

# Optics for laser measurements

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# Outline

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Geometrical optics

lenses

ray tracing

aberrations

Physical optics

diffraction and Fourier optics

Gaussian optics

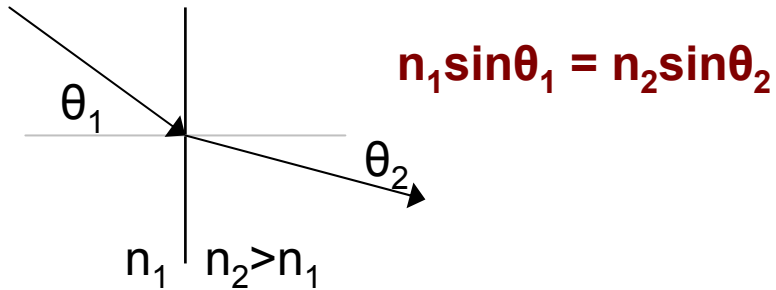
Reflections & coatings

Polarization, interference & coherence

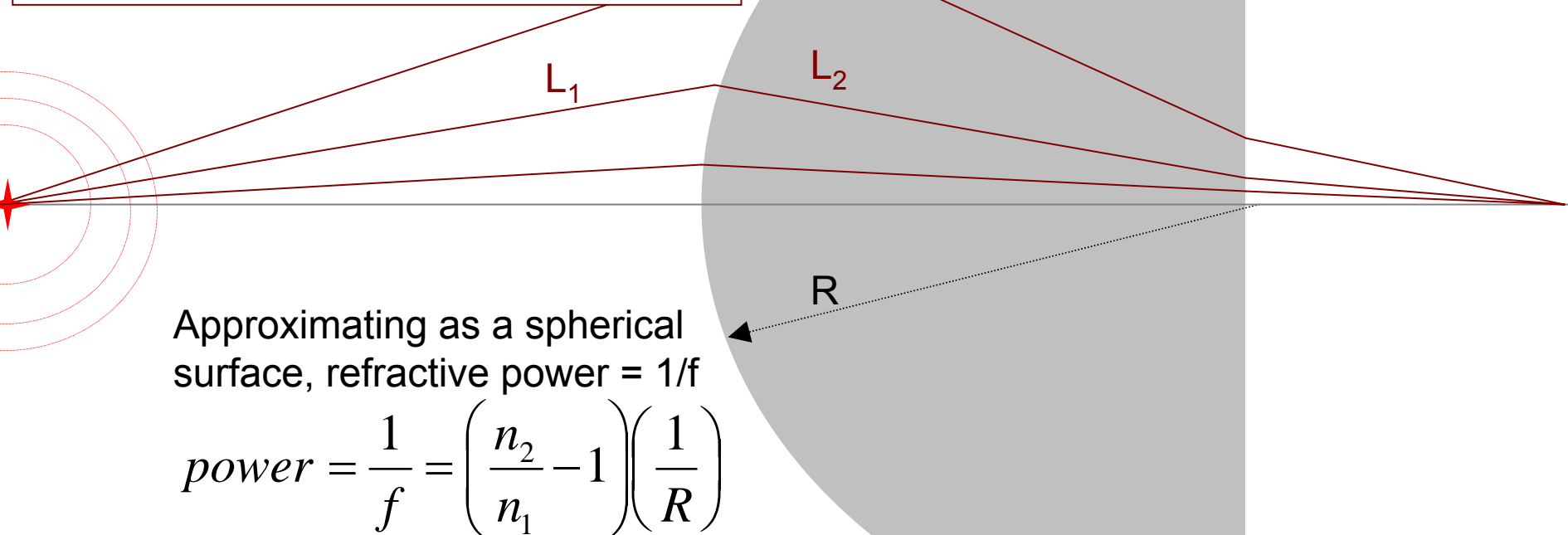
# Power of refracting surfaces

Fermat's principle of least time:  $n_1 L_1 + n_2 L_2$  is stationary ( $d(\Sigma nL)/dL = 0$ )

This principle leads to *Snell's law*

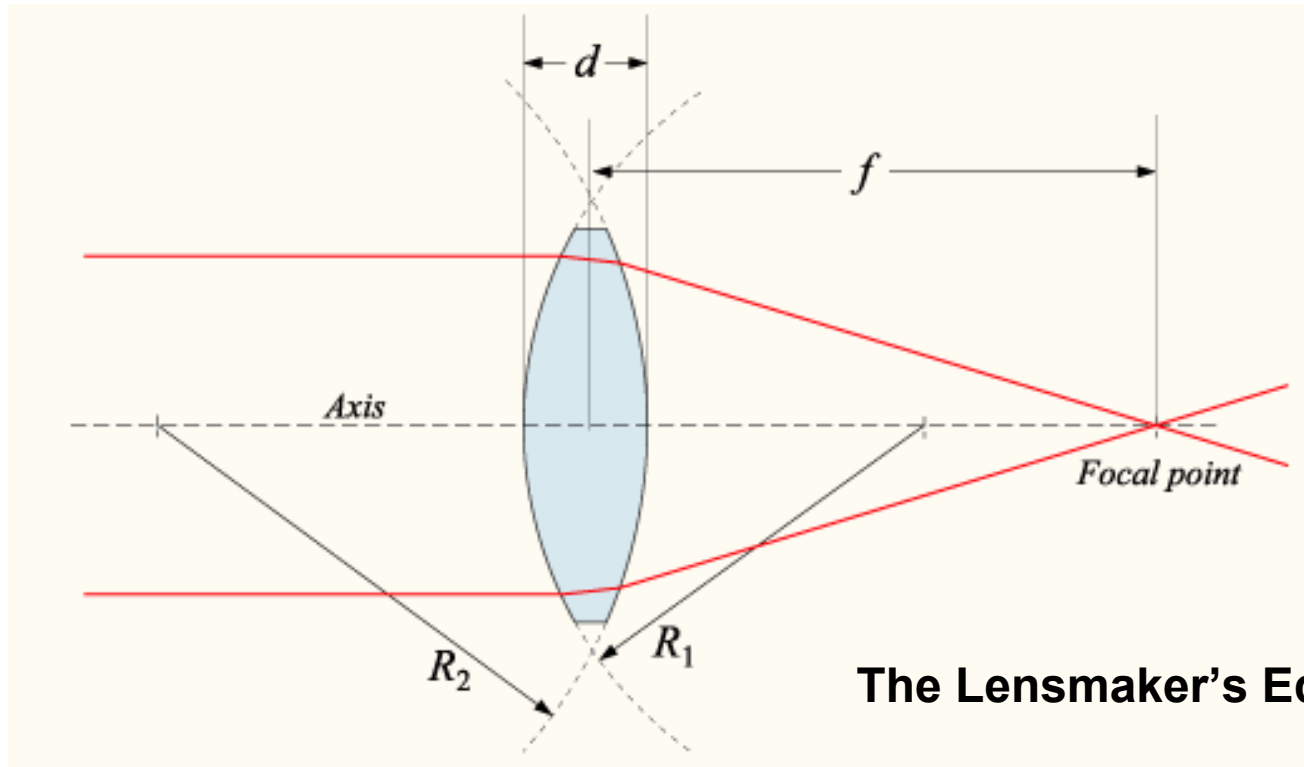


and gives curved surfaces focusing power



# Thick lens (and lensmaker's equation)

A lens is transmitting optic that has refractive power

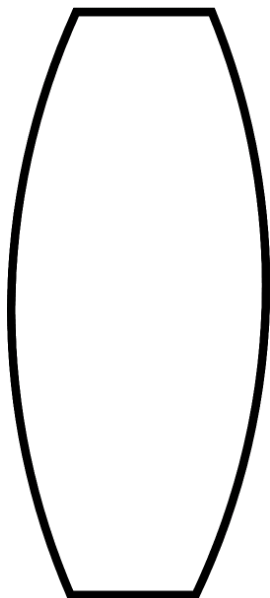


**A collimated input beam focuses to focal point  $f$**

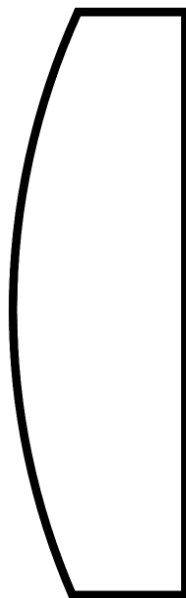
**The Lensmaker's Equation**

$$power = \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{(n - 1)d}{nR_1R_2} \right)$$

# Types of lenses



biconvex



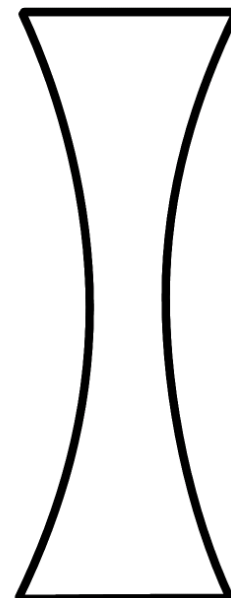
plano-convex



convex-concave



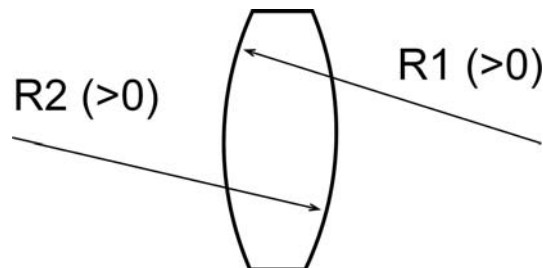
plano-concave



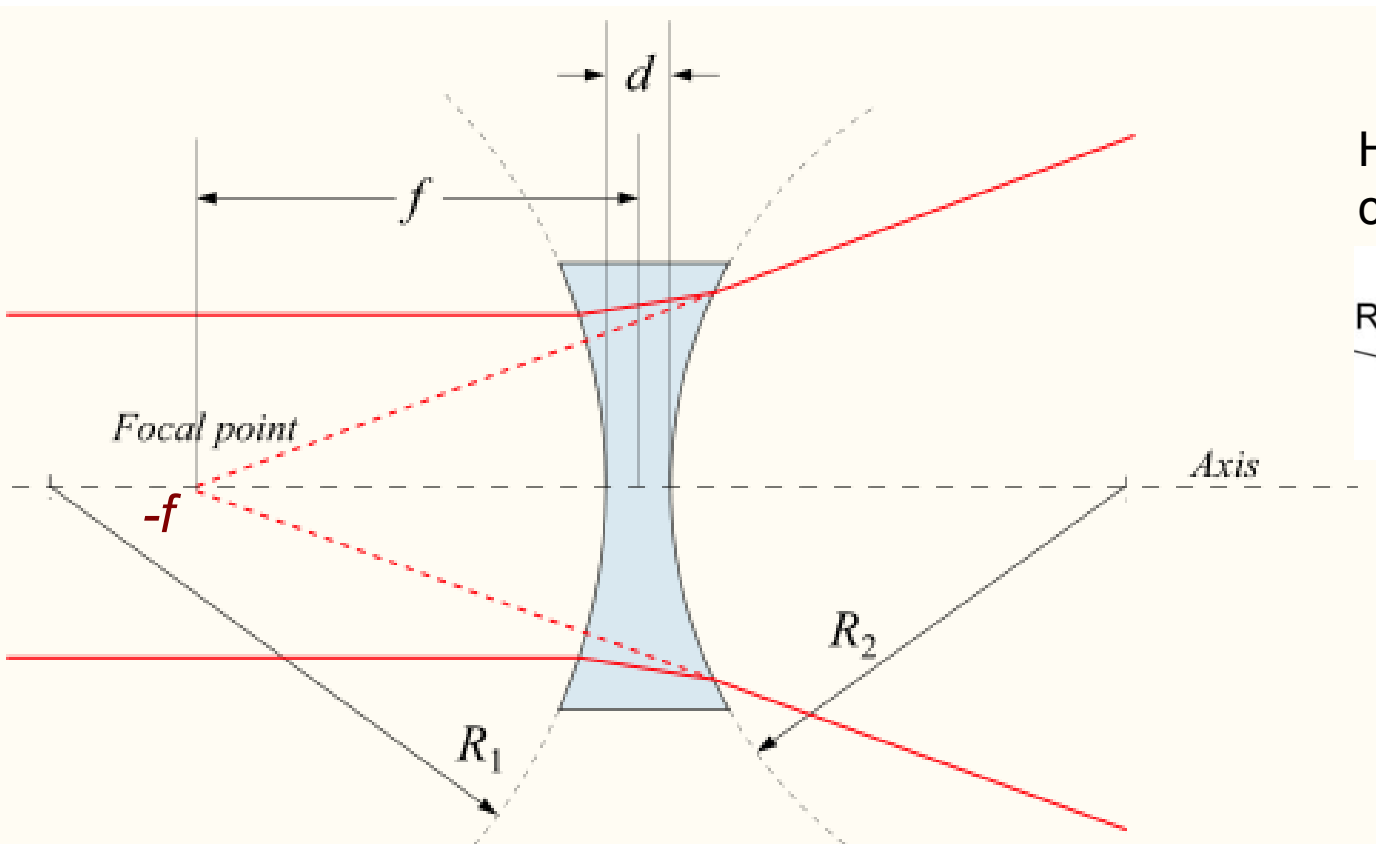
biconcave

Coddington  
shape factor

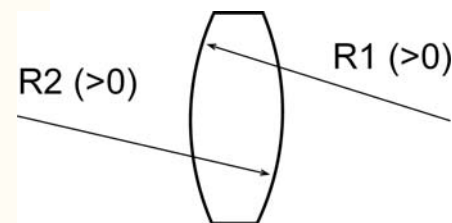
$$q = \frac{R_2 + R_1}{R_2 - R_1}$$



# A lens with negative power



Here the radii of curvatures are  $< 0$

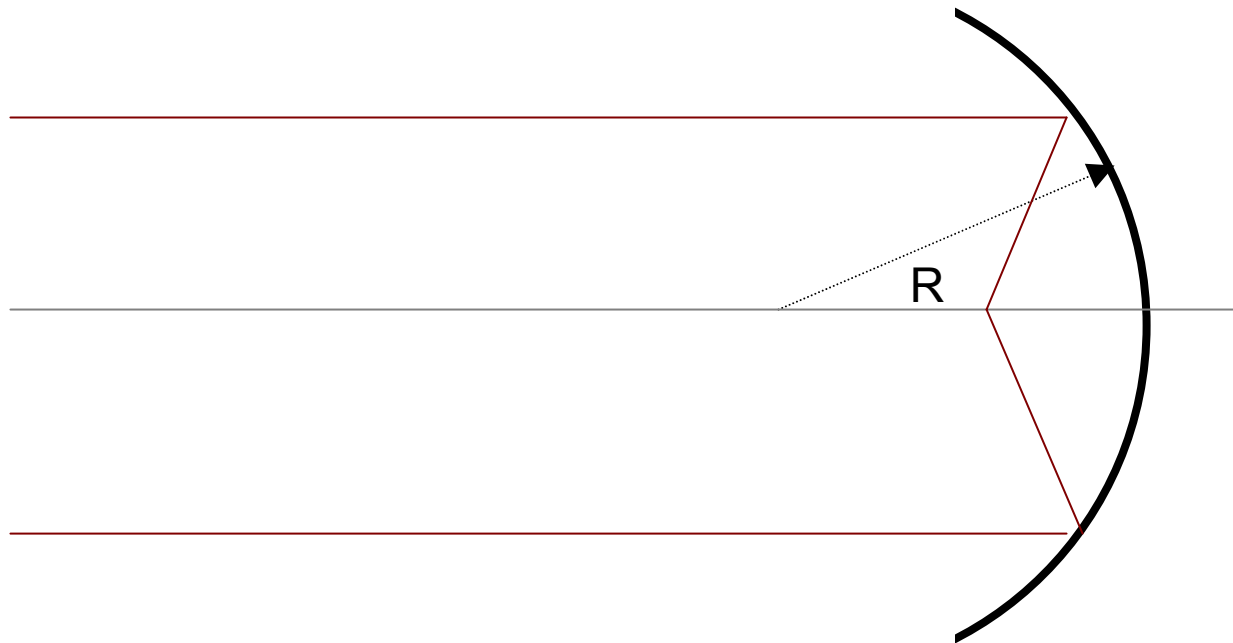


$$power = \frac{1}{f} = -(n-1) \left( \frac{1}{|R_1|} + \frac{1}{|R_2|} - \frac{(n-1)d}{nR_1R_2} \right)$$

# Curved mirrors have power

For a mirror,  $n = -1$

$$power = \frac{1}{f} = (n - 1) \left( \frac{1}{R} \right) \quad \text{so } f = -R/2$$

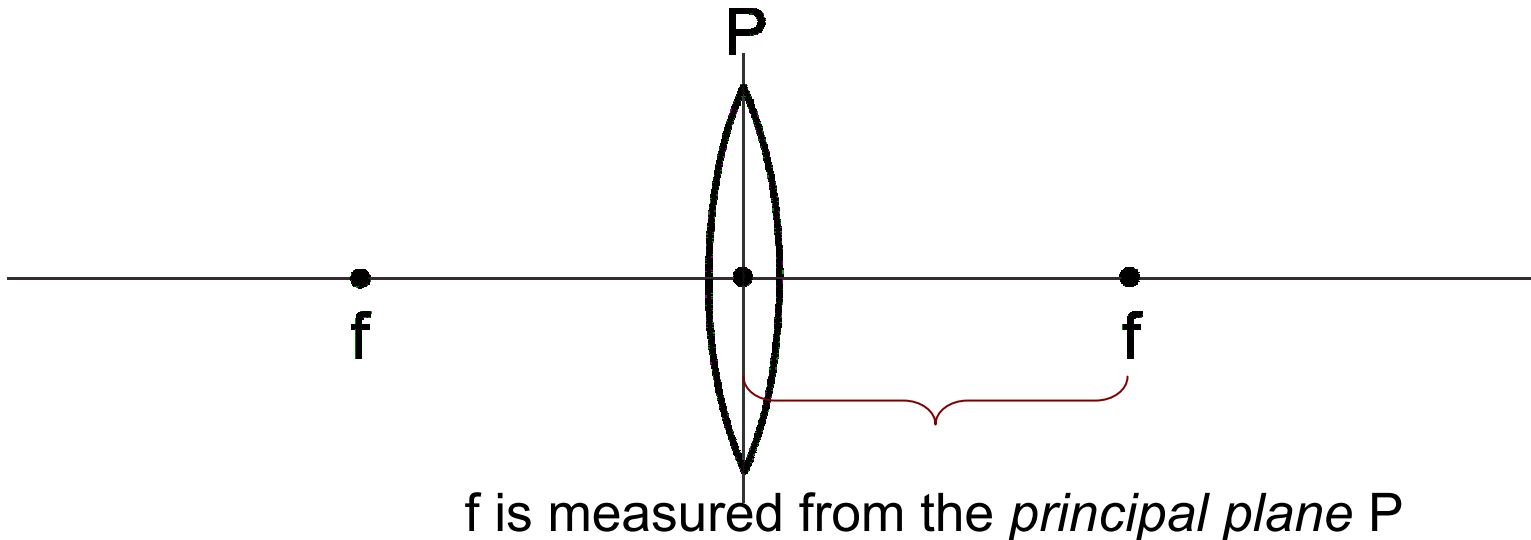


# The thin lens – a useful simplification

A thin lens has radii of curvatures greater than thickness ( $R_1, R_2 > d$ )

