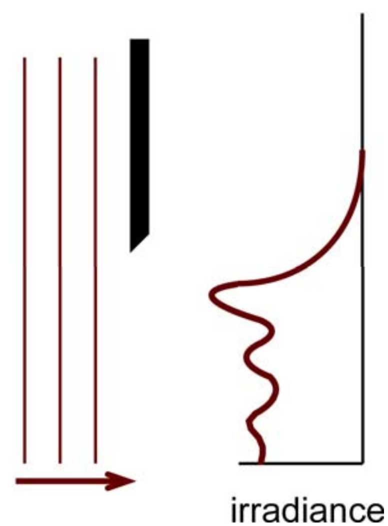


Diffraction: the interference phenomena that describes the alteration of wave propagation due to optics of finite size

Ray model (no diffraction)



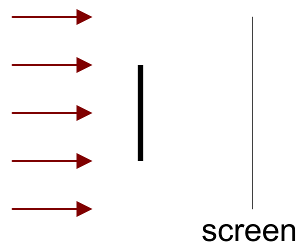
Wave model (with diffraction)



Wave optics

diffraction creates intensity distributions geometric optics cannot explain

What do you see behind an illuminated opaque disk?



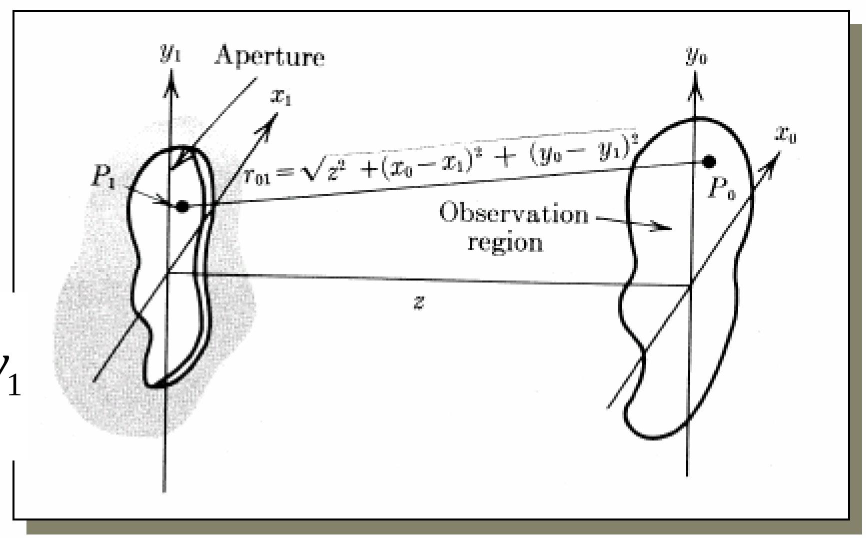
Arago's (or Poisson's) spot

To model the general propagation of a wave through space…

$$U(x,y,z) = E(x,y,z)\exp[-j(\omega t + \phi(x,y,z)]$$

$$U(x_0,y_0,z) = \frac{z}{j\lambda}\iint_{surface} U(x_1,y_1)\frac{\exp(jkr_{01})}{r_{01}^{\;2}}\,dx_1 dy_1$$

for $r_{01} >> \lambda$



Fourier optics

The diffraction pattern is very complicated and evolves with the distance z from the aperture…*need simplifications!!*

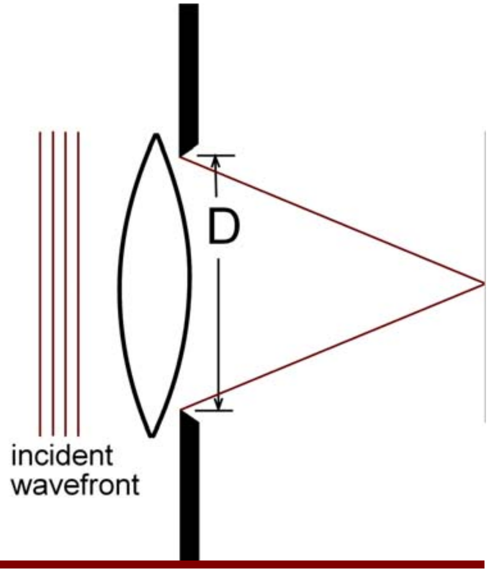
In the *far field* (or "Fraunhofer region") the field math is simplified:

Here the field is the Fourier transform of the wavefront in the aperture

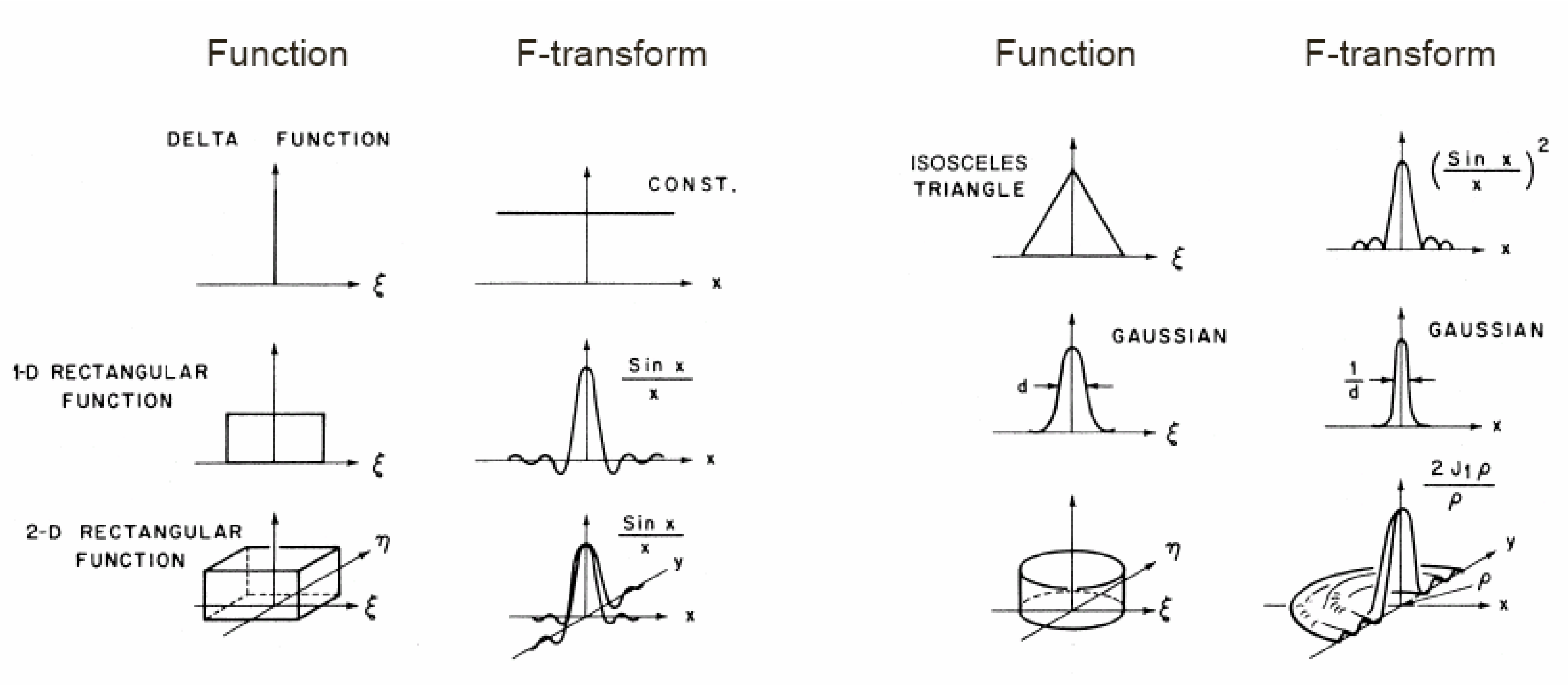
The far field exists at distances $z > \dfrac{2D^2}{\lambda}$

or

at the <u>focal plane of a lens</u>



Fourier transform pairs



Irradiance distribution is the *square* of the field distribution

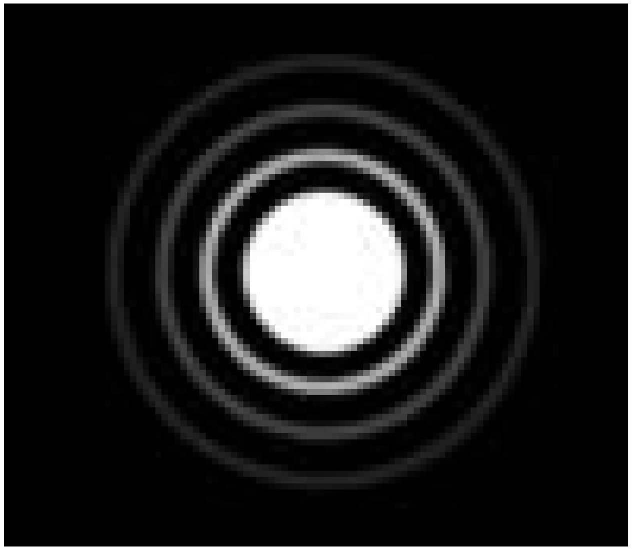
Airy function

The intensity distribution of a *plane wave* incident on a circular aperture with radius w is:

