PASSIVE CONTROL OF VIBRATION AND WAVE PROPAGATION IN SANDWICH PLATES WITH PERIODIC AUXETIC CORE

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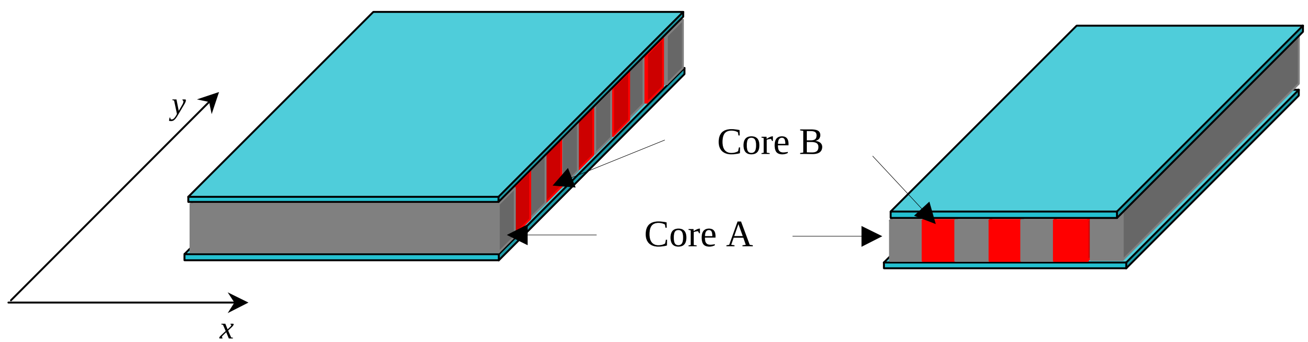
Outline

- Introduction: wave propagation in 1D and 2D periodic structures;
- Sandwich plate-rows with periodic honeycomb core:
  - Theoretical Modeling
  - Transfer Matrix
  - Propagation constants
  - Dynamic stiffness matrix
- Performance of periodic sandwich plate-rows:
  - Configuration of the unit cell
  - Propagation Patterns
  - Structural response
- Sandwich plates with periodic honeycomb core:
  - Finite Element Modeling of unit cell
  - Bloch reduction
  - Response to harmonic loading
- Performance of periodic sandwich plates:
  - Configuration of the unit cell
  - Phase constant surfaces
  - Contour plots
  - Harmonic response
- Conclusions

Motivation

Analysis of WAVE DYNAMICS in sandwich plates with core of two honeycomb materials alternating PERIODICALLY along the structure



Analysis is performed through the theory of 2-D PERIODIC STRUCTURES which are characterized by:

- Frequency bands where elastic waves do not propagate

  ⟹ STOP/PASS BANDS

- Directions where propagation of elastic waves does not occur

  ⟹ "FORBIDDEN" ZONES

Motivation

⇨ 2D periodic structures behave as DIRECTIONAL MECHANICAL FILTERS

GOAL: Evaluate characteristics of wave propagation for sandwich plates with periodic core configuration:

- Determine stop/pass band pattern;
- Determine directional characteristic and "forbidden zones" of response;
- Evaluate influence of the cell and core geometry;

Basic concepts

Sandwich plates with periodic auxetic core:

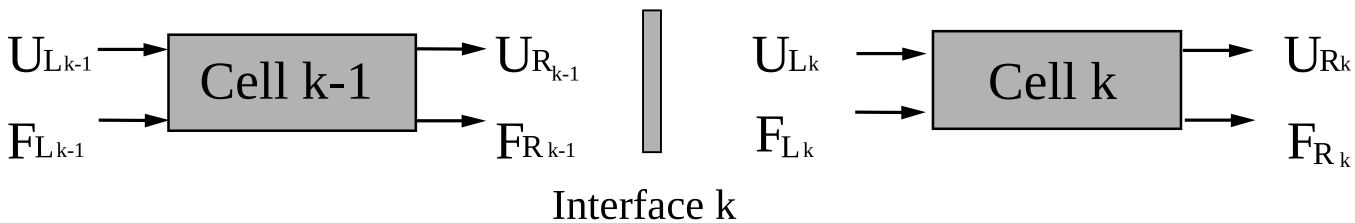
- Completely passive treatment
- Performance of traditional light-weight sandwich elements enhanced by directional filtering capabilities
- Improvement of the attenuation capabilities of periodic sandwich panels obtained through a proper selection of the core and cell configuration
- Stiffening geometric effect and change in mass density depending on core material geometry
- External dimensions and weight not significantly affected

Introduction: plate-rows

Periodic structure: assembly of identical elementary components, or cells, connected to one another in a regular pattern.

The plate-rows here considered are modeled as **quasi-one-dimensional multi-coupled** periodic systems

Transfer matrix formulation:



$$\begin{Bmatrix} U_L \\ F_L \end{Bmatrix}_k = [T_k] \cdot \begin{Bmatrix} U_L \\ F_L \end{Bmatrix}_{k-1} \quad \text{i.e.} \quad Y_k = [T_k] \cdot Y_{k-1}$$

Y_k : state vector

T_k : transfer matrix

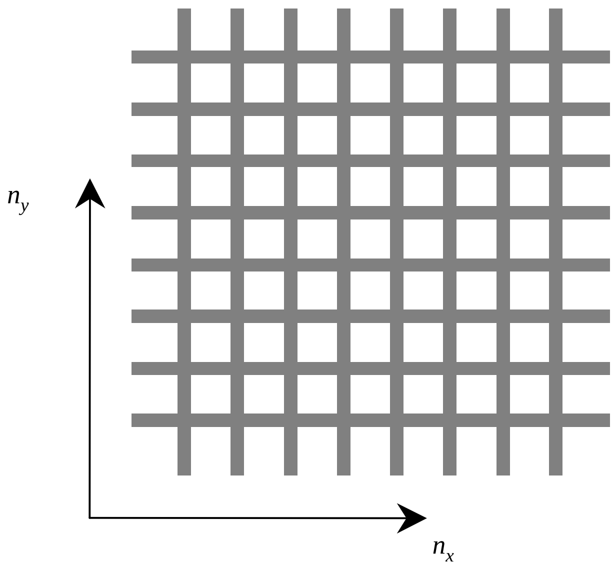
λ_i : i^{th} eigenvalue of Transfer Matrix T

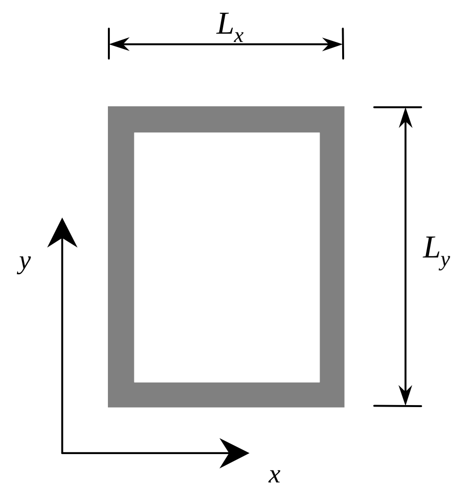
$\log(\lambda_i)$ = PROPAGATION CONSTANT

- $|\lambda_i| = 1$: pass band (wave propagation)
- $|\lambda_i| \neq 1$: stop band (wave attenuation)

Introduction: 2D plates

2D-periodic structure: cells connected to cover a plane





Unit cell

Wave motion in the 2-D structure (Bloch's Theorem):

$$\mathbf{w}(x,y,n_x,n_y) = \mathbf{g}(x,y)\cdot exp(\mu_x n_x + \mu_y n_y)$$

Motion of unit cell *Propagation Constants*

Introduction: 2D plates

Propagation Constants are complex numbers:

$$\underbrace{\mu_k = \delta_k}_{\text{Attenuation Constant}} + i\underbrace{\varepsilon_k}_{\text{Phase Constant}} \qquad (k=x,y)$$

Condition for wave propagation:

$$\delta_k = 0 \quad \& \quad \varepsilon_k = -\pi \div \pi$$

Imposing the propagation constants allows obtaining the corresponding frequency of wave propagation:

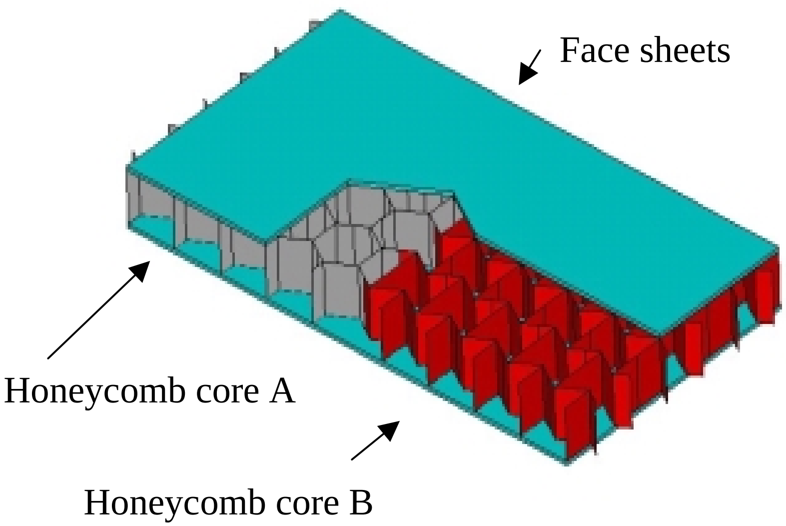
$$\omega = f(\varepsilon_x, \varepsilon_y)$$

