Plasma instabilities in the Anisotropic Quark Gluon Plasma

- Why they should (?) be expected
- Similarities and Differences with conventional plasmas
- Possible implications
- Directions for further research

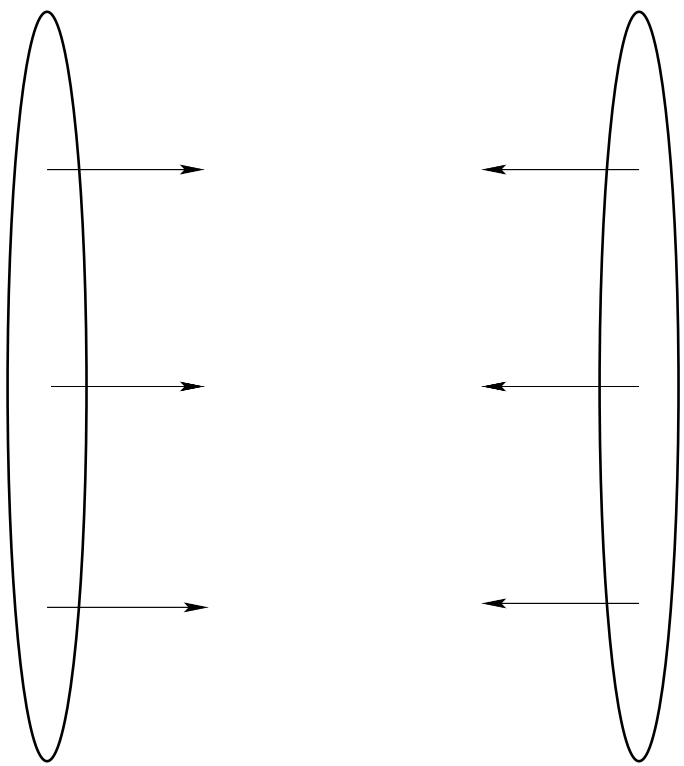
Is a weak-coupling treatment really in contradiction with hydrodynamic behavior as observed at RHIC?

Guy D. Moore P. Arnold, J. Lenaghan, L. Yaffe

Stanislaw Mrówczyński, Mike Strickland, P. Romatschke, A. Rebhan

Idealized Heavy Ion Collision: Before

Approaching Heavy Ions



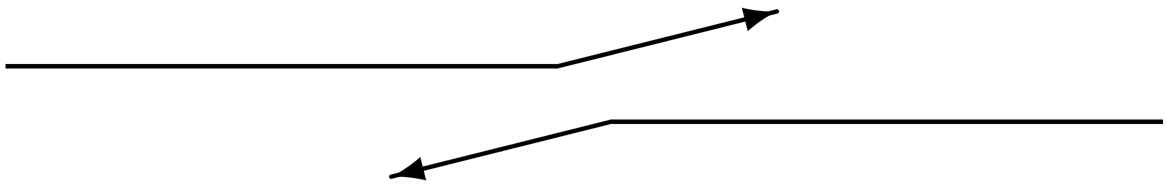
Flat — Lorentz contraction

Each made of $\sim$ 200 p, n

p, n are made of ~ 50 partons

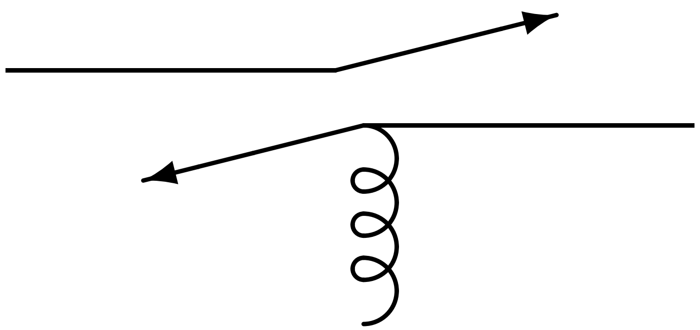
(Parton $=$ Quark or Gluon)

