Regulation with Competing Objectives, Self-Reporting, and Imperfect Monitoring

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Abstract
We model the optimal design of programs requiring firms to disclose harmful emissions when disclosure yields both direct and indirect benefits. The indirect benefit arises from the internalization of social costs and resulting reduction in emissions. The direct benefit results from the disclosure of previously private information which is valuable to potentially harmed parties. Previous theoretical and empirical analyses of such programs restrict attention to the former benefit while the stated motivation for such programs highlights the latter benefit. When disclosure yields both direct and indirect benefits, policymakers face a tradeoff between inducing truthful self-reporting and deterring emissions. Internalizing the social costs of emissions, such as through a Pigovian tax, will deter emissions, but may also reduce incentives for firms to truthfully report their emissions.

I. Introduction

Regulatory agencies, including the Environmental Protection Agency (EPA), commonly cite two categories of benefits associated with information disclosure programs. The first, an indirect benefit, arises from the internalization of the social costs of emissions (and consequent reductions in emissions) due to market responses to disclosures or regulatory instruments such as Pigovian taxes on disclosed emissions. The second, a direct benefit, results from the disclosure of previously private information. Referring to information disclosure programs in a recent report that describes the U.S. experience with various environmental policies, the EPA states "The environmental information embodied in these approaches has economic value...even in the absence of any changes in emissions by firms" (p. 153) [23].[1] Timely information about emissions may enable potential damages to be avoided or mitigated both by affected parties and public agencies. For example, disclosure may reduce consumption of contaminated water by alerting individuals of the need for avoidance or proper treatment. Disclosure may also decrease the environmental impacts of a toxic release by accelerating clean-up efforts.

Theoretical analyses have tended to represent the social cost of emissions as a function only of emissions levels, independent of whether the presence and magnitude of emissions are publicly disclosed. The empirical work has followed a similar convention by measuring program success in terms of reductions in emissions. Neither strand of the literature has yet to explicitly account for the possibility that disclosure of harmful emissions may be directly beneficial, outside of any indirect impacts of disclosure requirements on emissions. We develop a theoretical model that attempts to reconcile this

[1] In fact, the report refers to the benefits of disclosure from changes in consumer or producer behavior, such as reduced emissions, as "ancillary" (p. 153).

apparent inconsistency between the stated motivation for information disclosure programs and previous analyses of such programs.

In our model, disclosure of emissions is directly beneficial but actual emissions are imperfectly observable so policymakers face a tradeoff between inducing truthful self-reporting and deterring emissions.[2] Internalizing the social costs of emissions, such as through a Pigovian tax, will deter emissions, but it may also reduce incentives for firms to truthfully disclose their emissions.

When monitoring firm behavior (such as through an audit process) is costly, a policymaker must account for three factors when designing regulatory policy: (1) the benefit of reduced emissions arising from internalizing social costs, (2) the direct social benefit of disclosure of emissions that do occur, and (3) enforcement costs. Previous analyses of environmental compliance have addressed factors (1) and (3) by considering a regulator whose objective is to minimize emissions (Garvie and Keeler [4]; Macho-Stadler and Perez-Castrillo [18]) or to minimize enforcement costs for a given level of compliance (Livernois and McKenna [17]). We model the regulator's objective in a way that accounts for the reduction in social costs arising both from disclosure of emissions and a reduction in the quantity of emissions. This framework is both more general and more representative. In this paper our principal objective is to model the optimal policy choice in this context when the instruments at the regulator's discretion are a tax on

[2] This trade-off is present in other regulatory settings such as consumer product and food safety. Firms are required to disclose product failures and hazards, but the more costly such disclosure (either due to fines or liability exposure) the greater the incentive firms have to conceal such information. Reducing fines or limiting liability costs encourages disclosure but may dull incentives to reduce product defects. However, this tradeoff is not present in some other regulatory settings where information disclosure programs have traditionally been applied, such as income taxation.

(disclosed) emissions and the frequency (or probability) of auditing a firm's disclosure report.

In order to better understand the characteristics of the regulator's trade-off between inducing compliance with disclosure requirements and reducing emissions, we develop a model of firm behavior in the context of an imperfect audit. An imperfect audit reveals some percentage of the firm's actual emissions according to a known probability distribution. Firms then optimize their choice of how much of their true emissions to disclose in order to minimize their expected costs. Firms also choose how much to emit conditional on their expected emissions costs. The regulator in turn optimally chooses the policy parameters based on his expectations about how firms facing a particular regulatory environment will behave.