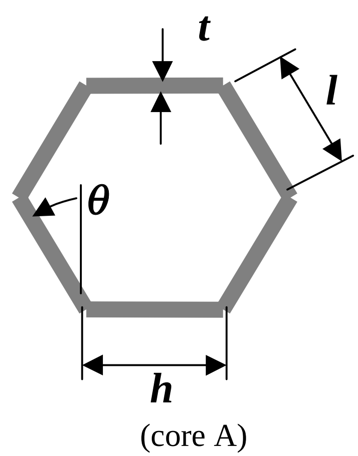
⇨ **Phase Constant Surfaces**

Honeycomb core

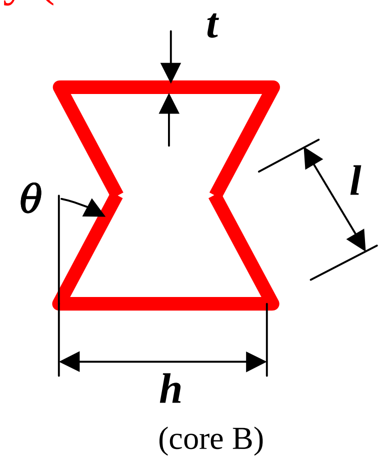
Geometric layout of regular (A) and auxetic (B) honeycomb structures

$\theta < 0$: Re-entrant geometry (AUXETIC SOLID)

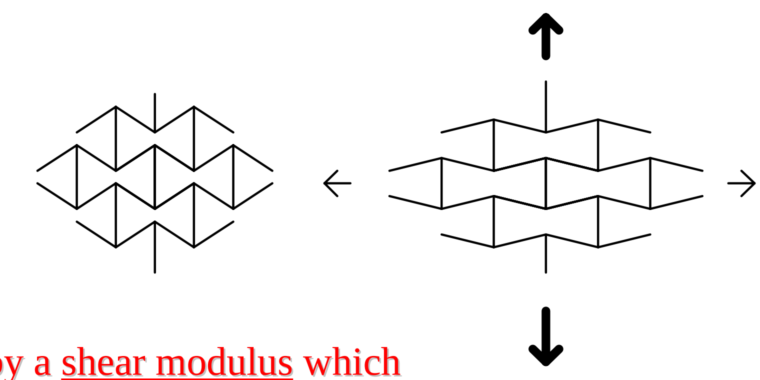




$\alpha = h/l$
$\beta = t/l$



Negative Poisson's ratio behavior:



Auxetic honeycombs with θ=-60°, α=2 are characterized by a <u>shear modulus</u> which outcast up to five times the shear modulus of a regular honeycomb of the same material

Theoretical Modeling of the sandwich plate

Strain energy: $U = U_1 + U_2 + U_3$

- Face sheets (*extension + bending*):

$$U_i = \frac{1}{2}\frac{E_i h_i}{\left(1-\nu_i^2\right)}\iint\left\{u_{ix}^2 + \nu_i u_{ix}v_{iy} + v_{iy}^2 + \nu_i v_{iy}u_{ix} + \frac{1-\nu_i}{2}\left(u_{iy}^2 + v_{ix}^2 + 2u_{iy}v_{ix}\right)\right\}dxdy +$$

$$+\frac{1}{2}\frac{E_i h_i^3}{12\left(1-\nu_i^2\right)}\iint\left\{w_{xx}^2 + 2\nu_i w_{xx}w_{yy} + w_{yy}^2 + 2\left(1-\nu_i\right)w_{xy}^2\right\}dxdy, \qquad (i=1,3)$$

- Core (*shear deformation):*

$$U_2 = \frac{1}{2}Gh_2\iint\left\{\begin{aligned}&\left(\frac{u_1-u_3}{h_2}\right)^2 + \left(\frac{v_1-v_3}{h_2}\right)^2 + \left(w_x^2+w_y^2\right)\left(\frac{d}{h_2}\right)^2 + \\ &-\frac{2d}{h_2}\left(w_x\frac{u_1-u_3}{h_2} + w_y\frac{v_1-v_3}{h_2}\right)\end{aligned}\right\}dxdy$$

Theoretical Modeling of the sandwich plate

Kinetic energy: $T = T_1 + T_2 + T_3$

- Face sheets (*translation* + *rotation*):

$$T_i = \frac{\rho_i h_i}{2}\iint \dot{w}^2 dxdy + \frac{1}{2}\iint\left(h_i \rho_i \dot{u}_i^2 + \dot{w}_x^2 \frac{\rho_i h_i^3}{12} + h_i \rho_i \dot{v}_i^2 + \dot{w}_y^2 \frac{\rho_i h_i^3}{12}\right) dxdy, \quad i = (1,3)$$

- Core (*translation + rotation):*

$$T_2 = \frac{\rho_2 h_2}{2}\iint \dot{w}^2 dxdy + \frac{1}{2}\iint \rho_2 h_2 \left\{\left(\frac{\dot{u}_1 + \dot{u}_3}{2} + \dot{w}_x \frac{h_3 - h_1}{4}\right)^2 + \left(\frac{\dot{v}_1 + \dot{v}_3}{2} + \dot{w}_y \frac{h_3 - h_1}{4}\right)^2\right\} dxdy$$

$$+ \frac{1}{2}\iint \frac{\rho_2 h_2}{2}\left\{\left(\dot{u}_1 - \dot{u}_3 - \frac{h_1 + h_3}{2}\dot{w}_x\right)^2 + \left(\dot{v}_1 - \dot{v}_3 - \frac{h_1 + h_3}{2}\dot{w}_y\right)^2\right\} dxdy$$

Theoretical Modeling of the sandwich plate

Equations of Motion:

1) $$\frac{E_1 h_1}{\left(1-\nu_1^2\right)}\left\{u_{1xx}+\frac{1}{2}\left(1+\nu_1\right)v_{1xy}+\frac{1}{2}\left(1-\nu_1\right)u_{1yy}\right\}+Gh_2\left\{\frac{d}{h_2^2}w_x-\frac{u_1-u_3}{h_2^2}\right\}-\rho_1 h_1 \ddot{u}_1-\rho_2 h_2\left(\frac{\ddot{u}_1}{3}+\frac{\ddot{u}_3}{6}+\ddot{w}_x\frac{h_3-2h_1}{12}\right)=0$$

2) $$\frac{E_1 h_1}{\left(1-\nu_1^2\right)}\left\{v_{1yy}+\frac{1}{2}\left(1+\nu_1\right)u_{1xy}+\frac{1}{2}\left(1-\nu_1\right)v_{1xx}\right\}+Gh_2\left\{\frac{d}{h_2^2}w_y-\frac{v_1-v_3}{h_2^2}\right\}-\rho_1 h_1 \ddot{v}_1-\rho_2 h_2\left(\frac{\ddot{v}_1}{3}+\frac{\ddot{v}_3}{6}+\ddot{w}_y\frac{2h_3-h_1}{12}\right)=0$$

3) $$\frac{E_3 h_3}{\left(1-\nu_3^2\right)}\left\{u_{3xx}+\frac{1}{2}\left(1+\nu_3\right)v_{3xy}+\frac{1}{2}\left(1-\nu_3\right)u_{3yy}\right\}-Gh_2\left\{\frac{d}{h_2^2}w_x-\frac{u_1-u_3}{h_2^2}\right\}-\rho_3 h_3 \ddot{u}_3-\rho_2 h_2\left(\frac{\ddot{u}_1}{3}+\frac{\ddot{u}_3}{6}+\ddot{w}_x\frac{2h_3-h_1}{12}\right)=0$$

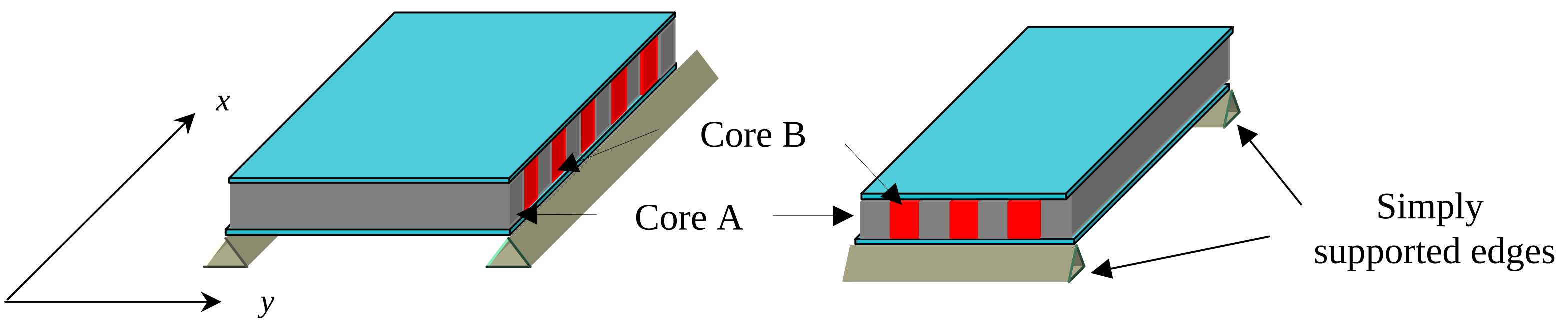
4) $$\frac{E_3 h_3}{\left(1-\nu_3^2\right)}\left\{v_{3yy}+\frac{1}{2}\left(1+\nu_3\right)u_{3xy}+\frac{1}{2}\left(1-\nu_3\right)v_{3xx}\right\}-Gh_2\left\{\frac{d}{h_2^2}w_y-\frac{v_1-v_3}{h_2^2}\right\}-\rho_3 h_3 \ddot{v}_3-\rho_2 h_2\left(\frac{\ddot{v}_1}{6}+\frac{\ddot{v}_3}{3}+\ddot{w}_y\frac{2h_3-h_1}{12}\right)=0$$