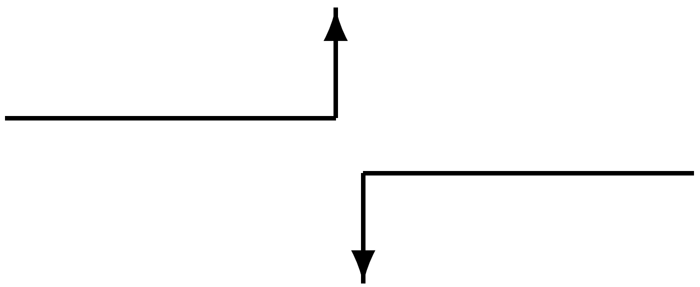
Most partons miss all other partons or make a glancing scattering



So most particles continue nearly forward

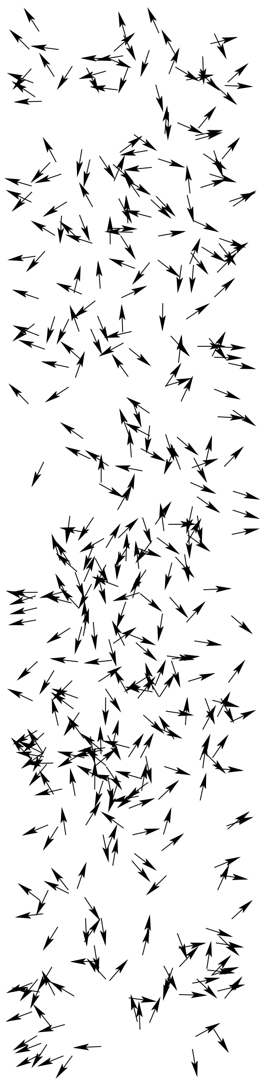
A few scatter or emit at closer to right angles.





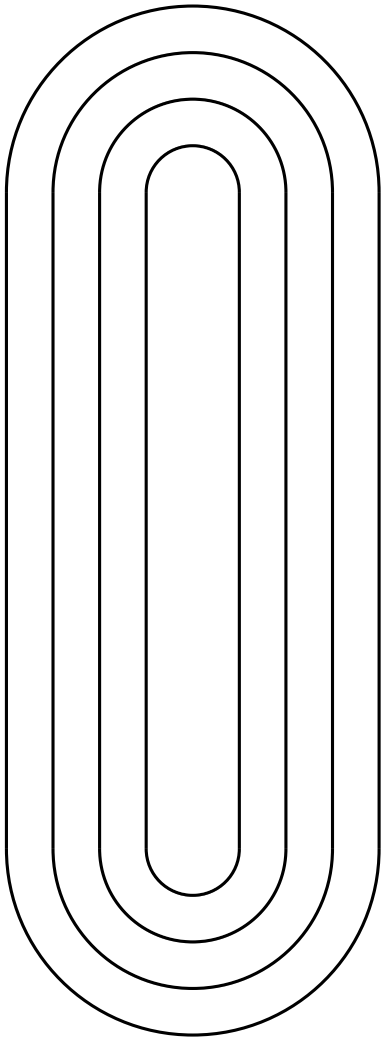
So considerable population at central rapidity