$$I(R) \propto \left[\frac{J_1(kwR/z)}{kwR/z} \right]^2$$

distance z from aperture, k=2π/λ

Diffraction-limited spot



Diameter of central lobe

$$d = 1.22 \frac{\lambda z}{w} \qquad for \quad z > \frac{2D^2}{\lambda}$$

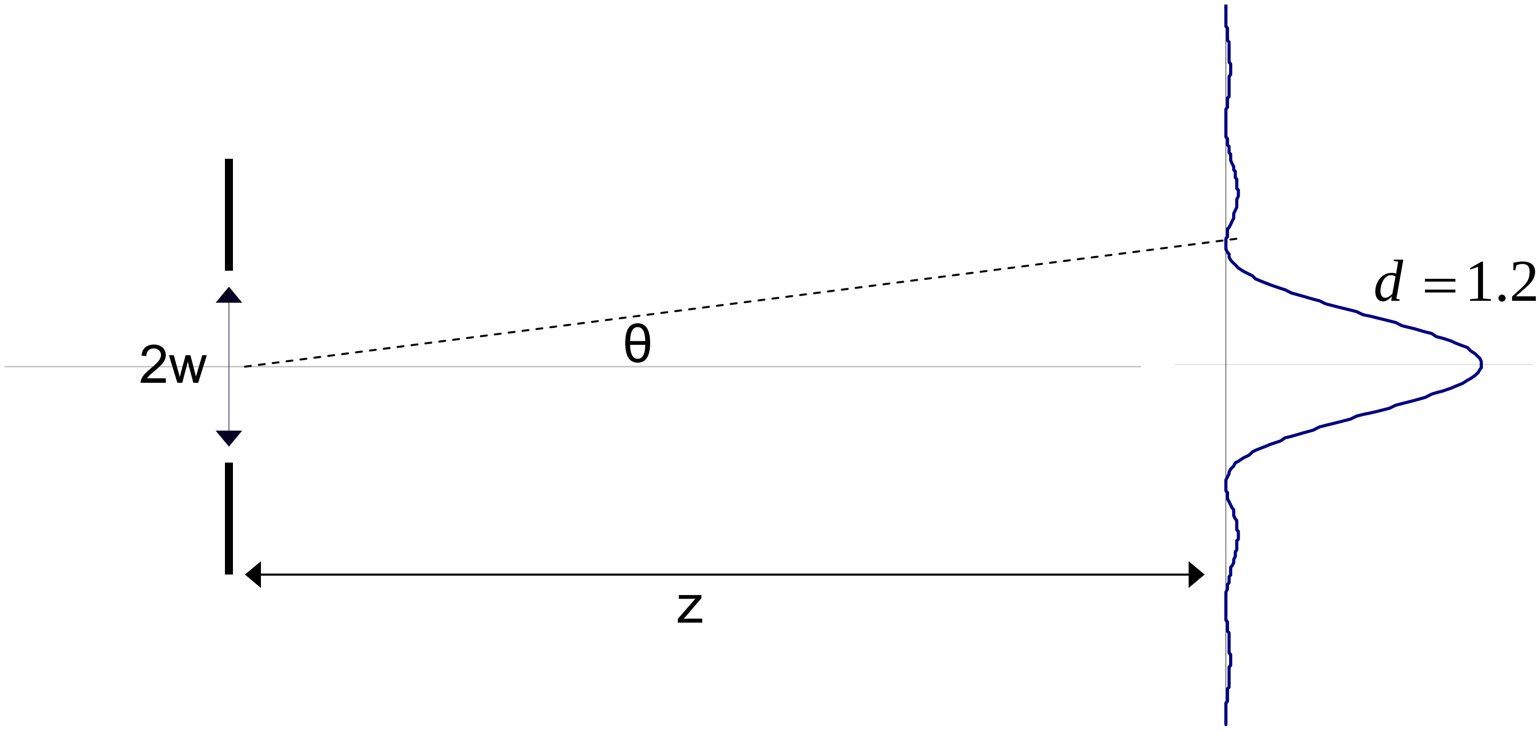
or

$$d = 1.22 \frac{\lambda f}{w}$$ at the focal plane

~86% of energy is in the central lobe

Beam divergence

Beyond the far field, the beam diverges at a constant angle

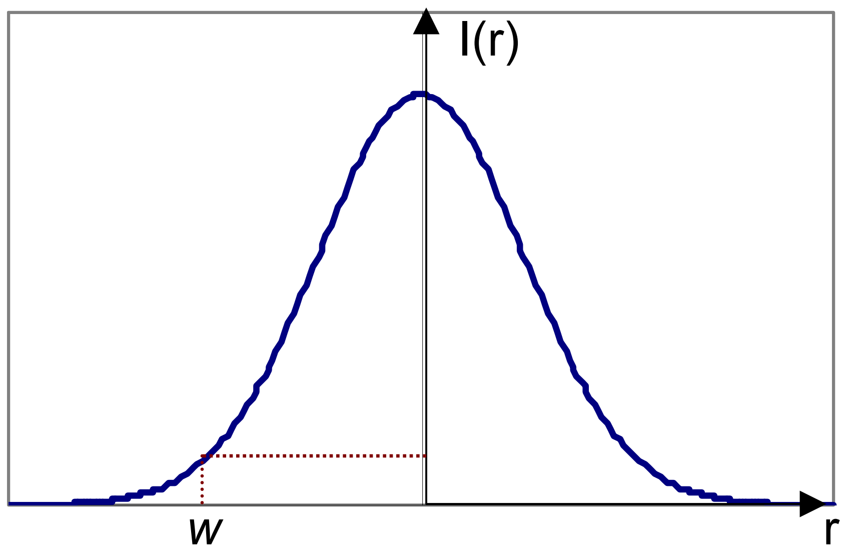


$$d = 1.22\frac{\lambda z}{w}$$

$$\Theta \cong 0.61\frac{\lambda}{w}$$ *in the far field*

Gaussian beams

- Spherical-mirror resonator modes are Hermite-Gaussian
- Lowest-order mode is a Gaussian beam
- Hermite-Gaussians functions are self-transforms

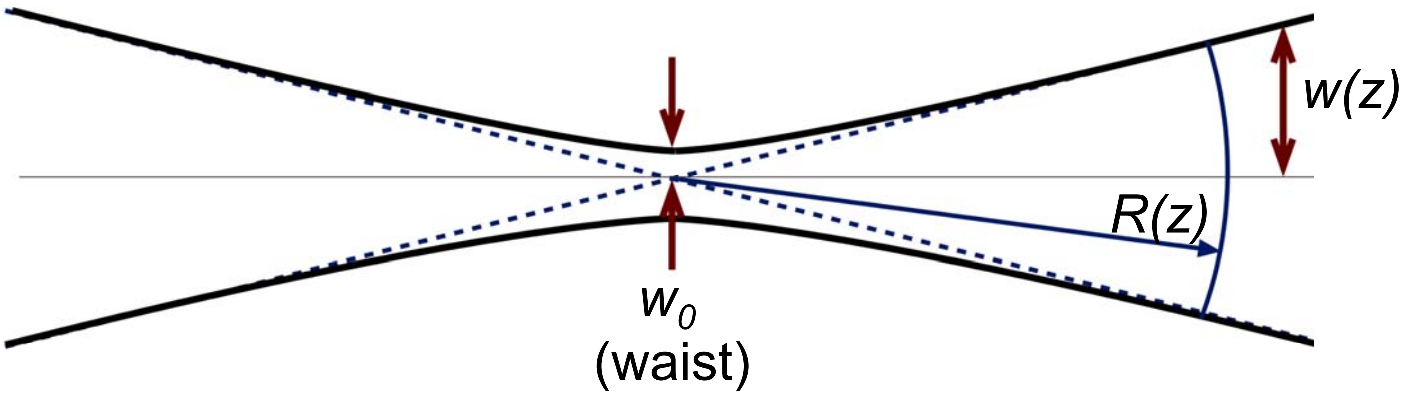


$$I(r) = \frac{2P}{\pi w^2} \exp\left[\frac{-2r^2}{w^2} \right]$$

w is the $1/e^2$ radius
(or the $1/e$ radius for the E field)

Gaussian beam propagation

Gaussian beam propagation is defined by the wavelength and w_0, the radius of the beam at the waist



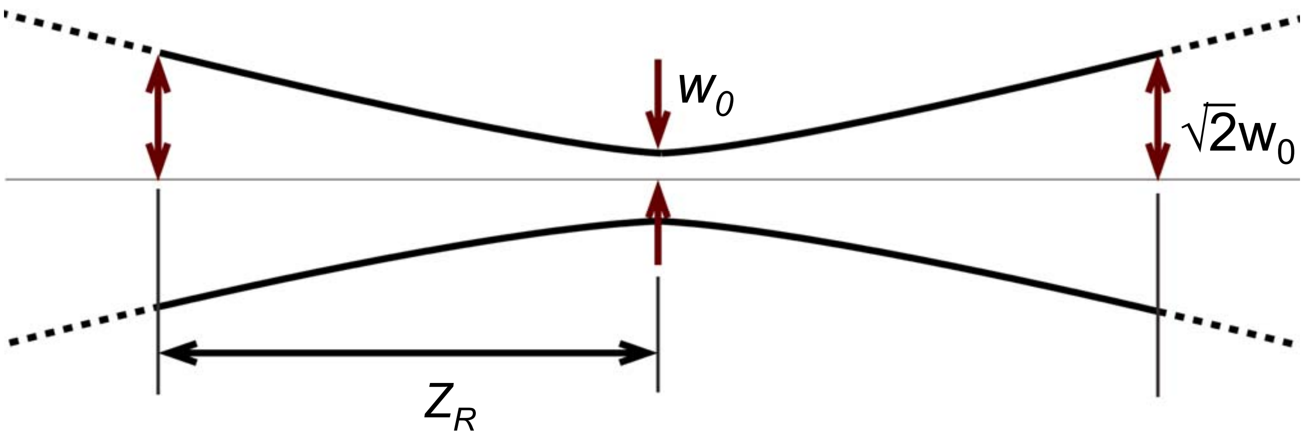
Beam radius at z	Radius of curvature at z	Rayleigh range
$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$	$R(z) = z + \frac{z_R^{\ 2}}{z}$	$Z_R = \frac{\pi w_0^{\ 2}}{\lambda}$