5) $$\left(D_1+D_3\right)\cdot\nabla^4 w-Gh_2\frac{d}{h_2^2}\left[d\left(w_{xx}+w_{yy}\right)-u_{1x}+u_{3x}-v_{1y}+v_{3y}\right]-\frac{1}{12}\left(\rho_1 h_1^3+\rho_3 h_3^3\right)\cdot\left(\ddot{w}_{xx}+\ddot{w}_{yy}\right)-\rho_{ts}\ddot{w}$$

$$-\rho_2 h_2\left\{\left[\frac{h_3-2h_1}{12}\cdot\left(\ddot{u}_{1x}+\ddot{v}_{1y}\right)+\frac{2h_3-h_1}{12}\cdot\left(\ddot{u}_{3x}+\ddot{v}_{3y}\right)\right]\right\}-\rho_2 h_2\left\{\left[\left(\frac{h_3-h_1}{4}\right)^2+\frac{\left(\left(h_1+h_3\right)/2\right)^2}{12}\right]\left(\ddot{w}_{xx}+\ddot{w}_{yy}\right)\right\}=0$$

Wave propagation in sandwich plate-rows

Plate-Rows



Outline of concepts described and methods applied

- SFEM is formulated from Transfer Matrix approach
- Transfer Matrix obtained from distributed parameter model of sandwich plate
- Transfer Matrix is recast to obtain the Dynamic Stiffness Matrix of plate element

TRANSFER MATRIX ⇨ **PASS / STOP BANDS**

ASSEMBLED DYNAMIC STIFFNESS MATRIX ⇨ **RESPONSE OF STRUCTURE**

Transfer Matrix formulation

- Only the dynamics along the x-axis has to be investigated
- The behavior along the y-axis is described by the harmonic $exp(jk_y y)$;
- $k_y = m\pi/L_y$ (with m integer) is the wave number along the y-axis
- The analysis is performed independently for each harmonic m for the deformations along the y-axis:

state space formulation:

$$\mathbf{Z_x}(x) = \mathbf{A} \cdot \mathbf{Z}(x) \Longrightarrow \mathbf{Z}(x) = e^{\mathbf{A} \cdot x} \cdot \mathbf{Z}(0)$$

Equations of motion

$$\mathbf{Z} = \{u_1 \quad u_3 \quad v_1 \quad v_3 \quad w \quad w_x \quad u_{1x} \quad u_{3x} \quad v_{1x} \quad v_{3x} \quad w_{xxx} \quad w_{xx}\}^T$$

$$\mathbf{Y} = [\underbrace{u_1 \quad u_3 \quad v_1 \quad v_3 \quad w \quad w_x}_{\text{Generalized Displacements}} \quad \underbrace{N_{x1} \quad N_{x3} \quad N_{xy1} \quad N_{xy3} \quad F \quad M}_{\text{Generalized Forces}}]^T$$

$$\mathbf{Y} = \mathbf{L} \cdot \mathbf{Z}$$

according to the **CLT** (Classical Laminated plate Theory).

$$\mathbf{Y}(L_k) = \mathbf{L} \cdot e^{\mathbf{A}_k \cdot L_k} \cdot \mathbf{L}^{-1} \cdot \mathbf{Y}(0) = \mathbf{T}_k \cdot \mathbf{Y}(0) \Longrightarrow \mathbf{T}_k = \mathbf{L} \cdot e^{\mathbf{A}_k \cdot L_k} \cdot \mathbf{L}^{-1}$$

Transfer Matrix

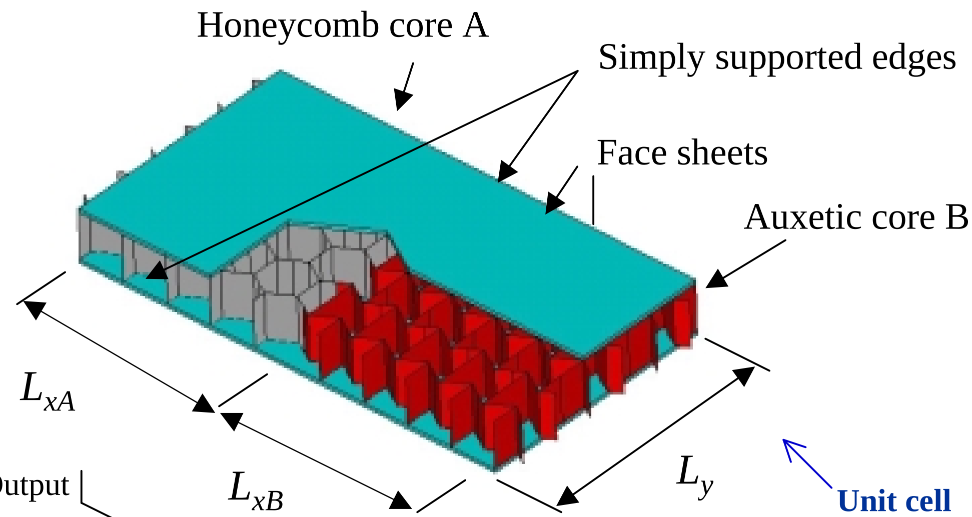
(k^{th} part of the cell)

Performance of periodic sandwich plate-rows

Cell configuration and Geometry:

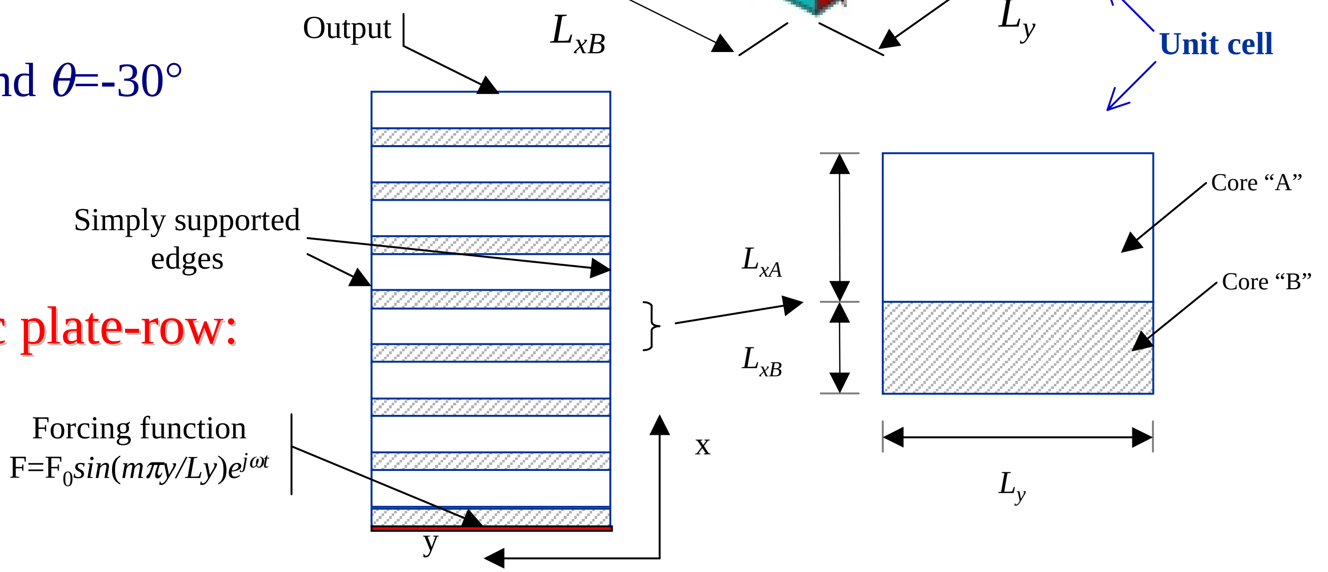
	material	thickness
Face sheet (1)	Al	5 mm
core (2)	Nomex	9 cm
Face sheet (3)	Al	5 mm

- Aspect ratio $L_x/L_y=1/2$
- Length ratio $L_{xA}/L_{xB}=1, 2, 1/2$
- Internal core geometry:
  Type A: $\alpha=1$, $\theta=30°$
  Type B: $\alpha=2$, $\theta=-60°$ and $\theta=-30°$
- $L_y=1$ m



