A new approach to inclusive decay spectra

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Plan of the talk

- Introduction: inclusive B decay spectra — the challenge
- Sudakov resummation with NNLL accuracy
- Divergence of perturbation theory
- The quark distribution in the meson — cancellation of renormalon ambiguities
- Dressed Gluon Exponentiation: renormalon resummation in the Sudakov exponent
- computed $\bar{B} \longrightarrow X_s\gamma$ spectrum — comparison to data

Inclusive B–decay Spectra and Infrared Renormalons

References

- Taming the $\bar{B} \longrightarrow X_s\gamma$ spectrum by Dressed Gluon Exponentiation, J.R. Andersen, E. Gardi, accepted for publication in JHEP [hep-ph/0502159].

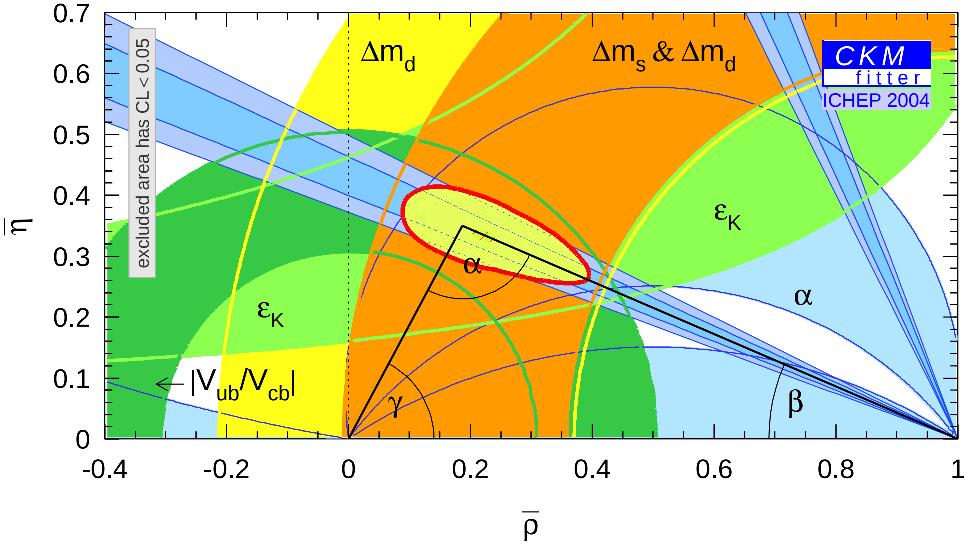
- On the quark distribution in an on-shell heavy quark and its all-order relations with the perturbative fragmentation function, E. Gardi, JHEP **0502**, 053 (2005) [hep-ph/0501257].

- Radiative and semi-leptonic B-meson decay spectra: Sudakov resummation beyond logarithmic accuracy and the pole mass, E. Gardi, JHEP **0404**, 049 (2004) [hep-ph/0403249].

Motivation: flavor physics

Identifying deviations from SM by

- (over)constraining the unitarity triangle
- branching ratios of rare decays



Decays are difficult to compute
since quarks are *confined* into mesons

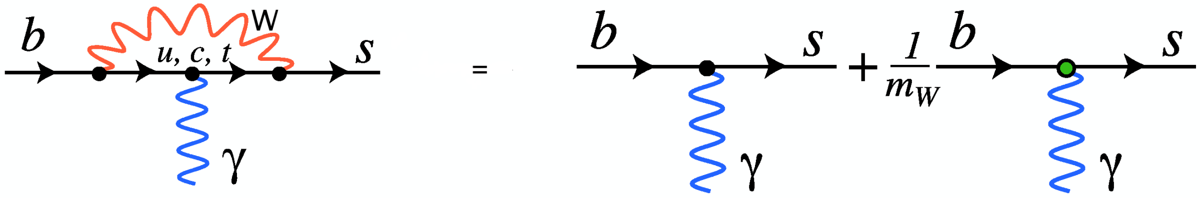
- both exclusive decays (**specific** final states) and inclusive ones (e.g. $\bar{B} \to X_c l \bar{\nu}$, $\bar{B} \to X_u l \bar{\nu}$, $\bar{B} \to X_s \gamma$) are useful.

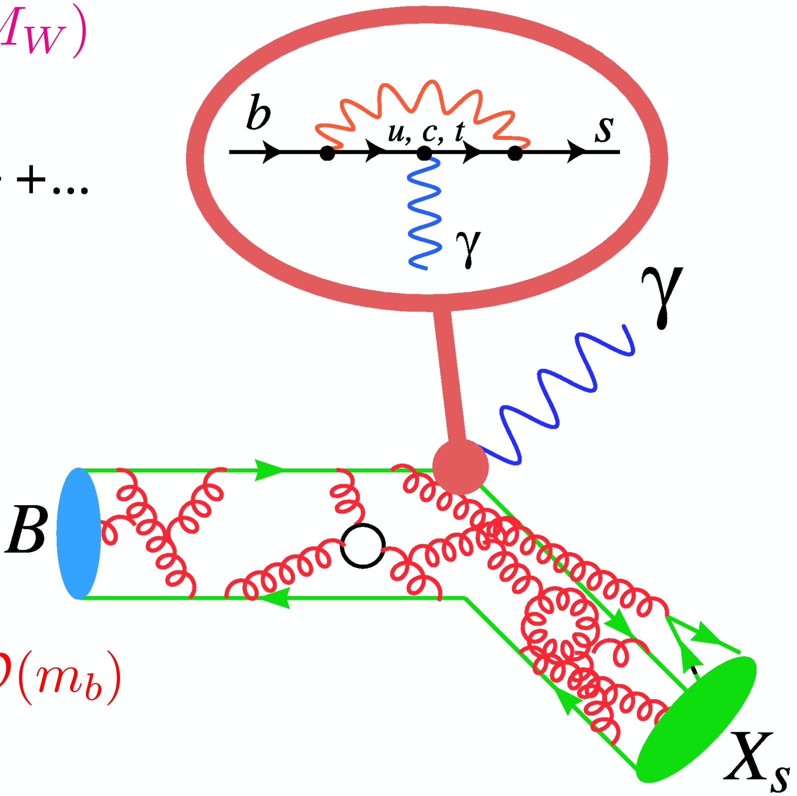
  But inclusive decays are **insensitive** to the details of the final and **initial** states. In particular, the absolute inclusive rate is **computable** in perturbation theory.