Rayleigh range

The Rayleigh range is distance from waist at which beam area doubles

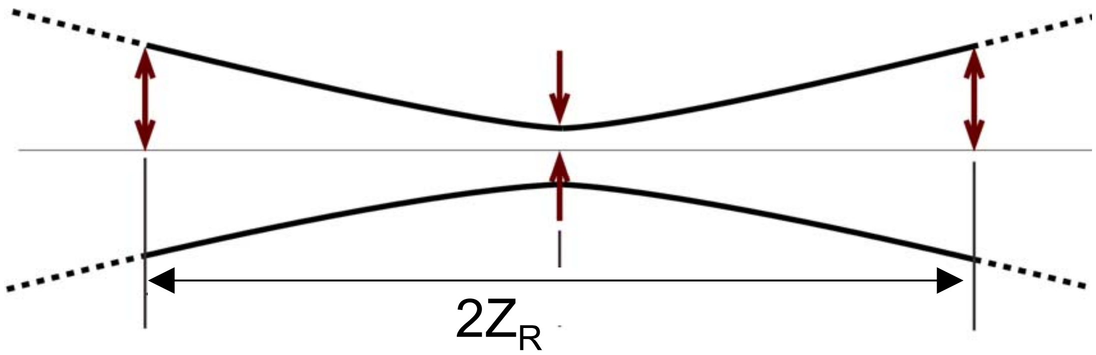
Rayleigh range

$$Z_R = \frac{\pi w_0^{\;2}}{\lambda}$$

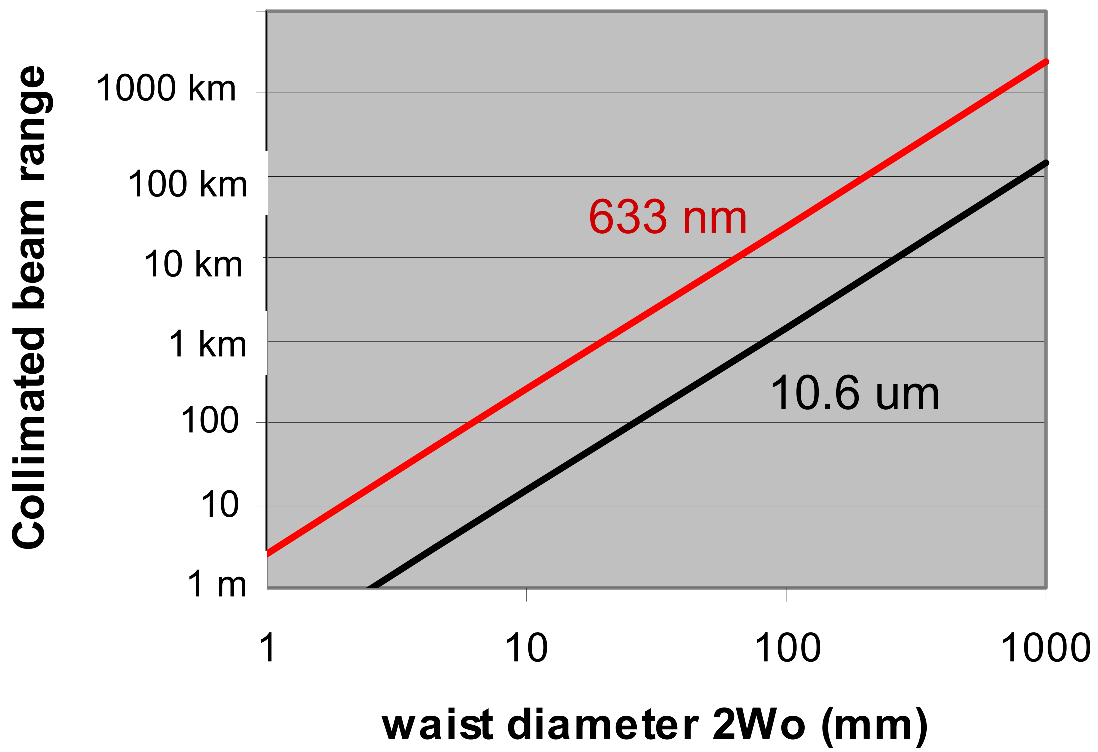


$2Z_R$ is know as the “confocal region” or “depth of focus”

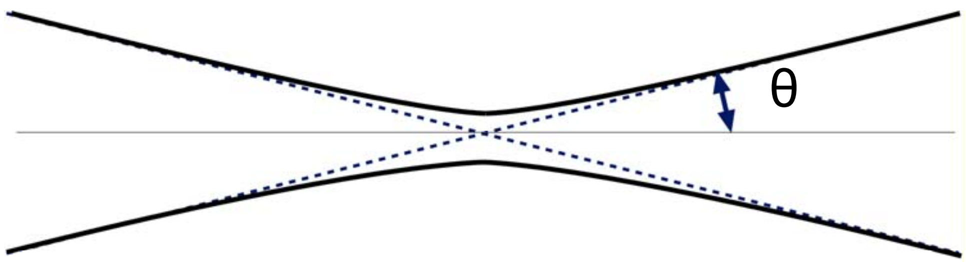
“collimated” beam range



Tighter focusing causes a smaller “collimated” range



Gaussian beam divergence



$$w(z) = w_0 \sqrt{1 + \left(\frac{z\lambda}{\pi w_0{}^2} \right)^2} \cong \frac{z\lambda}{\pi w_0}$$ for large *z* ($z \gg z_R$)

Beam divergence

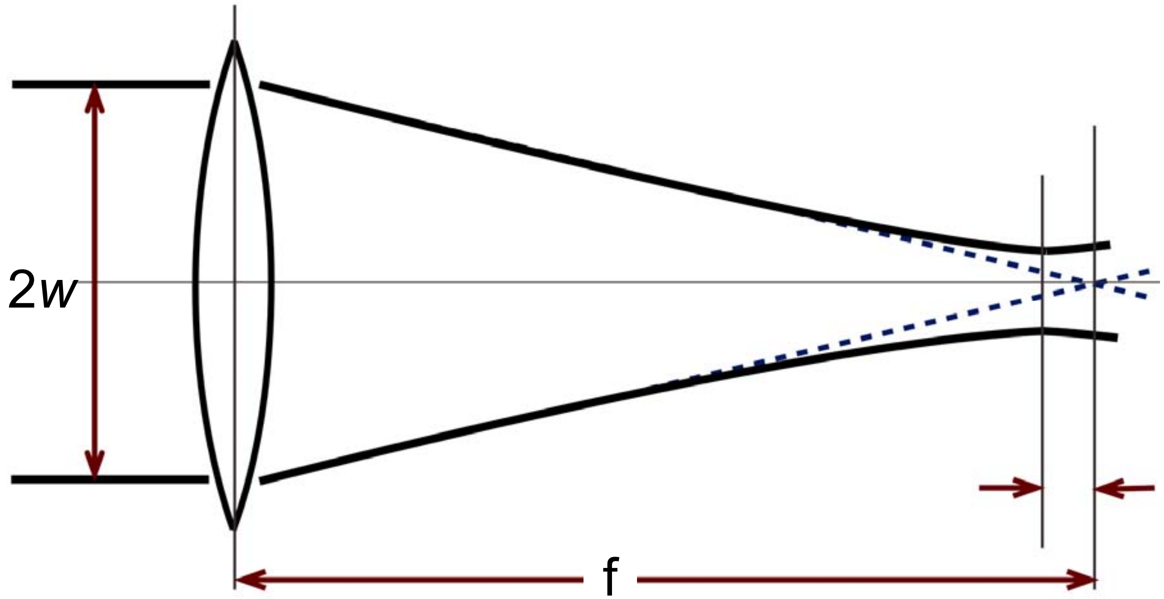
$$\theta_{1/e} = \lim_{z \to \infty} \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}$$

Divergence is inversely proportional to w

Focusing a Gaussian beam

Recall $w(z) \cong \dfrac{z\lambda}{\pi w_0}$

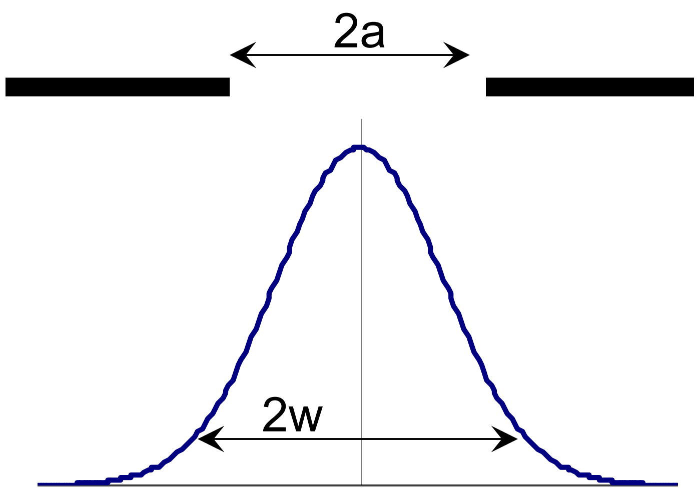
At focus, spot radius is $w_0 \cong \dfrac{\lambda f}{\pi w}$



$$\Delta f \cong \frac{z_R^{\ 2}}{f} = \frac{\pi^2 w_0^{\ 4}}{\lambda^2 f}$$

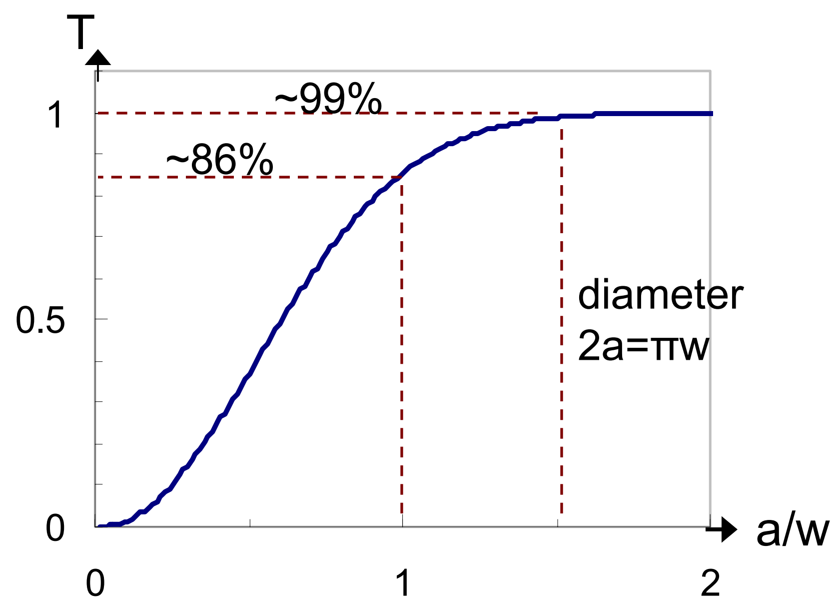
There is a (usually negligible) shift Δf between the waist position and geometric focus