Configuration of periodic plate-row:

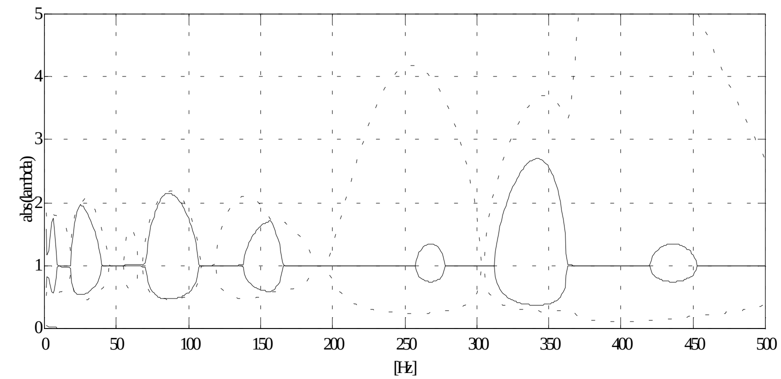
- 20 cells along S-S edges

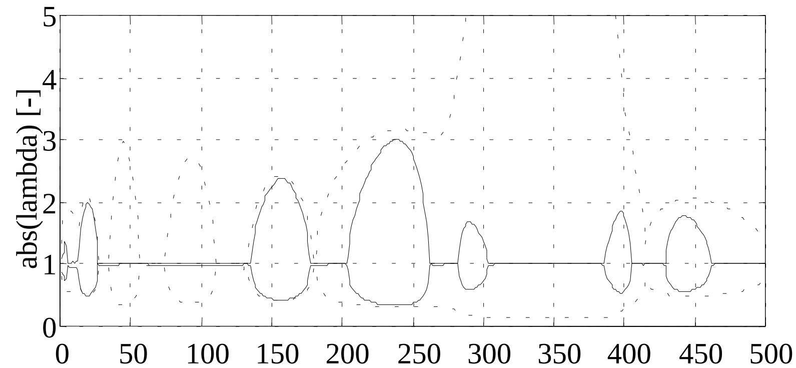


Performance of periodic sandwich plate-rows

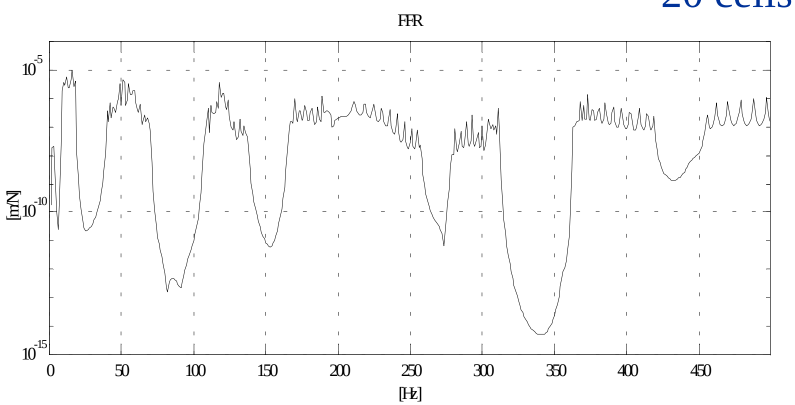
$k_y = \pi / L_y$

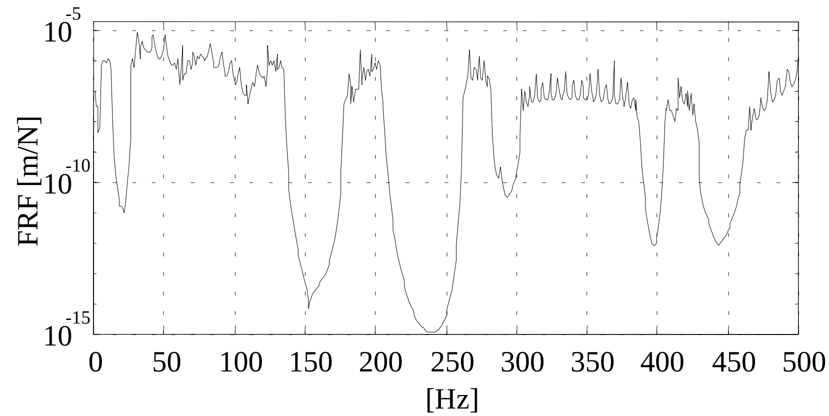
Single cell eigenvalues





20 cells plate-row FRF





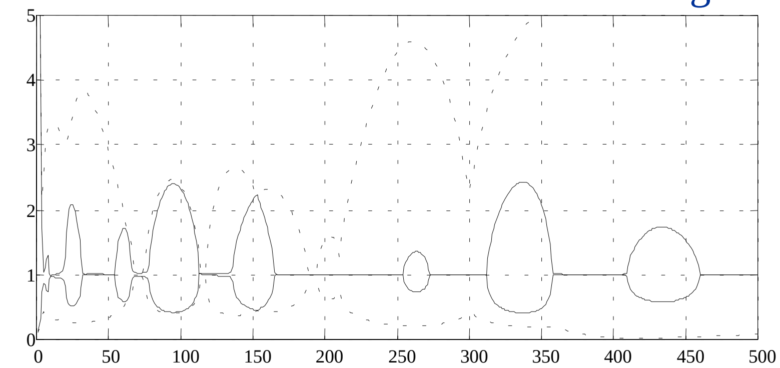
$L_{xA}/L_{xB}=1;\ \theta_B=-60^o$

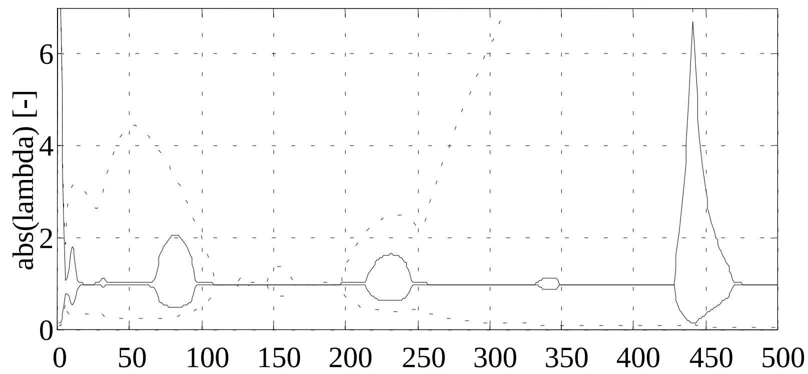
$L_{xA}/L_{xB}=1/2;\ \theta_B=-60^o$

Performance of periodic sandwich plate-rows

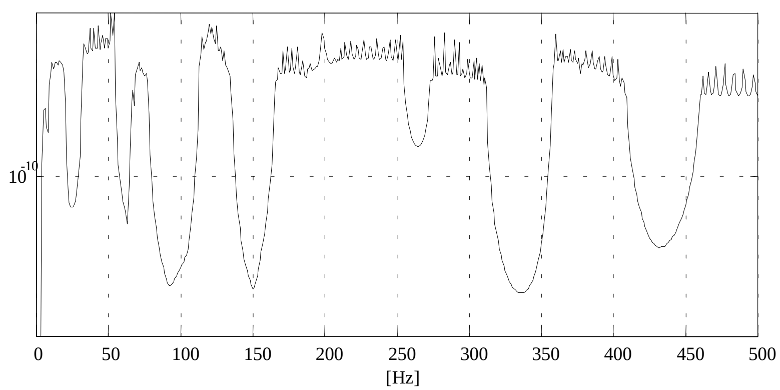
$k_y = 2\pi / L_y$

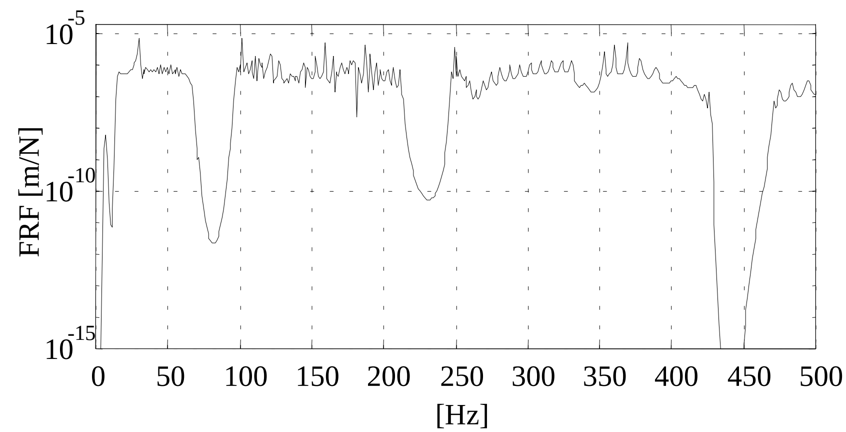