Hierarchy of scales and the Operator Product Expansion

separation between short– and long–distance physics

First step: integrate out fields with virtuality $\mathcal{O}(M_W)$





To leading order in $1/m_W$ and in α_s (only O_7):

$$H = \sqrt{2} G_F V_{tb} V_{ts}^* A(m_t^2/m_W^2)\,\frac{em}{16\pi^2}\,\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}$$

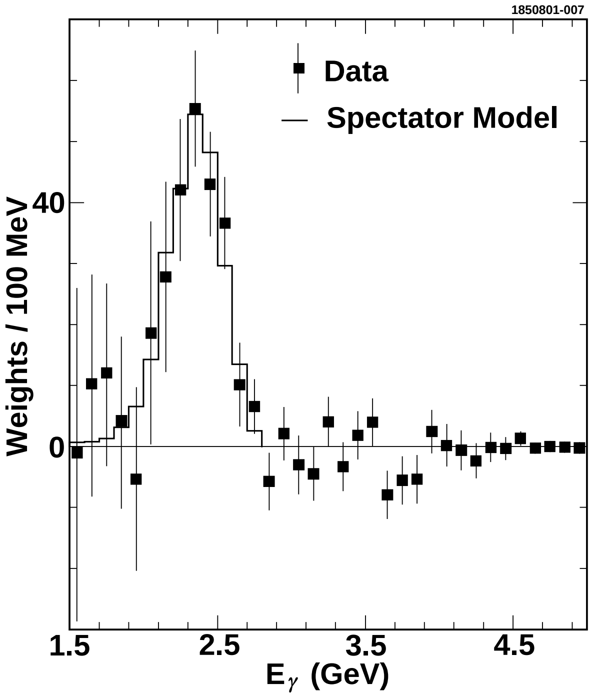
Second step: integrate out fields with virtuality $\mathcal{O}(m_b)$
$m_b \gg \Lambda \Rightarrow$ the b quark is close to its mass shell
Heavy–Quark Effective Theory

Expansion in Λ/m_b is useful for the calculation of total rates
but **insufficient** for spectra.

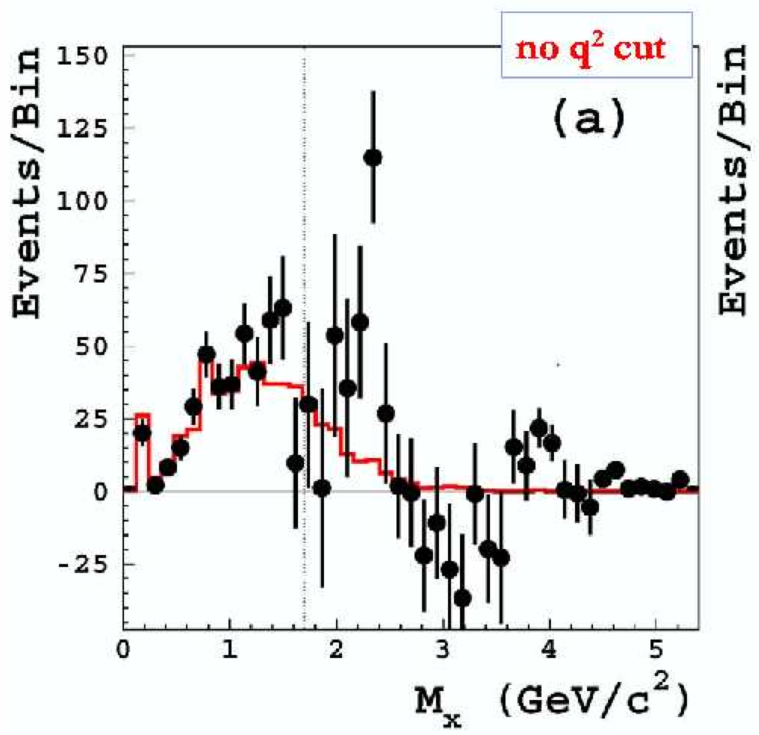
Inclusive B–decay Spectra

radiative decay: $\bar{B} \longrightarrow X_s\, \gamma$

semi-leptonic decay: $\bar{B} \longrightarrow X_u\, l \bar{\nu}_l$



CLEO



BELLE

The distribution peaks close to the endpoint $(E_\gamma \longrightarrow M_B/2; \quad \text{small } M_X)$

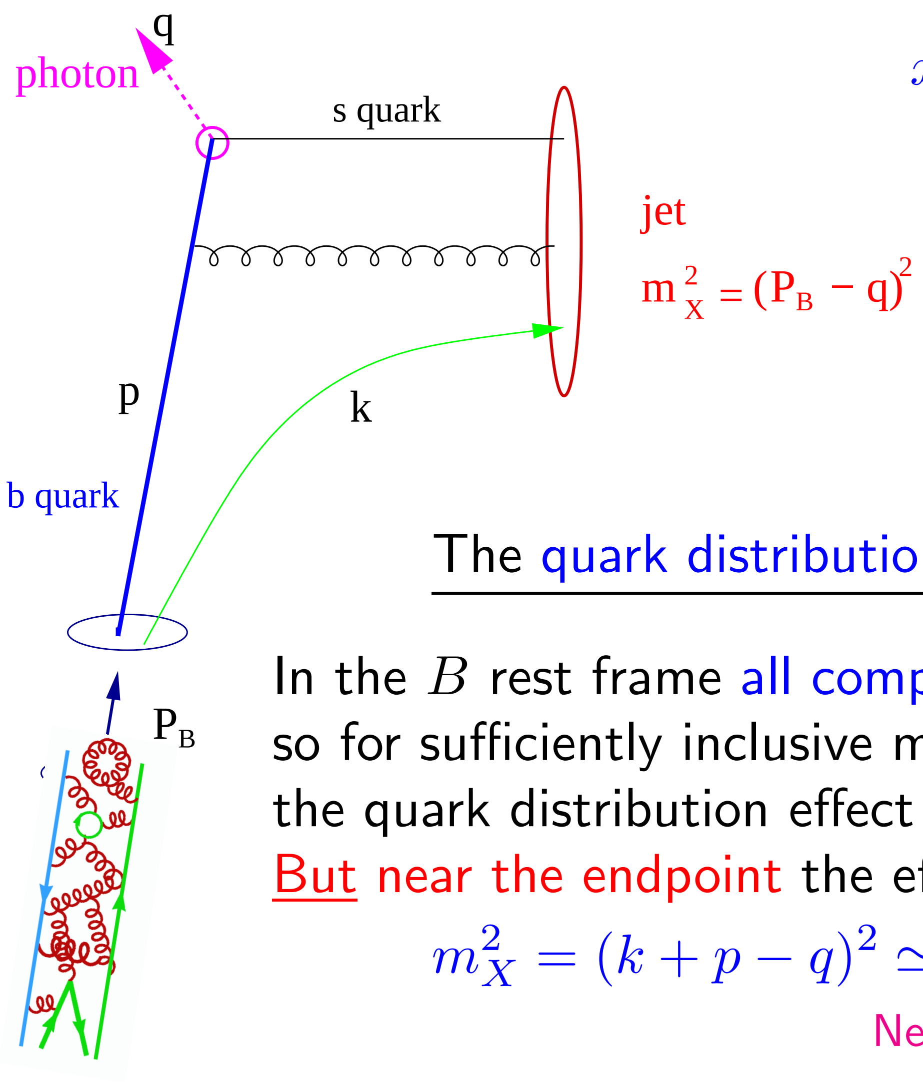
Example: extracting $|V_{ub}|$ from the semi-leptonic decay

Precise measurements are restricted to the small M_X region (charm background)

Determination of $|V_{ub}|$ relies on calculation of the spectrum.