The model we develop adds to the literature on the role of self reporting in environmental regulation. Malik [19], Swierzbinski [22] and others have shown that incentive-compatible mechanisms for self reporting (in which firms are induced to truthfully report their emissions) can achieve enforcement cost savings and increase social welfare. The benefit of self reporting in these models arises due to the regulator having incomplete information regarding the social costs or private benefits (i.e., abatement costs) of emissions by a particular firm. Unlike these previous models, we assume the regulator has full information in these respects.[3] The social benefit from self reporting in our model arises very differently (and more directly) from the fact that reported emissions cause less social damage than undisclosed emissions. In our model

[3] Section III of the paper presents a variant of our model in which firms have private information.

disclosure of emissions by firms is a desirable end in itself, rather than a mechanism to achieve desirable emissions reductions in a more cost effective manner.[4]

This paper is organized as follows. Section II develops our main model. We first consider the decision facing a representative firm required to disclose emissions subject to a tax enforced through imperfect audits. We then analyze the optimal policy choice of the regulator, who we assume has complete information. Section III relaxes the perfect information assumption and confirms that our main results continue to hold. Section IV concludes with discussion of the implications of our model and possible extensions.

II. The Model

A. The Firm's Problem

We first analyze the decision facing a firm subject to a mandatory information disclosure policy requiring the firm to report a level of emissions to the regulator. The compliance decision for a firm is defined by three factors: 1) the disclosure costs the firm incurs as a function of its reported emissions, 2) the penalty costs the firm incurs as a function of any emissions that are revealed in excess of the level it discloses, and 3) the nature of the auditing program.[5]

[4] Of course regulations requiring self reporting may serve a dual purpose, both to capture direct benefits of disclosure and to achieve enforcement cost savings from information revelation. We focus on the direct benefits of disclosure to keep our model fairly straightforward and make the implications of this regulatory motive most transparent.

[5] Becker's [2] "optimal penalty" model provides the theoretical basis for the literature on environmental compliance. The main insight from his model is that potential offenders respond to the probability of detection as well as the severity of the punishment. See Polinsky and Shavell [20] (and the citations within) for a general review of the enforcement literature. Cohen [3] and Heyes [10] provide reviews of the environmental compliance and enforcement literature.

Firms may face costs associated with emissions (whether disclosed or undisclosed) arising from a variety of sources.[6] Most directly, a firm may be subject to a Pigovian tax on disclosed emissions, and a subsequent penalty on unreported emissions that are later revealed. A firm may also face current or future liability costs associated with emissions, both of which may be reflected immediately in the market valuation of the firm upon the revelation of its emissions.[7] Finally, the firm may face costs associated with the revelation that it failed to disclose emissions when required. The revelation of under-reporting by a firm may be either a direct consequence of regulatory enforcement, or through other mechanisms such as internal whistleblowers, disclosures by the media or environmental watchdog groups, or simply due to random events that bring information into the public domain.

Most previous analyses of environmental compliance assume an error-free audit process (see for example Kaplow and Shavell [14] and Innes [11]), an assumption consistent with the tax compliance literature.[8] We define an audit to be error-free if it reveals, perhaps with some probability less than one, the exact degree of misreporting. Recently, Macho-Stadler and Perez-Castrillo [18] depart from the more common assumption in the literature of an audit that always reveals the exact degree of misreporting by allowing the probability of perfect revelation to be less than one. Notice however that the effect of this assumption is merely to decrease the probability of

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[6] Firms may fail to perfectly comply in some cases simply because it is costly to collect the necessary information (e.g., a firm may bear some cost of simply measuring its own emissions). We ignore the possibility here and simply assume the firm has perfect knowledge of its emissions.

[7] See Hamilton [5], Khanna et al. [15], and Konar and Cohen [16] for empirical evidence on market reactions to releases of the Toxics Release Inventory (TRI).

[8] Malik [19] is an exception. He models a binary compliance decision allowing for errors in auditing the firm's compliance status. In contrast, we model compliance with the information disclosure requirement as a continuous choice in order to focus our analysis on behavioral changes at the intensive, rather than extensive, margin.

detection (the firm now faces a compound probability). Heyes [8] considers a similar audit structure where the probability that an audit (perfectly) detects non-compliance is endogenous. In each of these models, provided an audit occurs, it reveals either no misreporting or the exact degree of misreporting and therefore is consistent with our definition of an error-free audit. The assumption of error-free audits seems best suited to situations where firms make dichotomous choices to comply with a regulation or not. However, in the case of environmental information disclosure requirements, where incurred penalties are likely to vary with the degree of noncompliance, the firm's decision may be more accurately modeled as choosing the optimal degree of compliance. Therefore, we model compliance as a continuous choice and assume the firm faces an imperfect audit, one that reveals a *percentage* of the firm's actual emissions.