Single cell eigenvalues





20 cells plate-row FRF





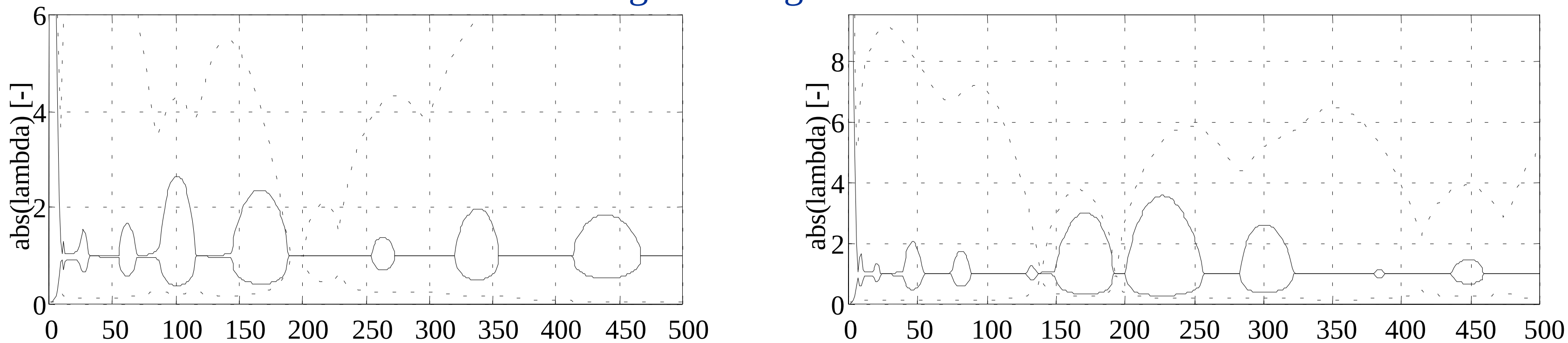
$L_{xA}/L_{xB}=1;\ \theta_B=-60^{\circ}$

$L_{xA}/L_{xB}=2;\ \theta_B=-60^{\circ}$

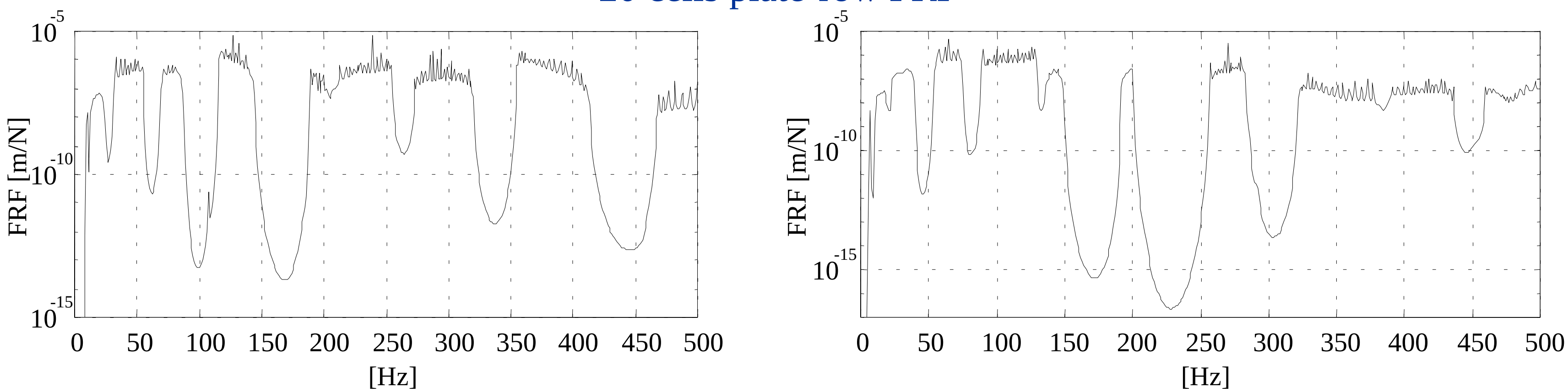
Performance of periodic sandwich plate-rows

$k_y = 3\pi / L_y$

Single cell eigenvalues



20 cells plate-row FRF



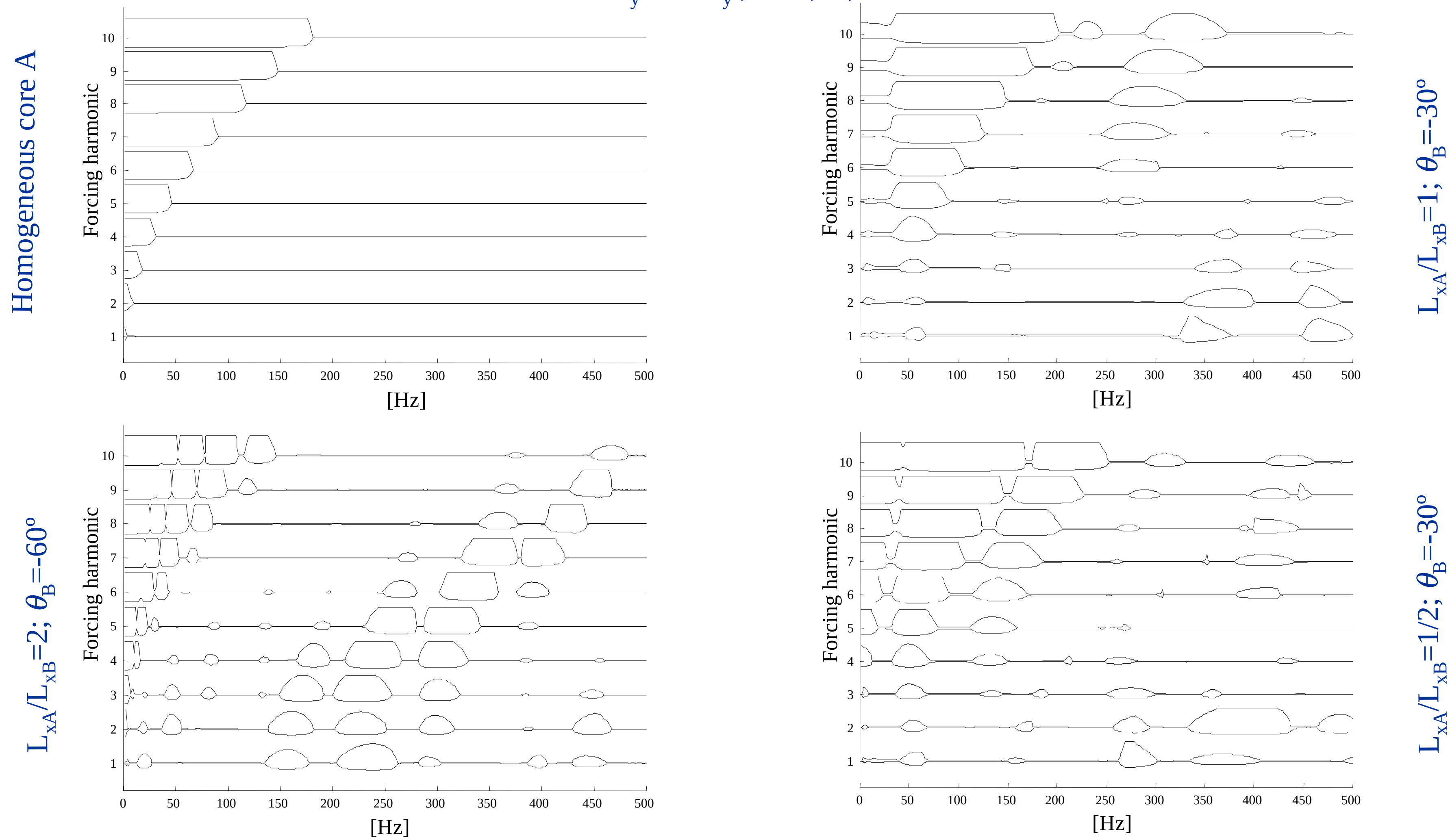
$L_{xA}/L_{xB}=1;\ \theta_B=-60^o$

$L_{xA}/L_{xB}=1/2;\ \theta_B=-60^o$

Performance of periodic sandwich plate-rows

Propagation patterns

$k_y = m\pi/L_y$; $m=1,\ldots,10$

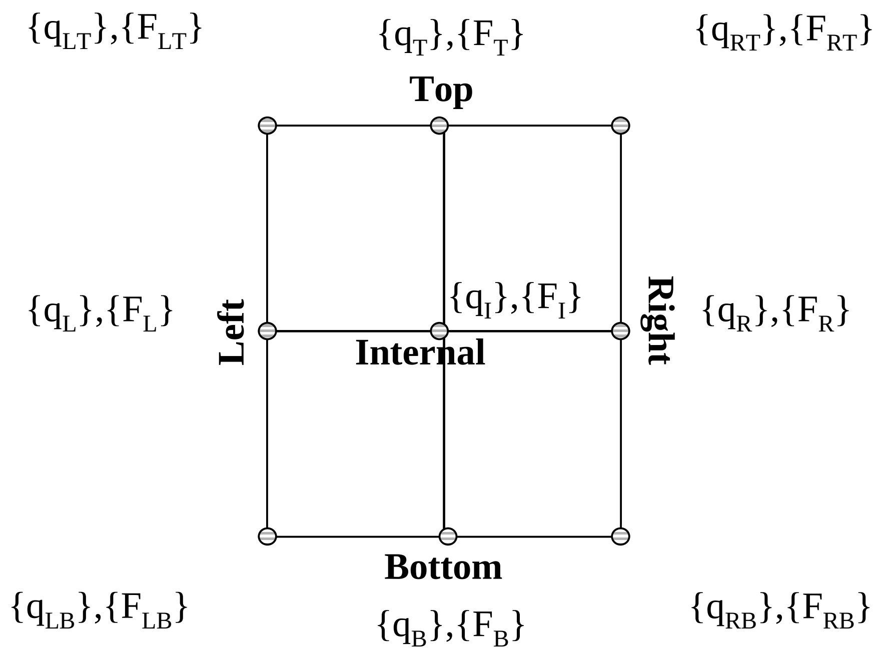


2D analysis: Bloch reduction

Cell analysis:

$\{q_{ij}\}$ generalized displacements at interface i,j

$\{F_{ij}\}$ generalized forces at interface i,j



Cell`s equation of motion:

$$\left([K]-\omega^2[M]\right)\{q\}=\{F\}$$

where:

$$\{q\}=\{q_{LB}\ q_{LT}\ q_{RT}\ q_{RB}\ q_L\ q_R\ q_B\ q_T\ q_I\ \}^T$$

$$\{F\}=\{F_{LB}\ F_{LT}\ F_{RT}\ F_{RB}\ F_L\ F_R\ F_B\ F_T\ F_I\ \}^T$$

$[K]$, $[M]$: cell`s stiffness and mass matrices

2D analysis: Bloch reduction

Bloch`s Theorem:

- Relation between interface displacements (compatibility conditions)

$$\{q_T\} = e^{\mu_y}\{q_B\} \qquad \{q_R\} = e^{\mu_x}\{q_L\}$$

$$\{q_{LT}\} = e^{\mu_y}\{q_{LB}\} \qquad \{q_{RB}\} = e^{\mu_x}\{q_{LB}\}$$

$$\{q_{RT}\} = e^{\mu_x+\mu_y}\{q_{LB}\}$$

- Relation between interface forces (equilibrium conditions)

$$\{F_T\} = -e^{\mu_y}\{F_B\} \qquad \{F_R\} = -e^{\mu_x}\{F_L\}$$

$$\{F_{LT}\} = -e^{\mu_y}\{F_{LB}\} \qquad \{F_{RB}\} = -e^{\mu_x}\{F_{LB}\}$$

$$\{F_{RT}\} = e^{\mu_x+\mu_y}\{F_{LB}\}$$

- Reduced Mass and Stiffness Matrices:

$$[M_{red}]=[A]\cdot[M]\cdot[A] \qquad [K_{red}]=[A]\cdot[K]\cdot[A]$$

with: $\{q\}=[A]\cdot\{q_{red}\}$ $\quad$ $\{q_{red}\}=\{q_{LB}\ q_L\ q_B\ q_I\}^T$

- Cell`s Equation of Motion is reduced at:

$$\left([K_{red}(\mu_x,\mu_y)]-\omega^2[M_{red}(\mu_x,\mu_y)]\right)\{q_{red}\}=\{0\}$$

$$\Downarrow$$

Frequency ω of wave motion for the assigned set of propagation constants μ_x, μ_y

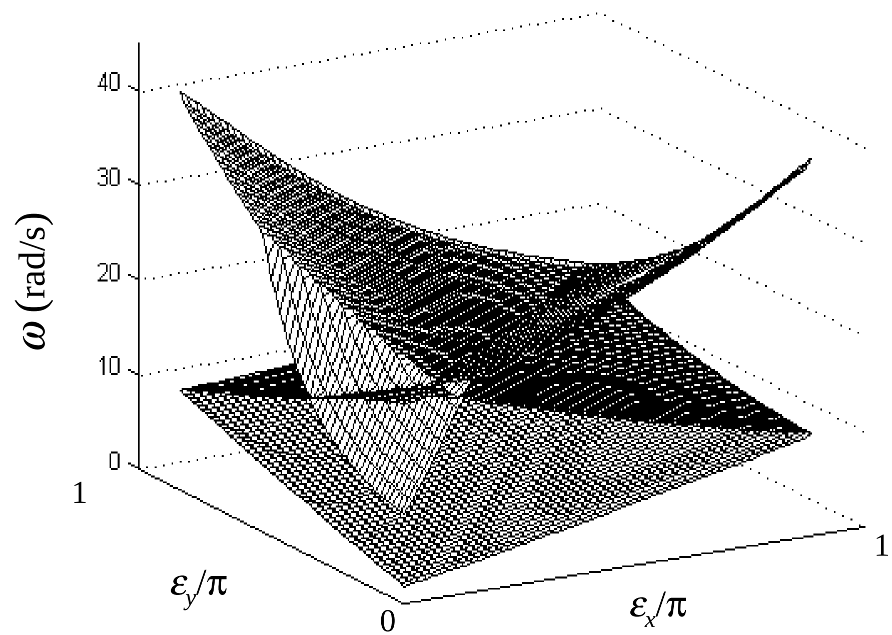
2D Wave propagation

Solution of Dispersion Relation:

Phase Constant Surfaces: $\omega=\omega(\varepsilon_x, \varepsilon_y)$

- Phase Constant Surfaces are symmetric with respect to both ε_x, ε_y

- Analysis can be limited to the first quadrant of the ε_x, ε_y plane, within the $[0,\pi]$ range for ε_x, ε_y .



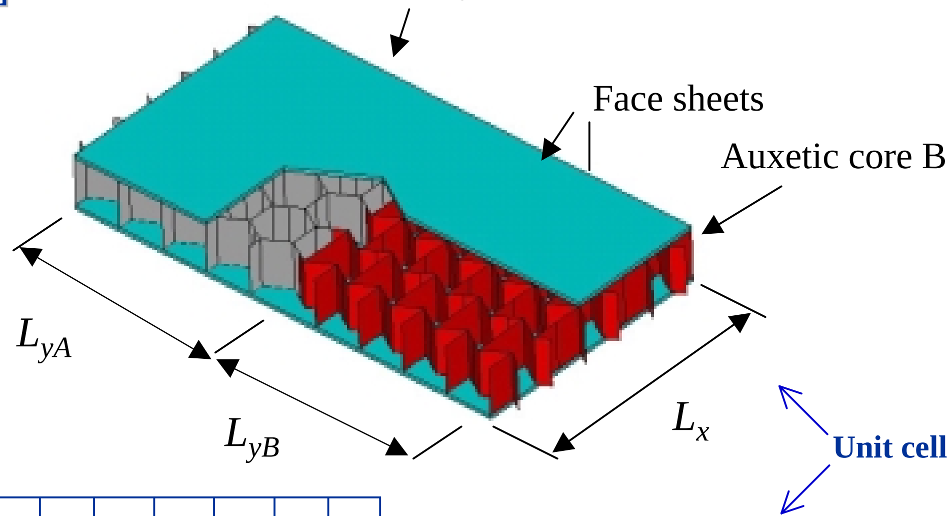
First three phase constant surfaces for a sandwich plate with uniform core (A) represented over the first propagation zone ($[0,\pi]$ range for ε_x, ε_y)

Performance of periodic sandwich plates

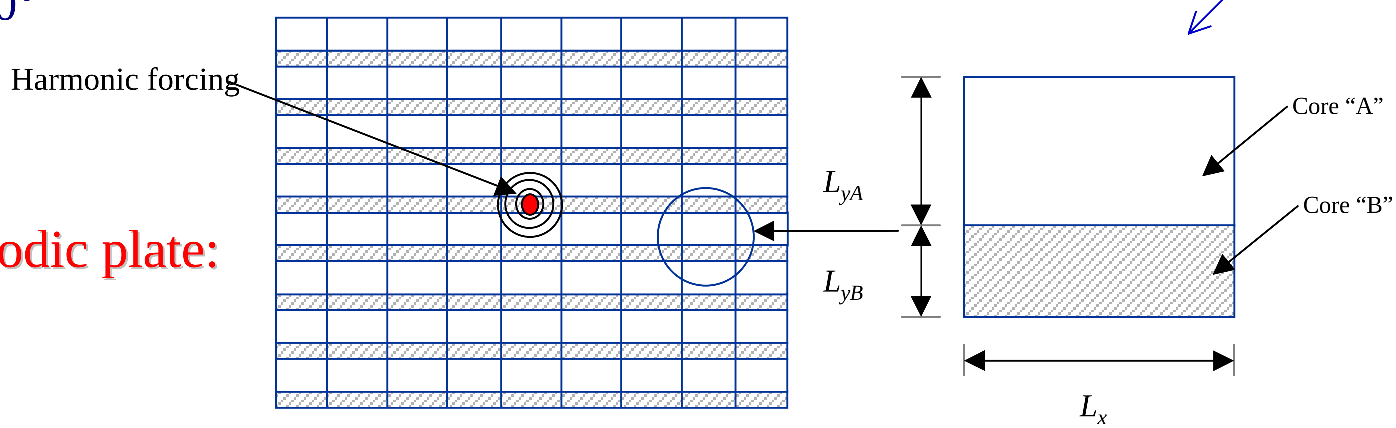
Cell configuration and Geometry:

	material	thickness [mm]
Face sheet (1)	Al	1
core (2)	Nomex	2
Face sheet (3)	Al	1

- Aspect ratio $L_x / L_{y\_} = 1$
- Length ratio $L_{yA} / L_{yB\_} = 1$ and $2/3$
- Internal core geometry:
  Type A: α=1, θ=30°
  Type B: α =2, θ=-60°