Conditions Just After a Collision



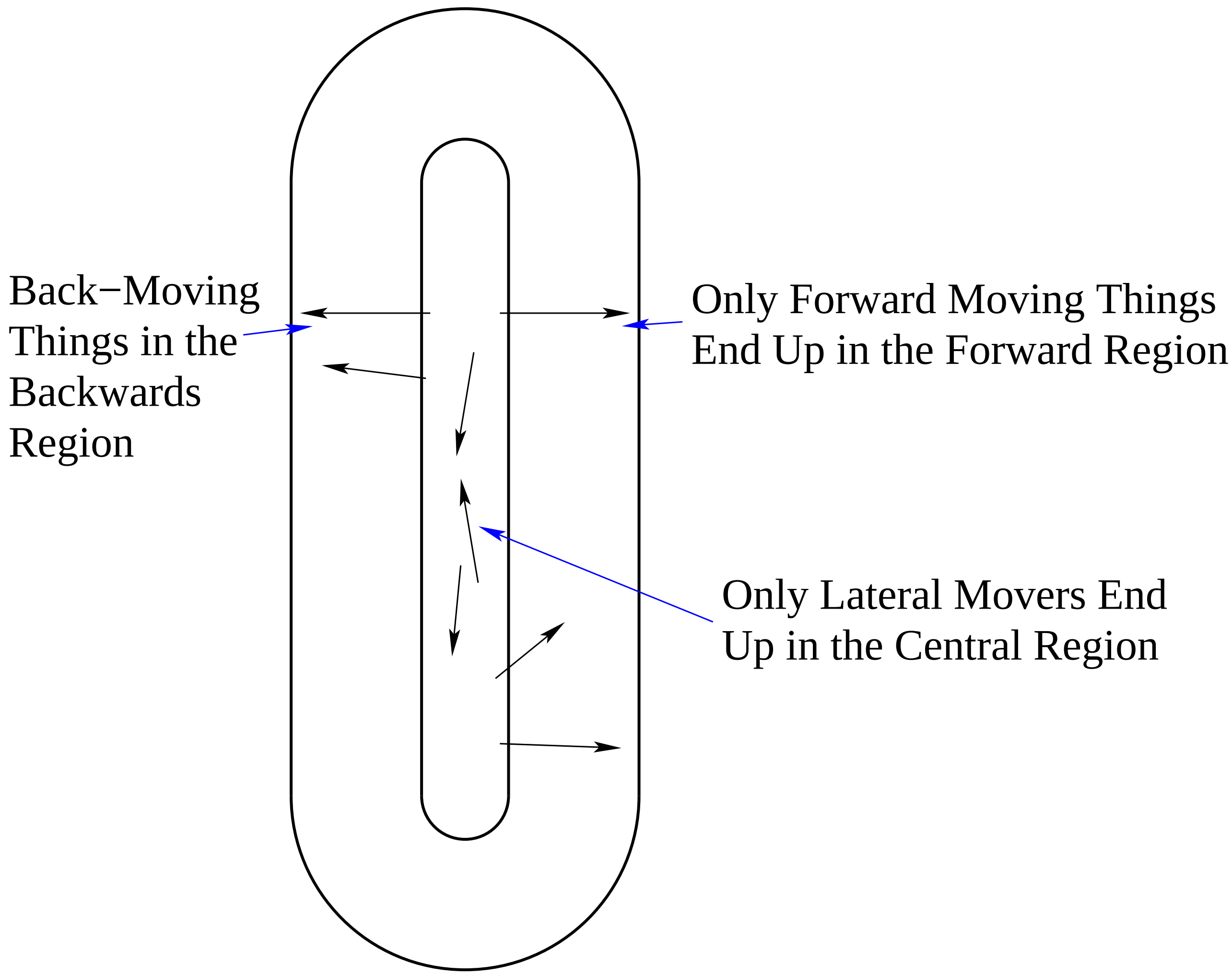
Middle region consists of Quarks and Gluons with Random directions (Quark Gluon Plasma, QGP).

System expands—but it is Pancake Shaped.



Expansion is quasi-1 dimensional–aspect ratio of the QGP is changing rapidly.

In the Absence of Further Scattering:



Central plasma is anisotropic (Oblate p distribution).

Quark-Gluon Plasma is –Sort Of– like conventional plasma

There are gluonic fields which obey Maxwell equations.

Small deBroigle wavelength quarks (and gluons!) act like point-particle charges for these gluonic fields

Classical particle + classical field description is justified at leading order in $\alpha_{\rm s}$ (and underlies Saturation/CGC picture)