**For a thin lens in air**

$$power = \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

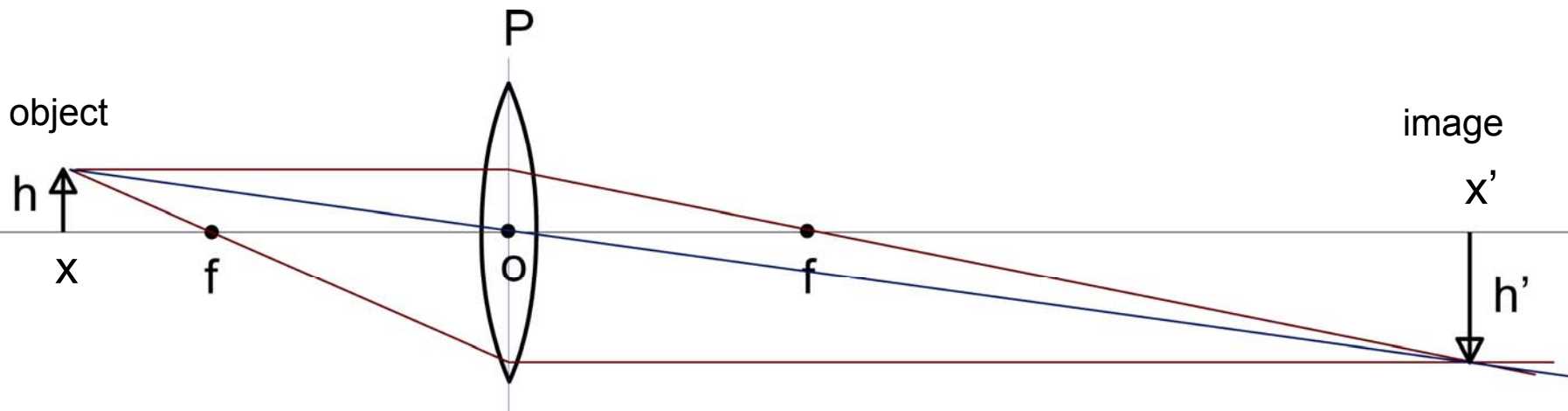


A thick lens may have two non-coincident principal planes



# Simple ray tracing

An **object**  $h$  units high creates an **image**  $h'$  units high



**Can map the object to the image using two of three rays:**

- Ray parallel to axis to principal plane, then through back focal point
- Ray through front focal point to principal plane, then parallel to axis
- Ray through nodal point

Points  $x$  and  $x'$  are called *conjugates*

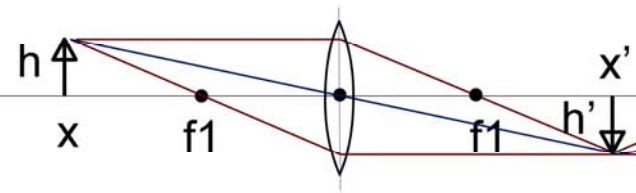
see <http://www.colorado.edu/physics/phet/simulations/lens/lens.swf>

# Simple two-lens system

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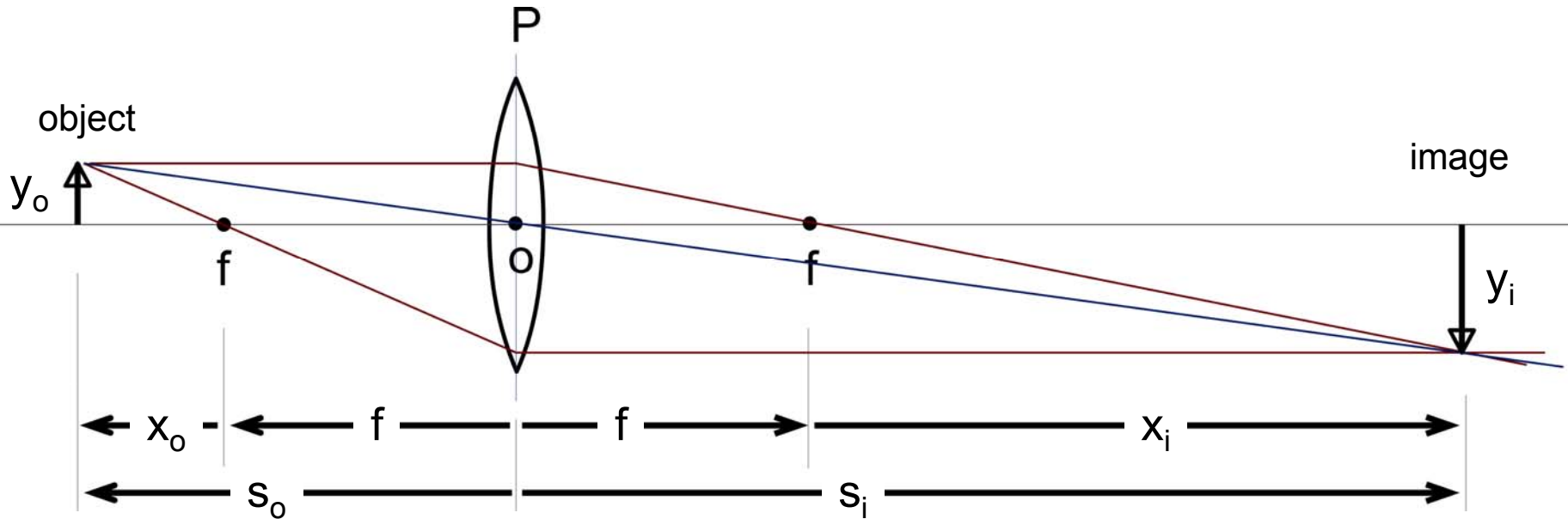
If lenses are separated by more than sum of focal lengths

- find an intermediate image
- map the intermediate image to the final image



**Question... How does the lens diameter affect the image???**

# Imaging equations



Gaussian lens equation: 
$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

Newtonian lens equation: 
$$x_o x_i = f^2$$

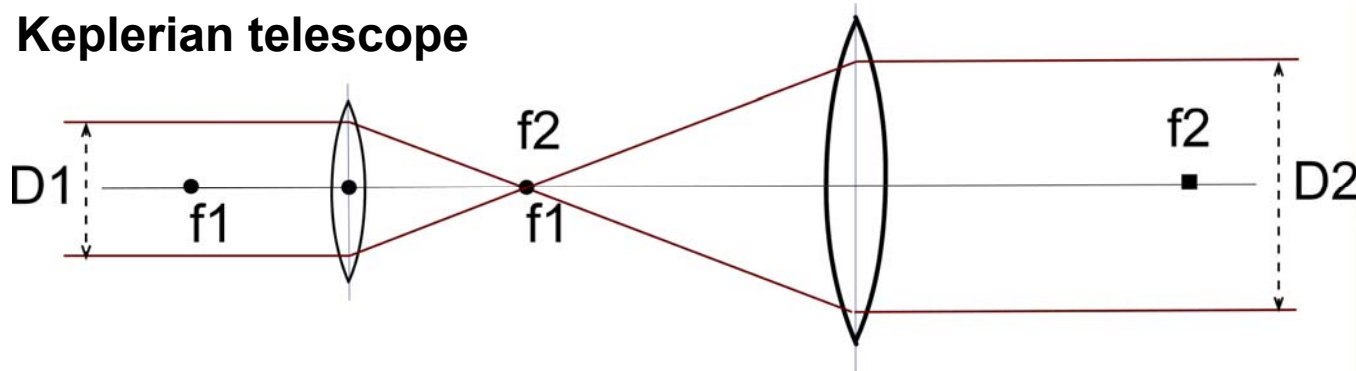
Transverse magnification

$$M = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

# Beam expanders

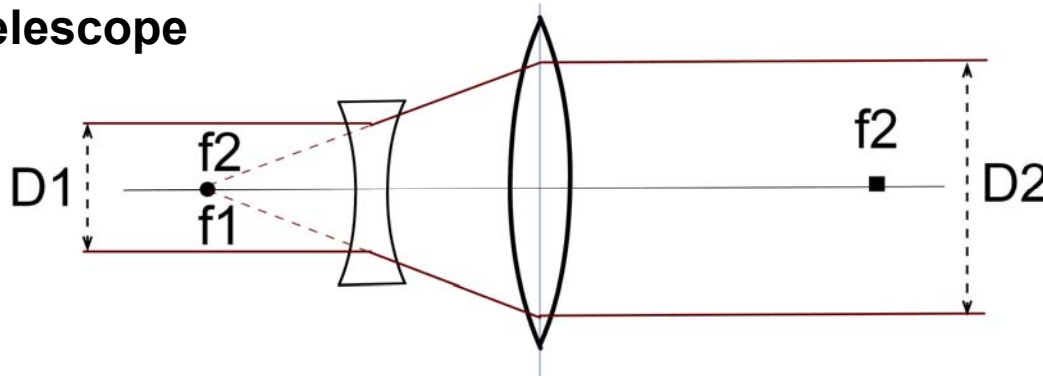
Can expand beam by separating two lenses by the sum of their focal lengths

## Keplerian telescope



$$\frac{D_2}{D_1} = \frac{f_2}{f_1}$$

## Galilean telescope



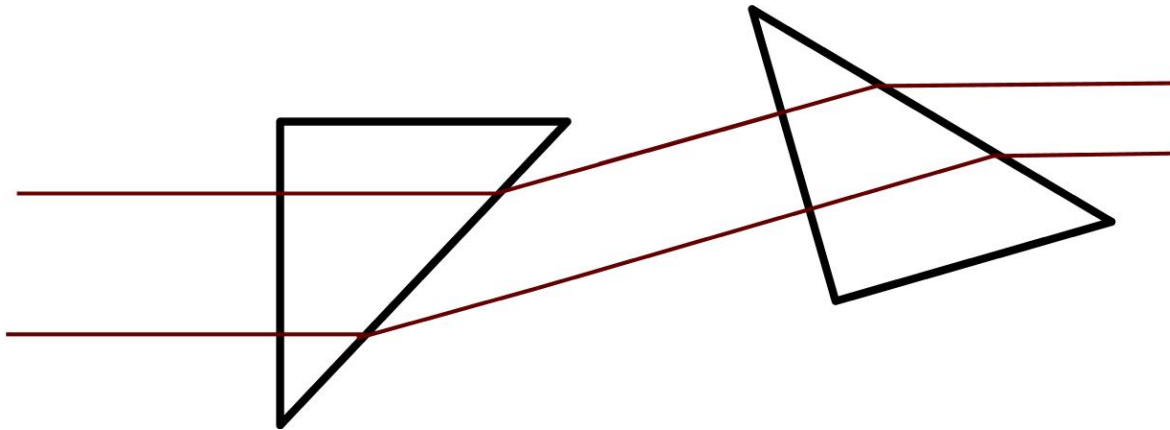
Does the lens diameter matter?

Which telescope is better for high pulse-energy lasers?

# Beam shaping

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Non-circular beams (e.g., laser diodes) can be “circularized” in one-dimension using cylindrical lenses or prisms

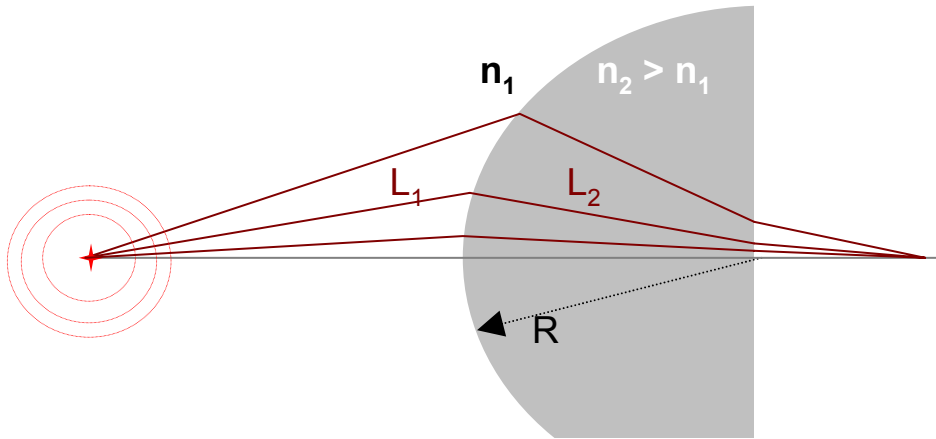


This method provides 1-d compression/expansion

To avoid the beam offset, use cylindrical lenses in a telescope

# Aberrations

The perfect (or “stigmatic”) imaging discussed before occurs for hyperboloids



Ray optics assumes spherical surfaces...

- *this only works in the “paraxial” (small angle, where  $\sin x \approx x$ ) limit*

- *we ignored  $x^3/3! + x^5/5! - \dots$  in the expansion of  $\sin x$*

Aberrations exist when the marginal rays don’t focus at the same place as the paraxial rays

The third-order aberrations are:

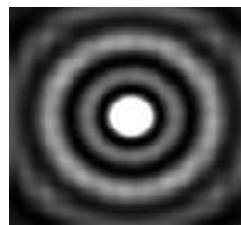
- spherical
- coma
- astigmatism
- field curvature
- distortion