Kinematics in $\bar{B} \longrightarrow X_s\,\gamma$

Most of the B meson momentum is carried by the b quark



$$x \equiv \frac{2E_\gamma}{m_b}; \qquad \frac{1}{\Gamma_{\text{tot}}}\frac{d\Gamma}{dx}\bigg|_{\text{LO}} = \delta(1-x)$$

Beyond LO: the peak is smeared.

Perturbative endpoint: $x = 1$

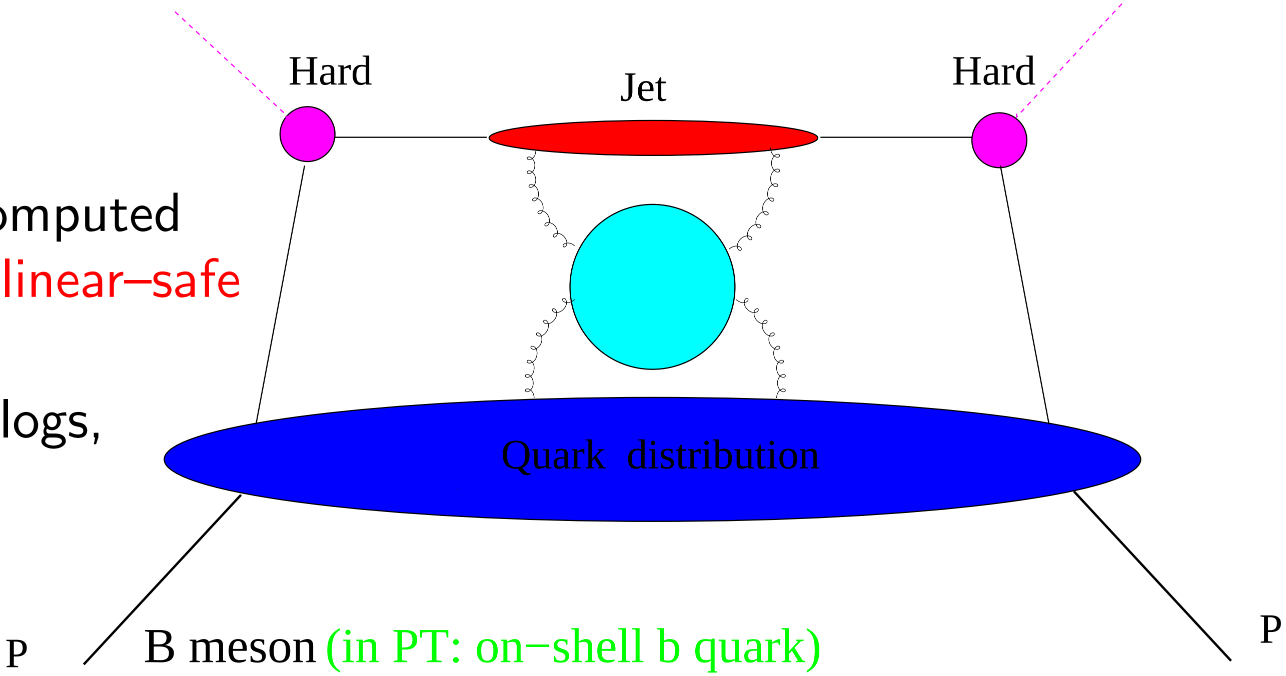
The quark distribution in the B meson $f(z;\mu)$

In the B rest frame all components of k are $\mathcal{O}(\Lambda) \ll m_b$
so for sufficiently inclusive measurement
the quark distribution effect is power suppressed
But near the endpoint the effect of $f(z;\mu)$ is $\mathcal{O}(1)$

$$m_X^2 = (k+p-q)^2 \simeq (p-q)^2 + 2k\cdot(p-q) = \mathcal{O}(\Lambda M)$$

Neubert; Bigi, Shifman, Uraltsev & Vainshtein (93)

Large–x factorization in inclusive B decays



The spectrum can be computed
in PT: infrared– and collinear–safe

Dominated by Sudakov logs,
$\ln(1-x)$

scales:

Hard: m
Jet: $m_X^2 = (P_b - q)^2 \simeq m^2(1-x) \Longrightarrow m^2/N$
Soft: $m(1-x) \Longrightarrow m/N$

Korchemsky & Sterman (94)

Spectral moments:

$$\begin{aligned} \Gamma_N^{\text{PT}} &\equiv \int_0^1 dx x^{N-1} \frac{1}{\Gamma_{\text{tot}}^{\text{PT}}} \frac{d\Gamma^{\text{PT}}}{dx} \\ &= H(m) J(m^2/N;\mu) S_{\text{PT}}(m/N;\mu) \;+\; \mathcal{O}(1/N) \\ &\equiv H(m)\,\text{Sud}(N,m) \qquad\quad + \; \mathcal{O}(1/N) \end{aligned}$$

Coefficients in the Sudakov exponent

$$\mathrm{Sud}(N,m) = \exp\left\{ -\sum_{n=1}^{\infty}\sum_{k=1}^{n+1} C_{n,k} \ln^k N \left(\frac{\alpha_s^{\overline{\mathsf{MS}}}(m^2)}{\pi}\right)^n \right\}$$

The coefficients $C_{n,k}$ are known **exactly** to **NNLL accuracy**. For $N_f = 4$ they are:

$k \longrightarrow$

$n \downarrow$

-1.564	0.667	0	0	0	0	0	0
3.837	-0.078	1.389	0	0	0	0	0
?	20.579	6.339	3.376	0	0	0	0
?	?	116.464	33.024	9.042	0	0	0
?	?	?	597.221	138.600	25.955	0	0
?	?	?	?	2859.284	548.170	78.492	0
?	?	?	?	?	13141.289	2129.058	247.233
?	?	?	?	?	?	58941.217	8238.359
?	?	?	?	?	?	?	260391.559

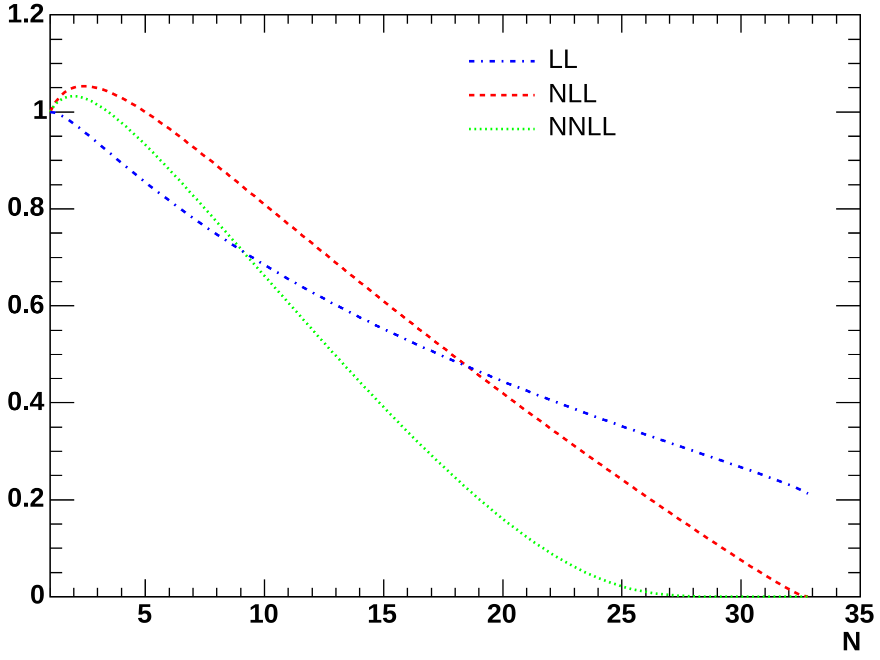
- At a given order in α_s the coefficients of **subleading logs** (lower k) get large...
- Is the fixed–logarithmic–accuracy approximation at LL / NLL / NNLL good?