Maxwell + Anisotropy $\Longrightarrow$ Weibel Instability

Key Differences between QGP & conventional plasma

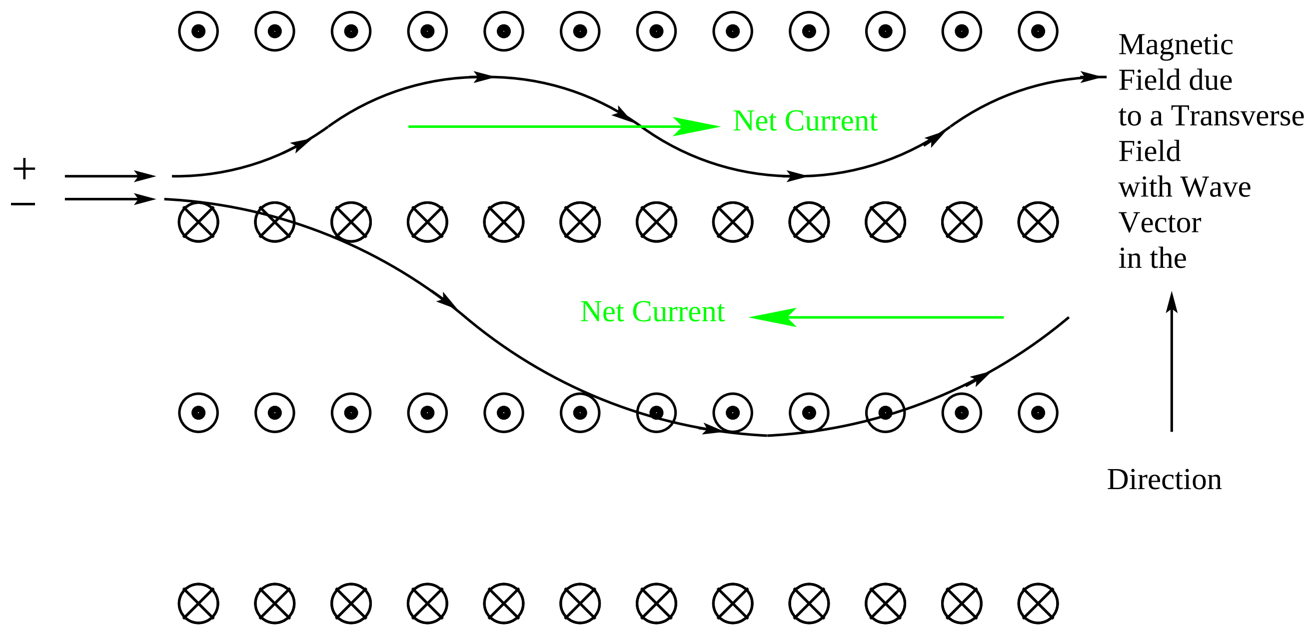
- Charges are Ultra-Relativistic
- Coupling is not small: $\alpha_{\rm s} \sim 1/3$ while $\alpha_{\rm EM} \sim 1/137$
- 3 types of charge and 8 electromagnetic (gluon) fields
- Color E&M fields have direct mutual interactions,

$$J_\mu^a = g f^{abc} A_b^\nu F_{\mu\nu}^c$$

Weibel instability may saturate differently than in normal plasmas.

Physical Origin of Weibel Instability

Trapping of charges by soft magnetic fields



$\vec{B}$ fields trap particles moving along $\vec{A}$ direction

Trapped particle's current reinforces trapping $\vec{B}$ field!

Formal approach to Weibel Instability

$$\begin{aligned} G^{-1}_{\mu\nu} &= G^{-1}_{\mu\nu,\mathrm{vac}} + \Pi_{\mu\nu,\mathrm{medium}} \\ &\simeq p^2\eta_{\mu\nu} - p_\mu p_\nu + \Pi_{\mu\nu,\mathrm{medium}}(p^0/|\mathbf{p}|, \hat{\mathbf{p}})\,. \\ \Pi_{\mu\nu} &\sim g^2 \int \frac{d^3k}{k} f(k) \propto m^2_\infty (\mathrm{or}\ m^2_{\mathrm{D}})\,. \end{aligned}$$