*increasing  
dependence on  
distance from  
image plane axis*

# Focused spots with aberration



Perfect focus



Spherical aberration

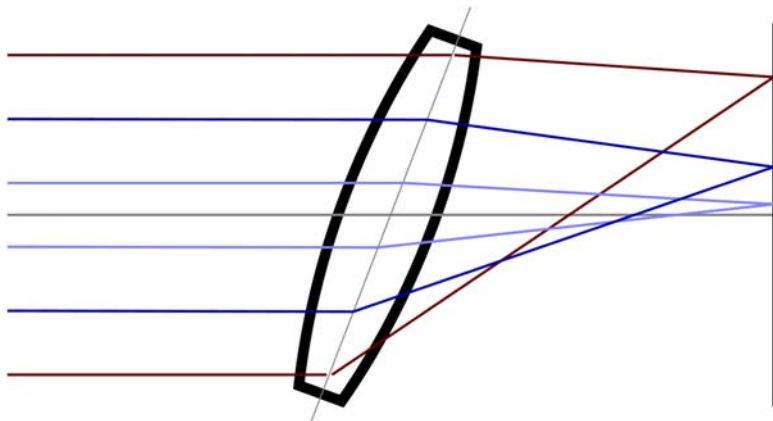


Coma

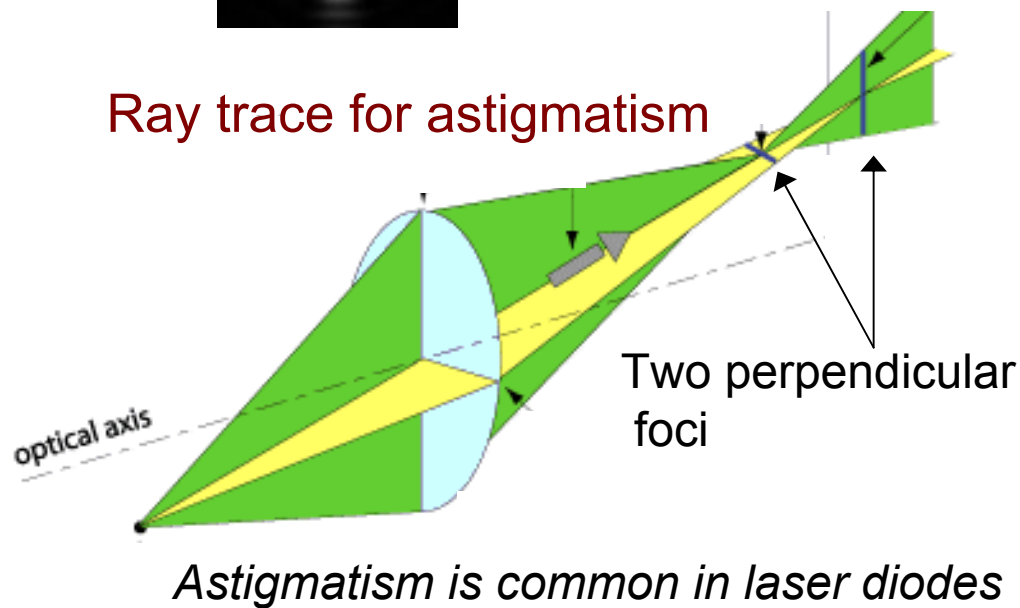


Astigmatism

Ray trace for coma

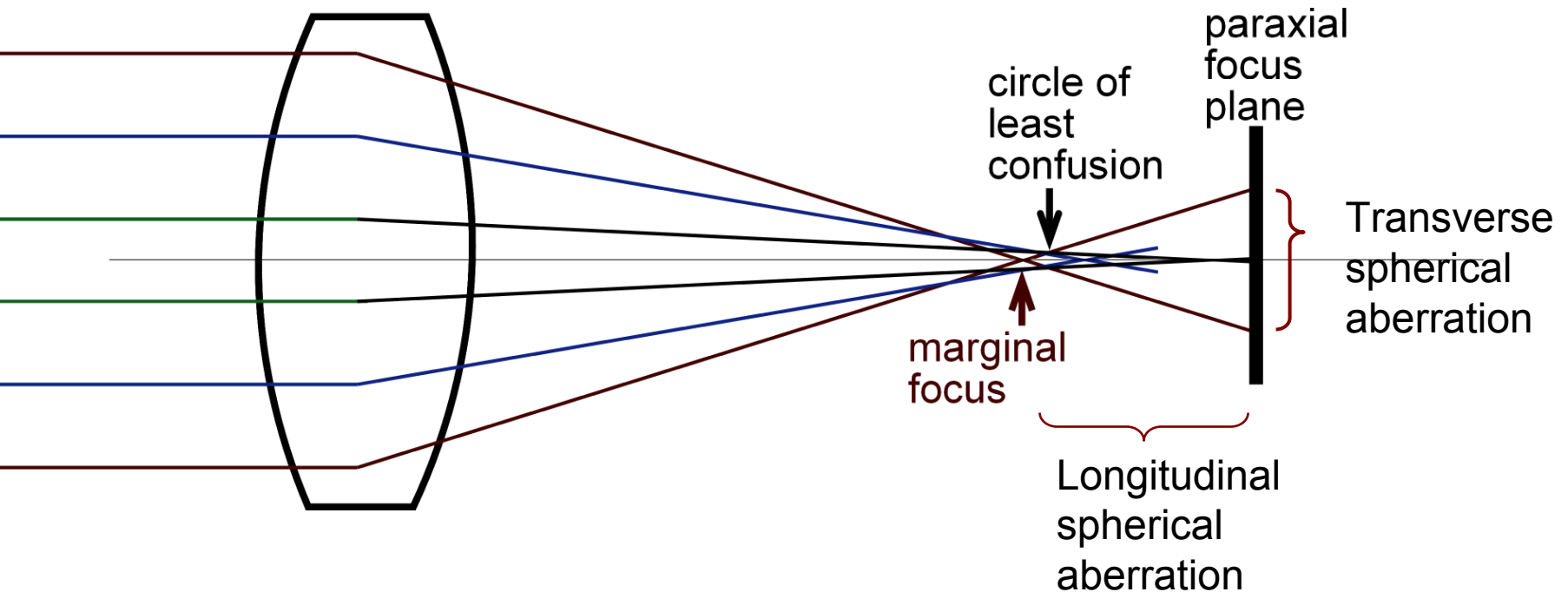


Ray trace for astigmatism



# Spherical aberration

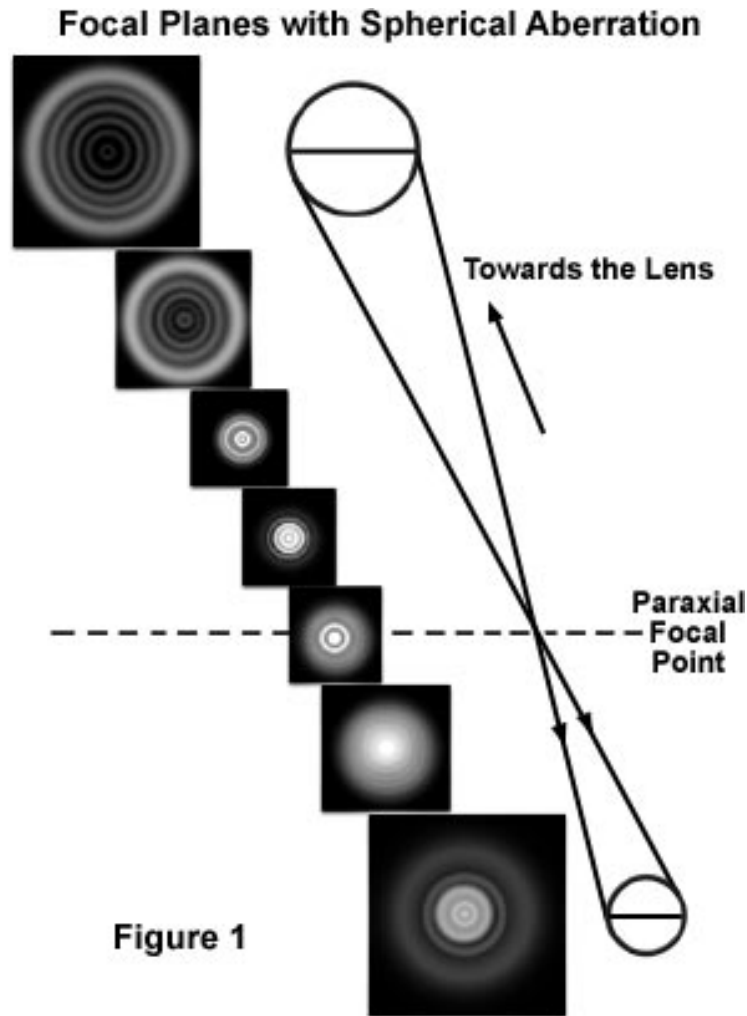
**Spherical aberration causes on-axis distortion**



- SA causes a radially symmetric spread of the focused point
- Smallest “spot” will occur in front of paraxial focus plane

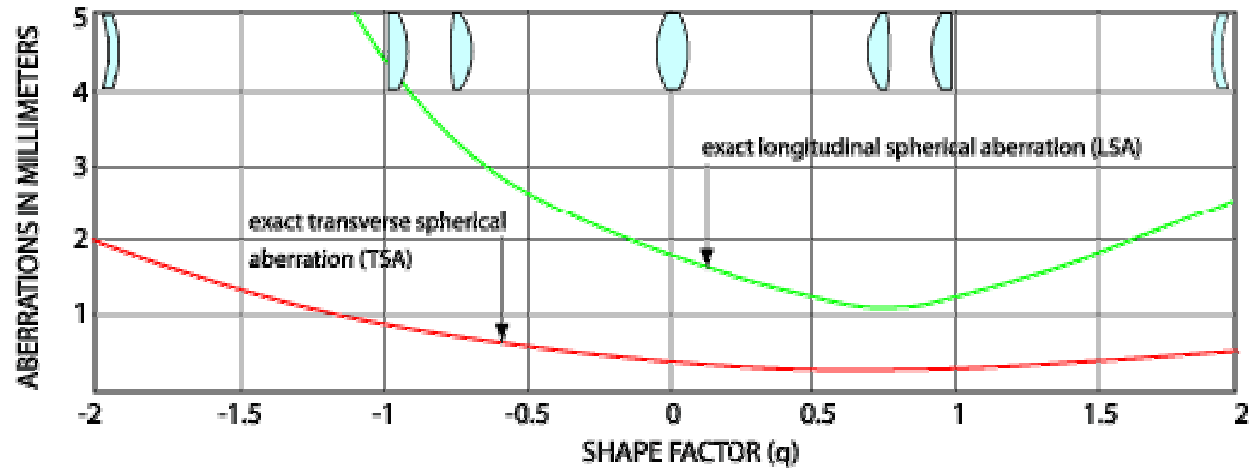


# Pictures of SA though focus



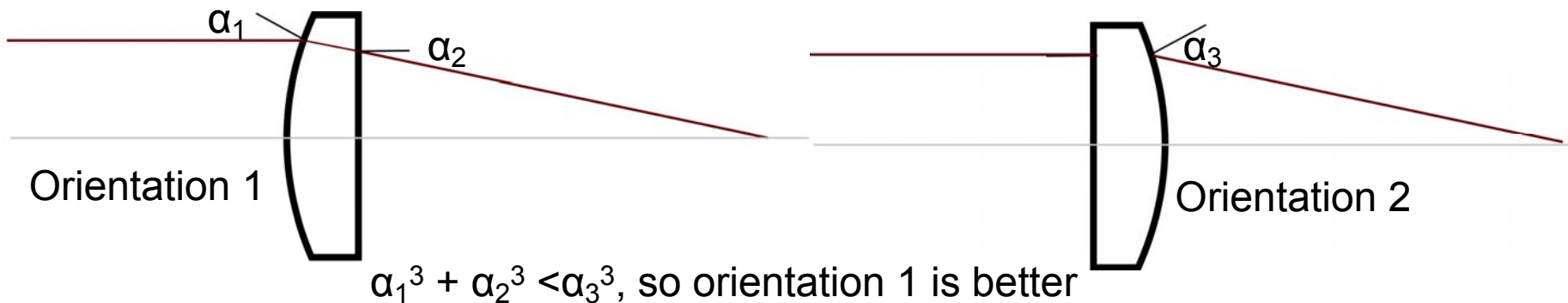
Java tutorial at <http://www.olympusmicro.com/primer/anatomy/aberrationhome.html>

# SA depends on lens shape



[http://optics.mellesgriot.com/opguide/fo\\_3\\_3.htm](http://optics.mellesgriot.com/opguide/fo_3_3.htm)

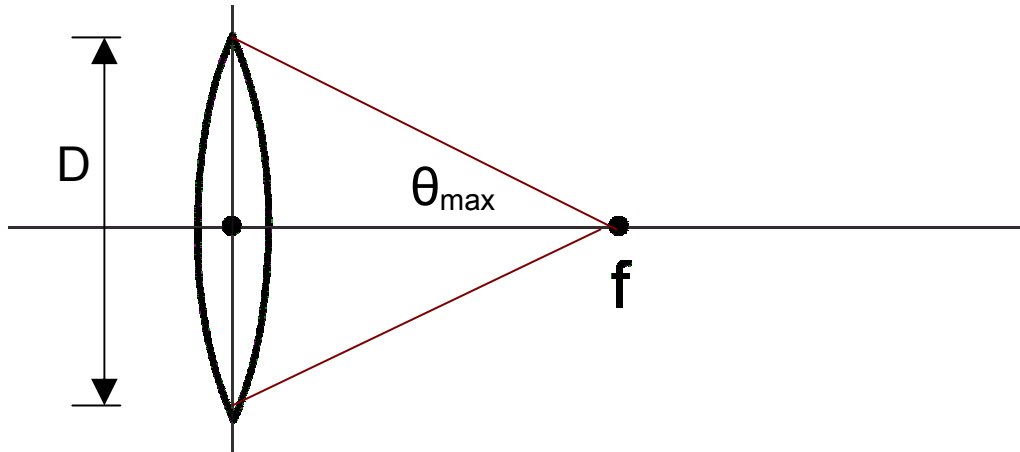
Aberration is minimized when the incident angles to the optical surface are minimized



*Rule of thumb: point most curved surface towards most distant conjugate*

# Numerical aperture & F-number

## Numerical aperture

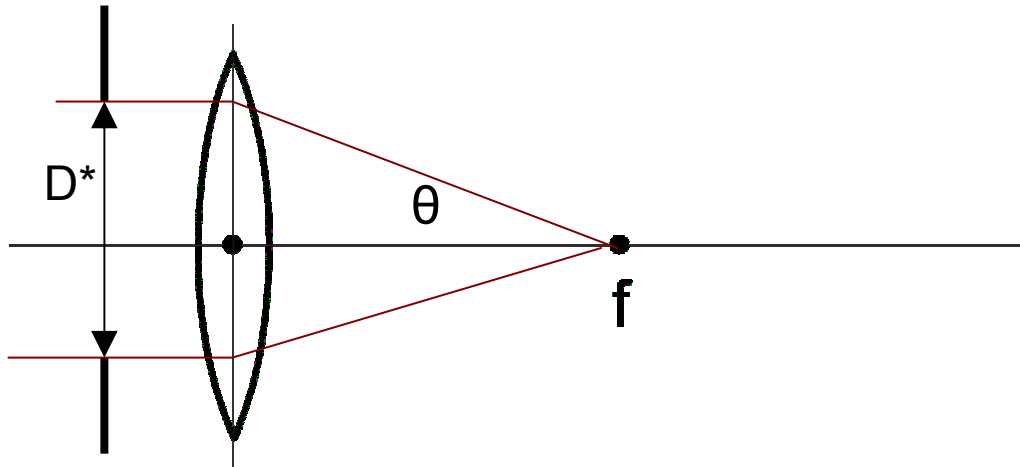


$$NA = n \sin \theta_{\max}$$

*$n = 1$  for lens in air*

$$F\# = f/D$$

One can “stop down” to reduce the NA or increase the  $F\#$



$$NA = n \sin \theta$$

$$F\# = f/D^*$$

# Aberration scaling

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The severity of aberrations generally depends on the aperture stop diameter ( $D$ ) and the off-axis distance of the image point ( $h'$ )

<u>Aberration</u>	Dependence on	
	<u><math>D</math></u>	<u><math>h'</math></u>
Spherical aberration	$D^3$	none
Coma	$D^2$	$h'$
Astigmatism	$D$	$h'^2$

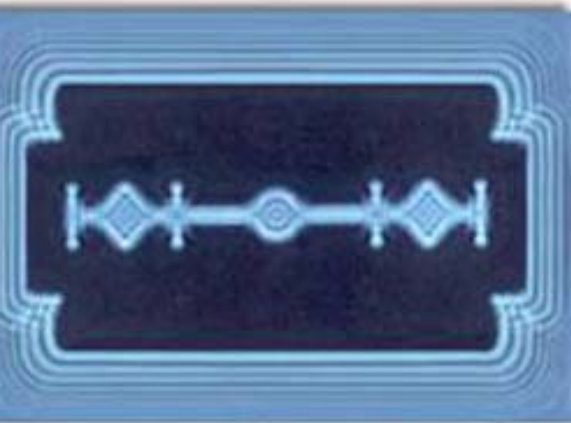
High NA or low F# systems have more SA than low NA / high F# systems

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## Physical optics

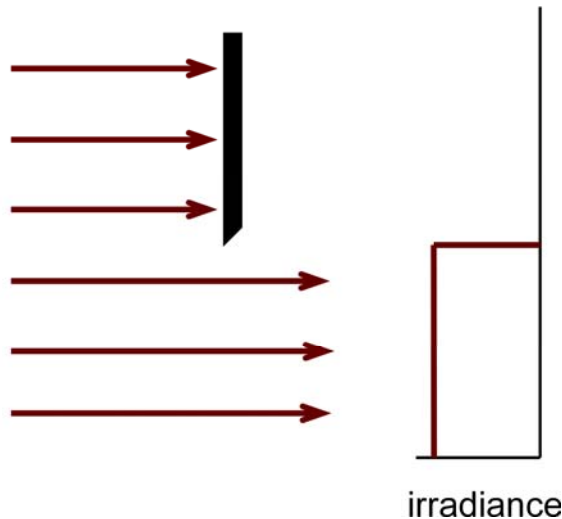
In geometric optics, a bundle of rays parallel to the axis will focus to a point  
*In real life, the spot size is finite even if aberrations are zero*

# Diffraction

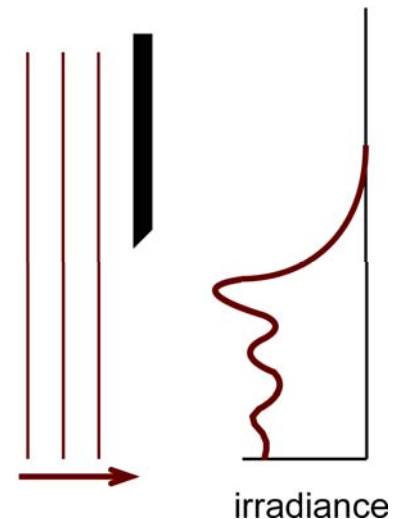


**Diffraction:** the interference phenomena that describes the alteration of wave propagation due to optics of finite size

Ray model (no diffraction)



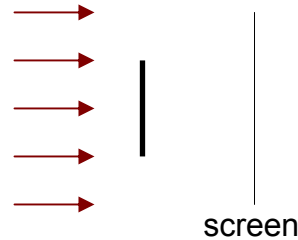
Wave model (with diffraction)



# Wave optics

diffraction creates intensity distributions geometric optics cannot explain

What do you see behind an illuminated opaque disk?



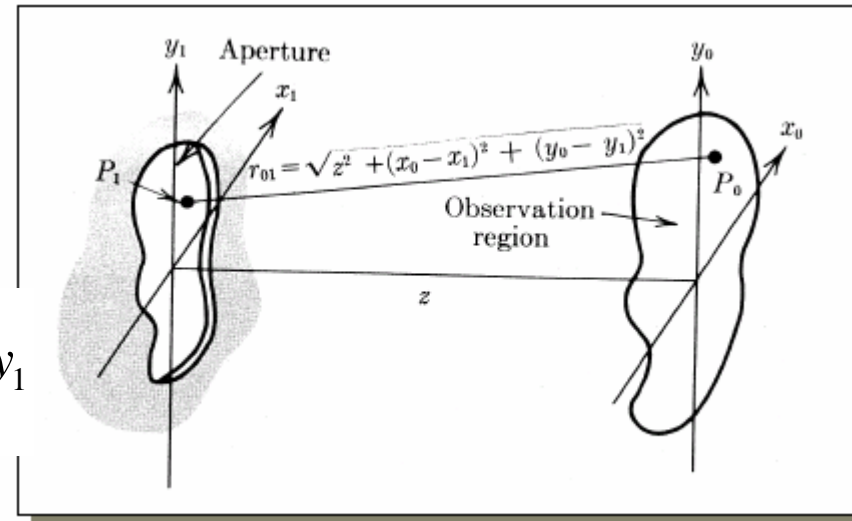
Arago's (or Poisson's) spot

To model the general propagation of a wave through space...

$$U(x, y, z) = E(x, y, z) \exp[-j(\omega t + \phi(x, y, z))]$$

$$U(x_0, y_0, z) = \frac{z}{j\lambda} \iint_{\text{surface}} U(x_1, y_1) \frac{\exp(jkr_{01})}{r_{01}} dx_1 dy_1$$

for  $r_{01} \gg \lambda$



# Fourier optics

The diffraction pattern is very complicated and evolves with the distance  $z$  from the aperture...*need simplifications!!*

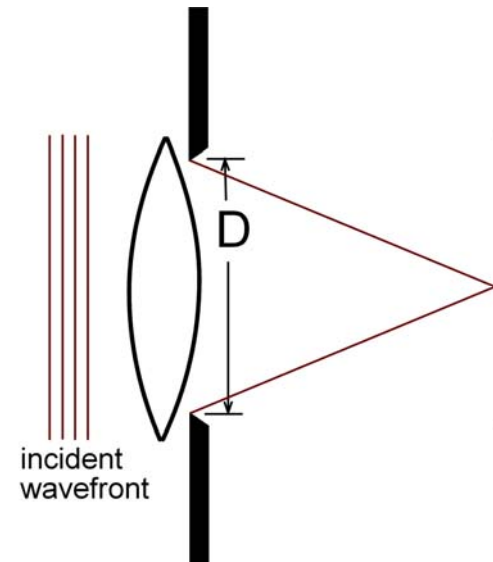
In the *far field* (or “Fraunhofer region”) the field math is simplified:

Here the field is the **Fourier transform** of the wavefront in the aperture

The far field exists at distances  $z > \frac{2D^2}{\lambda}$

or

at the focal plane of a lens





# Fourier transform pairs

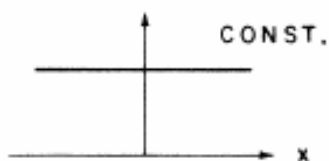
Function

F-transform

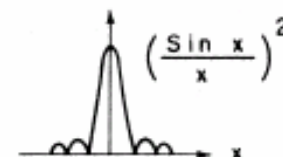
Function

F-transform

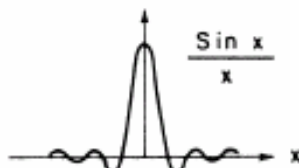
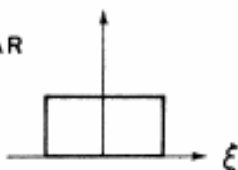
DELTA FUNCTION



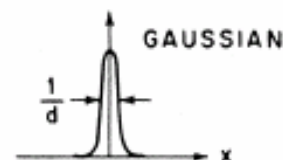
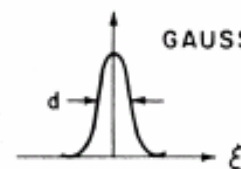
ISOSCELES TRIANGLE



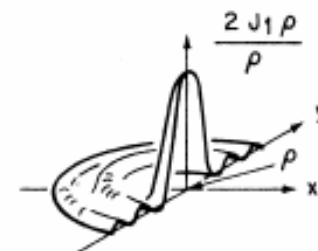
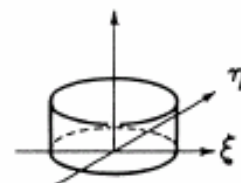
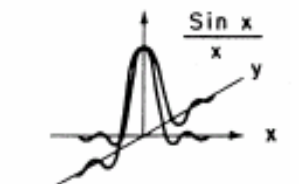
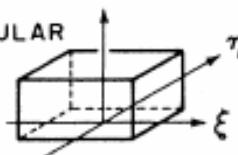
1-D RECTANGULAR FUNCTION



GAUSSIAN



2-D RECTANGULAR FUNCTION



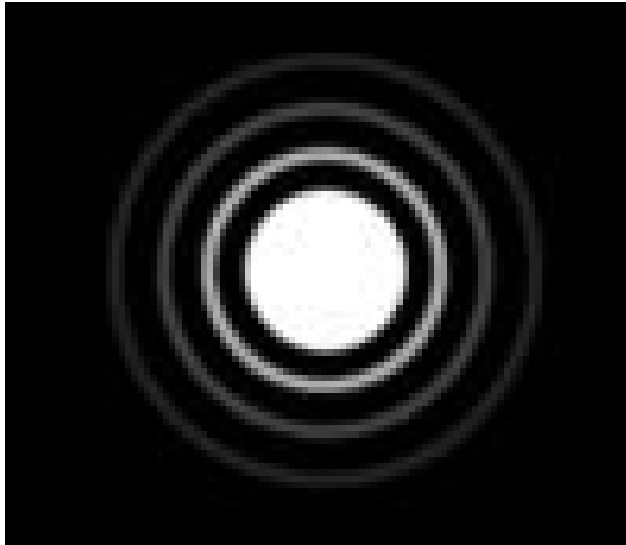
Irradiance distribution is the *square* of the field distribution

# Airy function

The intensity distribution of a *plane wave* incident on a circular aperture with radius  $w$  is:

$$I(R) \propto \left[ \frac{J_1(kwR/z)}{kwR/z} \right]^2 \quad \text{distance } z \text{ from aperture, } k=2\pi/\lambda$$

Diffraction-limited spot



Diameter of central lobe

$$d = 1.22 \frac{\lambda z}{w} \quad \text{for } z > \frac{2D^2}{\lambda}$$

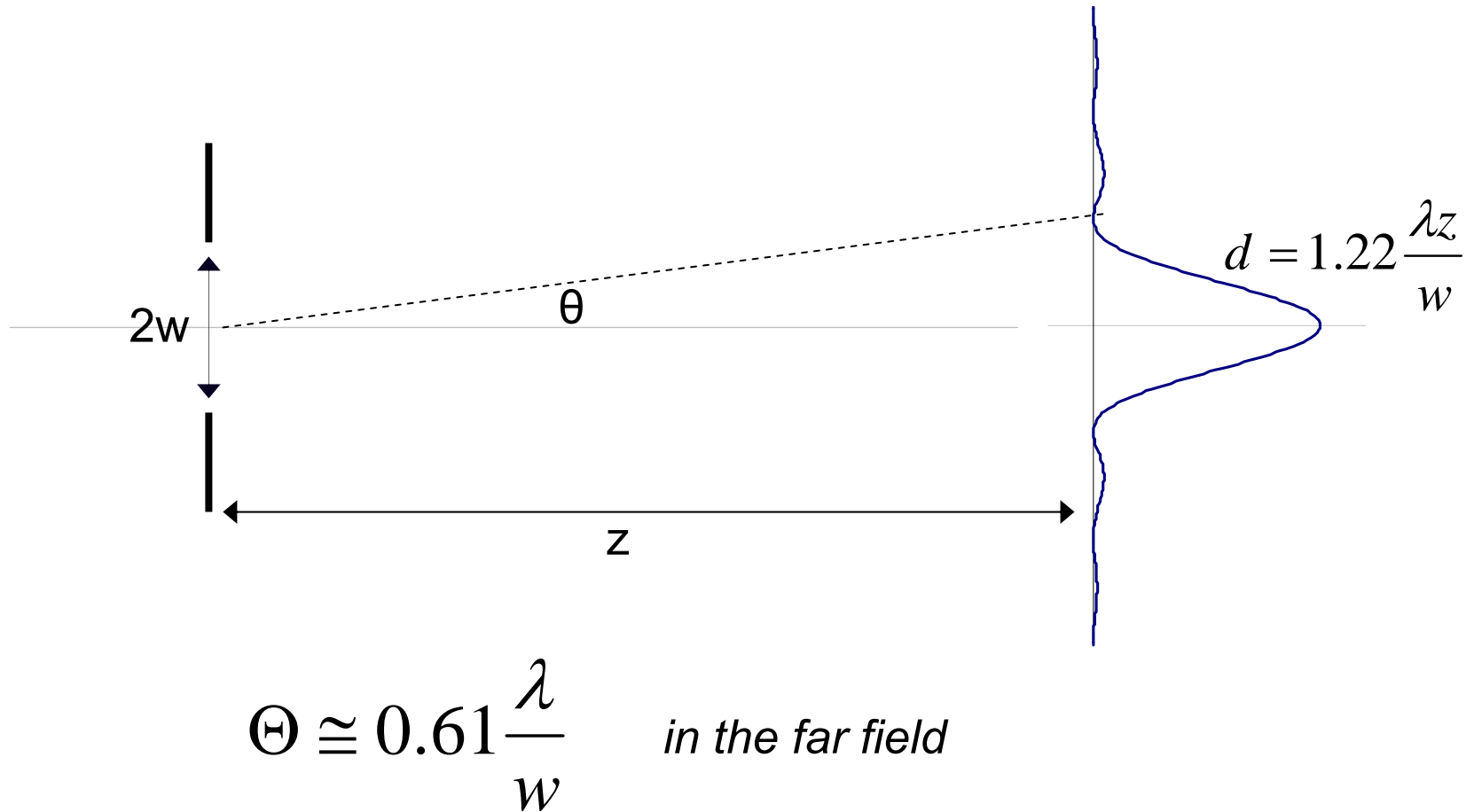
or

$$d = 1.22 \frac{\lambda f}{w} \quad \text{at the focal plane}$$

~86% of energy is in the central lobe

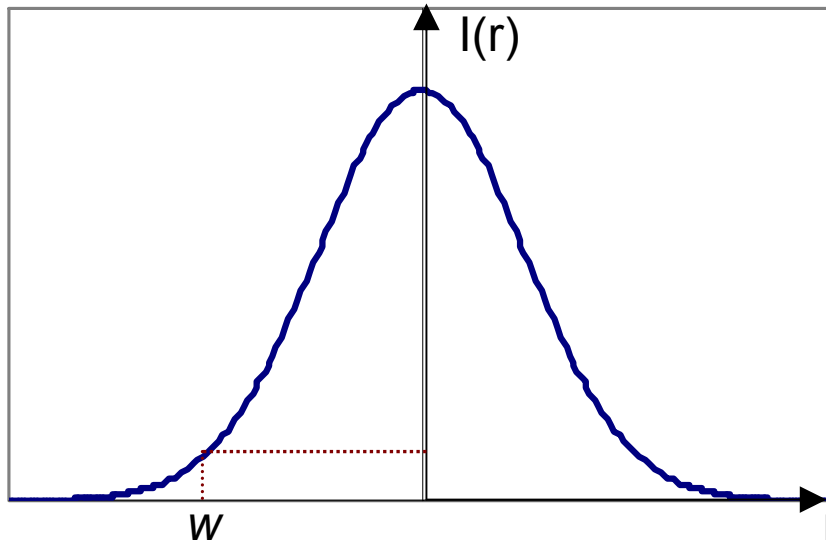
# Beam divergence

Beyond the far field, the beam diverges at a constant angle



# Gaussian beams

- Spherical-mirror resonator modes are Hermite-Gaussian
- Lowest-order mode is a Gaussian beam
- Hermite-Gaussians functions are self-transforms

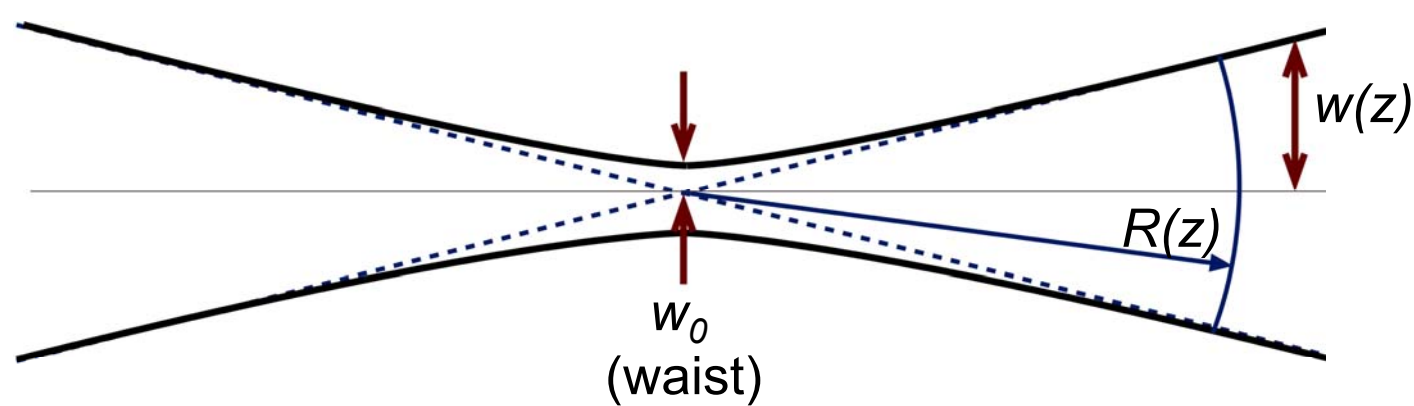


$$I(r) = \frac{2P}{\pi w^2} \exp\left[\frac{-2r^2}{w^2}\right]$$

$w$  is the  $1/e^2$  radius  
(or the  $1/e$  radius for the E field)

# Gaussian beam propagation

Gaussian beam propagation is defined by the wavelength and  $w_0$ , the radius of the beam at the waist



Beam radius at  $z$

$$w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2}$$

Radius of curvature at  $z$

$$R(z) = z + \frac{z_R^2}{z}$$

Rayleigh range

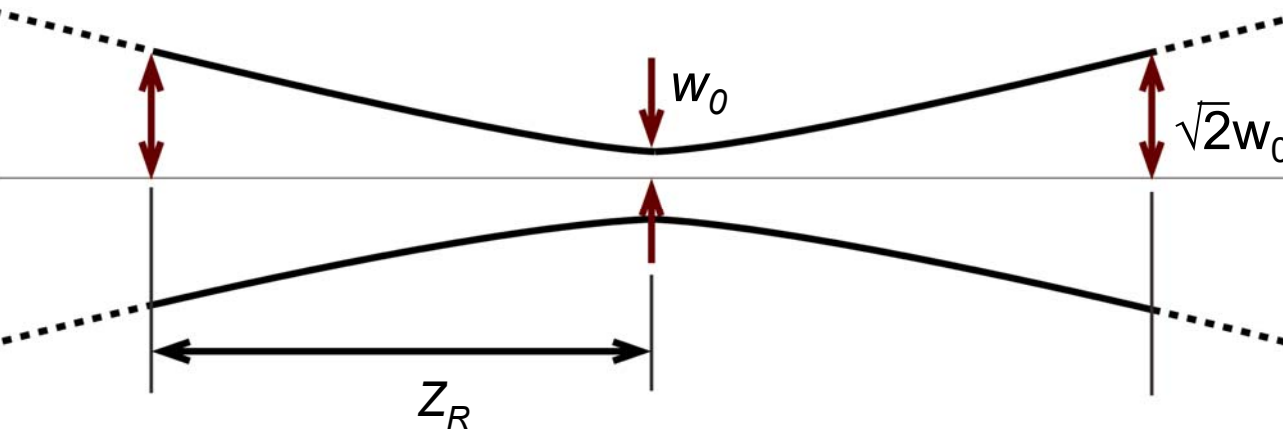
$$Z_R = \frac{\pi w_0^2}{\lambda}$$

# Rayleigh range

The Rayleigh range is distance from waist at which beam area doubles

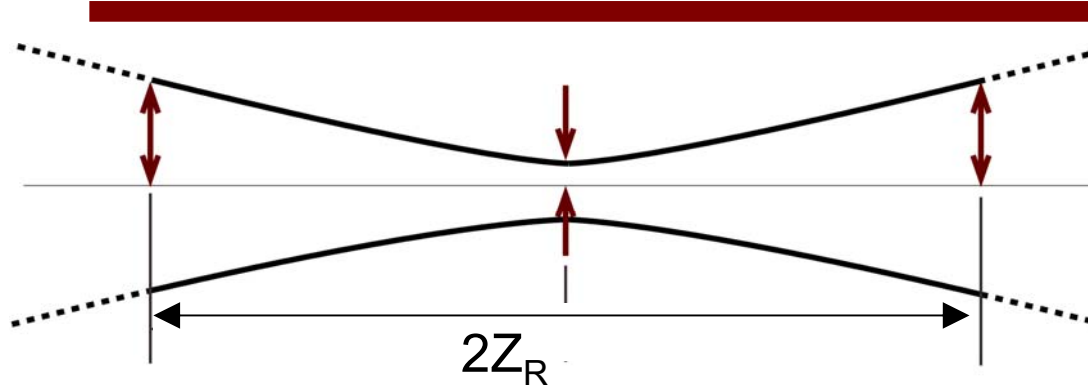
Rayleigh range

$$Z_R = \frac{\pi w_0^2}{\lambda}$$

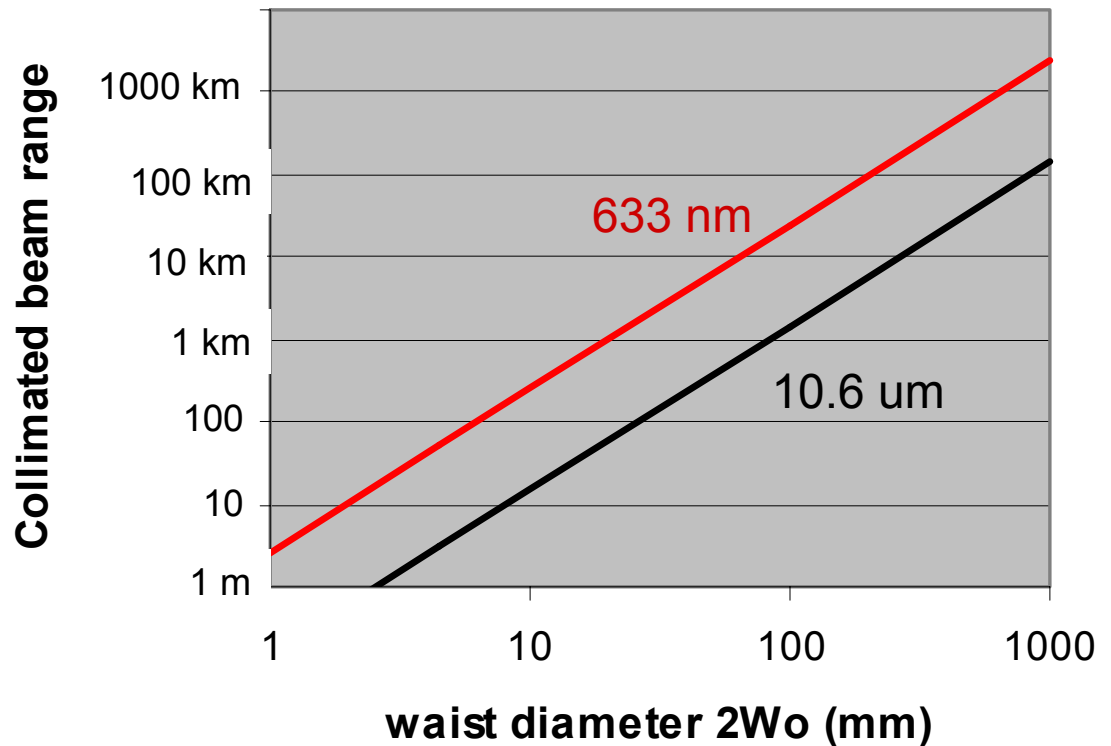


$2Z_R$  is known as the “confocal region” or “depth of focus”

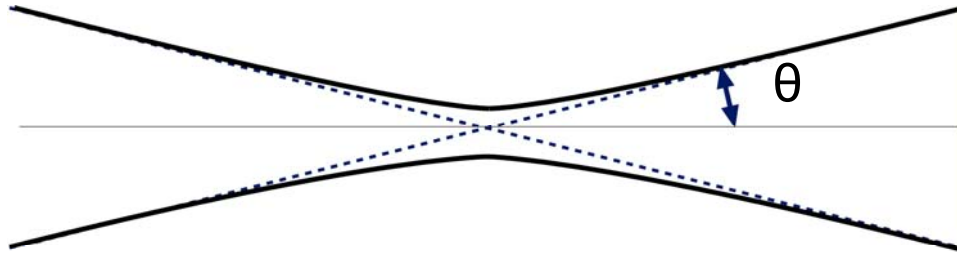
# “collimated” beam range



Tighter focusing  
causes a smaller  
“collimated” range



# Gaussian beam divergence



$$w(z) = w_0 \sqrt{1 + \left( \frac{z\lambda}{\pi w_0^2} \right)^2} \cong \frac{z\lambda}{\pi w_0} \quad \text{for large } z \text{ (} z \gg z_R \text{)}$$