Conventional Sudakov resummation with NNLL accuracy

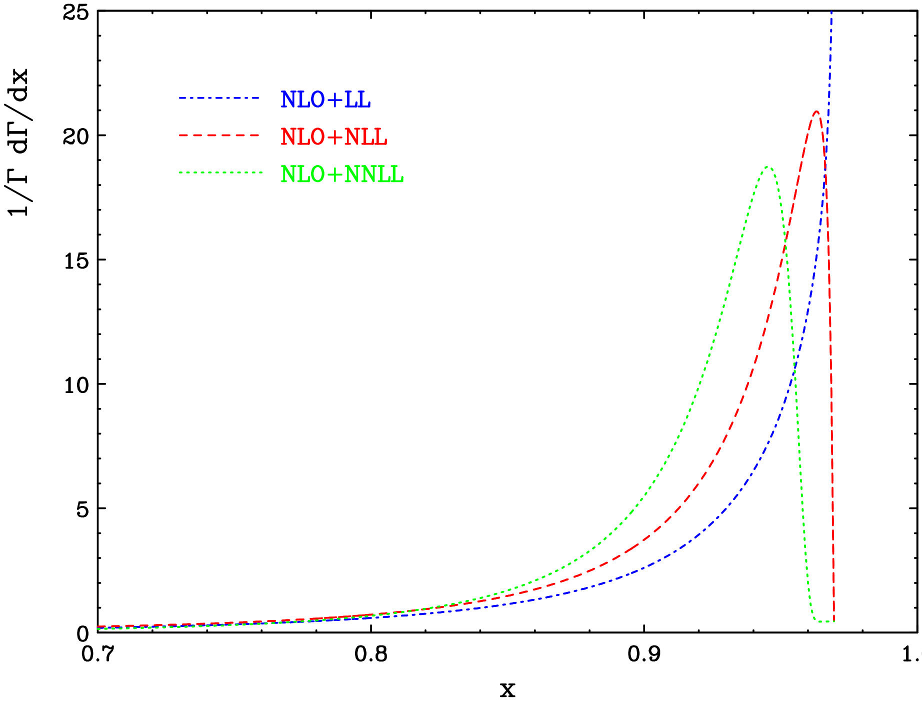
$$\mathrm{Sud}(N,m) \quad = \quad \exp\left\{\sum_{n=0}^{\infty} g_n(\lambda)\left(\frac{\alpha_s^{\overline{\mathrm{MS}}}(m^2)}{\pi}\right)^{n-1}\right\}; \qquad \lambda \equiv \frac{\alpha_s^{\overline{\mathrm{MS}}}(m^2)}{\pi}\beta_0 \ln N$$

$$g_0(\lambda) = \frac{C_F}{{\beta_0}^2}\left[(1-\lambda)\,\ln(1-\lambda) - \frac{1}{2}(1-2\lambda)\,\ln(1-2\lambda)\right]$$

$\mathrm{Sud}(N,m)$



Corresponding spectra



Coefficients in the Sudakov exponent in the large–β_0 limit

$$\mathrm{Sud}(N,m) = \exp\left\{ -\sum_{n=1}^{\infty}\sum_{k=1}^{n+1} C_{n,k} \ln^k N \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}(m^2)}{\pi}\right)^n \right\}$$

The part in $C_{n,k}$ that is proportional to $(\beta_0)^{n-1}$ is known to all orders:

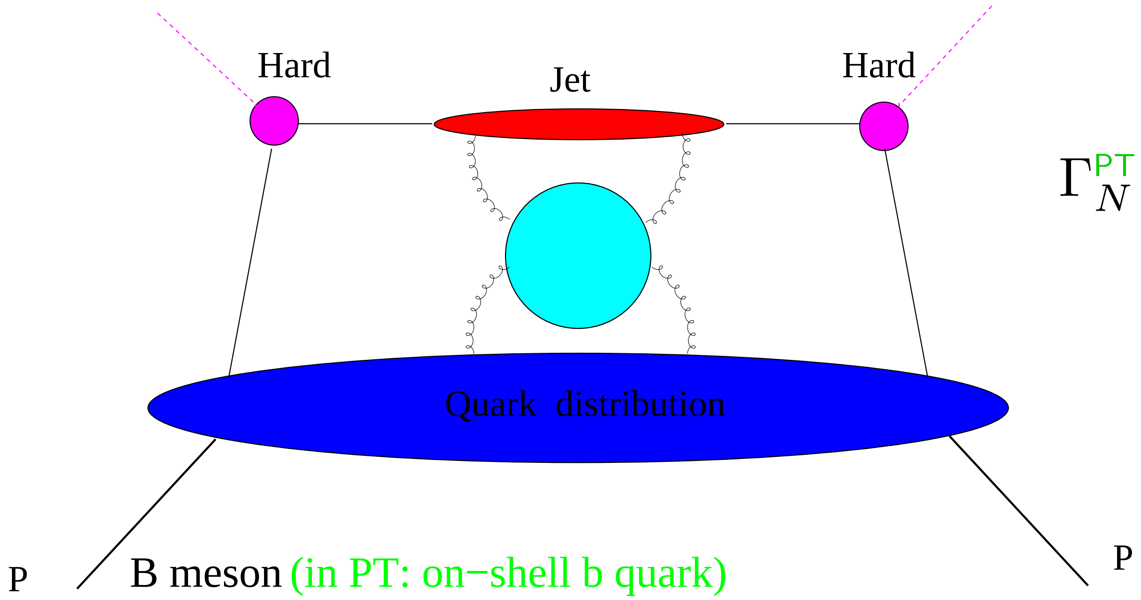
$$k \quad \longrightarrow$$

$$n \quad \downarrow$$

-1.56	0.67	0	0	0	0	0
1.24	0.90	1.39	0	0	0	0
61.17	28.32	8.28	3.38	0	0	0
1096.06	515.20	166.25	34.89	9.04	0	0
20399.23	10078.43	3231.40	793.25	131.33	25.95	0
444615.21	221481.03	73268.94	17791.58	3514.66	482.12	78.49
11342675.74	5665794.49	1883129.50	468180.33	91361.30	15080.79	1768.50
334032127.30	166960507.50	55609620.17	13867704.58	2760946.21	449959.01	63745.75

- $C_{n,k}$ **increase** for lower powers of $\ln N$, building up $\sum_{k=1}^{n+1} C_{n,k} \ln^k N \sim \mathbf{n!} f_n(N)$
- Truncation at fixed logarithmic accuracy is **not** a good approximation.
- **Renormalon divergence** sets in already at low orders — requires a prescription!

