# NUMERICAL METHODS
# FOR THE BIOT MODEL IN POROELASTICITY

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A Dissertation

Presented to

the Faculty of the Department of Mathematics

University of Houston

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

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By

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May 2002

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ABSTRACT

In this dissertation, efficient numerical methods for the Biot poroelastic model are considered. The model describes two basic mechanical processes in porous media: single-phase fluid flow and rock deformation. Its mathematical formulation is a coupled system of the Lamé and diffusion equations.

A mixed finite element and a mimetic finite difference scheme are used to solve simultaneously for the porous media displacement, fluid mass flux, and pore pressure. The resulting discrete problem is solved iteratively by the preconditioned Lanczos method. The preconditioner is a block diagonal matrix with as many blocks as there are physical variables. The diagonal blocks are preconditioners for the Lamé and diffusion operators. The problem is further simplified by the Korn inequality which states that the bilinear form associated with the elastic energy induces an $H^1$-norm in the space of displacements. Therefore, the spectral properties of the block diagonal preconditioner are solely dependent on spectral properties of a preconditioner for an elliptic operator. In this thesis, a few preconditioners for elliptic operators are studied theoretically and numerically. In addition, the first convergence estimates are proved for the mimetic finite difference scheme.

The preconditioners may be used in fully implicit time discretization schemes as an alternative to operator splitting algorithms.

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