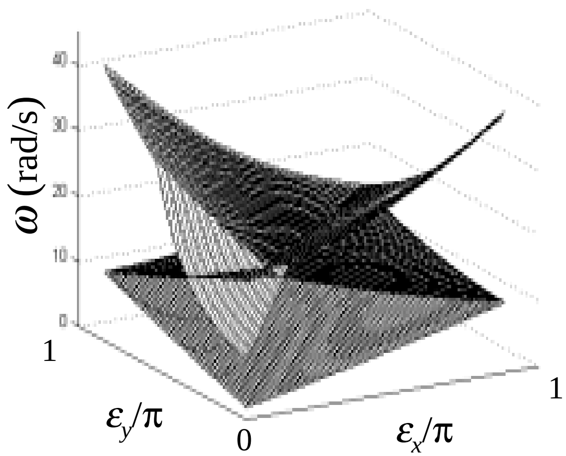


Configuration of periodic plate:

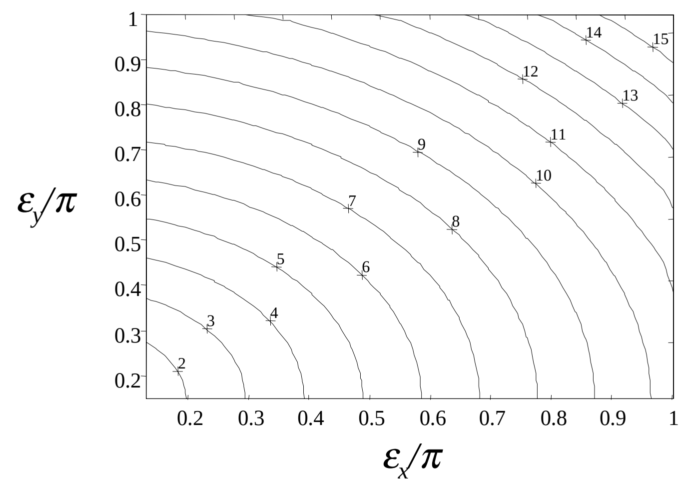


Phase Constant Surfaces

- First phase constant surface
- Contour plot



Homogeneous core, $L_x / L_{y\_} = 1$



The energy flow vector **P** at a given frequency ω lies along the normal to the corresponding iso-frequency contour line in the k_x, k_y space, where $k_i=\varepsilon_i/L_i$, $(i=x,y)$.

$$\mathbf{P} = E\left(\frac{\partial\omega}{\partial\varepsilon_x}L_x, \frac{\partial\omega}{\partial\varepsilon_y}L_y\right)$$

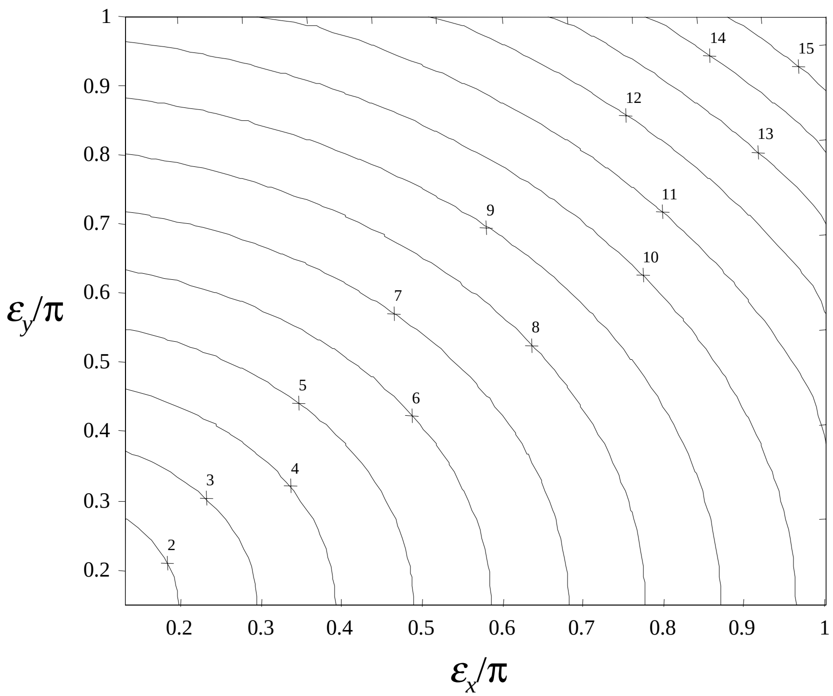
The perpendicular to a given iso-frequency line for an assigned pair ε_x , ε_y corresponds to the direction of wave propagation(*)

(*) **Langley R.S.**, "The response of two dimensional periodic structures to point harmonic forcing" *JSV* (1996) **197**(4), 447-469.

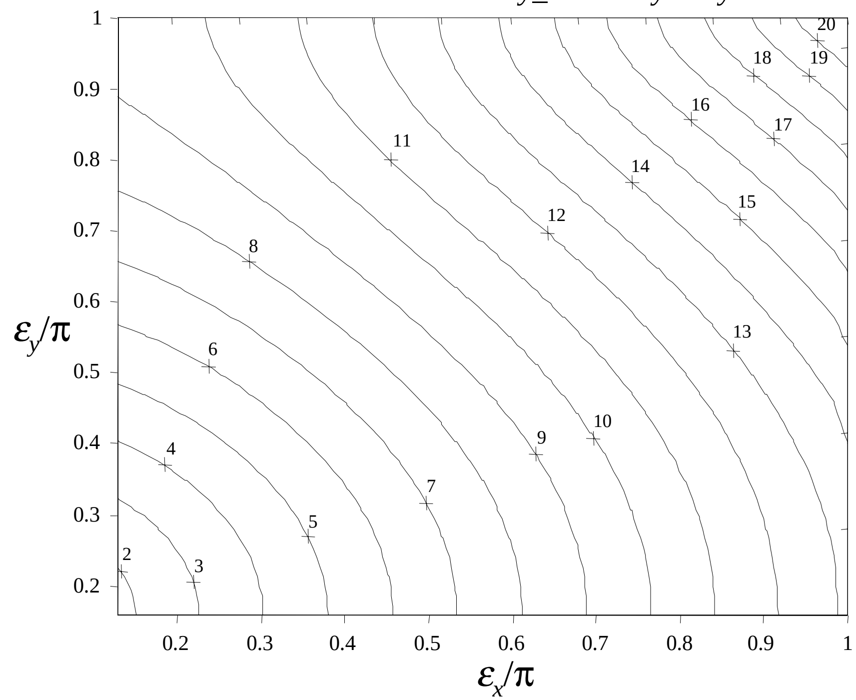
Contour plots

Influence of the periodic core:

Homogeneous core, $L_x / L_{y\_} = 1$



Periodic core, $L_x / L_{y\_} = 1$, $L_{yA}/L_{yB} = 1$



Homogeneous core:

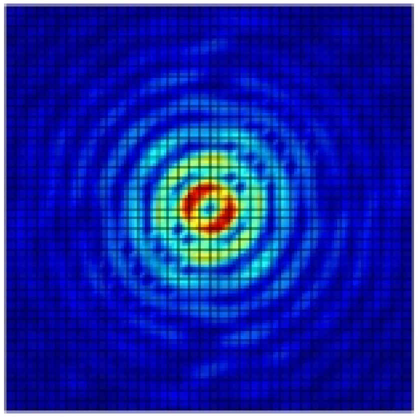
- No directional behavior expected

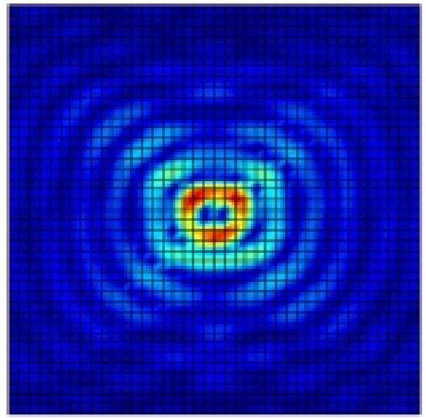
Periodic core (length ratio =1):

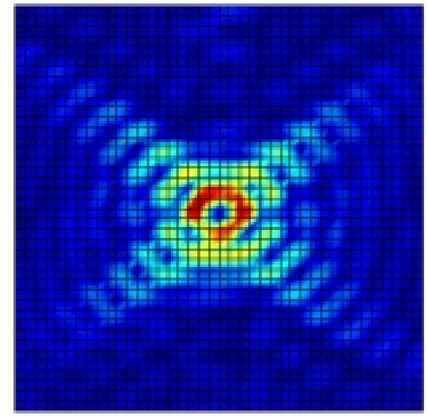
- Directional behavior expected above a “transition frequency”