Large–x factorization — going beyond PT



Perturbation theory (on-shell quark):

$$\begin{aligned}\Gamma_N^{\mathrm{PT}} &\equiv \int dE_\gamma \left(\frac{2E_\gamma}{m}\right)^{N-1} \frac{1}{\Gamma_{\mathrm{tot}}^{\mathrm{PT}}}\frac{d\Gamma^{\mathrm{PT}}}{dE_\gamma} \\ &\simeq H(m)J(m^2/N;\mu)S_{\mathrm{PT}}(m/N;\mu) \\ &= H(m)\,\mathrm{Sud}(N,m)\end{aligned}$$

Non-perturbatively (meson):

$$\begin{aligned}\Gamma_N &\equiv \int dE_\gamma \left(\frac{2E_\gamma}{M}\right)^{N-1} \frac{1}{\Gamma_{\mathrm{tot}}}\frac{d\Gamma}{dE_\gamma} \\ &\simeq H(m)J(m^2/N;\mu)S(M/N;\mu)\end{aligned}$$

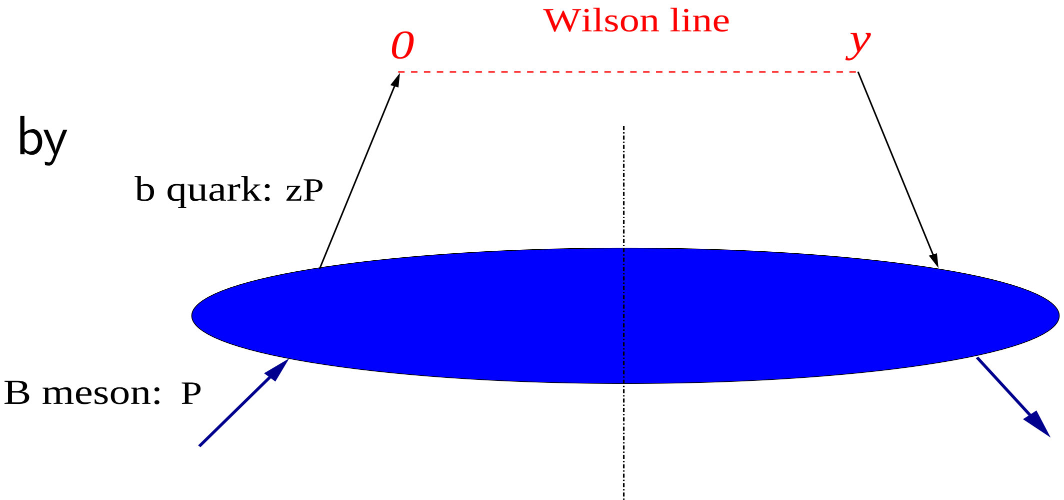
Quark distribution in the meson in terms of that in an on-shell heavy quark:

$$S(m/N;\mu) = S_{\mathrm{PT}}(m/N;\mu)\ \mathrm{e}^{-(N-1)\bar{\Lambda}/M}\,\mathcal{F}((N-1)\Lambda/M)$$

where $\bar{\Lambda} \equiv M - m$ captures the dependence on the mass; $\mathcal{F}$ has no linear term

The quark distribution function

Quark distribution $f(z;\mu)$
in the meson $({P_B}^2 = M^2)$ is defined by



$$f(z;\mu) \equiv \frac{1}{4\pi}\int_{-\infty}^{\infty} dy^- \, \mathrm{e}^{-izP_B^+y^-} \left\langle B(P_B) \middle| \left[\bar{\Psi}(y)\gamma^+ \, \Phi_y(0,y) \, \Psi(0)\right]_\mu \middle| B(P_B) \right\rangle$$

$$\text{where} \qquad \Phi_y(0,y) \equiv \mathbf{P}\exp\left(ig\int_0^{y^-} dx^- A^+(x^-)\right)$$

$$\text{Moments}: \qquad F(N;\mu) \equiv \int_0^1 dz z^{N-1} f(z,\mu) \qquad \Rightarrow \qquad iP_B^+ y^- \longrightarrow N$$

Perturbative quark distribution $f_{\text{PT}}(z;\mu)$ in an on-shell heavy quark $({p_b}^2 = m^2)$
is defined by the **same operator**, taken between quark states: $\langle b(p_b)|$.
It is infrared– and collinear–safe!

From the quark distribution function to power corrections

Quark distribution in an on-shell heavy quark $F_{\rm PT}(N;\mu)$ and in the meson $F(N;\mu)$:

The two have the **same** μ dependence.
They differ by power corrections.

The dependence on the soft scale m/N:

$$\begin{aligned} F_{\rm PT}(N;\mu) &= \left\langle b(p_b)\left|\left[\bar{\Psi}(y)\gamma^+\,\Phi_y(0,y)\,\Psi(0)\right]_\mu\right|b(p_b)\right\rangle\Big|_{ip_b^+y^-\longrightarrow N} + \mathcal{O}(1/N) \\ &= H_F(m)\,S_{\rm PT}(m/N;\mu)\ + \mathcal{O}(1/N) \\ F(N;\mu) &= \left\langle B(P_B)\left|\left[\bar{\Psi}(y)\gamma^+\,\Phi_y(0,y)\,\Psi(0)\right]_\mu\right|B(P_B)\right\rangle\Big|_{iP_B^+y^-\longrightarrow N} + \mathcal{O}(1/N) \end{aligned}$$

Using the OPE:

$$F(N;\mu) = H_F(m)\,\underbrace{S_{\rm PT}(m/N;\mu)\ {\rm e}^{-(N-1)\bar{\Lambda}/M}\ \mathcal{F}((N-1)\Lambda/M)}_{S\,(M/N;\,\mu)} + \mathcal{O}(1/N),$$

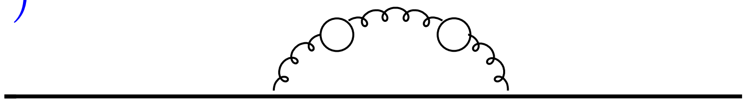
$\bar{\Lambda}\equiv M-m$ while $\mathcal{F}$ is independent of the quark–mass definition.
$\mathcal{F}$ sums power corrections on the soft scale starting at $\mathcal{O}\Big(((N-1)\Lambda/M)^2\Big)$.

Renormalon ambiguity in the pole mass

Infrared renormalons: a perturbative probe of large–distance effects

The propagator: $\dfrac{i}{\not p - m_{\overline{\rm MS}} - \Sigma(p, m_{\overline{\rm MS}})}$

Computed in the large–N_f limit



Off shell $\Sigma(p, m_{\overline{\rm MS}})$ has no renormalons

But applying the on–shell condition (inverse propagator vanishes at $p^2 = m^2$):

$$\frac{m}{m_{\overline{\rm MS}}} = 1 + \frac{C_F}{\beta_0}\int_0^\infty du \left(\frac{\Lambda^2}{m_{\overline{\rm MS}}^2}\right)^u \left[3e^{\frac{5}{3}u}\,\frac{(1-u)\Gamma(1+u)\Gamma(-2u)}{\Gamma(3-u)} + \frac{3}{4u} - R_{\Sigma_1}(u)\right].$$

Beyond PT the pole mass is ambiguous...