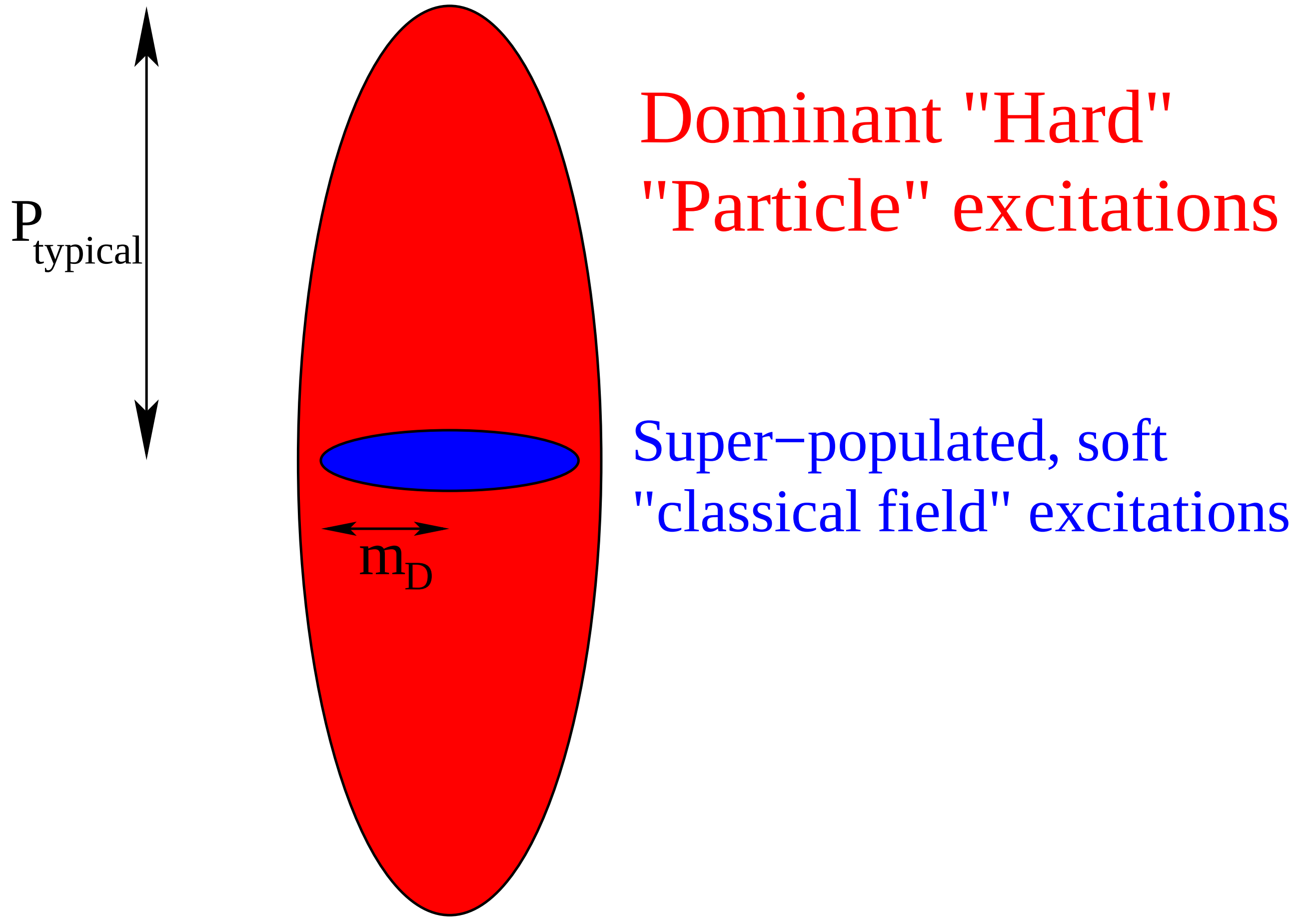
For most $p^0, \mathbf{p}$, $\Pi_{\mu\nu}$ is positive–but for $p^0 = 0$ (plasma frame) the average over $\hat{\mathbf{p}}$ directions of Π_T is zero.

Anisotropy: $\Pi_T < 0$ for some $\hat{\mathbf{p}}$ at small p^0.

Small p: Π dominates free G. Pole at imaginary ω.