Truncated Gaussian beams



Transmittance T through aperture is

$$T = 1 - \exp\left[-\frac{2a^2}{w^2}\right]$$



Choose lens diameter > πw
to transmit > 99% of energy

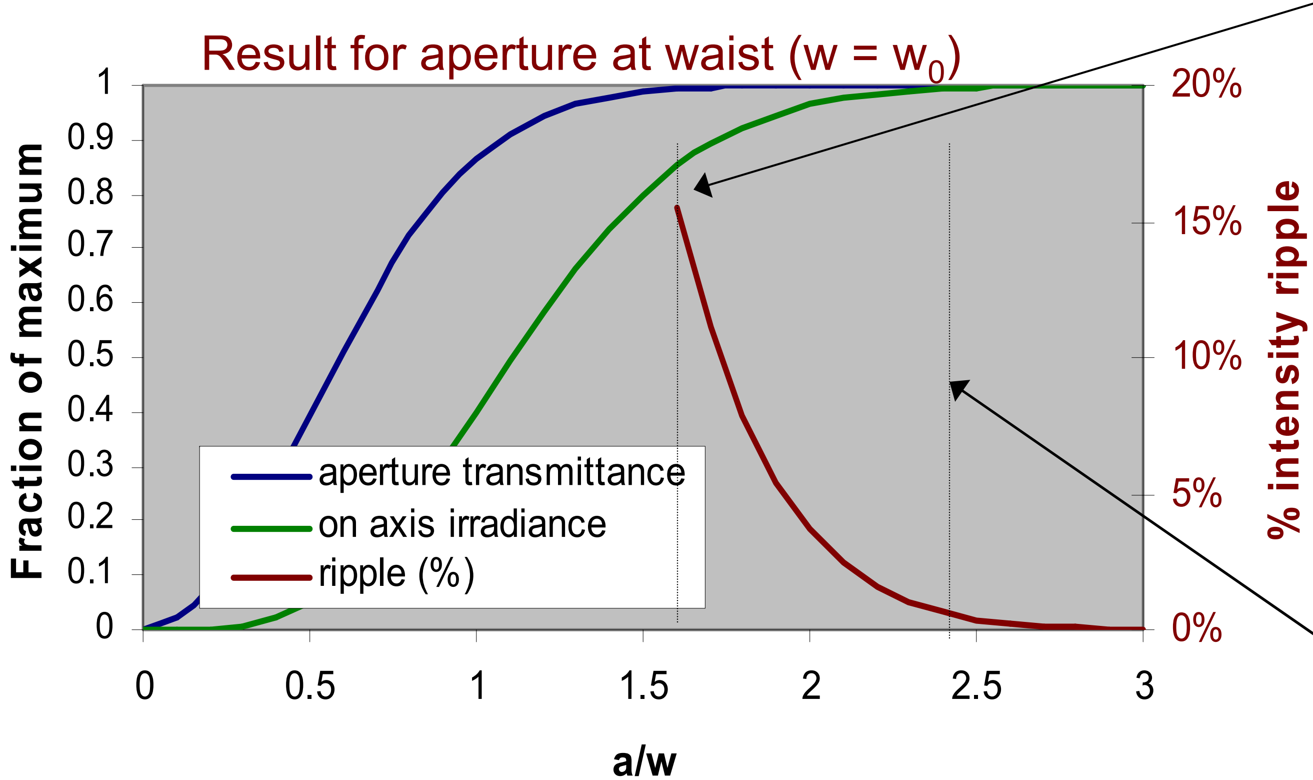
Truncated Gaussian problems

Truncating the Gaussian beam causes

- Decrease in total power transmitted
- **Diffraction!!!**

  Decreases on-axis irradiance

  Increases intensity ripple



From last slide…

When $a = \pi w/2$

T = 99%

P_{axis} =84%

Ripple=17%

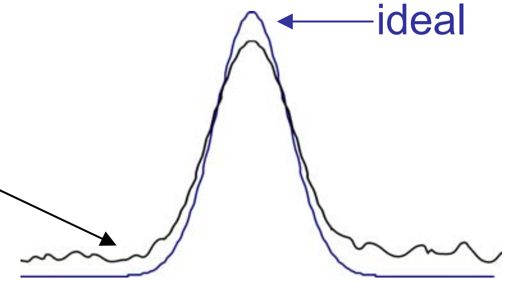
When a = 2.3w

P_{axis} = 99%

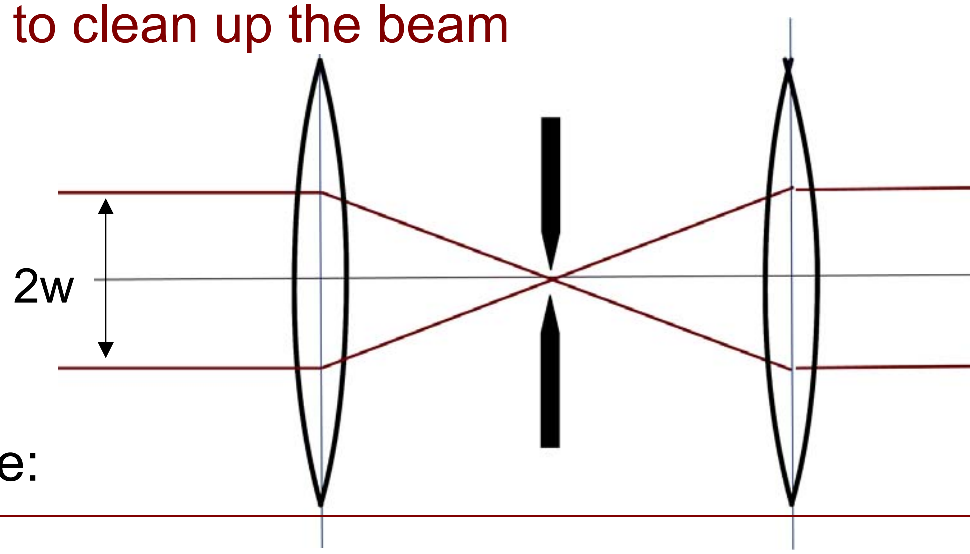
Ripple=1%

Spatial filtering

Surface roughness, dust, etc., adds high-spatial-frequency noise to beam profiles



Place pinhole at common foci of telescope to clean up the beam



For 1% ripple, pinhole diameter D ~ $4.6w_0$

Example:

f = 50 mm
λ = 633 nm
w = 1 mm

$$\longrightarrow \quad w_0 \cong \frac{\lambda f}{\pi w} \quad \longrightarrow$$

w_0 = 10 μm, so choose D = 50 μm

rounded up!

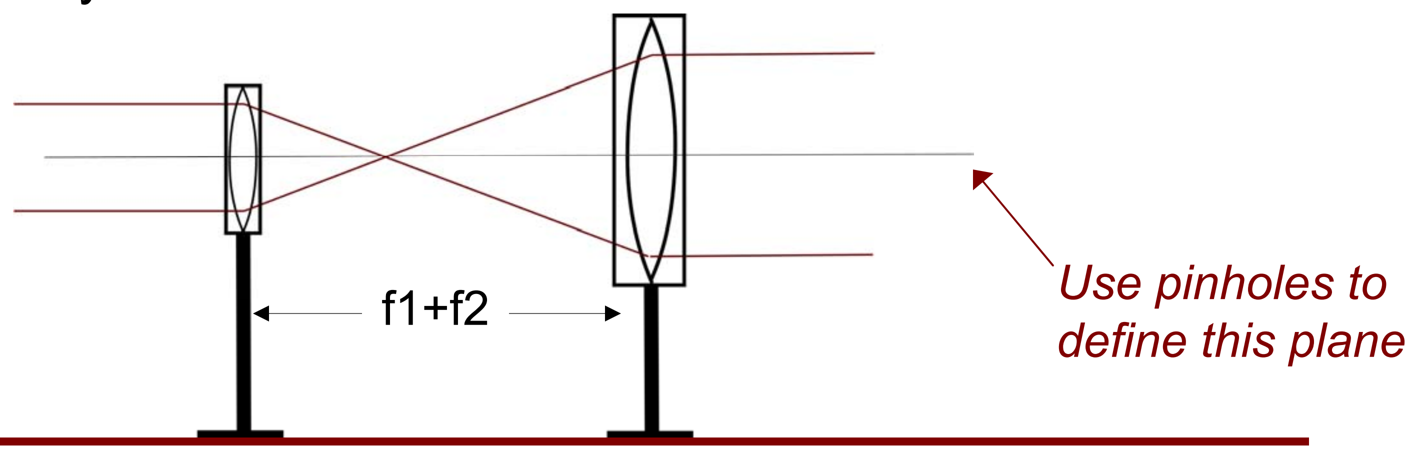
Beam expander alignment

Step 1. Adjust the beam to be parallel to be the optical table
- choose an “operating height”
- have several pinholes at this height

Step 2. Center lens 1 in the beam path

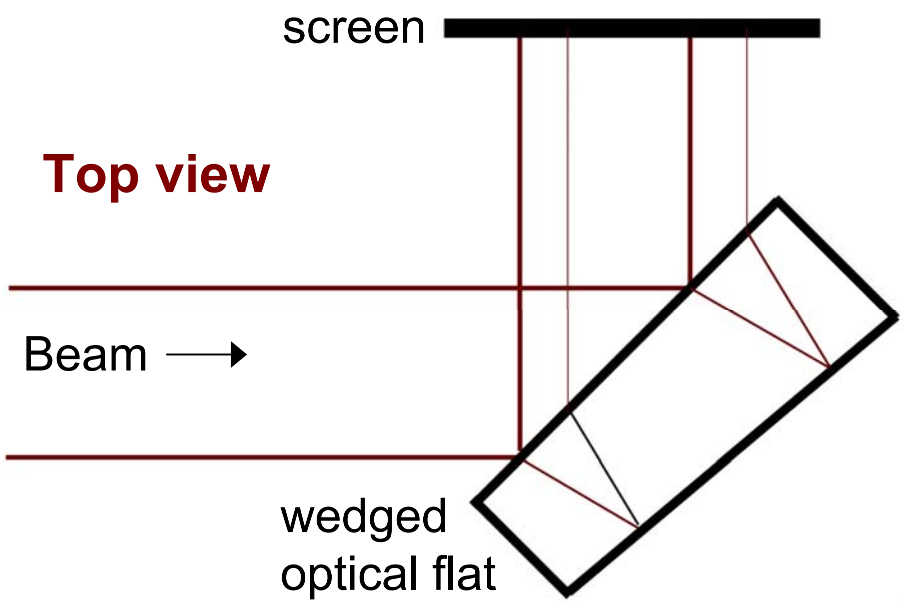
Step 3. Center lens 2 in the beam path, ~f1+f2 from lens 1

Step 4. Adjust lens 2 position along beam until beam diameter is approximately constant between lens 2 and the wall

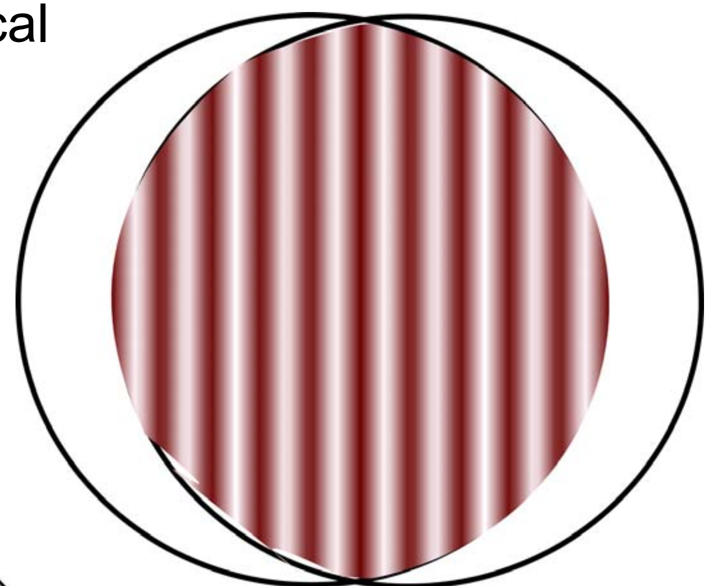


Collimation test with a shearing plate

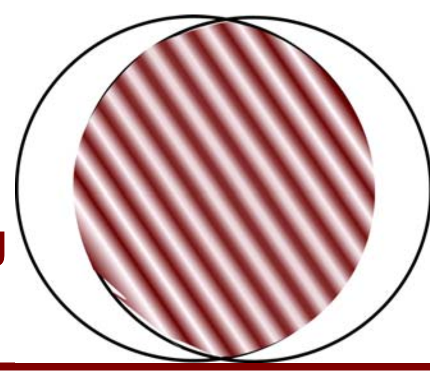
Step 5. Do a fine adjustment of lens 2 position with a shearing plate