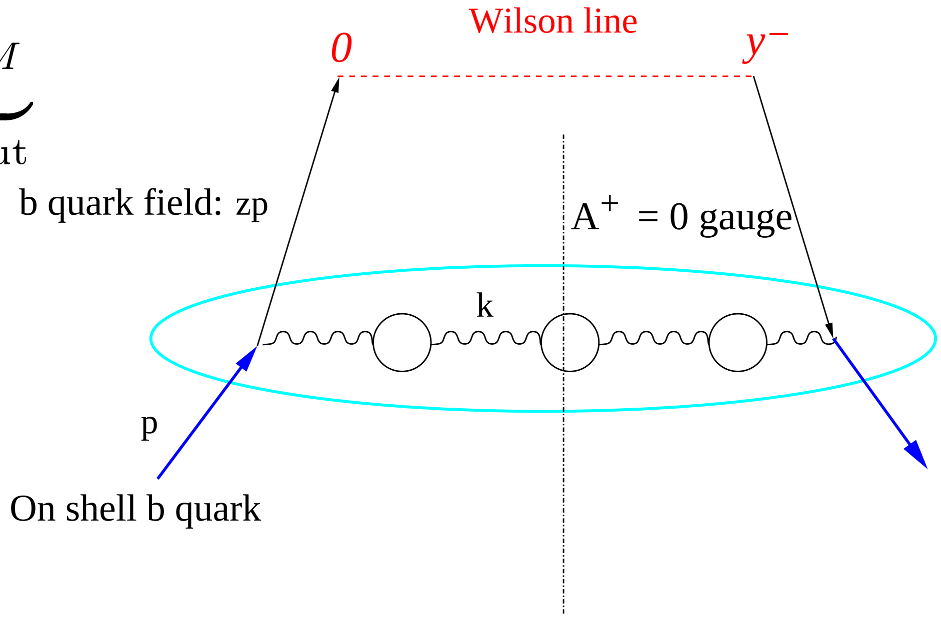
Beneke & Braun; Bigi, Shifman, Uraltsev & Vainshtein (94)

and so is $\bar\Lambda = M - m$.

Cancellation of the leading renormalon ambiguity

The non-perturbative quark distribution function is **renormalon free**:

$$S(m/N;\mu) \simeq \underbrace{S_{\mathrm{PT}}(m/N;\mu)\ \mathrm{e}^{-(N-1)\bar{\Lambda}/M}}_{\text{leading}-\text{renormalon cancels out}}$$



Dressed Gluon Exponentiation (DGE):

$$S_{\mathrm{PT}}(m/N;\mu) = \exp\left\{\int_0^\infty du\, T(u)\left(\frac{\Lambda^2}{m^2}\right)^u \right.$$

$$\left. \times\frac{1}{u}\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right)+\left(\frac{m^2}{\mu^2}\right)^u B_{\mathcal{A}}(u)\ln N\right]\right\}$$

$B_{\mathcal{S}}(u)=\dfrac{C_F}{\beta_0}\mathrm{e}^{\frac{5}{3}u}\,(1-u)\,+\mathcal{O}(1/\beta_0^2);$ $\mathcal{A}$ is the cusp anomalous dimension.

The renormalon at $u=\frac{1}{2}$ **cancels** between $S_{\mathrm{PT}}(m/N;\mu)$ and $\mathrm{e}^{-(N-1)\bar{\Lambda}/M}$!

Renormalons at work — going beyond the large–β_0 limit

The cancellation mechanism is general and it can be **used**:

Calculation of the physical moments using the Principal Value prescription:

$$\begin{aligned}\Gamma_N &\equiv \int dE_\gamma \left(\frac{2E_\gamma}{M}\right)^{N-1} \frac{d\Gamma}{dE_\gamma} \\ &= H(m)\, \underbrace{\mathrm{Sud}(N,m)\big|_{\mathrm{PV}}\; \mathrm{e}^{-(N-1)\bar{\Lambda}_{\mathrm{PV}}/M}}_{u=\frac{1}{2}\ \text{prescription independent}}\, \mathcal{F}((N-1)\Lambda/M)\end{aligned}$$

where $\bar{\Lambda}_{\mathrm{PV}} \equiv M - m_{\mathrm{PV}}$;

$\mathrm{Sud}(N,m)\big|_{\mathrm{PV}}$ is **real–valued**: $\mathrm{Sud}(m,N)\big|_{\mathrm{PV}} = \left[\mathrm{Sud}(m,N^*)\big|_{\mathrm{PV}}\right]^*$.

Calculation of the E_γ spectrum:

$$\frac{d\Gamma(E_\gamma)}{dE_\gamma} = \frac{M}{2}\int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}\,\Gamma_N \left(\frac{2E_\gamma}{M}\right)^{-N}$$

This spectrum is **free** of the leading renormalon ambiguity!

Sudakov resummation beyond logarithmic accuracy

$$\mathrm{Sud}(m,N)\big|_{\mathsf{PV}} = \exp\left\{\mathrm{PV}\int_0^\infty du\,T(u)\,\left(\frac{\Lambda^2}{m^2}\right)^u \right.$$
$$\left. \times\frac{1}{u}\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) - B_{\mathcal{J}}(u)\Gamma(-u)\left(N^{u}-1\right)\right]\right\}.$$

What do we know about $B_{\mathcal{S}}(u)$?