Anisotropic, Perturbative $\Longrightarrow$ exponential growth

Momentum space distribution of excitations:



Oblate “hard” distribution, prolate classical field “soft” distribution

Does instability matter?

If I assume

1. Saturation picture, $f(Q_s) \sim \alpha_s^{-1}$ at $t \sim Q_s^{-1}$
2. $\alpha_s \ll 1$
3. Bjorken hydrodynamics (1-dimensional expansion)
4. No (other) fast isotropization mechanism

Then exponentiation timescale for instability **always** shorter than age of plasma for all $t > Q_s^{-1}$. Probably.

Arnold, Lenaghan, GDM hep-ph/0307325 (JHEP0308:002)

Key Dynamical Question

Do soft B fields grow to the scale

$$A_\mu \sim \frac{p_{\rm typical}}{g} (= Q_s/g), \quad B \sim \frac{m_{\rm D}\, p_{\rm typical}}{g}$$

the scale where $R_{\rm Lamor} \sim m_{\rm D}^{-1}$ the coherence length, and where $B^2 = \rho_B \sim \rho_{\rm total}$,

Or do they saturate at

$$A_\mu \sim \frac{m_D}{g}, \qquad B \sim \frac{m_{\rm D}^2}{g}$$

where nonabelian interactions become important (gauge fields mutually interact, charges color-rotate on the A field coherence length, etc)?

Nonabelian system: nonabelian interaction

$$J^a_\mu \sim g f^{abc} A^b_\nu F^c_{\mu\nu}$$

as large as linear Maxwell term for $A \sim m_{\rm D}/g$.

(Think of soft gauge fields as extra particles–but ones with a prolate momentum distribution!)

May stop growth, and may scatter around direction of other B fields to be random.

Steady state–pumping of soft modes, up-scattering of momentum to scales above $m_{\rm D}$–would probably follow.

Peter Arnold and Jonathan Lenaghan hep-ph/0408052

1 space $\times$ 3v toy model

Naive treatment of self-energy

System "Abelianizes"–2 of 8 color fields grow 2 commuting directions

These 2 suppress growth of other 6–they stay small and do not feed back

Strickland, Romatschke, and Rebhan hep-ph/0412016

1 space $\times$ 3v but full Yang-Mills with ballistic (Eikonalized) charges

Confirm Arnold&Lenaghan behavior

Implication If True

Assume plasma becomes strongly anisotropic

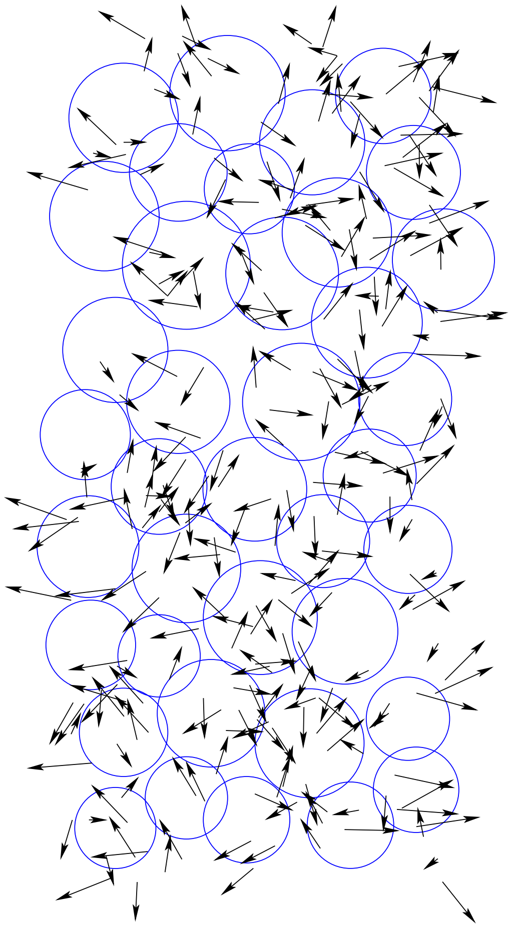
- B grows until $B^2 \sim \rho$
- $R_{\rm Lamor} \sim m_{\rm D}^{-1} \ll \tau$ (τ the system age)
- System isotropizes

Contradicts assumption of strong anisotropy

System must remain near-isotropic at all times $t > Q_s^{-1}$

Sufficient to ensure $T_{ij} = P\delta_{ij}$, hence Hydrodynamic behavior, radial and elliptic flow.