Beam divergence

$$\theta_{1/e} = \lim_{z \rightarrow \infty} \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}$$

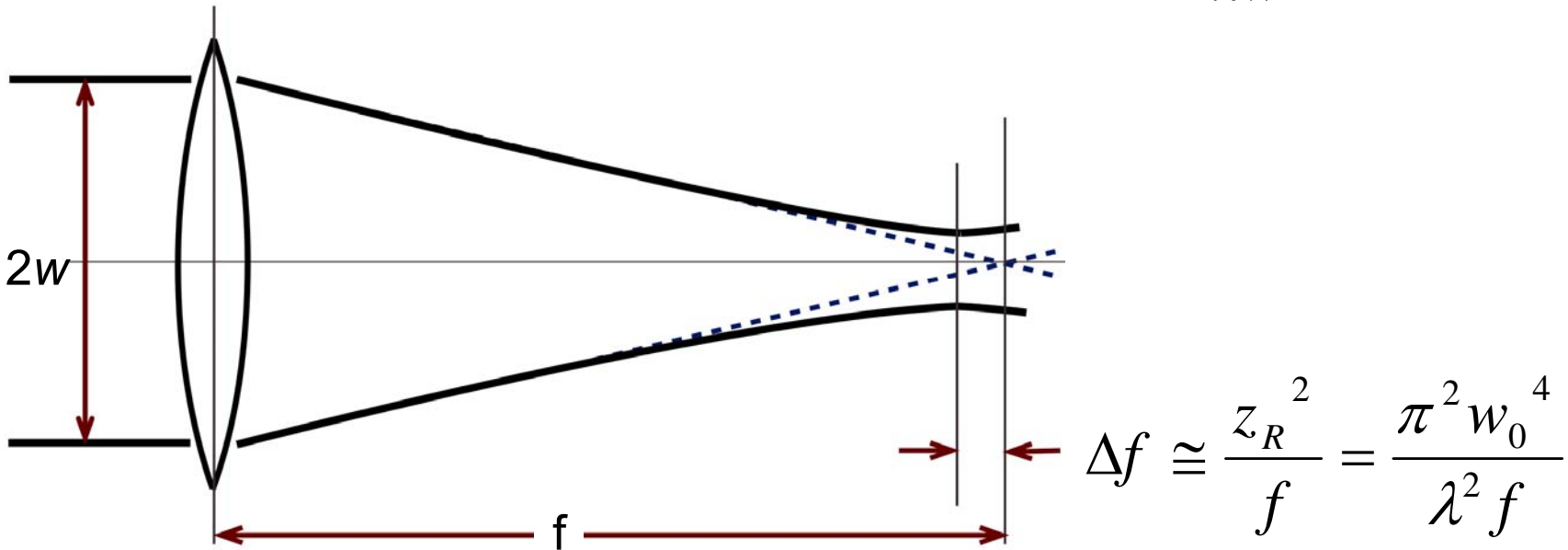
Divergence is inversely proportional to  $w$



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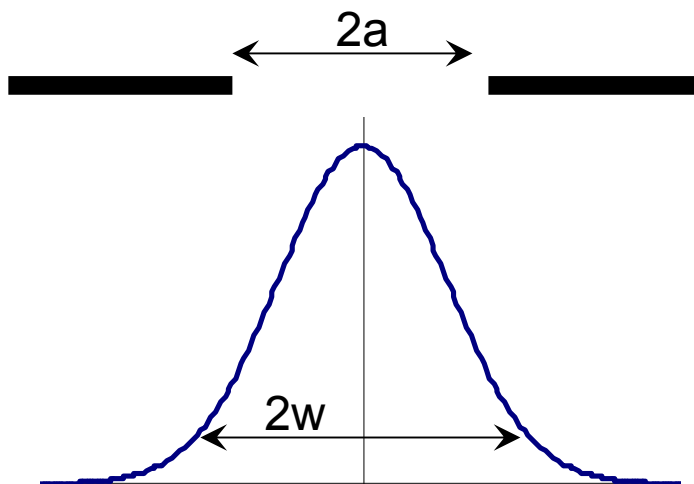
Recall  $w(z) \cong \frac{z^2}{\pi w_0}$

At focus, spot radius is  $w_0 \cong \frac{\lambda f}{\pi w}$



There is a (usually negligible) shift  $\Delta f$  between the waist position and geometric focus

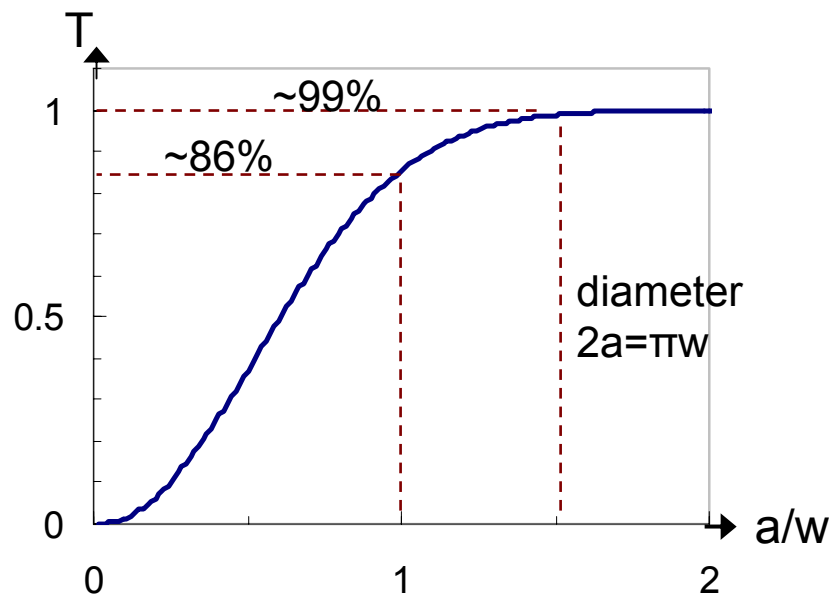
# Truncated Gaussian beams



Choose lens diameter  $> \pi w$   
to transmit  $> 99\%$  of energy

Transmittance  $T$  through aperture is

$$T = 1 - \exp\left[-\frac{2a^2}{w^2}\right]$$



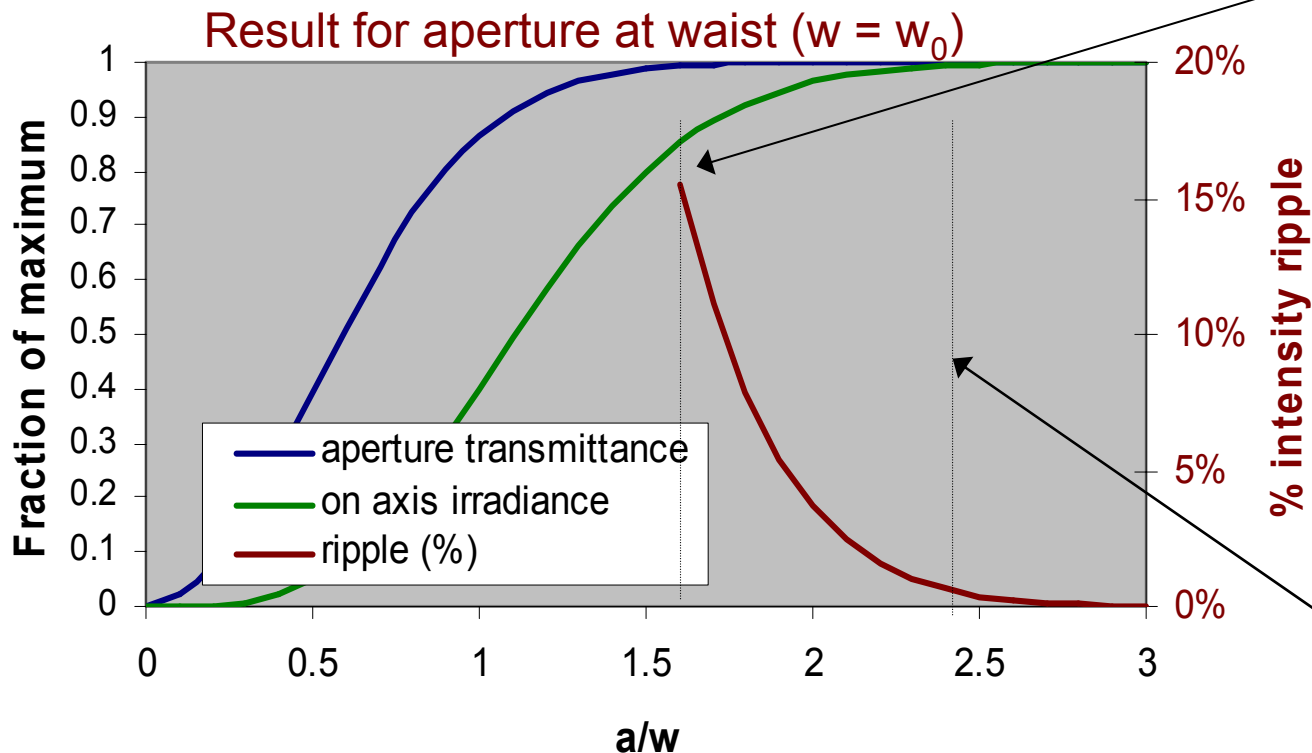
# Truncated Gaussian problems

Truncating the Gaussian beam causes

- Decrease in total power transmitted
- **Diffraction!!!**

Decreases on-axis irradiance

Increases intensity ripple



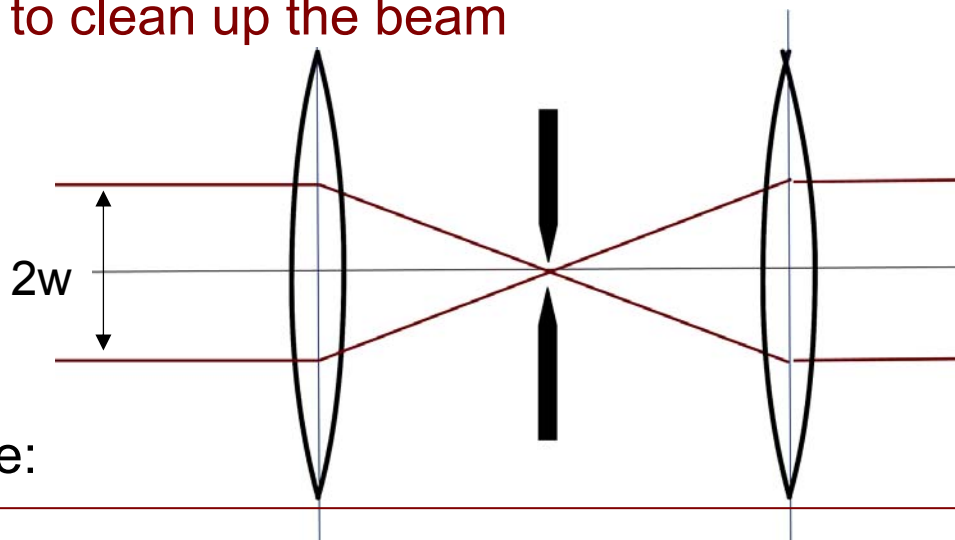
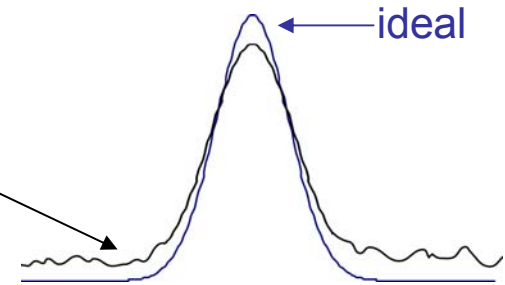
From last slide...  
When  $a = \pi w/2$   
 $T = 99\%$   
 $P_{\text{axis}} = 84\%$   
Ripple = 17%

When  $a = 2.3w$   
 $P_{\text{axis}} = 99\%$   
Ripple = 1%

# Spatial filtering

Surface roughness, dust, etc., adds high-spatial-frequency noise to beam profiles

Place pinhole at common foci of telescope to clean up the beam



For 1% ripple, pinhole diameter  $D \sim 4.6w_0$

Example:

$$\begin{aligned} f &= 50 \text{ mm} \\ \lambda &= 633 \text{ nm} \\ w &= 1 \text{ mm} \end{aligned} \quad \longrightarrow \quad w_0 \cong \frac{\lambda f}{\pi w} \quad \longrightarrow \quad w_0 = 10 \text{ } \mu\text{m}, \text{ so choose } D = 50 \text{ } \mu\text{m}$$

↑  
*rounded up!*

# Beam expander alignment

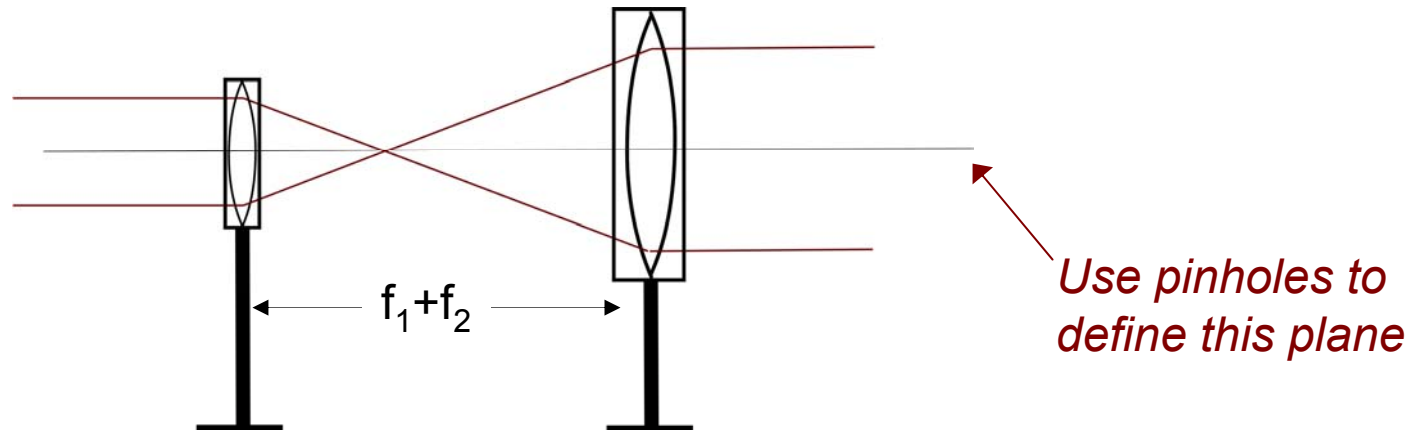
Step 1. Adjust the beam to be parallel to be the optical table

- choose an “operating height”
- have several pinholes at this height

Step 2. Center lens 1 in the beam path

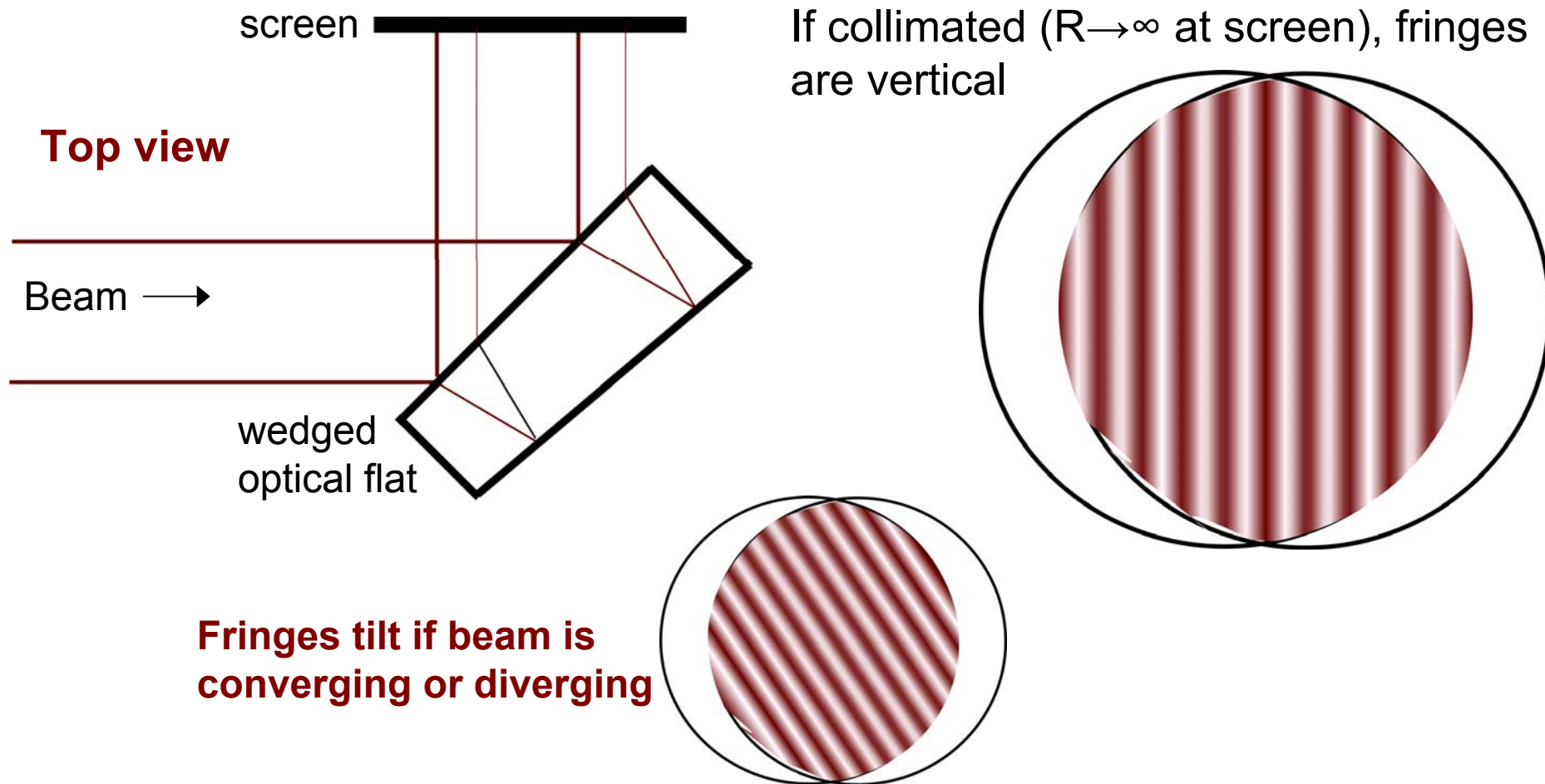
Step 3. Center lens 2 in the beam path,  $\sim f_1 + f_2$  from lens 1