We assume firms are homogeneous and consider the problem facing a representative firm. Let e represent the firm's emissions and denote the firm's benefit of emitting as $B(e)$ where $B'(e) > 0$ and $B''(e) < 0$. Let z denote the share of actual emissions reported by the firm, so the reported quantity of emissions is ze. For clarity and tractability, we assume that for each unit of reported emissions, the firm incurs a constant per unit cost, denoted α, which we characterize as the "tax" on emissions. Similarly, if the audit reveals a level of emissions that exceeds reported emissions, the firm incurs a constant per unit cost, denoted β, on the revealed but unreported emissions. We refer to β as the "penalty."[9]

[9] Both disclosure and penalty costs could of course be non-linear. For example, the penalty cost function might increase at an increasing rate with the magnitude of the violation if regulators take the view that large infractions should be punished severely while minor infractions receive a much milder treatment. The linearity assumption renders the model much more tractable and avoids issues associated with the optimal size of a firm as a function of the regulatory environment, which is beyond the scope of our analysis.

The firm is audited with probability *p*. If an audit occurs it reveals a quantity of emissions, denoted *x*. We assume $x = eu$ where *u* is a random variable with cumulative distribution function $F(u)$ and probability density function $f(u)$, which is strictly positive on the interval $[0, b]$ with $b \geq 1$.[10] We assume $f(u)$ has a single mode at one. The model thus allows for the possibility that an audit reveals less or perhaps more than was actually emitted. We do not require that audits be unbiased (i.e., that $E[u] = 1$) or that $f(u)$ be symmetrically distributed around one, but the model encompasses these possibilities. We assume that the audit distribution *F* is independent of the firm's actual emissions. That is, the scale of the firm or its emissions level does not impact the effectiveness of audits, so the audit is equally likely to reveal any given percentage of actual emissions regardless of the firm's true emissions level.

The firm's problem is to choose *e* and *z* to maximize the expected net benefit of emitting. Given our assumptions and the values of α, *p*, and β, the firm faces a constant per unit cost of emitting, denoted μ, with

$$\mu(\alpha, \beta, p) = \alpha z + p\beta \int_z^b (u - z) f(u) dt . \tag{1}$$

Therefore the firm's expected net benefit is given by:

$$B(e) - C(e, z) = B(e) - e \cdot \mu(\alpha, \beta, p) = B(e) - e \cdot \left[\alpha z + p\beta \int_z^b (u - z) f(u) dt \right]. \tag{2}$$

[10] Because the audit process has two-sided errors yielding the possibility that emissions are "revealed" in excess of the actual level (as in Harford [6]), it is possible that a firm would find it optimal to over comply, reporting emissions in excess of its actual level. As we discuss below, in our model the regulator will never find it optimal to induce overcompliance from a representative firm. Arora and Gangopadhyay [1], Shimshack and Ward [21], among others explicitly focus on overcompliance with environmental regulations.

It is clear from equation (2) that with a constant tax and penalty and independence between the audit effectiveness and actual emissions levels, the firm's optimal choice of z is independent of e. Thus, our assumptions allow us to decouple the choices of e and z. We begin by analyzing the firm's optimal choice of z. The first order condition for an interior solution on z is given by:

$$\alpha = p\beta \int_{z^*}^{b} dF(u) = p\beta\left[1 - F(z^*)\right] \tag{3}$$

where z^* denotes the optimal reported share of emissions. The first order condition indicates that the firm's optimal report, z^*, equates the marginal cost of reported emissions, α, and the expected marginal benefit of reported emissions. The expected marginal benefit reflects the expected avoided per unit penalty on revealed but unreported emissions. Using equation (3), we can solve for z^* as a function of the policy parameters:

$$z^* = F^{-1}\left(1 - \frac{\alpha}{p\beta}\right)$$

With this we state the following proposition characterizing the firm's optimal choice of z. All proofs are given in the appendix.

Proposition 1. Given α, β, and p, the firm's optimal choice of z will be such that

(i) $z^* = 0$ if $\alpha \geq p\beta$

(ii) For $p\beta > \alpha$ an interior solution exists with z^* defined by expression (3) above.

(iii) For an interior solution, the firm's optimal report, z^*, is decreasing in the tax on reported emissions, α; increasing in the probability of audit, p; and increasing in the penalty on revealed but unreported emissions, β.

Note that $p\beta > \alpha$ is required for an interior solution on z*. That is, in order to elicit reporting in our model, the tax on reported emissions must be below the expected penalty on revealed but unreported emissions.[11] We assume this condition is satisfied and focus attention on an interior solution for z^*.