What does system "look like"? In this picture,



Patchwork of domains, $m_{\mathrm{D}} \sim 1\mathrm{fm}$ across

Particles bend coherently within each domain

B fields dominate *bending* but not *energy*

Possible implications: hydrodynamic behavior, enhanced bremsstrahlung (γ production, jet quenching)

Needed: 3 space $\times$ 3v simulations

Method AMY, in preparation: treat A, f on 3-D lattice:

- Gauge field: lattice (link variable) method
- $f(\mathbf{p}) = f_0 + \delta f$, with f_0 uncolored and δf adjoint color fluctuations, f_0 background and δf dynamical
- $\delta f(\mathbf{p})$ angular dependence expanded in Y_{lm}'s truncated at some l_{max}, probably $\sim$ 6–12

Perturbative in *deflection angle*, Nonperturbative in A field

Can tell if saturation occurs at $A \sim m_{\mathrm{D}}/g$ or $A \sim p_{\mathrm{typical}}/g$.

Similar to Bödeker GDM Rummukainen hep-ph/9907545 (PRD61:056003)

$f(x,p)$ really $f^{ab}(x,p)$ an $\bar{R} \times R$ tensor

$$f^{ab} = f^{\rm singlet} + f^a + f^{\rm higher}$$

Maxwell equation involves f^a,

$$D_\mu F_a^{\mu\nu} = g \int \frac{d^3k}{(2\pi)^3} v^\nu f^a(k)$$

$$D_t f + \vec{v} \cdot \vec{D} f = g v_\mu F^{\mu\nu} \partial_\nu^k f(x,p)$$

Expand in $f^{\rm singlet} \gg f^a$ (small-angle approximation!)

$$(v^\mu D_\mu) f^a = g v_\mu F_a^{\mu\nu} \partial_\nu^p f^{\rm singlet}$$

Ultra-relativistic–free to integrate over $|k|$, f^a a function only of $\vec{v}$ (2 internal dimensions)

Strengths:

- Fully nonperturbative in nonabelian interactions
- Full 3 space and 3v (Probably essential)

Limitations:

- Treats particles in Eikonal approximation (small deflection angle)
- No hard collisions (Not an Issue)
- All founded on weak coupling (As usual)

Can resolve $scale$ of Weibel saturation, but probably not subsequent evolution.

Preliminary results

A appears to saturate at the nonabelian scale

Subsequent cascade of energy from m_{D} scale to larger k gauge fields

Totally different dynamics than in Abelian plasmas

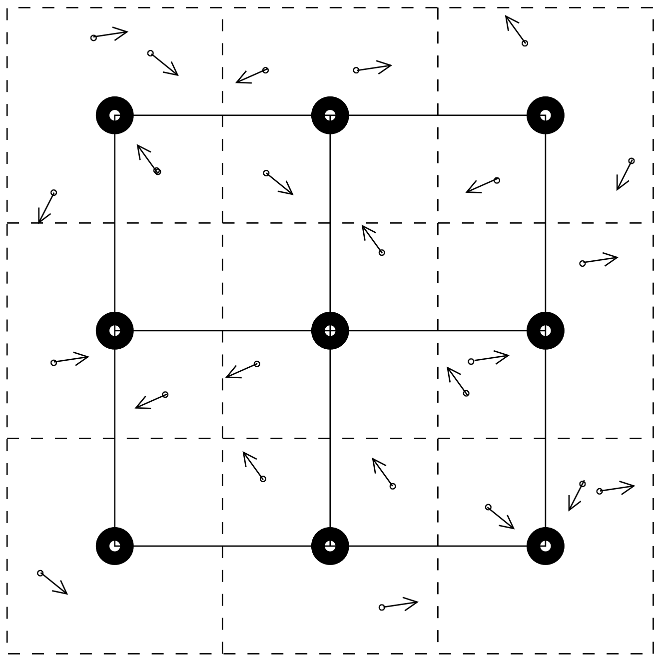
Enough for isotropy at all timescales? Not clear.