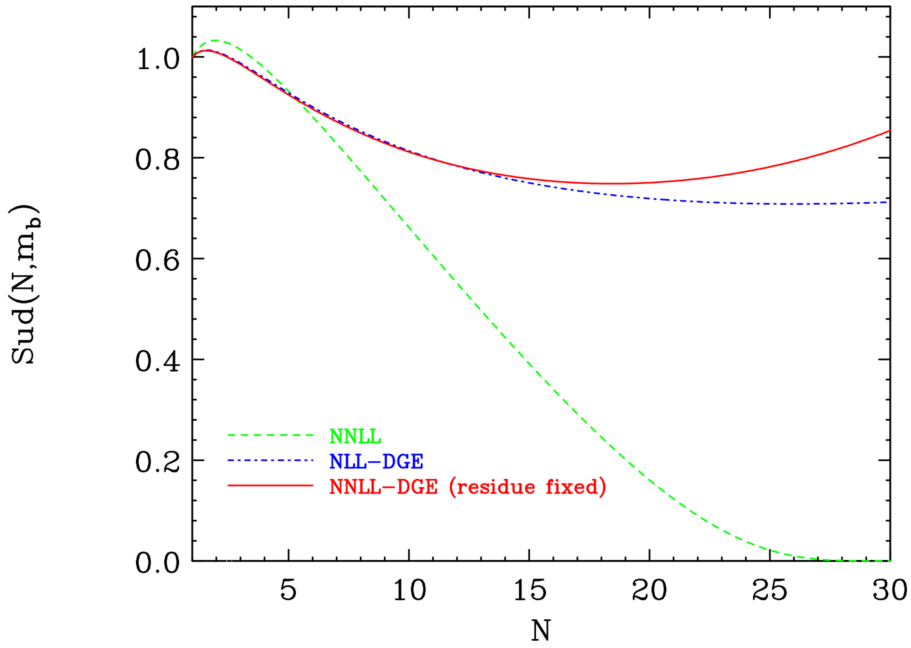
- All orders in the large–β_0 limit: $B_{\mathcal{S}}(u) = \frac{C_F}{\beta_0}\mathrm{e}^{\frac{5}{3}u}\,(1-u)\;+\mathcal{O}(1/\beta_0^2)$
- NNLO in the full theory: $B_{\mathcal{S}}(u) = 1 + s_1\frac{u}{1!} + s_2\frac{u^2}{2!} + \cdots$
- Renormalon cancellation in $\mathrm{Sud}(m,N)\,\mathrm{e}^{-(N-1)\bar{\Lambda}/M}$ implies:
  $B_{\mathcal{S}}(u=1/2)$ is equal in magnitude and opposite in sign to the residue of the $u=1/2$ renormalon in $m/m_{\overline{\mathsf{MS}}}$, which can be determined from the known NNLO expansion in $\overline{\mathrm{MS}}$ within a few percent.

Dressed Gluon Exponentiation: Results

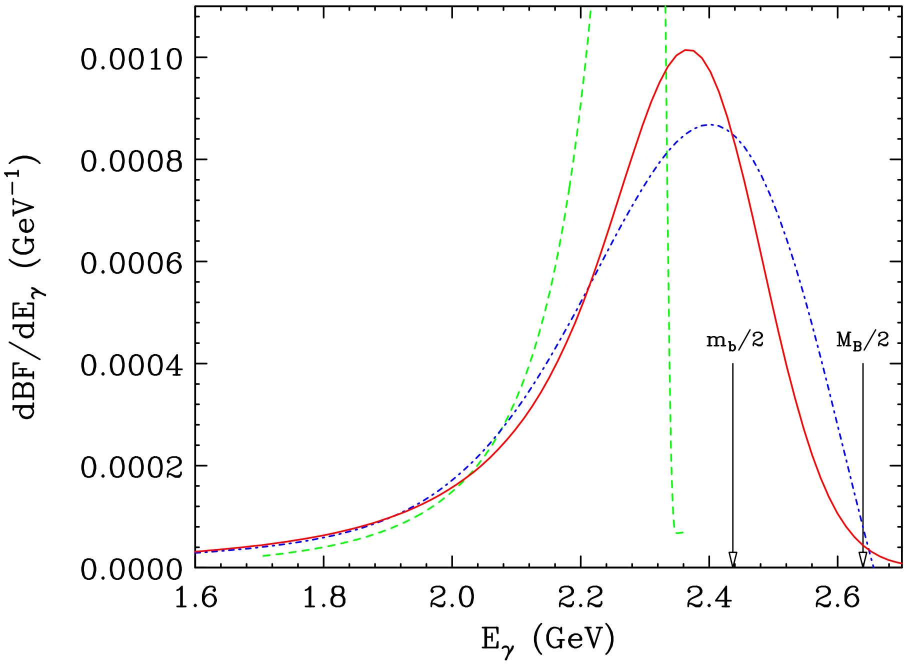
$$\mathrm{Sud}(m,N)\big|_{\mathrm{PV}} = \exp\left\{\mathrm{PV}\int_0^\infty du\, T(u)\, \left(\frac{\Lambda^2}{m^2}\right)^u \times\frac{1}{u}\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) - B_{\mathcal{J}}(u)\Gamma(-u)\left(N^{u}-1\right)\right]\right\}.$$

$$\frac{d\Gamma(E_\gamma)}{dE_\gamma} = \frac{m_{\mathrm{PV}}}{2}\int_{c-i\infty}^{c+i\infty}\frac{dN}{2\pi i} H(m)\ \mathrm{Sud}(m,N)\big|_{\mathrm{PV}}\left(\frac{2E_\gamma}{m_{\mathrm{PV}}}\right)^{-N}$$

$\mathrm{Sud}(N,m)\big|_{\mathrm{PV}}$ with various approx. for $B_{\mathcal{S}}(u)$

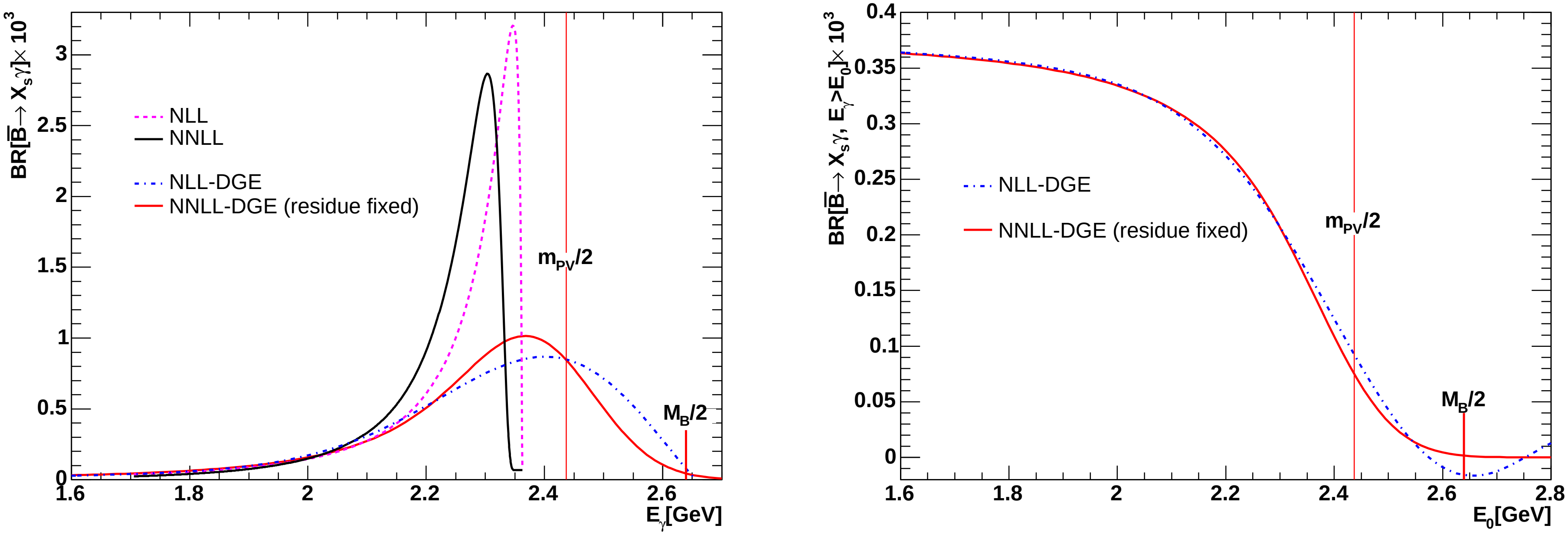


Corresponding spectra



Summary

- Resummed perturbation theory **predicts** the photon energy spectrum in $\bar{B} \longrightarrow X_s\gamma$.