We now consider the firm's optimal choice of emissions. Given z^*, the firm will choose e^* to maximize $B(e) - C(e, z^*) = B(e) - e \cdot \mu^*$ where $\mu^* = \alpha z^* + p\beta \int_{z^*}^{b} (u - z^*) f(u) dt$. The first order condition with respect to the choice of e is given by:

$$\alpha z^* + p\beta \int_{z^*}^{1} (u - z^*) f(u) du = B'(e^*) \text{ or } \mu^* = B'(e^*) \tag{4}$$

which simply states that the optimal level of emissions occurs where the marginal cost and marginal benefit of emitting are equal. Equation (4) implicitly defines the firm's demand for emissions, as a function of the marginal cost of emitting (given z^*), which we denote $e(\mu^*) = B'^{-1}(\mu^*)$, where $e'(\mu^*) < 0, e''(\mu^*) \geq 0$. Proposition 2 states the comparative static results for the optimal level of emissions, e^*.

Proposition 2. The firm's optimal level of emissions, e^*, decreases with the tax on reported emissions, α; the penalty on revealed but unreported emissions, β; and the probability of audit, *p*.

Proposition 2 confirms the intuitive result that emissions decrease with increases in those factors that raise μ^*, namely the tax, the penalty, and the frequency of audits.

[11] Heyes [9], Innes [11] and Kambhu [13], among others, present models in which fines set below their maximal levels are optimal, which is analogous to setting the tax sufficiently low to induce disclosure in our model. For example, in Kambhu [13] higher penalties lead to lower compliance because they induce regulated firms to take actions that obstruct the enforcement process.

The next section considers the policymaker's problem conditional on the firm responding to changes in policy parameters according to Proposition 2.

In the model of optimal regulatory policy developed below we will employ the fact that the firm's optimized *net* benefit of emitting is $B(e^*) - C(e^*, z^*) = \int_{\mu^*}^{\mu_c^*} e(\rho) d\rho$ where μ_c^* represents the choke price for emissions. This expression simply states that the firm's net benefit of emitting (given z^*) is the area under the firm's emissions demand curve above μ^*. This is denoted area *A* in Figure 1.

B. The Regulator's Problem

The regulator's objective function must account for (1) the welfare loss from emissions in excess of the socially optimal quantity, (2) the direct benefit of information disclosure, and (3) the costs associated with auditing firms.

Let *m* denote the per unit social cost of undisclosed emissions. Let *s* represent the difference between the unit cost of undisclosed emissions and the unit cost of disclosed emissions. We assume $s < m$, allowing for disclosure to increase the range of available private and public mitigation strategies and therefore decrease the social cost of emissions. For a particular level of disclosure, *z*, the per unit social cost of emissions is then given by $m - sz$.

When we assume, as we do in this section, that the regulator has complete information about the effectiveness of the audit process and the firm's demand for emissions, he can infer the firm's true emissions. However, this inference is no longer possible in a model with heterogeneity in the distribution of audit outcomes among firms, and incomplete information on the part of the regulator. Section III confirms that our main results continue to hold under these conditions. We maintain the complete information, homogeneous firms assumptions in this section for ease of exposition and because they allow us to develop a model which is somewhat more general in other respects.[12]

We model the situation facing the regulator as a minimization problem and assume his objective function, denoted *V*, is comprised of three terms: (1) the total damages from emissions net of expected taxes and fines paid by the firm; (2) enforcement costs; (3) the firm's net benefit from emitting. Based on our assumptions, the total social cost of emissions is equal to $e(\mu^*)\cdot(m - sz^*)$. The firm pays expected taxes and fines equal to $e(\mu^*)\cdot \mu^*$. Therefore, the total damages from emissions net of payments by the firm, the first component of *V*, is $e(\mu^*)[m - sz^* - \mu^*]$. We denote the cost of an audit to be *w*, so enforcement costs, the second component, are simply *pw*. As described earlier, the firm's optimized *net* benefit from emissions is represented by $\int_{\mu^*}^{\mu_c} e(\rho)d\rho$. This is the final component of *V*.

Given the three components, the regulator's objective function is:

$$V = e(\mu^*)[m - sz^* - \mu^*] + pw - \int_{\mu^*}^{\mu_c^*} e(\rho)d\rho \tag{5}$$

We assume the regulator minimizes *V* with respect to his choice of α, the tax on reported emissions, and *p*, the audit probability.[13] Therefore, we assume β, the marginal penalty

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[12] In particular, the model with heterogeneous firms developed later relies on assuming linear demand for emissions among firms to obtain comparable results.