If collimated (R→∞ at screen), fringes are vertical



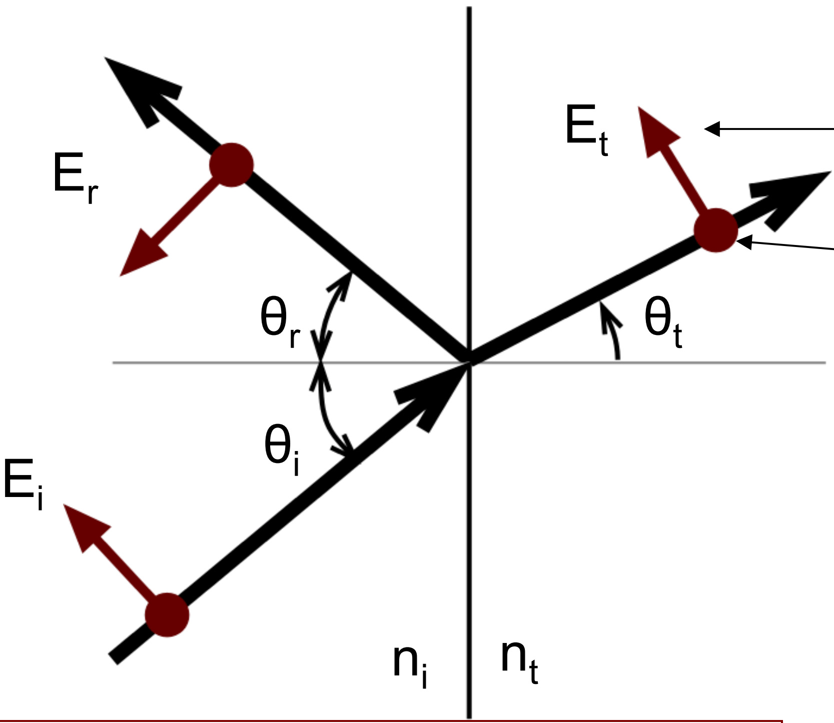
Fringes tilt if beam is converging or diverging



Reflections, coatings, polarization, and stuff

Fresnel reflection

An incident wave (E_i) will create a reflected wave (E_r) and transmitted wave (E_t)
The fields are decomposed with repect to the plane of incidence



p-polarization is in-plane

s-polarization is out-of-plane

Define reflectance and transmittance as

$$R = \left[\frac{E_r E_r^{\,*}}{E_i E_{i^*}}\right] = \frac{I_r}{I_i} \qquad T = 1 - R$$

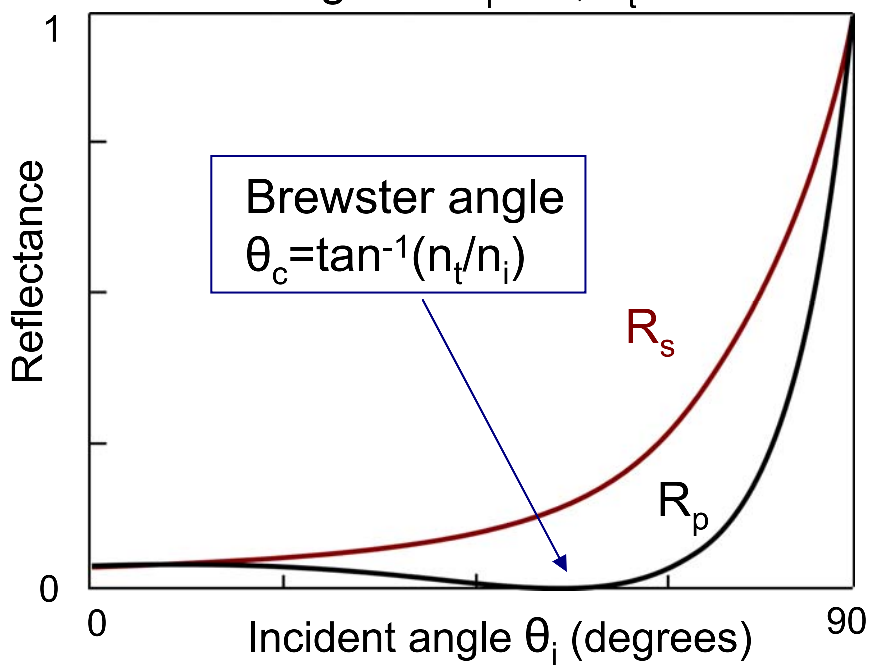
<u>Intensity</u> reflection coefficients depend on polarization, angles, and refractive indices

$$R_p = \left[\frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}\right]^2$$

$$R_s = \left[\frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}\right]^2$$

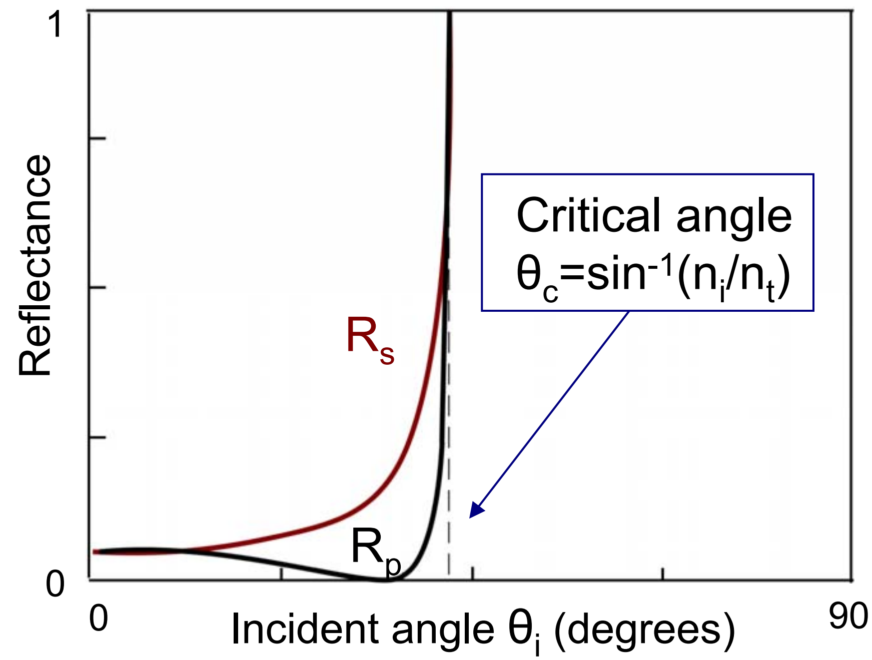
Fresnel coefficients

air-to-glass: n_i = 1, n_t = 1.5



Brewster angle incidence provides perfect transmission for p-polarized

glass-to-air: n_i = 1.5, n_t = 1

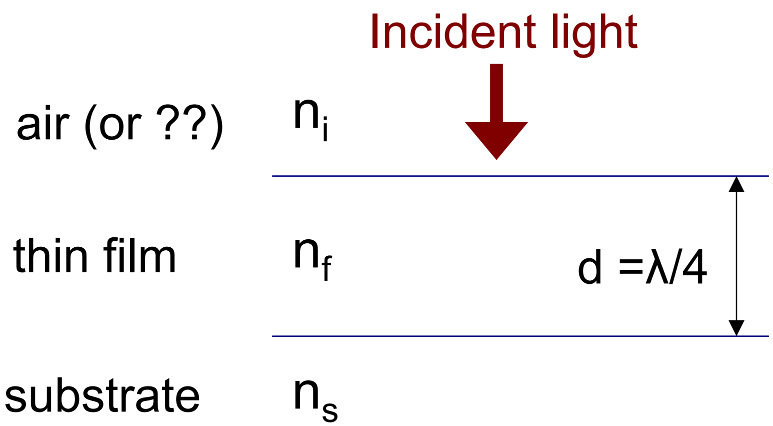


Reflectance is total for $\theta_i > \theta_c$

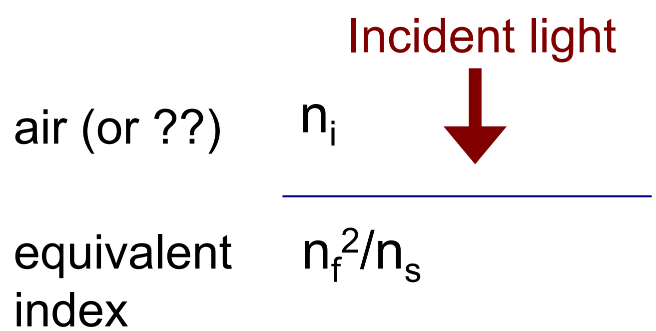
One can modify reflectance by applying thin films to the optic