Step 4. Adjust lens 2 position along beam until beam diameter is approximately constant between lens 2 and the wall



# Collimation test with a shearing plate

Step 5. Do a fine adjustment of lens 2 position with a shearing plate

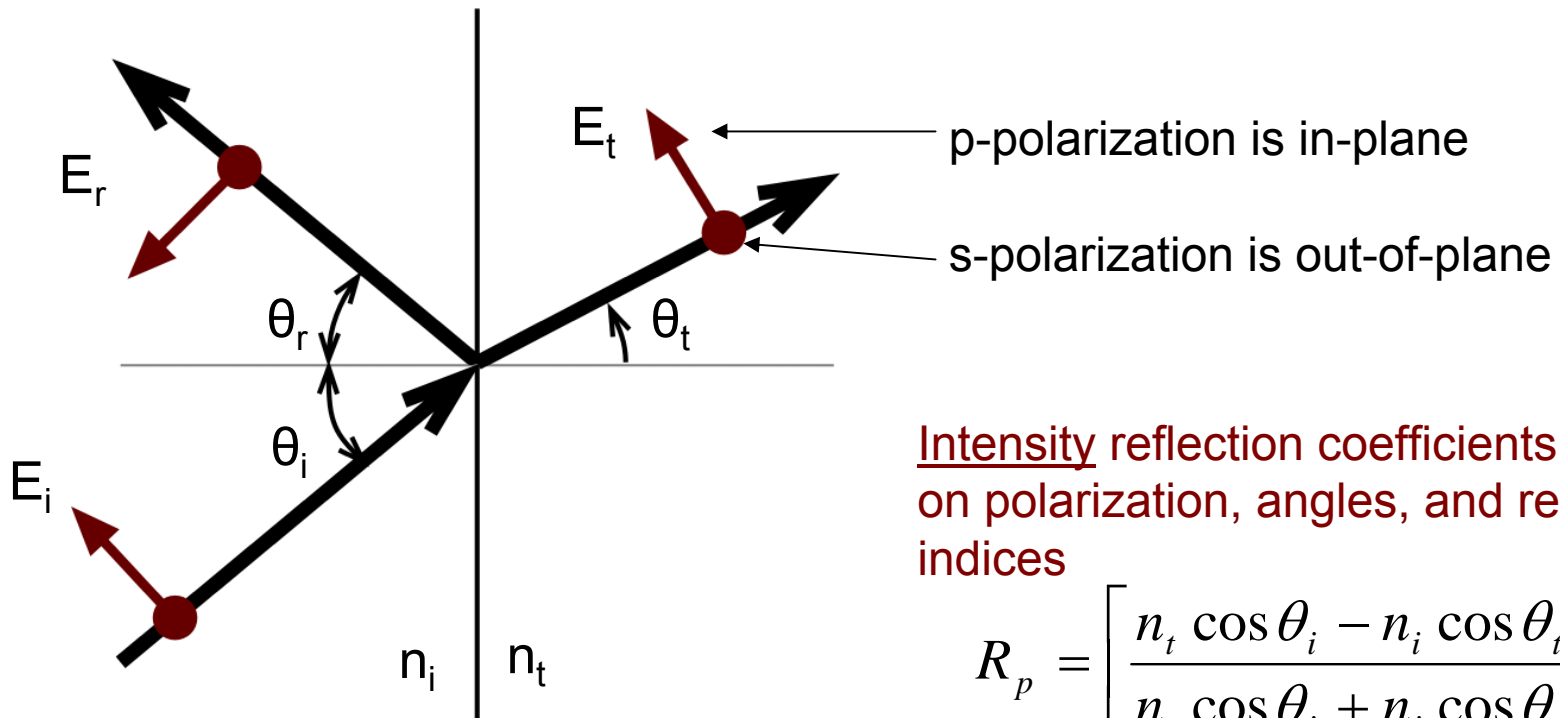


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Reflections, coatings, and stuff

# Fresnel reflection

An incident wave ( $E_i$ ) will create a reflected wave ( $E_r$ ) and transmitted wave ( $E_t$ )  
The fields are decomposed with respect to the plane of incidence



Intensity reflection coefficients depend on polarization, angles, and refractive indices

$$R_p = \left[ \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_t \cos \theta_i + n_i \cos \theta_t} \right]^2$$

$$R_s = \left[ \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \right]^2$$

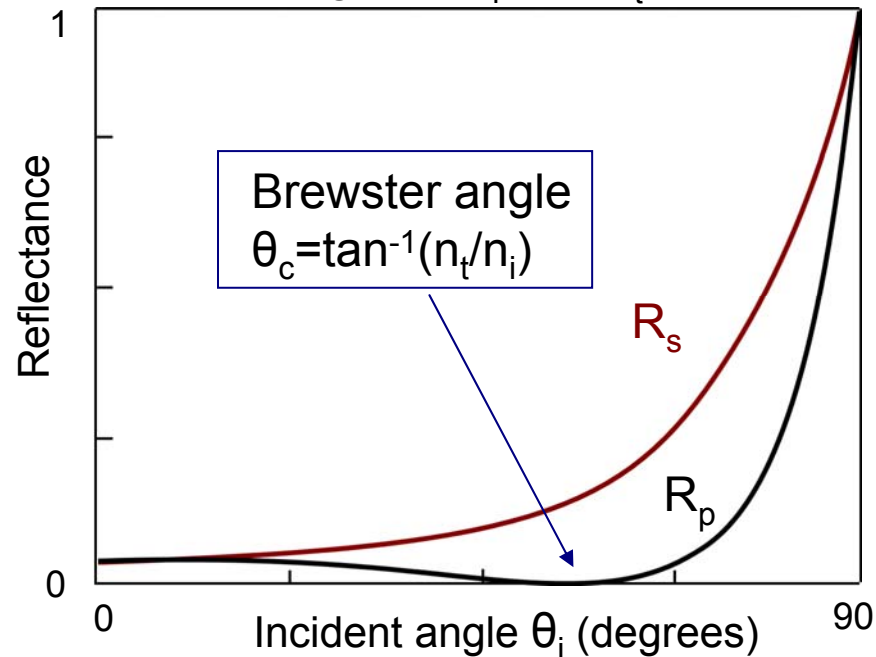
Define reflectance and transmittance as

$$R = \left[ \frac{E_r E_r^*}{E_i E_i^*} \right] = \frac{I_r}{I_i} \quad T = 1 - R$$



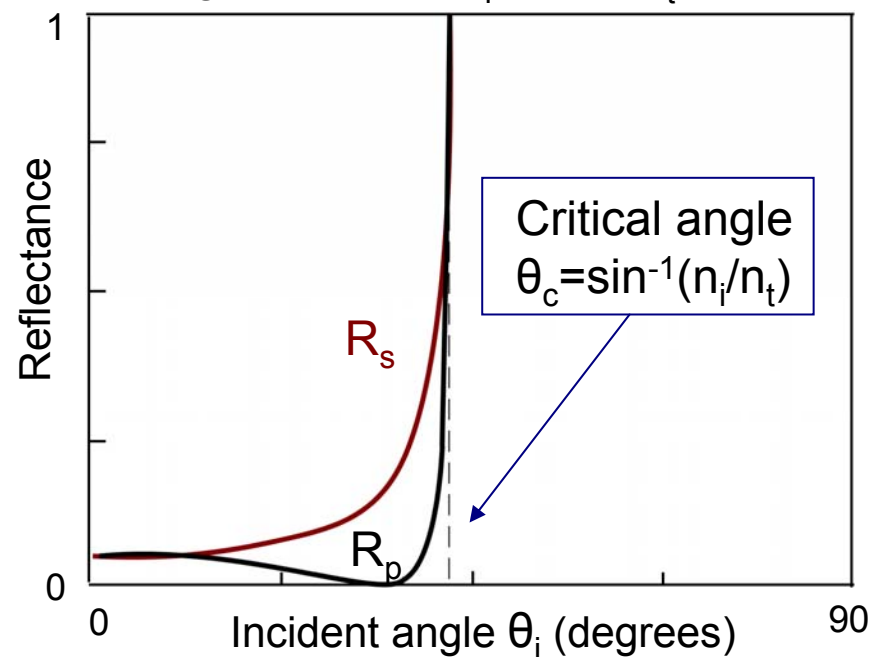
# Fresnel coefficients

air-to-glass:  $n_i = 1$ ,  $n_t = 1.5$



Brewster angle incidence provides perfect transmission for p-polarized

glass-to-air:  $n_i = 1.5$ ,  $n_t = 1$

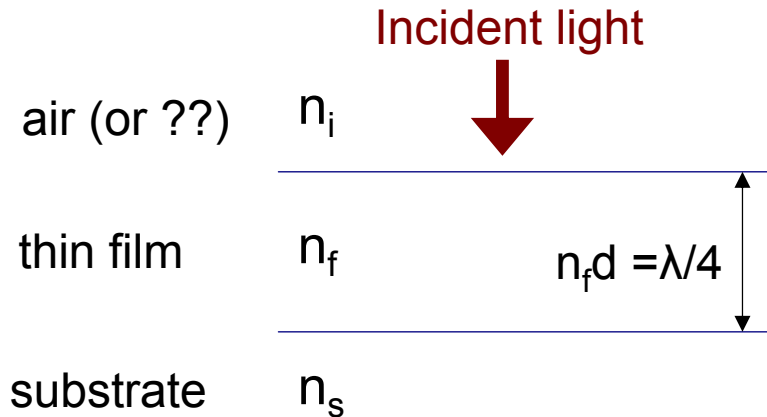


Reflectance is total for  $\theta_i > \theta_c$

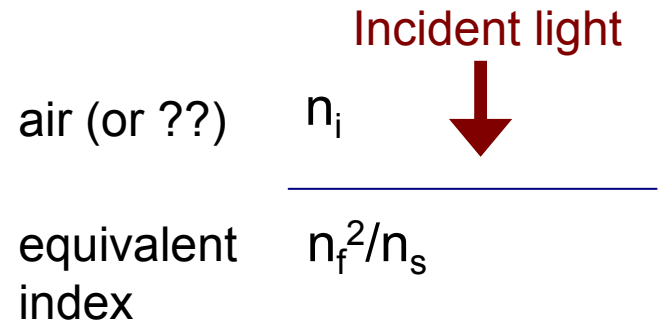
One can modify reflectance by applying thin films to the optic

# Optical coatings – quarter wave rule

Consider a thin film of quarter-wave optical thickness

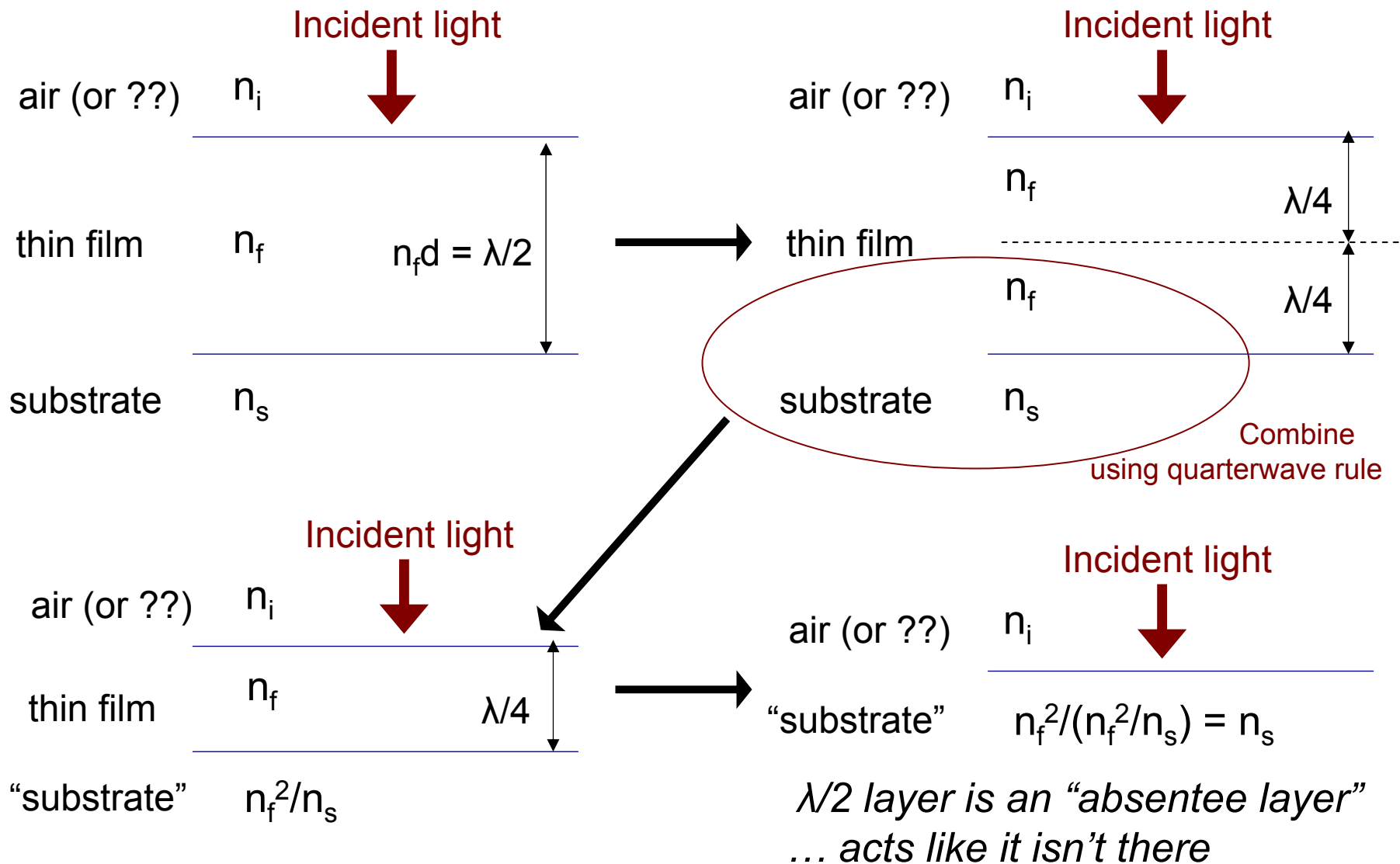


If film is exactly  $\lambda/4$  thick ( $=\lambda_0/(4n_f)$ ) can transform to an exact two-layer system



*Now can use Fresnel reflection equations for the equivalent interfaces*

# Optical coatings – half wave rule



# Antireflection coatings

For normal incidence, Fresnel reflectance is

$$R = \left[ \frac{n_i - n_s}{n_i + n_s} \right]^2$$

If the substrate is coated with a  $\lambda/4$  film, we can use the quarter-wave rule...

$$R = \left[ \frac{n_i - n_f^2 / n_s}{n_i + n_f^2 / n_s} \right]^2$$

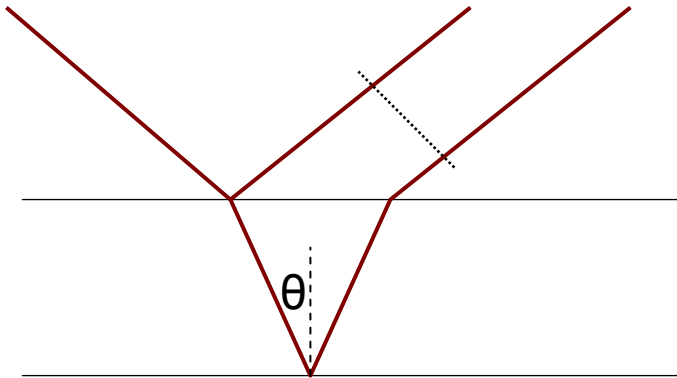
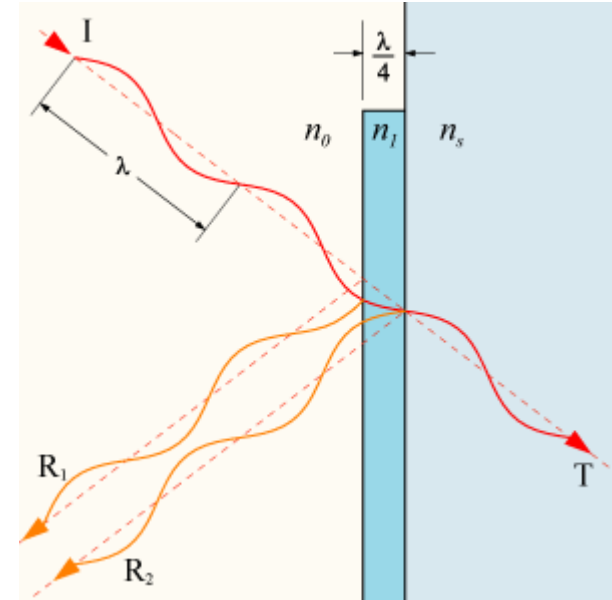
Reflectance is zero when numerator is zero (when  $n_f = \sqrt{n_i n_s}$ )

example:  $n_s = 1.5$  (glass),  $n_f = 1.38$  (MgF),  $n_i = 1$ ;  $R \sim 1.3\%$

# AR coatings – what's the trick?

$$\delta = \frac{2\pi n d}{\lambda_0}$$

When path difference  $d = \lambda/2$ , the phase difference  $\delta = \pi$   
.... so the two reflected waves destructively interfere