[13] In modeling the policy choices available to the regulator we have not allowed the regulator to choose a deposit-refund instrument in lieu of a tax. Swierzbinski [22] finds a deposit-refund system to be optimal in

on revealed but unreported emissions, is exogenous. In the context of our model the regulator would always do best to set this penalty as high as possible because doing so achieves the highest compliance given any tax with the least enforcement costs. This fairly standard result leads us to simply assume that the regulator faces some constraint on the magnitude of the penalty that can be imposed.[14]

The first order conditions for an interior solution to the regulator's problem are given by:

$$\frac{\partial V}{\partial \alpha} = 0 \Leftrightarrow e'(\mu^*)\frac{\partial \mu^*}{\partial \alpha}(m - sz^* - \mu^*) = e(\mu^*)s\frac{\partial z^*}{\partial \alpha}. \tag{6}$$

$$\frac{\partial V}{\partial p} = 0 \Leftrightarrow e(\mu^*)s\frac{\partial z^*}{\partial p} - e'(\mu^*)\frac{\partial \mu^*}{\partial p}(m - sz^* - \mu) = w. \tag{7}$$

Equation (6) indicates that the regulator chooses α^* to equate the marginal benefit of a higher tax (due to lower emissions) with the marginal cost of a higher tax (due to less truthful reporting). Similarly equation (7) illustrates that p^* equates the marginal benefit of increased audit frequency (greater disclosure and reduced emissions) and the marginal cost (additional audit resources, w).

Both a higher tax and higher audit probability achieve greater internalization of social costs (and thus a reduction in emissions), but each is costly in a different way. A

a model of regulation with self reporting. However, as discussed earlier, the role of self reporting in Swierzbinski's model is quite different than in ours because it arises as a result of the regulator's uncertainty about a firm's pollution abatement costs (absent any direct benefits of disclosure). A deposit-refund scheme would not be optimal in general in our context because it raises the enforcement cost of internalizing social damages. Although a deposit-refund scheme could be optimal in our context under certain conditions, we've chosen to constrain the regulator to using a Pigovian tax both for simplicity and because deposit-refund mechanisms are not broadly utilized in environmental regulation (particularly in the U.S., see EPA [23])

[14] See, for example, Becker [2] and Harrington [7]. This assumption can also be grounded in the argument that the marginal penalty may include factors which are outside the regulator's control such as the market's reaction to news that a firm underreported its actual emissions or explicit fines and increased liability resulting from an independent judiciary process (Garvie and Keeler [4]).

higher tax reduces disclosure, which is costly when disclosure has direct benefits. A higher audit probability is directly costly as more resources are devoted to enforcement. To understand the interplay between these choices, consider the two extreme cases regarding the value of disclosure. First, suppose disclosure has no direct benefit so $s = 0$. In this case there is no interior solution on α; it is optimal to set $\alpha^* \geq p\beta$ (in which case the firm discloses nothing). This achieves the greatest internalization of social costs (arising entirely through fines rather than taxes) with the least expenditure on enforcement. The optimal audit probability, p^*, will reflect the marginal benefit of reduced emissions resulting from internalization relative to the marginal cost of auditing, and an interior solution will exist for w sufficiently large. At the other extreme, suppose that once emissions are disclosed they are no longer socially harmful, so $s = m$. In such a case the optimal policy involves zero tax on reported emissions. Full compliance with the disclosure requirement can then be achieved with a negligible audit probability. Although this extreme case may seem unrealistic, it conveys important intuition: as s approaches m the optimal policy may be minimal taxation and infrequent auditing. Auditing is costly for the regulator and high compliance rates can still be achieved with a low probability of audit when the tax on reported emissions is also low.

An interior solution in both dimensions of the regulator's choice will exist if s is sufficiently large but strictly less than m (i.e., the costs of emissions are sufficiently reduced but not completely eliminated by disclosure) and if the cost of auditing, w, is sufficiently large. We henceforth assume this is the case and focus our analysis on the comparative statics at an interior solution.

Proposition 3. The regulator's optimal tax, α^*, is increasing in *m*, the per unit social cost of undisclosed emissions and decreasing in *s*, the difference between the per unit social costs of undisclosed and disclosed emissions. The optimal audit probability p^* is decreasing in the cost of auditing, *w*.

The comparative static results regarding the optimal tax are broadly intuitive. The regulator trades-off internalizing social costs with a higher tax against the consequent reduction in disclosure; the more valuable is disclosure (due to higher *s*), the lower the optimal tax. Conversely, the more socially costly all emissions are (as represented by *m*), the higher the optimal tax in order to achieve greater internalization of these costs and lower resulting emissions. The effect of the cost of auditing, *w*, on α^* is ambiguous. A higher cost of auditing, *w*, does not directly affect the optimal tax but will of course reduce the optimal audit probability, p^*. Whether the optimal tax increases or decreases with an increase in *w* depends on how the decrease in the audit probability affects the marginal benefit and cost of the tax. The expression for $\frac{\partial \alpha^*}{\partial w}$ is provided in the appendix.