Conclusions

- Heavy ion collisions—in the idealization of high energy—contain all conditions for (color) plasma instabilities
- May be relevant to good hydrodynamic behavior observed at RHIC
- Instability may saturate early due to nonabelian interactions

Just in Case Slides

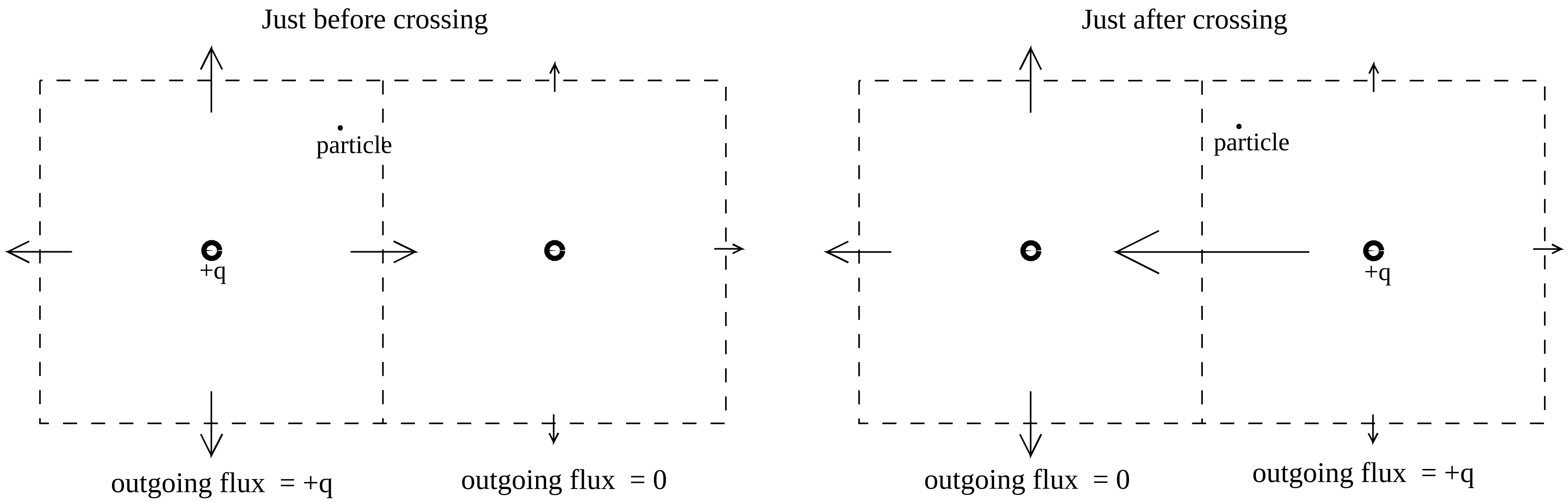
Why don't we perform a Particle and Cell Simulation?



Gauge fields on links, particles in space, their charges on the nearest site, B on plaquettes and E on links

GDM, B. Müller, C. Hu, hep-ph/9710436, PRD58:045001

Particle changes "color" when it crosses between "boxes"



$Q \to e^{iA\cdot l}Q$. Unlike abelian, A is a matrix

Requires local update (cannot perform smearing) to conserve Charge, Energy and Gauss' Law

Particle velocity: $v = c$ (must be at least close!)

Field velocities: lattice dispersion

$$v_{\rm group} = \frac{dE}{dp} \sim c\frac{\sin p}{2\sin p/2} < 1$$

[Really, $E^2 = \sum_i (4/a^2)\sin^2(ap_i/2)$]

Cherenkov condition met for UV lattice modes

Spurious particle–UV lattice field interactions

Extremely numerically expensive to make this effect small

Not as important for applications of MMH paper