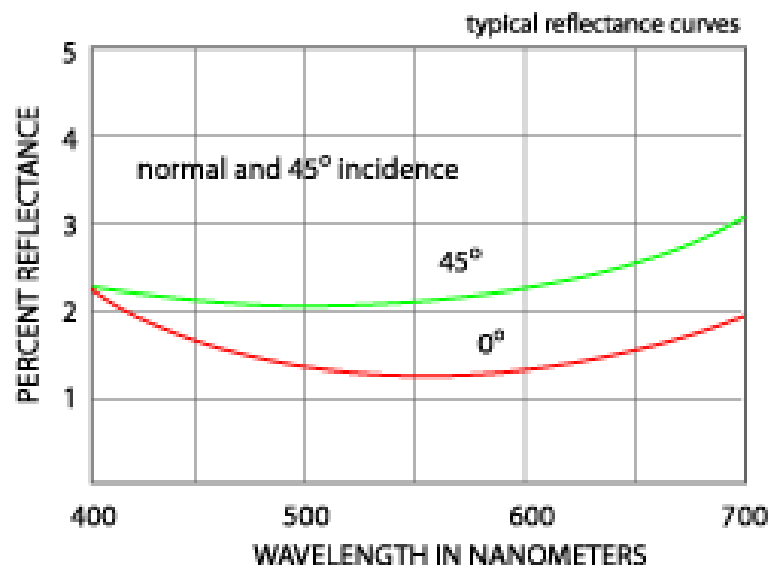


For non-normal incidence, require  $\delta = \pi$   
for the refracted paths

$$\delta = \frac{2\pi n d \cos \theta}{\lambda_0}$$

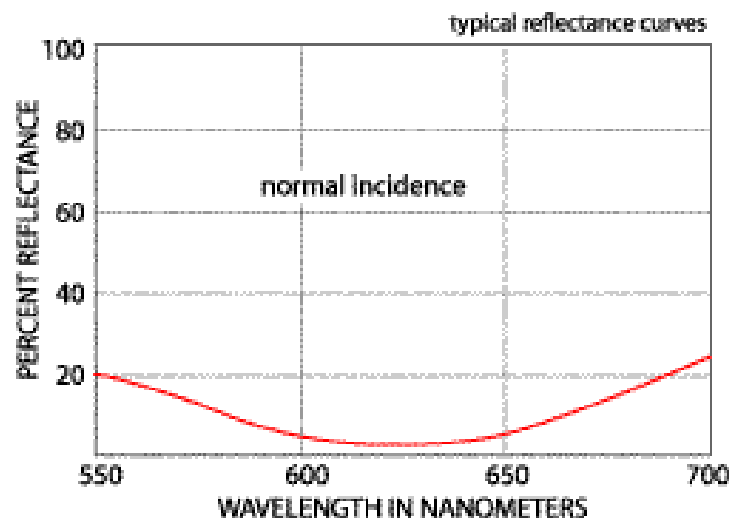
Non-normal incidence shifts minimum reflectance to shorter wavelength

# AR coating performance



Single-layer MgF coating

Multi-layer "V-coating"



# High-reflectance coatings

Incident light

air (or ??)  $n_i$

thin film  $n_f$   $\lambda/4$

substrate  $n_s$

$$R = \left[ \frac{n_i - n_f^2 / n_s}{n_i + n_f^2 / n_s} \right]^2$$

To increase R, make  $n_f$  large

Incident light

$n_i$

$n_H$

$n_L$

$n_H$

$n_L$

$n_H$

$n_s$

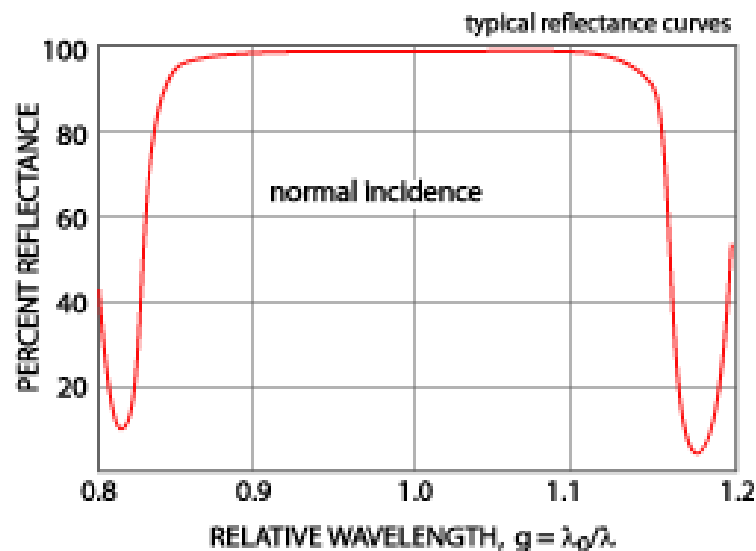
To get higher reflectance, alternate  $n_H$  and  $n_L$   $\lambda/4$  layers

for  $x$  layers on  $n_H$  and  $x-1$  layers  $n_L$

$$R = \left[ \frac{n_i - \frac{n_H^{2x}}{n_L^{2x-2} n_s}}{n_i + \frac{n_H^{2x}}{n_L^{2x-2} n_s}} \right]^2$$

maximize  $n_H/n_L$

# Multilayer reflector performance

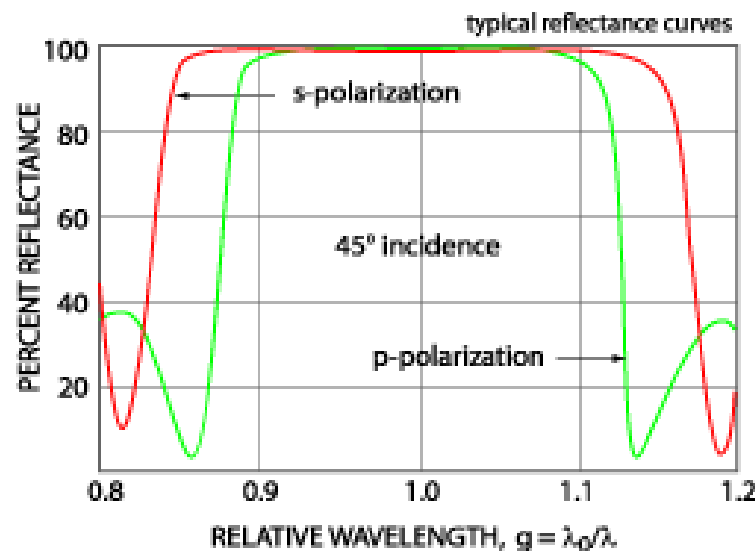


Can achieve  $R > 99.9\%$  off-the-shelf  
*five-9's possible... limited by losses*

Damage thresholds  $> 10 \text{ J/cm}^2$  (10 ns pulse)

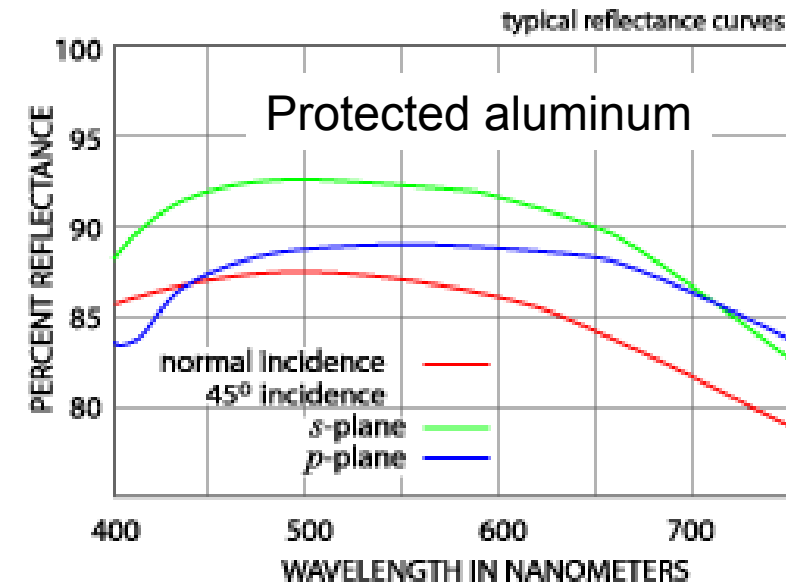
Non-normal incidence shows  
polarization dependence

*Caution: may not be suitable for  
ultrafast lasers... why??*

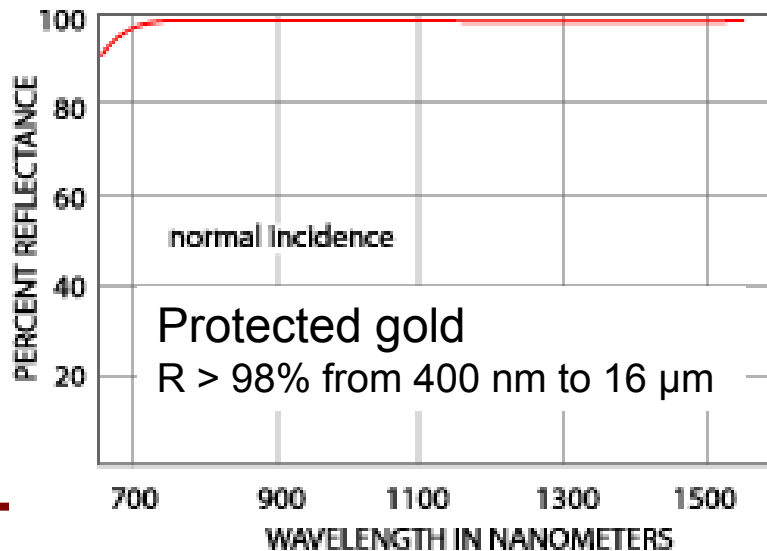
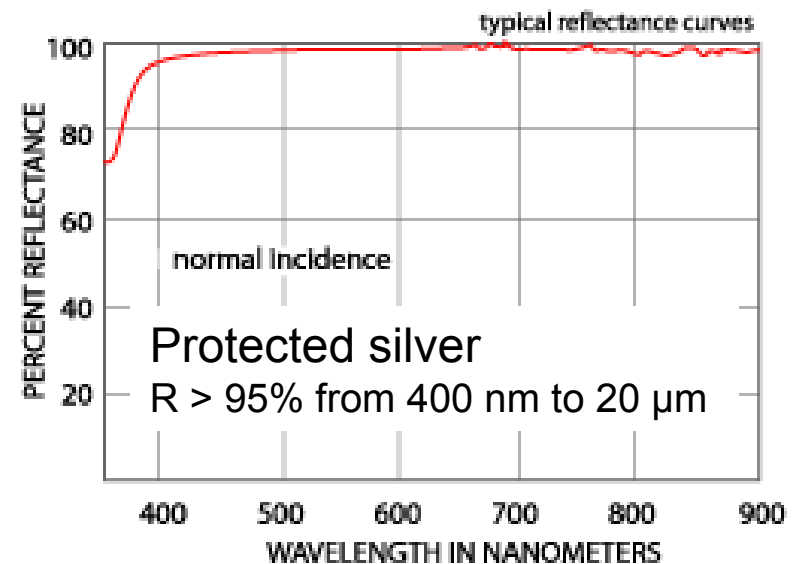




# Metallic mirror performance



Damage thresholds 10x to 100x lower than dielectric stacks

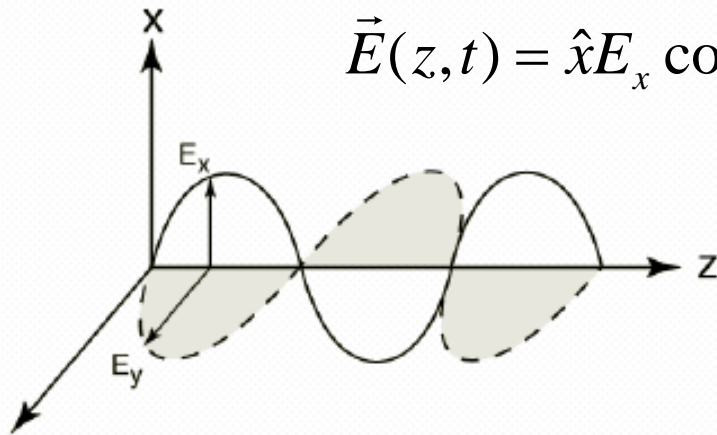


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Polarization, interference & coherence

# Wave polarization

The polarization state is defined by the direction of the transverse E-field



$$\vec{E}(z, t) = \hat{x}E_x \cos(\omega t - k_0 z + \phi_x) + \hat{y}E_y \cos(\omega t - k_0 z + \phi_y)$$

Any polarization can be decomposed into orthogonal components

## Polarization state

Linear vertical

Linear horizontal

Linear @ 45°

Right circular

Left circular

E<sub>x</sub>

1

0

1/√2

1/√2

1/√2

E<sub>y</sub>

0

1

1/√2

1/√2

1/√2

phase

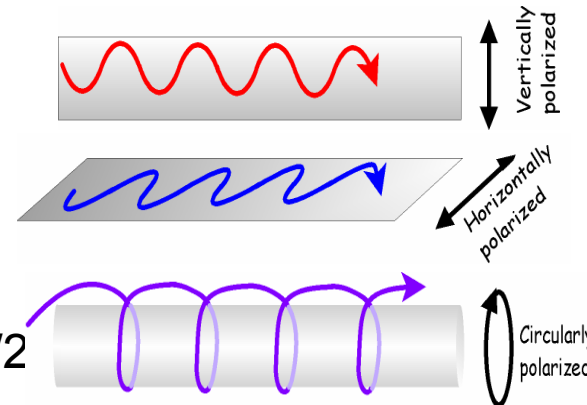
$\Phi_x = \Phi_y = 0$

$\Phi_x = \Phi_y = 0$

$\Phi_x = \Phi_y = 0$

$\Phi_x = 0, \Phi_y = \pi/2$

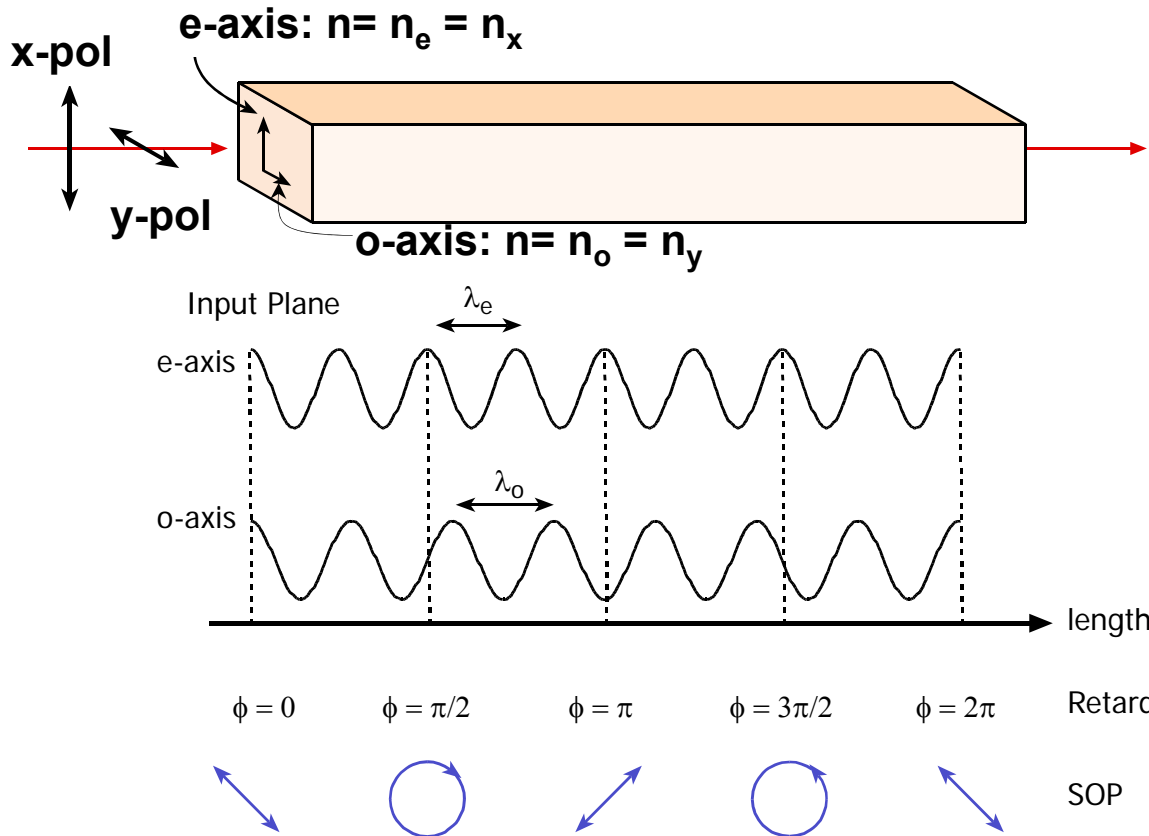
$\Phi_x = 0, \Phi_y = -\pi/2$



# Eigenpolarizations

Isotropic materials: refractive index and loss is independent of direction / polarization

Anisotropic materials: properties depend on direction & polarization



An **eigenpolarization** is a polarization state that propagates unchanged through optically anisotropic materials.