Unlike the comparative statics for the optimal tax, the directions of the effects of *m* and *s* on the optimal audit frequency are in general ambiguous. Consider first the effect of *m*. As the social cost of emissions rises (holding constant the reduction that occurs due to disclosure, *s*) the marginal benefit of internalizing emissions costs rises. For this reason it seems intuitive that the optimal audit probability would rise as well, since raising *p* increases the internalized cost of emitting. However, an increase in *m* increases the optimal tax α^* as stated in Proposition 3. This in turn increases μ^* and reduces emissions *ceteris paribus*. A reduction in emissions reduces the marginal benefit of achieving a higher percentage of emissions disclosure. This reduces the value of auditing with regards to achieving higher rates of disclosure. If the firm's elasticity of demand for emissions is very high, then the optimal response to an increase in *m* may be to raise the tax to reduce emissions but reduce the audit probability. The comparative static result shows that we cannot exclude the possibility that $\frac{\partial p^*}{\partial m} < 0$. However, were the regulator restricted to choosing only *p*, with α fixed, then we find unambiguously $\frac{\partial p^*}{\partial m} > 0$.

The ambiguity of the effect of an increase in *s* on the optimal audit probability is more easily understood. An increase in *s* has opposing effects on the value of auditing. A higher *s* increases the value of disclosure, which increases the marginal benefit of auditing. However, the higher *s* decreases the value of internalizing the social costs of emissions because the higher *s* reduces the social cost of emissions. This decreases the marginal benefit of auditing. Either effect may dominate. The expression which determines the sign of $\frac{\partial p^*}{\partial s}$ is stated in the appendix.

III. Heterogeneous Firms and Incomplete Information

Our model in the previous section assumes a single firm representative of a homogeneous industry, and complete information on the part of the regulator. While these assumptions greatly simplify the analytics of our model, they also imply that the regulator can infer the firm's actual emissions.[15] In this section, we discuss the issues

[15] Optimal regulatory policy in the context of the tradeoff between deterring emissions and eliciting truthful disclosure is, of course, determined at the margin. Assuming, as we do in section II, that the regulator has complete information about the firm's demand for emissions and about the firm's incentives to truthfully disclose (arising from the effectiveness and probability of audits) implies that the regulator

arising from inference of emissions levels, and relax our assumptions to allow for firm heterogeneity and incomplete information.

Any model that captures the trade-off faced by a regulator between reducing emissions and eliciting truthful disclosure of emissions must entail the regulator's forming some inference regarding firms' behavior. That is, the regulator must infer actual emissions and the extent to which firms' disclosures are untruthful in order to evaluate the marginal benefits and costs of policy changes that affect actual emissions and disclosure. This leads to something of a paradox: why does the regulator value disclosure if he can infer how much a firm will emit?

Most fundamentally, we argue that the reduction of social costs arising from a firm's disclosure of emissions is different from what can be achieved from inferring their presence. While we model disclosed emissions simply as a quantity, in practice emissions disclosure is likely to involve additional, directly beneficial but difficult to infer information involving the nature of emissions, the time and location of releases, etc.[16] The ability to mitigate the harm caused by emissions is likely to be very sensitive to these specific details, perhaps most importantly the immediate knowledge of a release (or even

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also knows exactly what level of actual emissions is optimal for the firm, in addition to knowing what percentage of emissions the firm will optimally disclose. However, the model can be thought of as simply a framework for understanding how a regulator would evaluate policy choices at the margin. In applying the model what is required is that the regulator form beliefs regarding how the truthfulness of disclosure and cost of emitting are affected at the margin by the policy parameters, and how the level of emissions is affected by the cost of emitting (i.e., the elasticity of demand for emissions). A regulator may well be able to estimate these marginal responses without actually having complete information. For example, the regulator may be able to estimate the elasticity of demand for emissions without knowing the entirety of the demand curve.

[16] This suggests several possible extensions that are beyond the scope of the current analysis. For example, one could permit firms to report more detailed information about the characteristics of their emissions and allow the social cost of disclosed emissions to vary with the nature of the information. As noted by an anonymous reviewer, one could also consider a model in which undiscovered and un-inferred emissions are most costly, followed by undiscovered but inferred emissions, and finally disclosed emissions. Both extensions would add additional complexity (and choice variables for the firm and regulator). We've restricted the model to capture what we believe are the most central aspects of policy choice in our context.

prior knowledge in the case of planned releases). A regulator's belief (or even certainty) that a firm is emitting more than it discloses may very well be insufficient to enable mitigation. Furthermore, the regulator presumably could not act to penalize the firm based on inferred emissions since penalties could not be legally enforced on inferred emissions that have not actually been revealed by the audit.