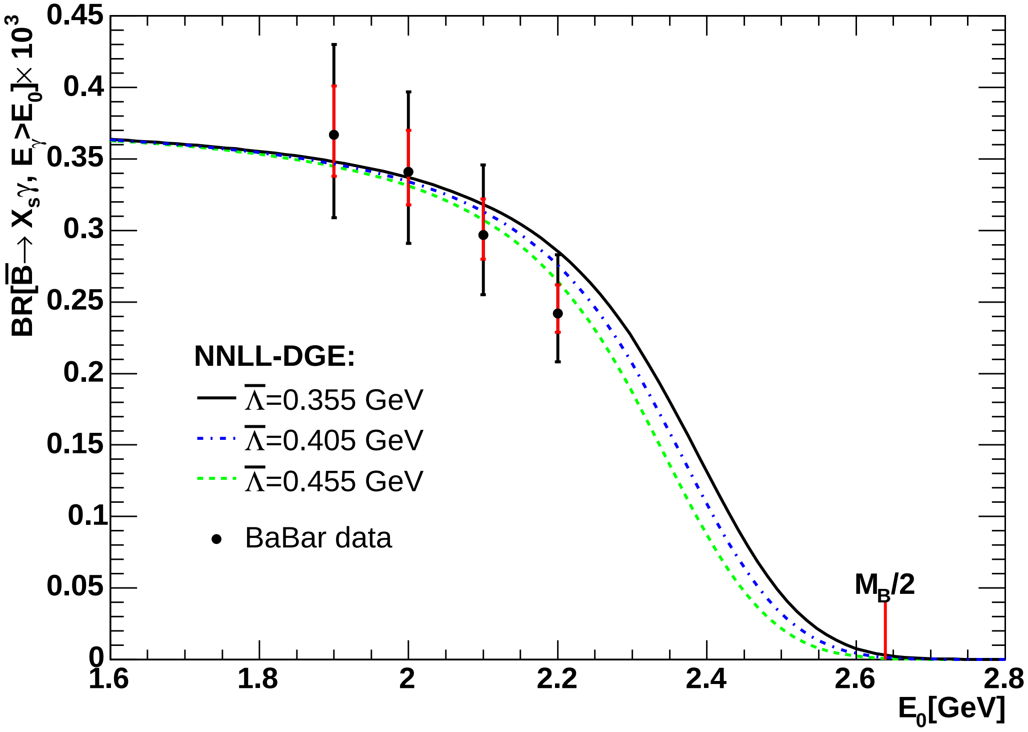
- Renormalon cancellation $(u = 1/2)$ is respected by using the **same prescription** for both the Sudakov exponent and the pole mass.

- Contrary to Sudakov resummation with fixed logarithmic accuracy, DGE is **stable**. This is owing to resumming running–coupling effects with a definite prescription.

- The DGE spectrum smoothly extends beyond the perturbative endpoint and tends to zero for $E_\gamma = (m + \mathcal{O}(\Lambda))/2$, close to the physical endpoint $E_\gamma = M/2$.

Comparison to data: branching fraction

- Theoretical uncertainty on the total BF $\sim 10\%$
- Experimental cuts on E_γ *do not* significantly increase the overall uncertainty.
- The measured BF is consistent with the Standard Model.

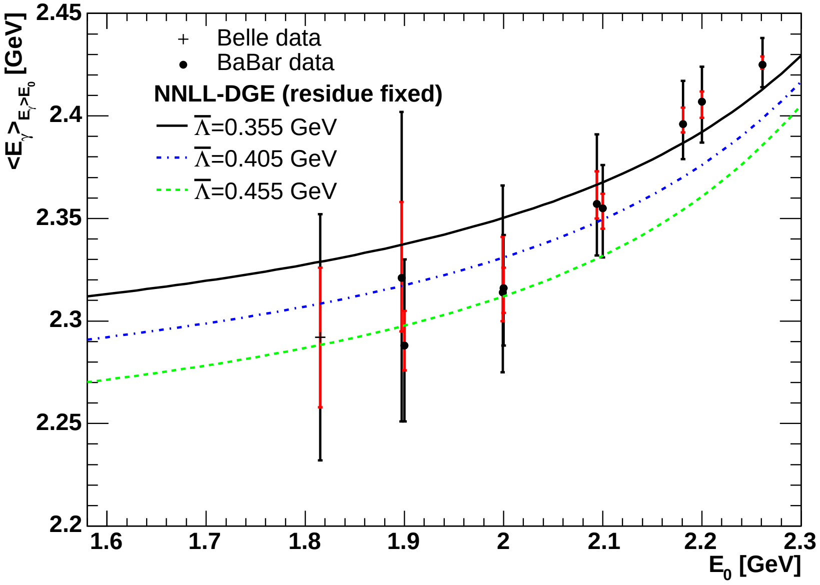


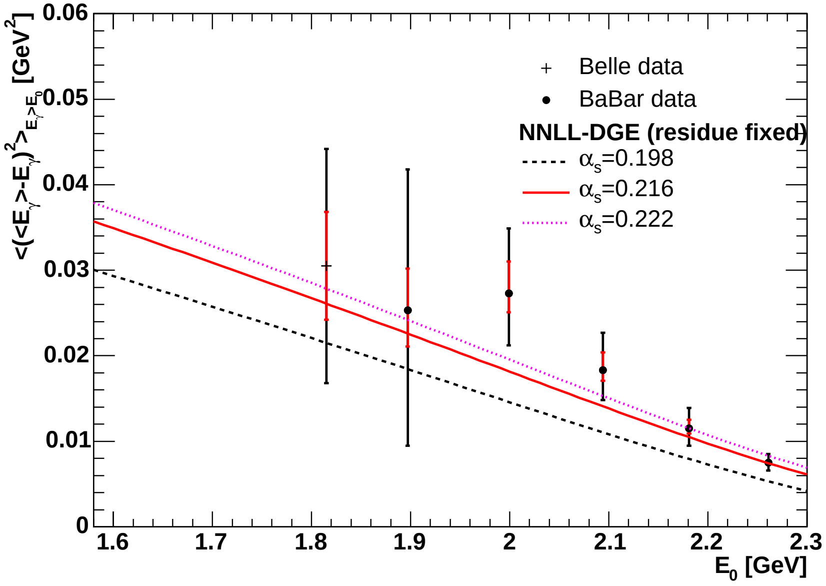
- Possible determination of m_b!

Comparison to data: cut moments

$$\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \; {}_{E_0} \, dE_\gamma \frac{d\Gamma(E_\gamma)}{dE_\gamma} E_\gamma$$

$$\left\langle \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \; {}_{E_0} \, dE_\gamma \frac{d\Gamma(E_\gamma)}{dE_\gamma} \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n .$$





- Measurement of power corrections.
- Good prospects for determination of V_{ub} from **charmless semileptonic decays**.