Any incident polarization can be decomposed into orthogonal eigenpolarizations

# Retarders (aka waveplates)

Retarders induce a phase difference  $\Delta\Phi$  between orthogonally polarized components of light

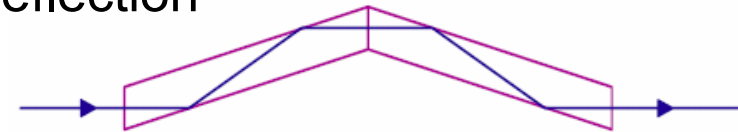
Linear retarders produce  $\Delta\Phi = \Phi_x - \Phi_y$

- waveplates use optically anisotropic materials with *linear birefringence*  $\Delta n = n_x - n_y$
- electro-optic materials (Pockel's effect):  $\Delta n = f(E)$
- rhombs use retardance from total-internal-reflection



Circular retarders produce  $\Delta\Phi = \Phi_{\text{rcp}} - \Phi_{\text{lcp}}$

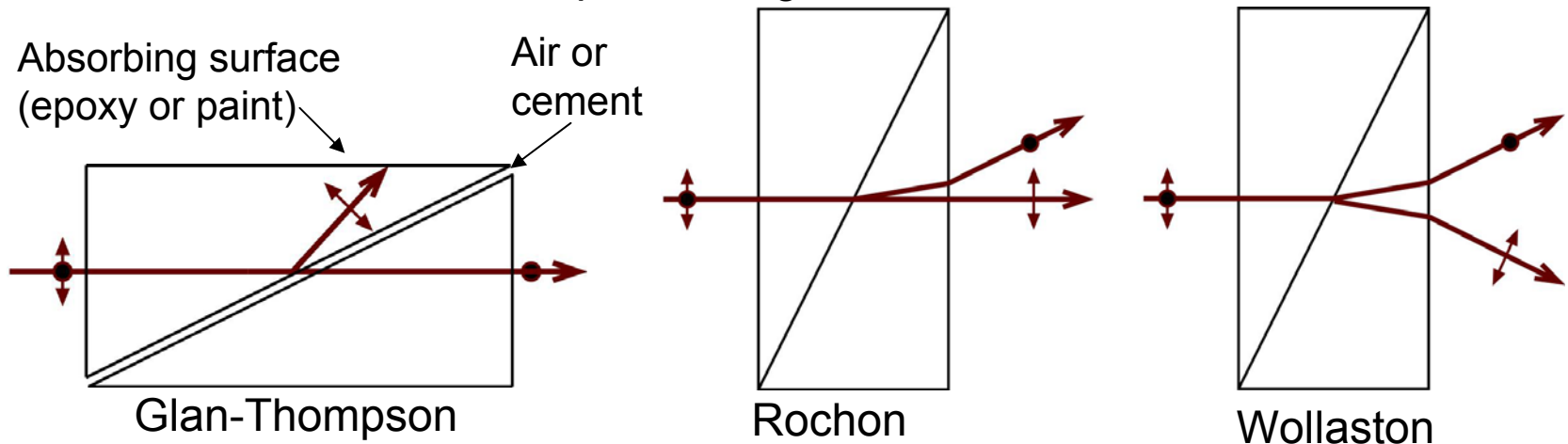
- optically active chiral materials (sugar solution) with circular birefringence  $\Delta n = n_{\text{rcp}} - n_{\text{lcp}}$
- magneto-optic effects:  $n_{\text{rcp}} - n_{\text{lcp}} = f(H)$



**Retarders can rotate or transform the polarization state**

# Polarizers

Birefringent crystal polarizers – extinction ratios of  $10^{-5}$  to  $10^{-7}$  are possible but have limited acceptance angle



Dichroic polarizers –  
cheap, limited power-handling capability  
extinction ratio  $\sim 10^{-2}$  to  $10^{-5}$  in visible



*How do you determine the polarizer's transmission axis?*  
look at the glare from a shiny surface...  
s-polarization "skips", p-polarization "plunges"



# Jones calculus

Jones calculus is a tool for calculating the evolution of completely polarized beams

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{out} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{in}$$

## Jones vectors (for fields)

Linear along x  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Linear along y  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Linear at 45°  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Right-circular  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$

Left circular  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

## Jones matrices (for components)

Polarizer at 0°  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

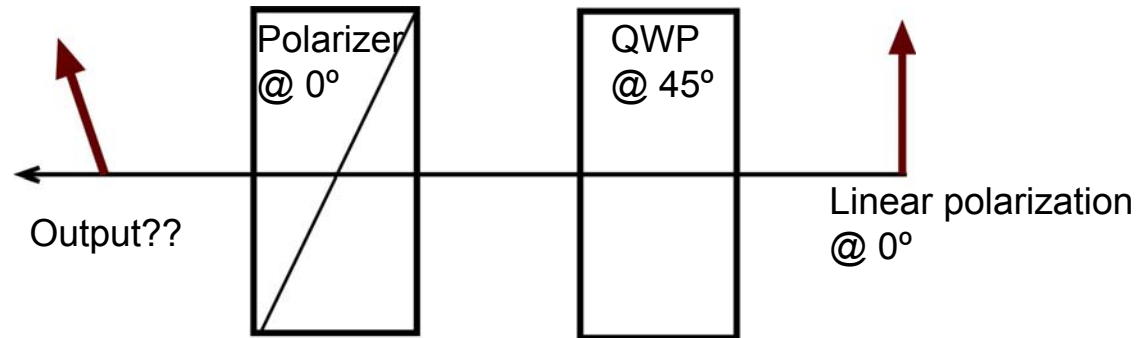
Polarizer at  $\theta^\circ$   $\begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$

Quarter wave retarder at 0°  $\begin{bmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{bmatrix}$

Quarter wave retarder at 45°  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$

Half wave retarder at 0°  $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

# Jones calculus example



$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{out} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{in}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Output polarization  
is linear vertical

$$I = E E^* = \frac{1}{2}$$

Optical power  
is halved

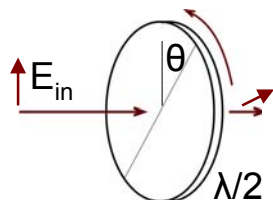


# Polarization control

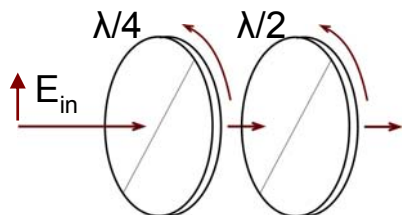
## polarization control

### Linear-to-linear

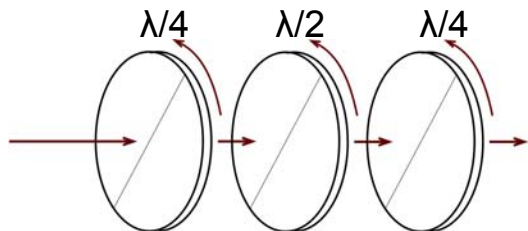
$\lambda/2$  plate rotates state by  $2\theta$



### Linear-to-arbitrary



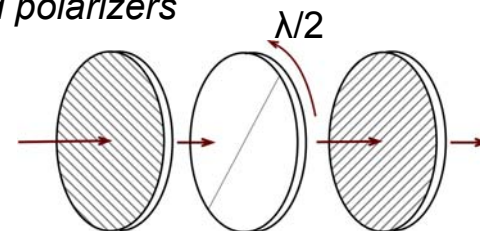
### Arbitrary-to-arbitrary



## power control

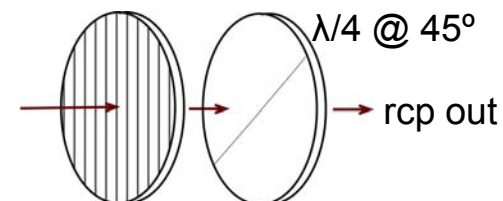
### Linear input attenuator

$\lambda/2$  between crossed polarizers  
 $T = \sin^2\theta$



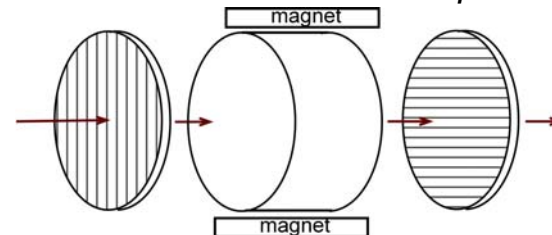
### Linear input “pseudo-isolator”

requires unchanged reflection polarization



### Optical isolator

magneto-optic rotator between crossed polarizers

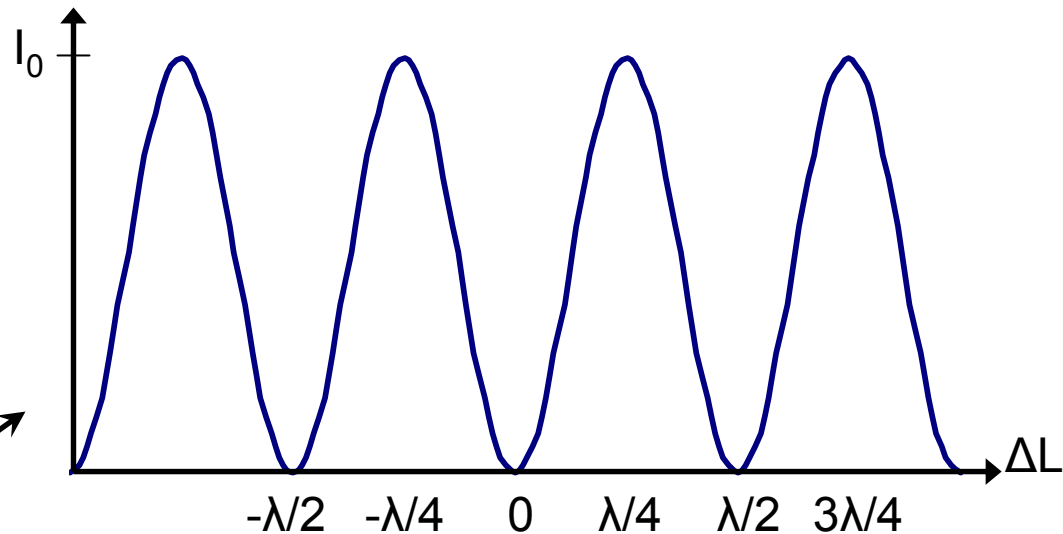
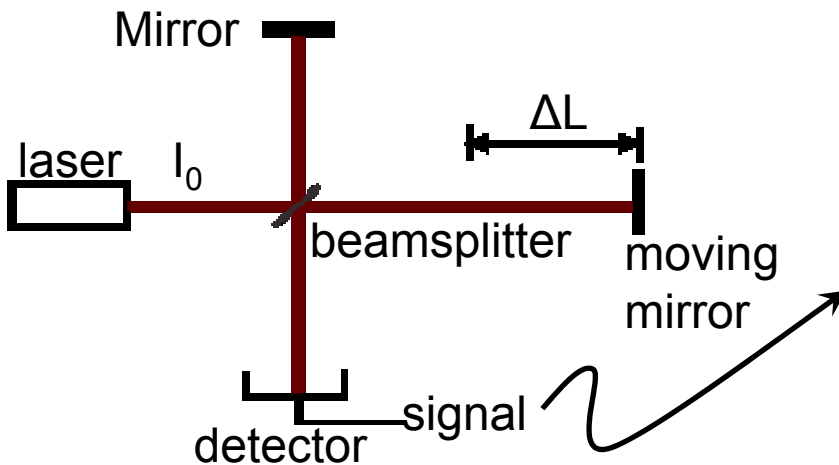


# Interference

## Monochromatic light waves can interfere

- constructive interference when waves interfere in phase
- destructive interference when wave interfere  $\pi$  out of phase

### Michelson interferometer



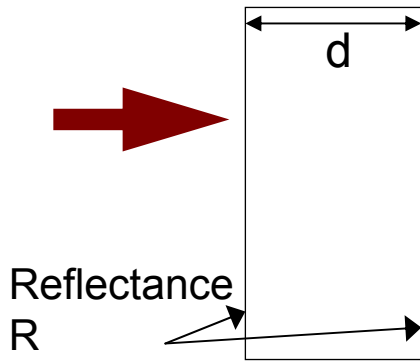
$$I(\Delta L) = \frac{I_0}{2} \left( 1 - \cos \left( \frac{2\pi 2\Delta L}{\lambda} \right) \right)$$

Why is interference constructive when  $\Delta L=0$ ??

# Etalon interference

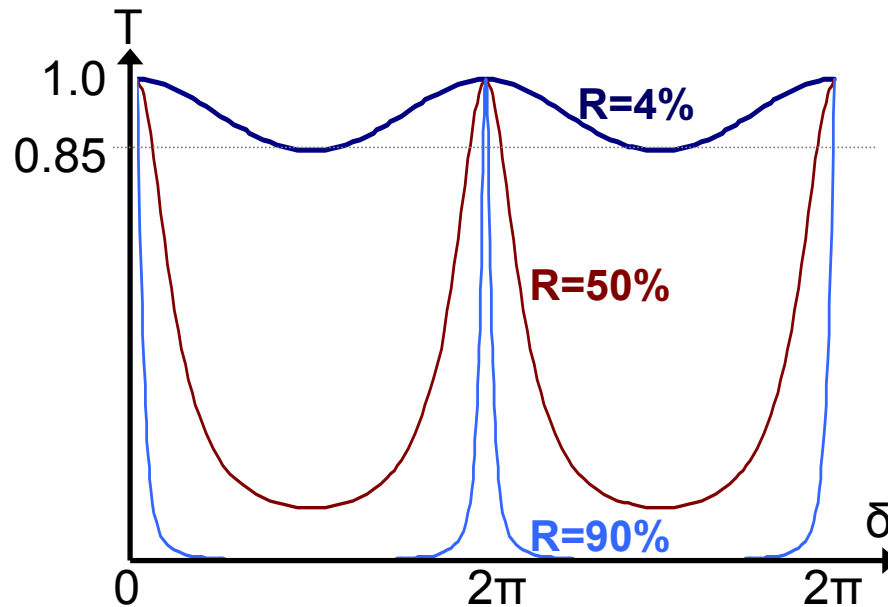
Flat, parallel surfaces may exhibit unintended interference effects

**Fabry-Pérot  
interferometer**



$$T_E(\delta) = \frac{(1-R)^2}{1+R^2-2R\cos\delta}$$

$$\delta = \frac{2\pi}{\lambda} 2nd \cos \theta$$



$$R_E \max = \frac{4R}{(1+R)^2}$$

**To avoid this effect:**

- use wedged flats
- use non-normal incidence
- use short coherence length sources

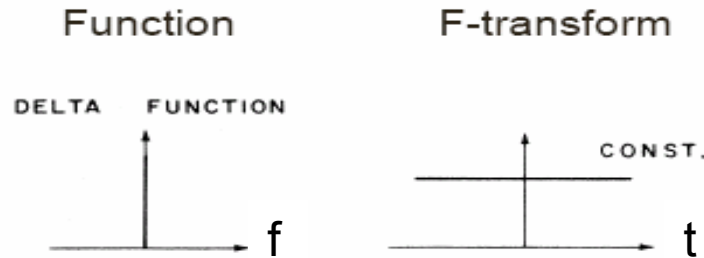
# Monochromatic light and coherence

## Monochromatic light is coherent

$$\vec{E}(z, t) = \hat{x}E_x \cos(\omega t - k_0 z + \phi_x) + \hat{y}E_y \cos(\omega t - k_0 z + \phi_y)$$

- For strictly monochromatic light  $E_x$ ,  $E_y$ ,  $\phi_x$ , &  $\phi_y$  are constant

*Remember Fourier transform pairs*



## What if laser beam is not constant (e.g., pulsed laser)?

- Light is not monochromatic...  $E_x$ ,  $E_y$ ,  $\phi_x$ , &  $\phi_y$  are constant
- For  $\Delta\omega \ll \omega$ , use “quasi-monochromatic” approximation
- Amplitude and phase vary about mean frequency  $\bar{\omega}$  and mean wavenumber  $\bar{k}_0$

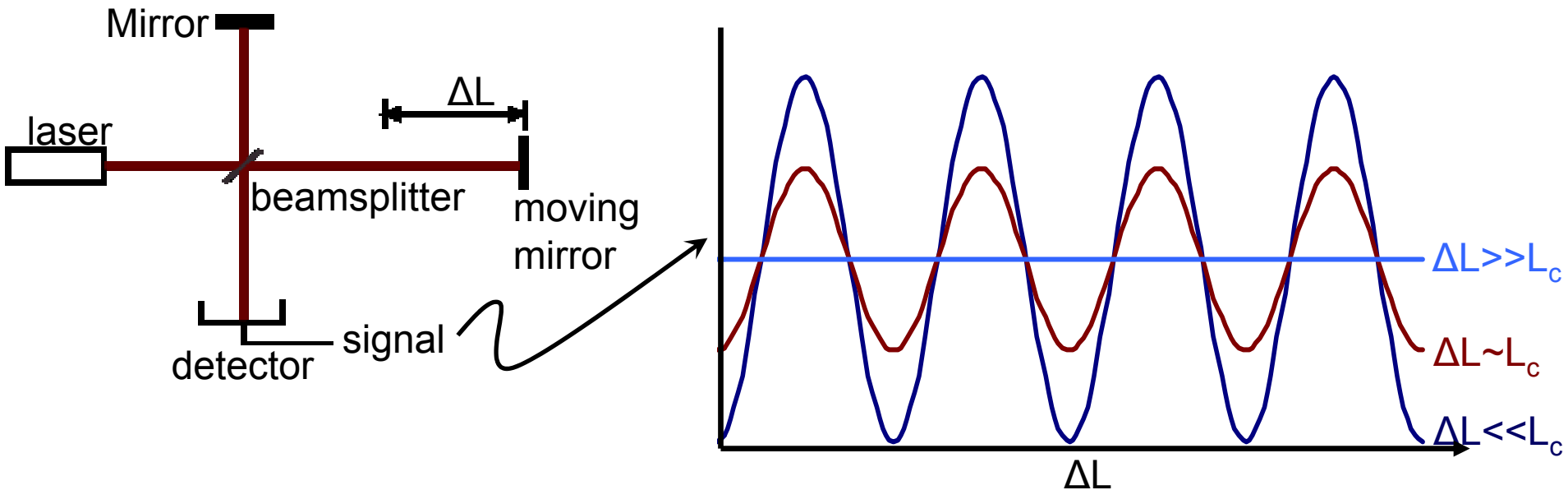
$$\vec{E}(z, t) = \hat{x}E_x(z, t) \cos(\bar{\omega} t - \bar{k}_0 z + \phi_x(z, t)) + \hat{y}E_y(z, t) \cos(\bar{\omega} t - \bar{k}_0 z + \phi_y(z, t))$$

# Coherence time and length

**Coherence time**  $\tau_c \sim 2\pi / \Delta\omega \sim \lambda^2 / c\Delta\lambda$

**Coherence length**  $L_c = c \tau_c$

*The laser beam will be mutually coherent over times (lengths)  $< \tau_c, L_c$*



# “Unpolarized” light

## Monochromatic light is polarized

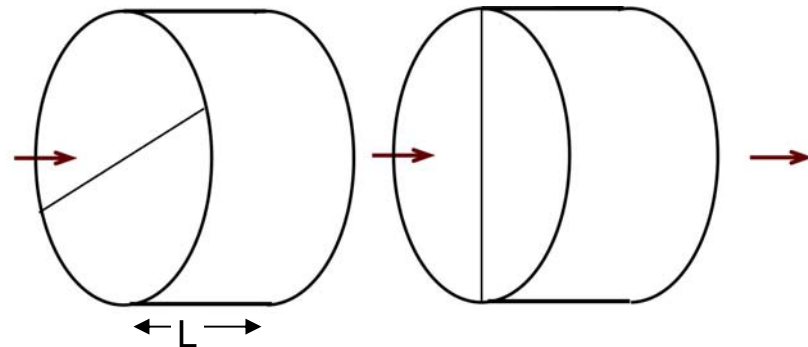
- $E_x$ ,  $E_y$ ,  $\phi_x$ , &  $\phi_y$  are constant
  - in other words, one cannot make an unpolarized monochromatic laser beam
  - exception is *spatial* depolarization using ground glass or integrating spheres

## For non-monochromatic light

- pairs  $E_x$  &  $E_y$ , and  $\phi_x$  &  $\phi_y$  have statistical correlations
- Correlation depends on the spectral bandwidth  $\Delta\omega$  of the light
  - Light is correlated only within the coherence time  $\tau_c$  or coherence length  $L_c$

## Can “depolarize” broadband light with a Lyot depolarizer

- Create delay  $\gg L_c$  between eigenpolarizations
- Need thick birefringent retarders with  $\Delta n L > L_c$



# References

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