The representative firm model employed in section II implies that the regulator's inference is applicable to a specific firm. We develop a more general model here which entails firm heterogeneity. In this framework the regulator forms inference regarding aggregate industry emissions and average disclosure behavior, but cannot infer any specific firm's emissions level. This allows meaningful analysis of policy tradeoffs but enhances the distinction between disclosed and inferred emissions. In such a context it is clear that the disclosure of emissions by individual firms would enable mitigation of social costs that could not be achieved by inference regarding aggregate industry emissions. We show that in an industry with heterogeneous firms, in which the regulator is able to infer only average industry emissions, the main results of our model continue to hold.

Assume that each firm has private information, represented by the parameter *k*, regarding the distribution of audit outcomes if it is audited.[17] That is, if a *k*-type firm is audited, the audit reveals a quantity of emissions equal to $e \cdot (u+k)$ where u is a random variable with probability density function $f(u)$ and cumulative distribution function $F(u)$ on the interval $[1-d, 1+d]$. We assume $f(u)$ is unimodal and symmetric around 1.

[17] There are several other ways in which we might add firm heterogeneity. For example, we could assume that firms differ in their perceived penalties for non-reporting or in their probabilities of being found noncompliant as in Innes [12]. We thank an anonymous referee for suggesting these possibilities to us.

The value of k varies across firms and the regulator knows only the distribution of k, denoted $G(k)$ with support $[-\varepsilon, \varepsilon]$. The expected value of k is assumed to be zero so that on average across firms audits are unbiased. An additional assumption, that $d + \varepsilon < 1$, is required to ensure an interior solution on z.

An individual firm's objective remains unchanged—choose the report, z, and emissions, e, to maximize the expected net benefits of emitting:

$$\underset{e,z}{Max}\left\{ B(e) - e\left[\alpha z + p\beta \int_{z-k}^{d} (u + k - z) f(u) du \right] \right\}.$$

We can solve the first order condition on z to obtain an expression for z^*:

$$z^* = F^{-1}\left(1 - \frac{\alpha}{p\beta}\right) + k. \tag{9}$$

Given z^*, the first order condition on e can be stated as follows:

$$\alpha z^* + p\beta \int_{z^*-k}^{d} (u + k - z^*) f(u) du = B'(e^*) \text{ or } \mu^* = B'(e^*)$$

where $\mu^* = \alpha z^* + p\beta \int_{z^*-k}^{d} (u + k - z^*) f(u) du$ denotes the marginal cost of emitting given the optimal report. The form of firm heterogeneity we have introduced enters the model fairly simply; the firm-specific audit parameter simply shifts the optimal report, z^*. The unit-cost of emissions, μ^*, for a particular firm depends both directly on k and on the resulting z^* (with μ^* of course increasing in k). Note however that taking expectations across the industry $E[z^*] = F^{-1}\left(1 - \frac{\alpha}{p\beta}\right)$ and $E[\mu^*] = \alpha E[z^*] + p\beta \int_{E[z^*]}^{d} (u - E[z^*]) f(u) du$.

The fact that the expected values of these key firm choice variables parallel the expressions for z^* and μ^* in the representative firm model of section II will enable us to model the optimal policy of the regulator very similarly. The effects of policy parameters on $E[z^*]$ and $E[\mu^*]$ (and therefore expected or average total emissions) precisely parallel the results for the representative firm model on z^* and μ^* described in Propositions 1 and 2.

Before turning our attention to the problem facing the regulator, note that the regulator is unable to infer a particular firm's true emissions, e^*, in this context. To see this, let x^* represent the *level* of emissions the *k*-type firm (optimally) reports to the regulator where

$$x^* = z^* e^* = \left[F^{-1}\left(1-\frac{\alpha}{p\beta}\right)+k\right]\cdot e(\mu^*) \tag{10}$$

with $\mu^* = \alpha z^* + p\beta \int_{z^*-k}^{d}(u+k-z^*)f(u)du$. The presence of *k* in the above expression breaks the inference—each x^* value is associated with more than one value of *k*.[18] To understand the intuition, consider two firms, one with a high value of *k* (audits are biased against it) and one with a low value of *k* (audits are biased in its favor). The firm with the high value of *k* will report a higher percentage of its emissions, possibly even more than 100%, but will emit less because its cost of emitting will be higher. The firm with the low

[18] Consider the case where the demand for emissions is linear: $e = a - c\mu$. With a linear demand for emissions, $x^* = \left[F^{-1}\left(1-\frac{\alpha}{p\beta}\right)+k\right][a-c\mu^*]$. The following two values of *k* yield the same value of x^*:

$$k = -F^{-1}\left(1-\frac{\alpha}{p\beta}\right) - \frac{(a-c\gamma)+\sqrt{(a-c\gamma)^2-4c\alpha x^*}}{-2x^*}, -F^{-1}\left(1-\frac{\alpha}{p\beta}\right) + \frac{(a-c\gamma)+\sqrt{(a-c\gamma)^2-4c\alpha x^*}}{-2x^*}$$

where $\gamma \equiv p\beta \int_{F^{-1}\left(1-\frac{\alpha}{p\beta}\right)}^{b}\left(u - F^{-1}\left(1-\frac{\alpha}{p\beta}\right)\right)f(u)du$.

value of *k* will report a smaller share of actual emissions but will emit more. Because the level of emissions reported to the regulator is given by the product of z^* and e^*, both firms could report the same x^* thus breaking the inference.[19] While the regulator is unable to infer a particular firm's emissions based on its report, he can still infer average emissions since he knows the expected value of *k*.