Plate harmonic response

Plate deformed configuration for excitation at ω=7.5 rad/s



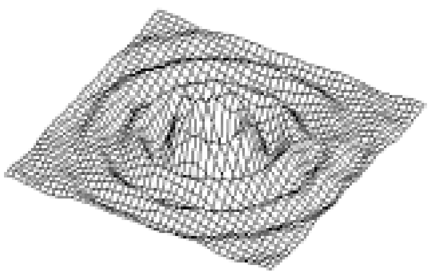


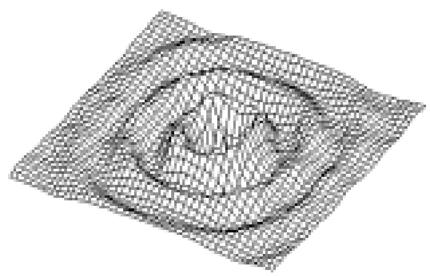


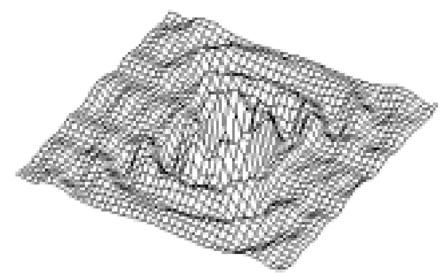
Homogeneous

$L_{yA}/L_{yB}=1$

$L_{yA}/L_{yB}=2/3$



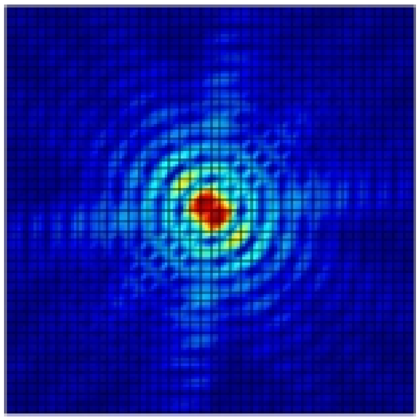




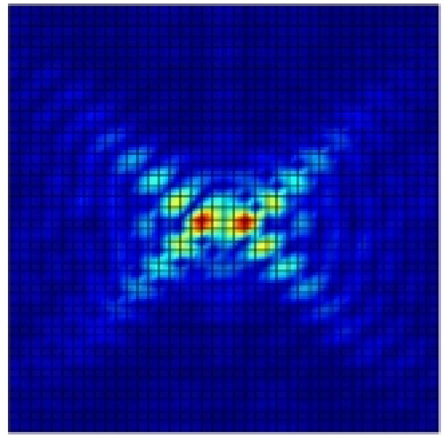
Number of cells: N_x= 20, N_y= 20; 40x40 finite element grid

Plate harmonic response

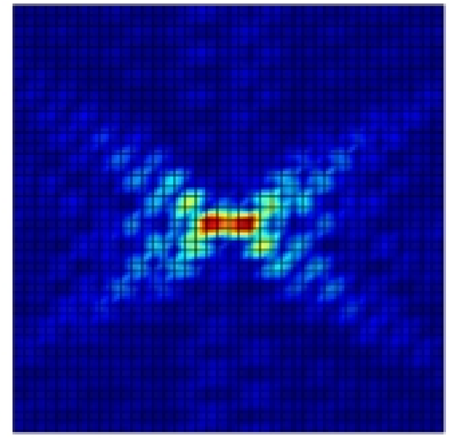
Plate deformed configuration for excitation at ω=9.5 rad/s



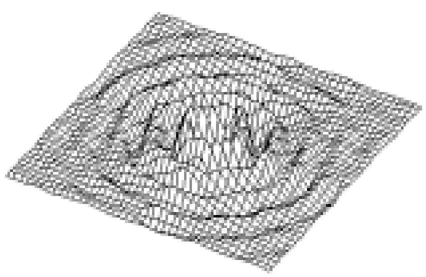
Homogeneous

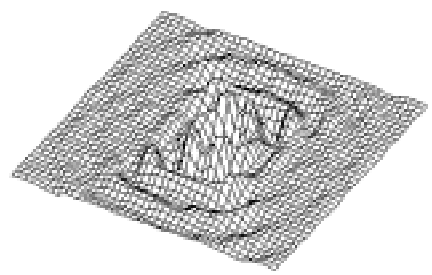


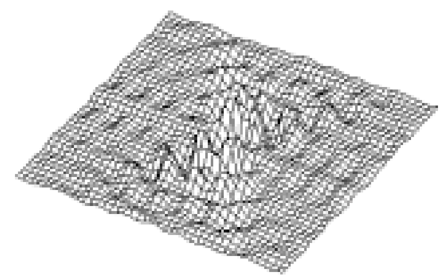
L_{yA}/L_{yB}=1



L_{yA}/L_{yB}=2/3



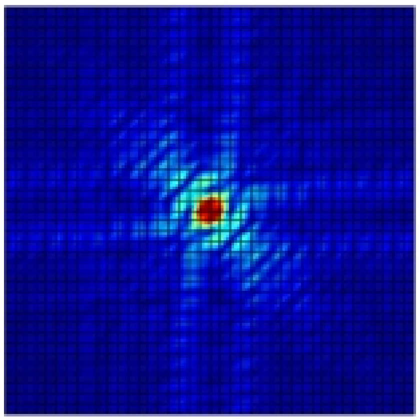
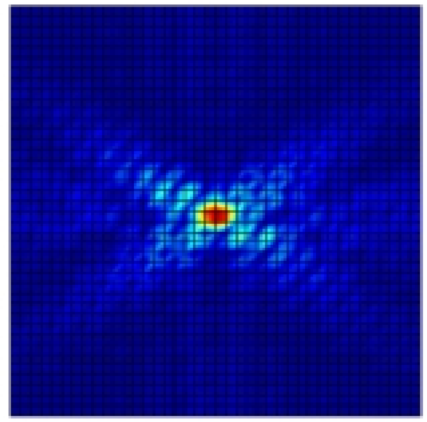
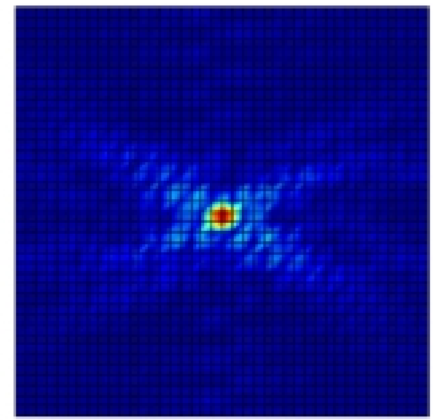




Number of cells: N_x= 20, N_y= 20; 40x40 finite element grid

Plate harmonic response

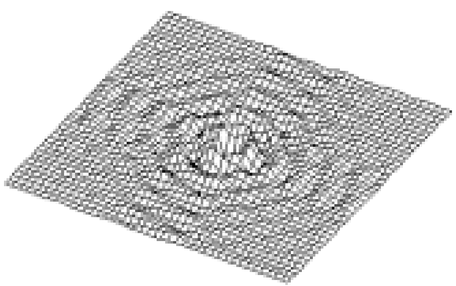
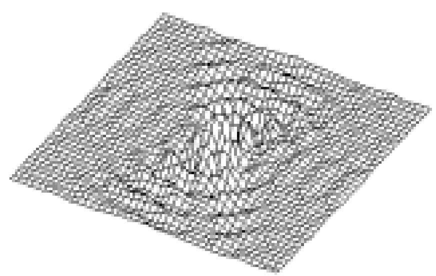
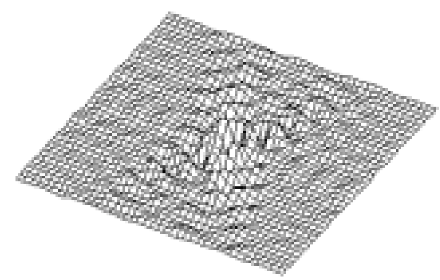
Plate deformed configuration for excitation at ω=13 rad/s

Homogeneous

$L_{yA}/L_{yB}=1$

$L_{yA}/L_{yB}=2/3$

Number of cells: N_x= 20, N_y= 20; 40x40 finite element grid

Conclusions

- Wave propagation in periodic sandwich plates and plate-rows is analyzed;
- Auxetic and regular honeycomb cellular solids are utilized as core materials to generate impedance mismatch zones;
- Analysis is performed through the combined application of the theory of periodic structures, the FE method, and the Transfer matrix and Spectral FE methods
- The capability of the periodic core to generate stop bands for the propagation of waves along the plate-rows, and directional patterns for the propagation of waves along the plate plane has been assessed;
- Analysis allows evaluating pass/stop bands propagation patterns, and the phase constant surfaces for the estimation of directional characteristics for wave propagation
- The filtering capabilities are influenced by the geometry of the periodic cell;
- Harmonic response shows directionality at specified frequencies, and confirms the propagation patterns
- Completely passive treatment;
- External dimensions and weight not significantly affected;
- Improvement of the attenuation capabilities of periodic sandwich panels obtained through a proper selection of the core and cell configuration;