Optical coatings – quarter wave rule

Consider a thin film of quarter-wave optical thickness

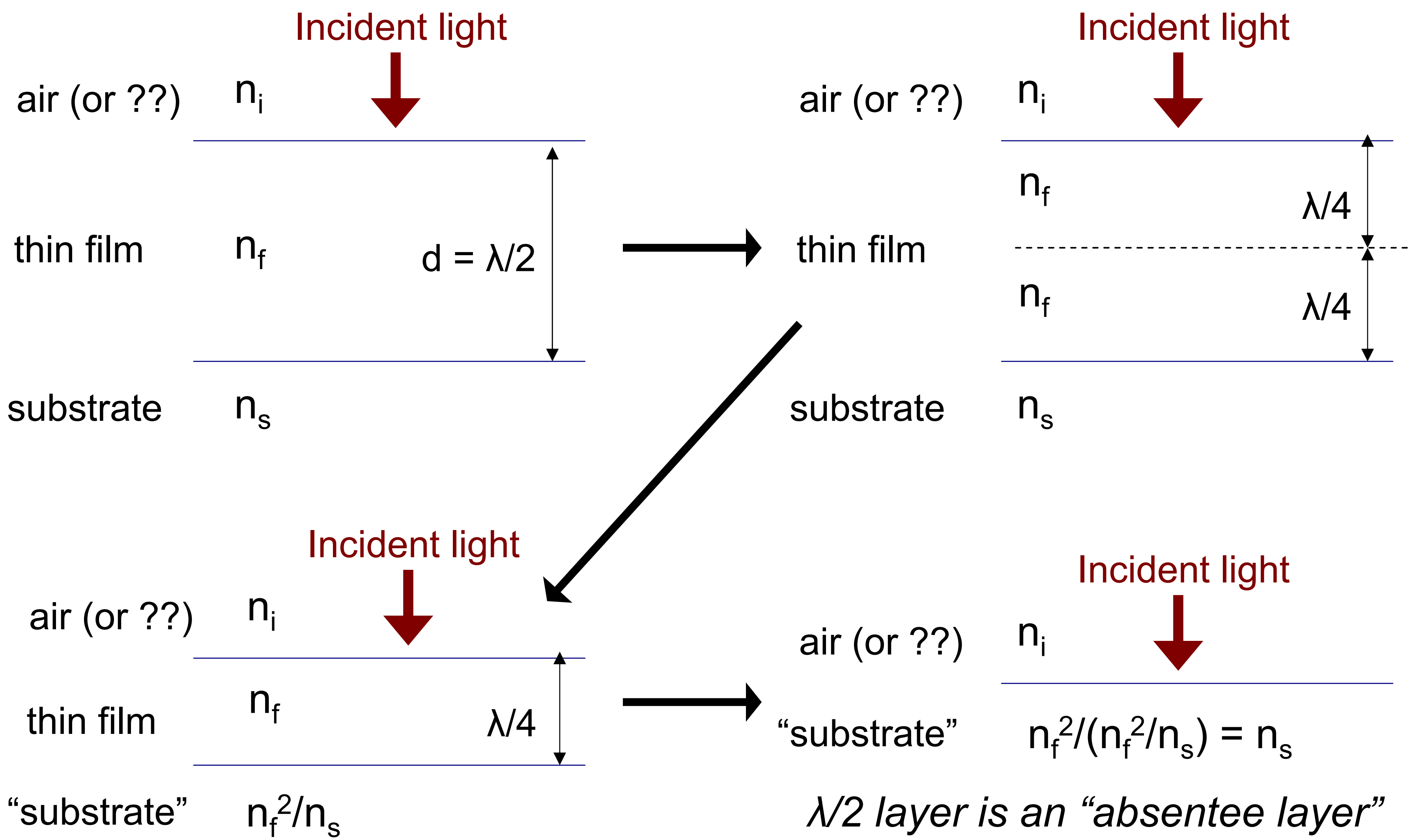


If film is exactly $\lambda/4$ thick $(=\lambda_0/(4n_f))$ can transform to an exact two-layer system



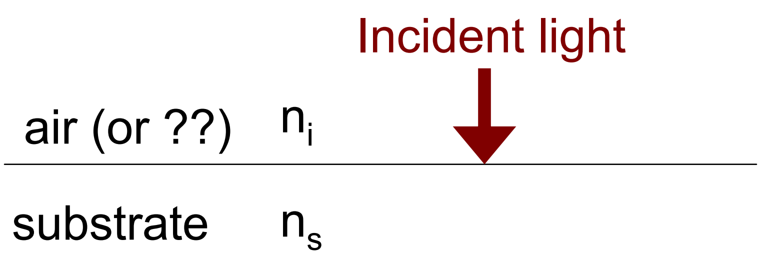
Now can use Fresnel reflection equations for the equivalent interfaces

Optical coatings – half wave rule



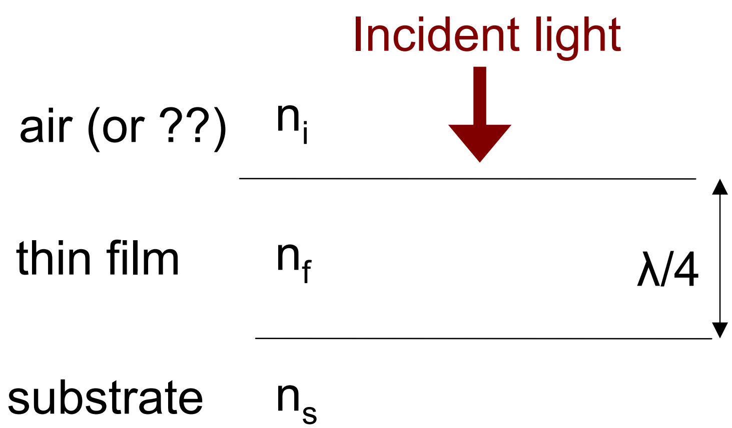
$\lambda/2$ layer is an "absentee layer"
… acts like it isn't there

Antireflection coatings



For normal incidence, Fresnel reflectance is

$$R = \left[\frac{n_i - n_s}{n_i + n_s} \right]^2$$



If the substrate is coated with a λ/4 film, we can use the quarter-wave rule…

$$R = \left[\frac{n_i - n_f^{\,2} / n_s}{n_i + n_f^{\,2} / n_s} \right]^2$$

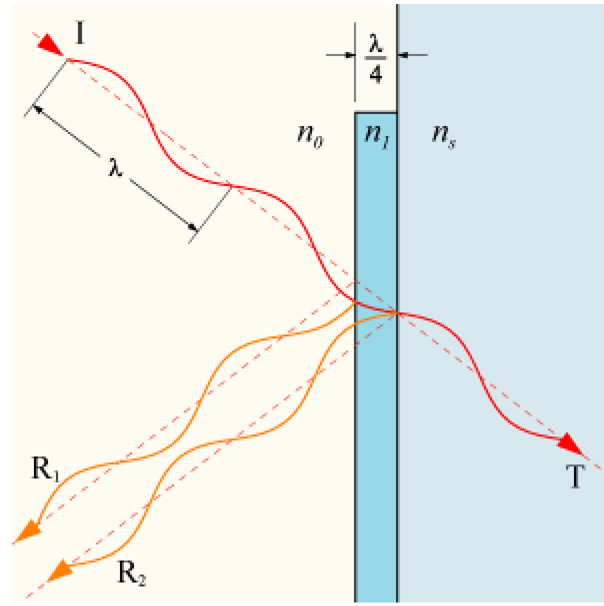
Reflectance is zero when numerator is zero (when $n_f = \sqrt{n_i n_s}$)

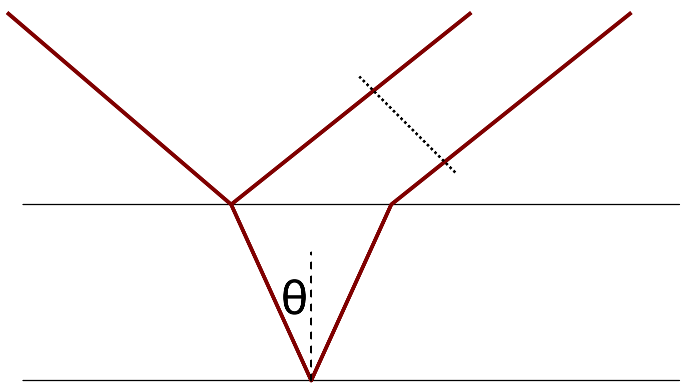
example: n_s = 1.5 (glass), n_f = 1.38 (MgF), n_i =1; R ~ 1.3%

AR coatings – what's the trick?

$$\boxed{\delta = \frac{2\pi\, n\, d}{\lambda_0}}$$

When path difference d = λ/2, the phase difference δ= π
…. *so the two reflected waves destructively interfere*



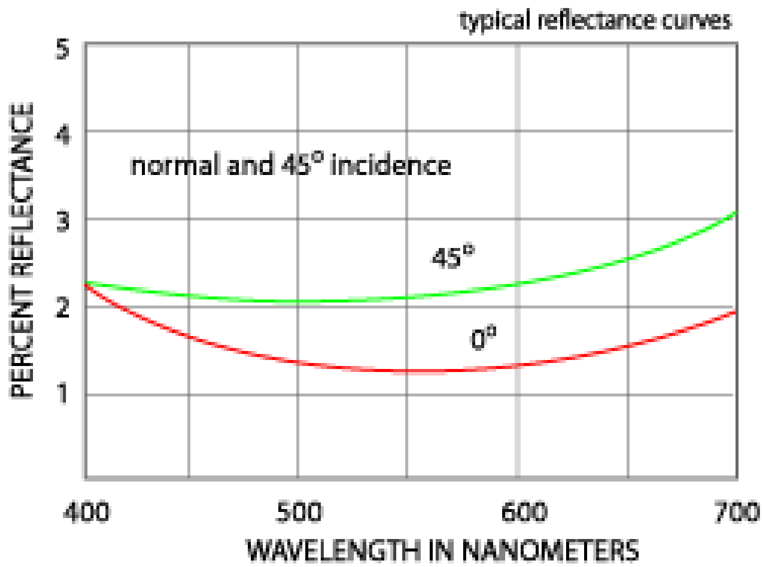


For non-normal incidence, require δ = π for the <u>refracted paths</u>

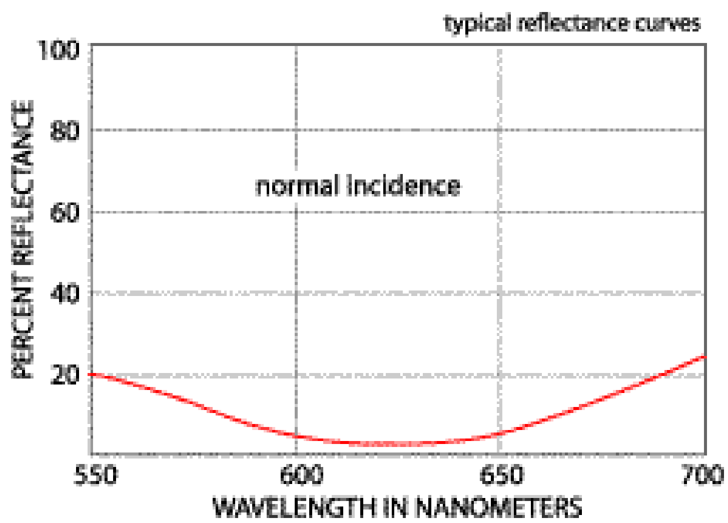
$$\delta = \frac{2\pi\, n\, d \cos\theta}{\lambda_0}$$

Non-normal incidence shifts minimum reflectance to shorter wavelength

AR coating performance

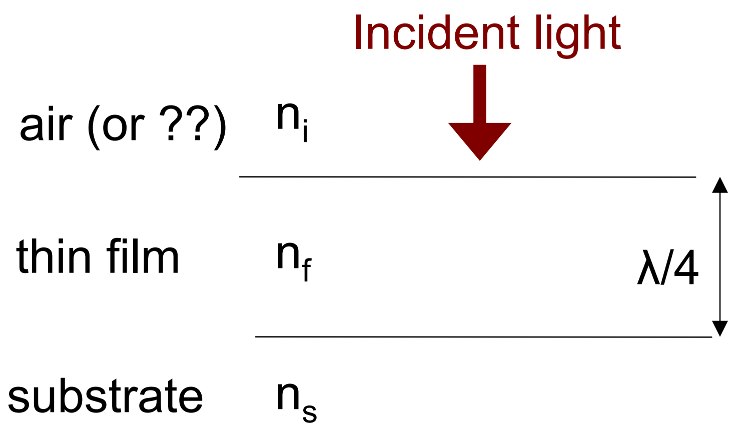


Single-layer MgF coating



Multi-layer “V-coating”

