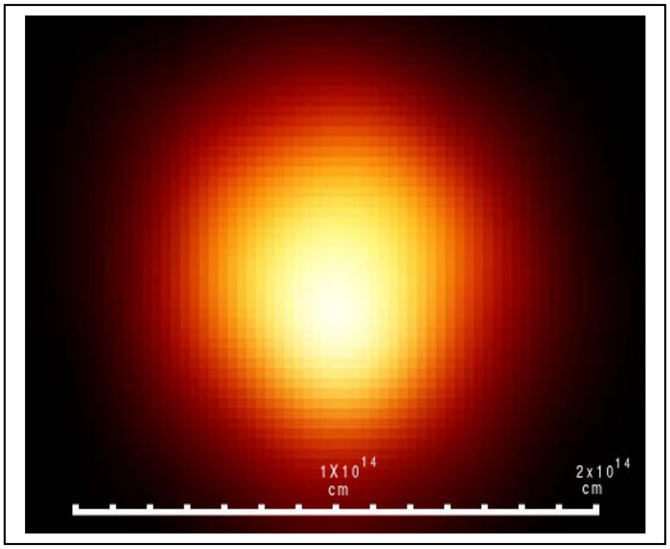
Stellar Temperature, Size and Power



The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun!

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4 \pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzman radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in degrees Kelvin. The formula then becomes,

$$L = 4 \pi R^2 \sigma T^4$$

Problem 1 - The sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

Problem 2 - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

Problem 3 - The nearby star, Sirius, has a temperature of 9,200 K and a radius of 1.76 times our sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our sun?

Advanced Math:

Problem 4 - Compute the total derivative of L(R,T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Problem 1 - We use $L = 4 (3.141) R^2 (5.67 \times 10^{-5}) T^4$ to get L (ergs/sec) = 0.00071 $R(cm)^2$ $T(degreesK)^4$ then,
A) L(ergs/sec) = 0.00071 x (696,000 km x 10^5 cm/km)^2 (5700)^4 = **3.6×10^{33} ergs/sec**
B) L(watts) = 3.6×10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6×10^{25} watts**.

Problem 2 - A) The radius of Antares is 700 x 696,000 km = 4.9×10^8 km.
L(ergs/sec) = 0.00071 x (4.9×10^8 km x 10^5 cm/km)^2 (3500)^4 = **2.5×10^{38} ergs/sec**
B) L(Antares) = (2.5×10^{38} ergs/sec) / (3.6×10^{33} ergs/sec) = **71,000 L(sun).**

Problem 3 - Sirius-A radius = 1.76 x 696,000 km = 1.2×10^6 km
L(Sirius-A) = 0.00071 x (1.2×10^6 km x 10^5 cm/km)^2 (9200)^4 = **7.3×10^{34} ergs/sec**
L = (7.3×10^{34} ergs/sec) / (3.6×10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = 0.00071 x (4900 km x 10^5 cm/km)^2 (27,400)^4 = **9.5×10^{31} ergs/sec**
L(Sirius-B) = 9.5×10^{31} ergs/sec / 3.6×10^{33} ergs/sec = **0.026 L(sun).**

Advanced Math:

Problem 4 *(*Note: In the discussion below, the symbol d represents a partial derivative)

$$dL(R,T) = \frac{d L(R,T)}{d R} dR + \frac{d L(R,T)}{d T} dT$$

$$dL = [4 \pi (2) R \sigma T^4] dR + [4 \pi (4) R2 \sigma T^3] dT$$

$$dL = 8 \pi R \sigma T^4 dR + 16 \pi R^2 \sigma T^3 dT$$

To get percentage changes, divide both sides by $L = 4 \pi R^2 \sigma T^4$

$$\frac{dL}{L} = \frac{8 \pi R \sigma T^4}{4 \pi R^2 \sigma T^4} dR + \frac{16 \pi R^2 \sigma T^3}{4 \pi R^2 \sigma T^4} dT$$

Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40**
The star's luminosity will increase by 40%.

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0.
This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 71,000 L(sun) or 2.5×10^{38} ergs/sec. If I increase its radius by 10% from 4.9×10^8 km to 5.4×10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = 0.00071 x (5.4×10^8 km x 10^5 cm/km)^2 (3325)^4 = **2.5×10^{38} ergs/sec**