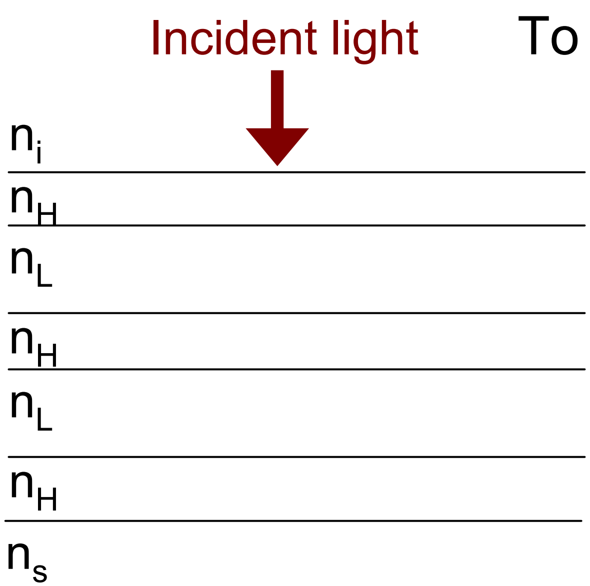
High-reflectance coatings



$$R = \left[\frac{n_i - n_f^{\,2} / n_s}{n_i + n_f^{\,2} / n_s} \right]^2$$

To increase R, make n_f large

To get higher reflectance, alternate n_H and n_L λ/2 layers

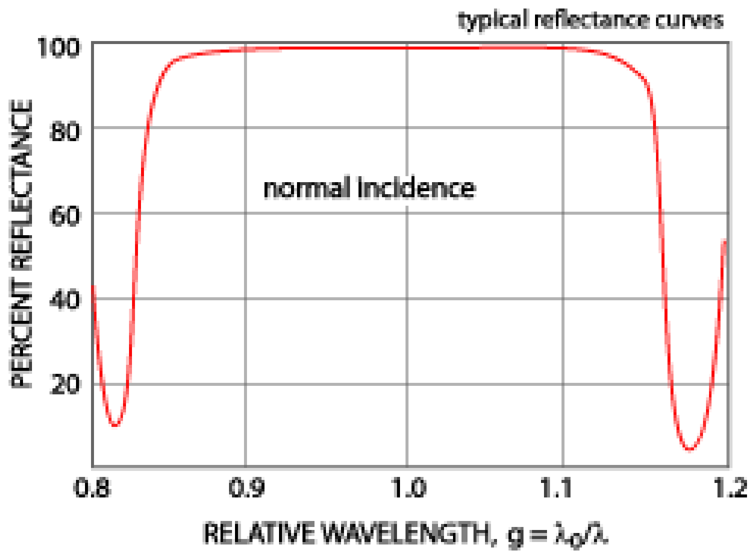


for x layers on n_H and x-1 layers n_L

$$R = \left[\frac{n_i - \dfrac{n_H^{\,2x}}{n_L^{\,2x-2}\, n_s}}{n_i + \dfrac{n_H^{\,2x}}{n_L^{\,2x-2}\, n_s}} \right]^2$$

maximize n_H/n_L

Multilayer reflector performance



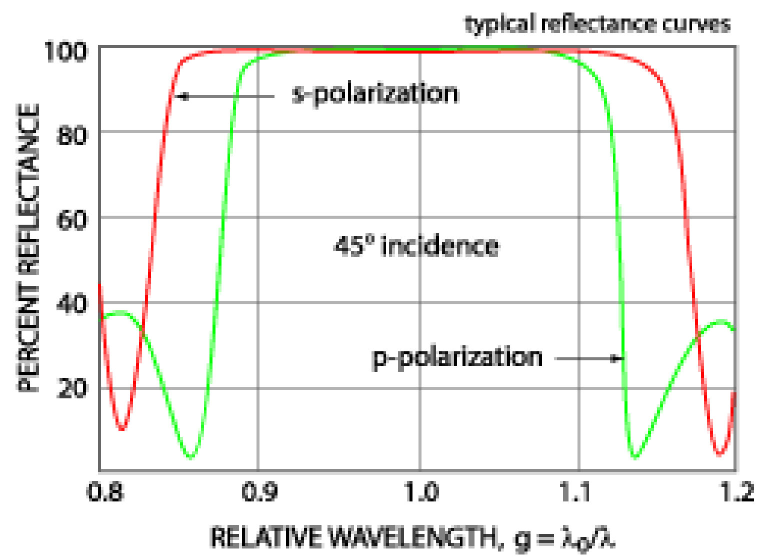
Can achieve R> 99.9% off-the-shelf

five-9's possible… limited by losses

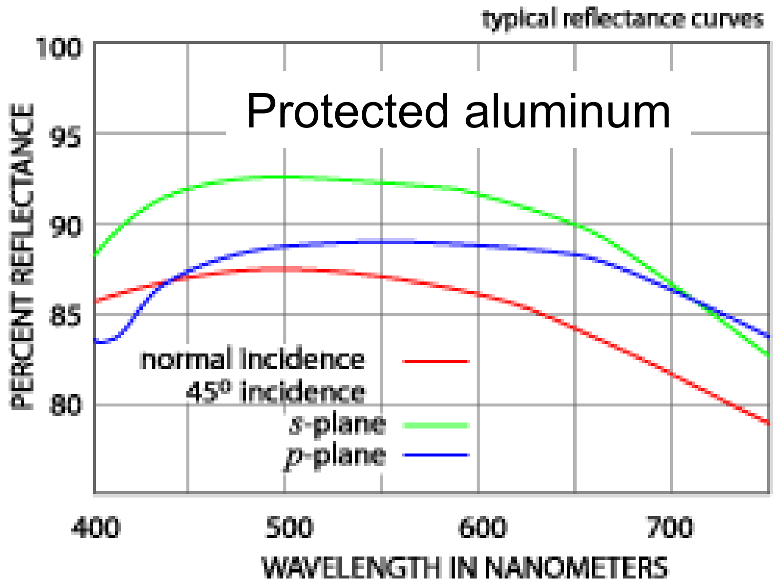
Damage thresholds> 10 J/cm^2 (10 ns pulse)

Non-normal incidence shows polarization dependence

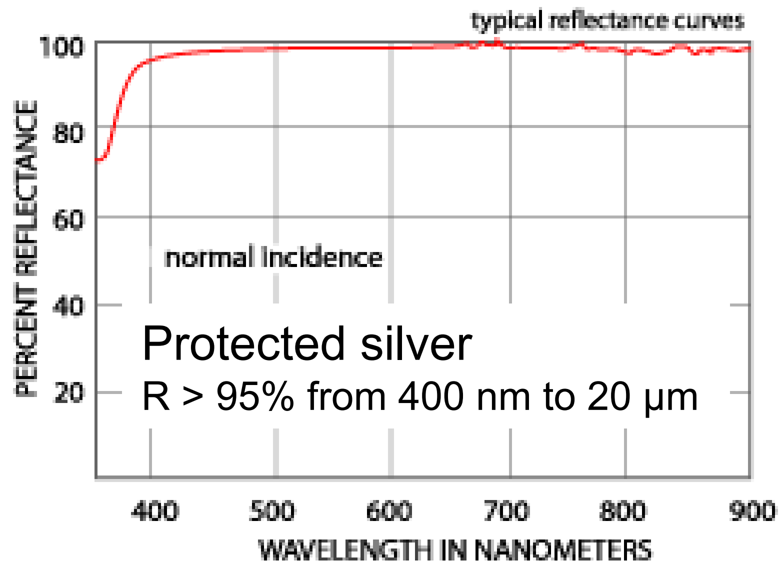
Caution: may not be suitable for ultrafast lasers

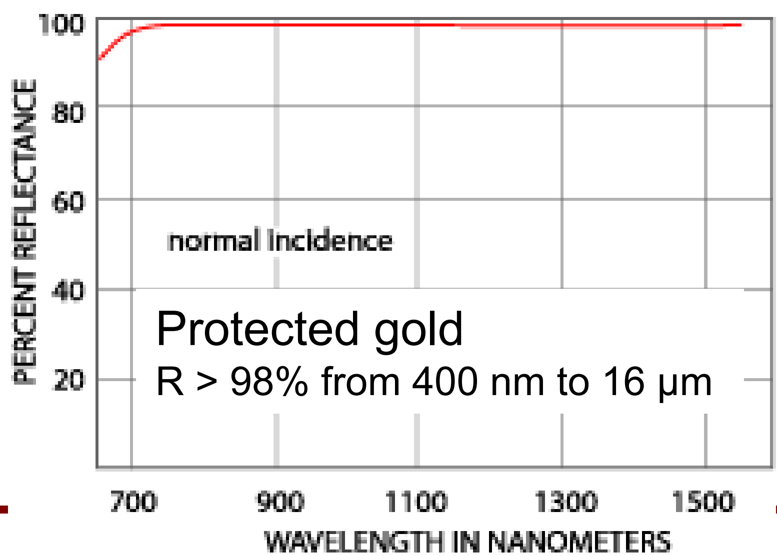


Metallic mirror performance



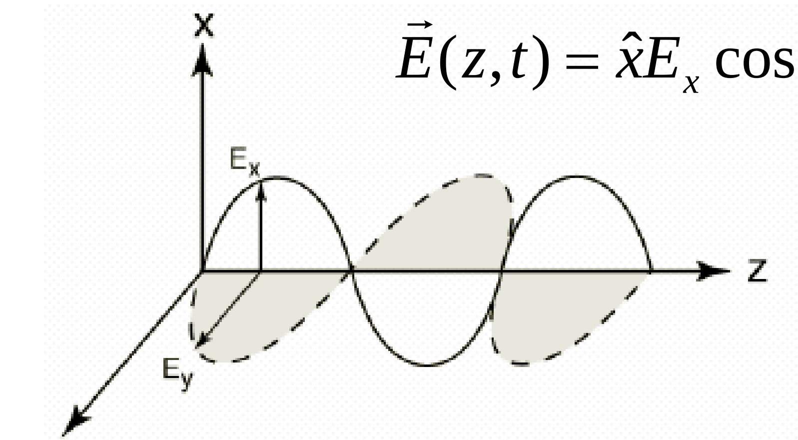
Damage thresholds 10x to 100x lower than dielectric stacks





Wave polarization

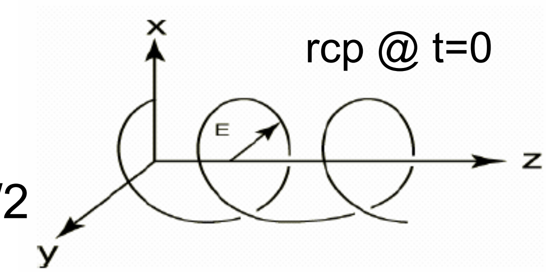
The polarization state is defined by the direction of the transverse E-field



$$\vec{E}(z,t) = \hat{x}E_x \cos(\omega t - k_0 z + \phi_x) + \hat{y}E_y \cos(\omega t - k_0 z + \phi_y)$$

Any polarization can be decomposed into orthogonal components

Polarization state	Ex	Ey	phase
Linear vertical	1	0	$\Phi_x = \Phi_y = 0$
Linear horizontal	0	1	$\Phi_x = \Phi_y = 0$
Linear @ 45º	$1/\sqrt{2}$	$1/\sqrt{2}$	$\Phi_x = \Phi_y = 0$
Right circular	$1/\sqrt{2}$	$1/\sqrt{2}$	$\Phi_x = 0,\ \Phi_y = \pi/2$
Left circular	$1/\sqrt{2}$	$1/\sqrt{2}$	$\Phi_x = 0,\ \Phi_y = -\pi/2$

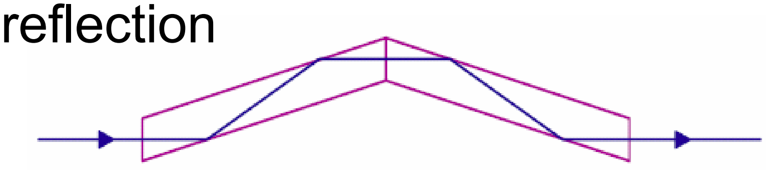


Retarders (aka waveplates)

Retarders induce a phase difference ΔΦ between orthogonally polarized components of light

Linear retarders produce $\Delta\Phi = \Phi_x - \Phi_y$

- waveplates use optically anisotropic materials with *linear birefringence* $\Delta n = n_x - n_y$
- electro-optic materials (Pockel's effect): $\Delta n = f(E)$
- rhombs use retardance from total-internal-reflection



Circular retarders produce $\Delta\Phi = \Phi_{rcp} - \Phi_{lcp}$

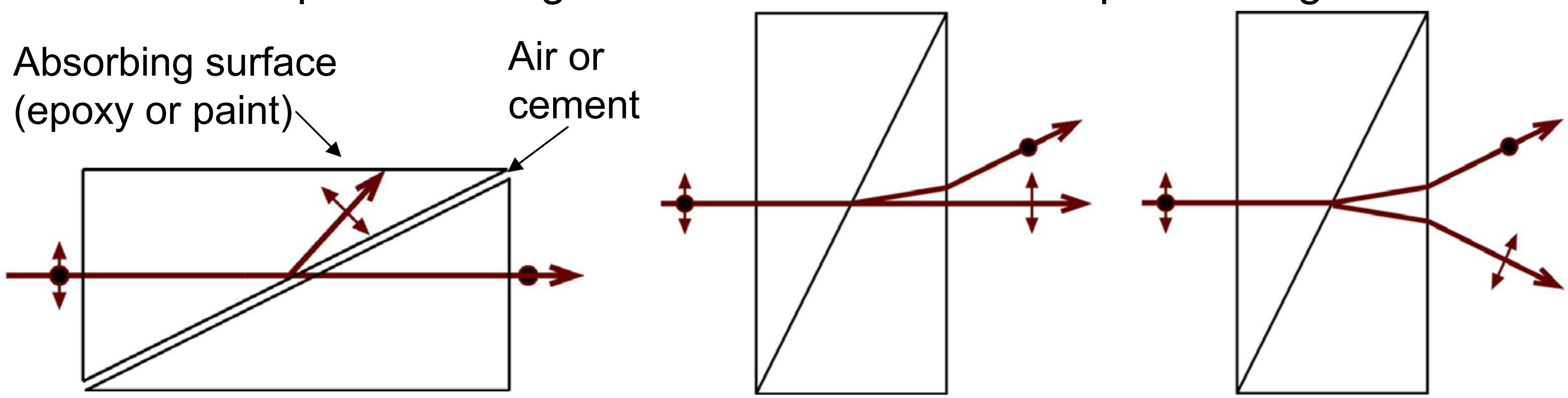
- optically active chiral materials (sugar solution) with circular birefringence $\Delta n = n_{rcp} - n_{lcp}$
- magneto-optic effects: $n_{rcp} - n_{lcp} = f(H)$

Retarders can rotate or transform the polarization state

Polarizers

Birefringent crystal polarizers – extinction ratios of 10^{-5} to 10^{-7} are possible

these operate through TIR so have limited acceptance angle



Glan-Thompson Rochon Wollaston

Dichroic polarizers –

cheap, limited power-handling capability

extinction ratio ~ 10^{-2} to 10^{-5} in visible

How do you determine the polarizer's transmission axis?

look at the glare from a shiny surface…

s-polarization “skips”, p-polarization “plunges”

Jones calculus

Jones calculus is a tool for calculating the evolution of completely polarized beams

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{out} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{in}$$

Jones vectors (for fields)

Linear along x $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Linear along y $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Linear at 45° $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Right-circular $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$

Left circular $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

Jones matrices (for components)

Polarizer at 0° $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

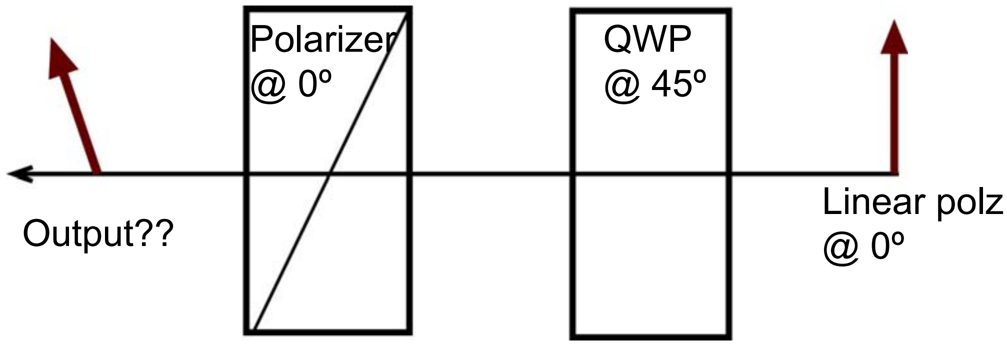
Polarizer at θ° $\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$

Quarter wave retarder at 0° $\begin{bmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{bmatrix}$

Quarter wave retarder at 45° $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$

Half wave retarder at 0° $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Jones calculus example



$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{out} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{in}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad I = E\,E^* = \frac{1}{2}$$

Output polarization is linear vertical

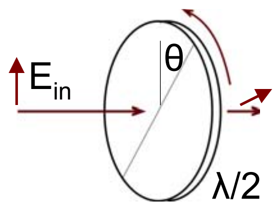
Optical power is halved

Polarization control

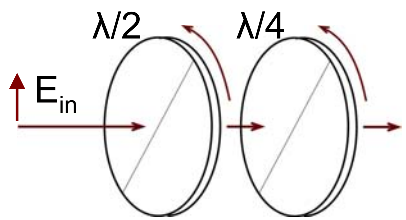
Desired polarization control

Linear-to-linear

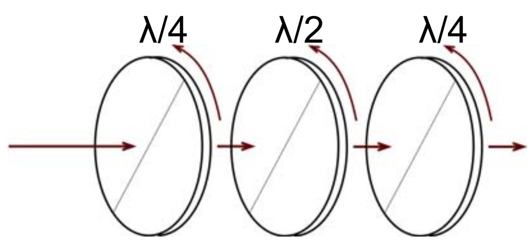
λ/2 plate rotates state by 2θ



Linear-to-arbitrary



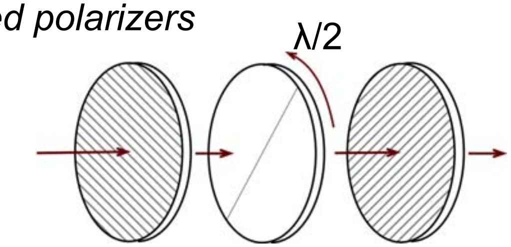
Arbitrary-to-arbitrary



Desired power control

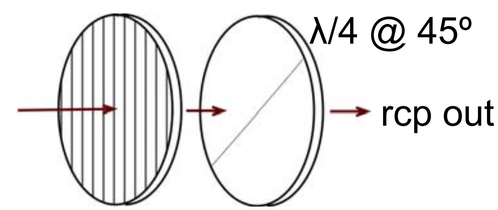
Linear input attenuator

λ/2 between crossed polarizers
$T = \sin^2\theta$



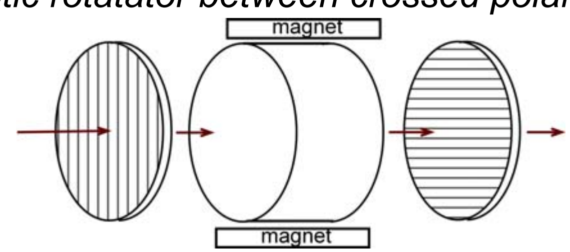
Linear input “pseudo-isolator”

requires unchanged reflection polarization



Optical isolator

magneto-optic rotatator between crossed polarizers



"Unpolarized" light

Monochromatic light is polarized

$$\vec{E}(z,t) = \hat{x}E_x \cos(\omega t - k_0 z + \phi_x) + \hat{y}E_y \cos(\omega t - k_0 z + \phi_y)$$

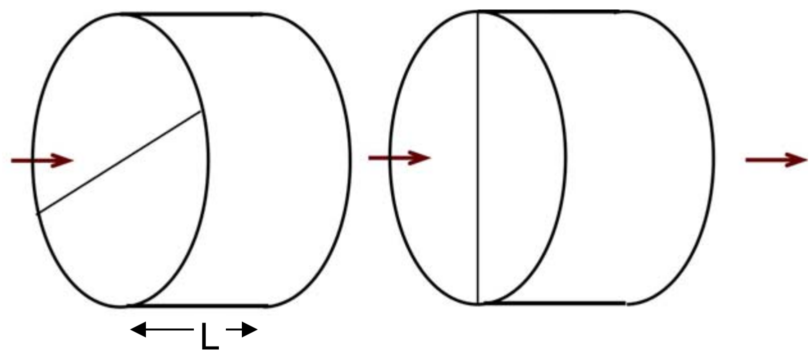
- For strictly monochromatic light Ex, Ey, φx, & φy are <u>constant</u>
  - in other words, one cannot make an unpolarized monochromatic laser beam
  - exception is *spatia*l depolarization using ground glass or integrating spheres

For non-monochromatic light, pairs E_x & $E_{y,}$ and $\varphi_{x,}$ & φ_y have statistical correlations that depend the spectral bandwidth Δω of the light

For quasi-monochromatic (with bandwidth Δω around ω_0) light the time scale of this correlation is the coherence time τ_c~2π/ Δω (or coherence length $L_c = c\, \tau_c$)

Can "depolarize" broadband light with a Lyot depolarizer

Requires thick birefringent retarders with $\Delta nL > L_c$



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