When firms are heterogeneous and the regulator has incomplete information, the regulator is assumed to choose the optimal tax and audit probability based on his knowledge of expected (or average) firm behavior. That is, the regulator minimizes the expected value of the social welfare function described in Section II:

$$E(V) = E\left[e(\mu^*)[m - sz^* - \mu^*] + pw - \int_{\mu^*}^{\mu_c^*} e(\rho)d\rho \right].$$

This problem is made far more tractable by assuming each firm faces linear demand for emissions:

$$e(\mu^*) = a - c\mu^*.$$

Given this assumption, the regulator's objective function becomes:

$$E(V) = E\left\{ (a - c\mu^*)(m - sz^* - \mu^*) + pw - \frac{1}{2}(a - c\mu^*)\left(\frac{a}{c} - \frac{1}{c}\mu^*\right) \right\}$$

The regulator minimizes $E[V]$ with respect to his choices of the tax, α, and audit probability, *p*. The fact that the respective forms of $E[z^*]$ and $E[\mu^*]$ resemble those of z^* and μ^* in the homogeneous firm model, together with linearity of demand, makes the solution to the regulator's problem in this context closely parallel that discussed in section II. In particular, the comparative static results obtained for an interior solution to

[19] More generally, a firm's reported level of emissions will not be a monotonic function of its *k* parameter.

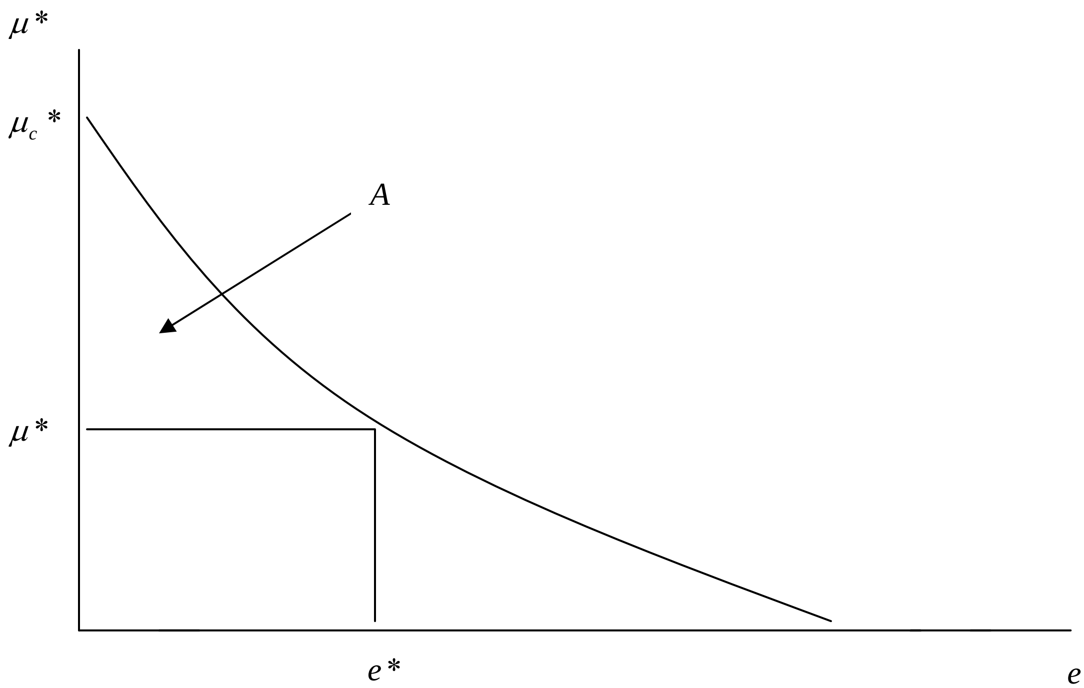
the regulators' problem hold with heterogeneous firm of the type modeled here. These results are formalized in the appendix.

IV. Conclusion

When information disclosure has direct social benefits but is costly for a firm and enforcement is costly and imperfect a regulator must confront the competing objectives of inducing disclosure and internalizing social costs. This tension is clearly present in many environmental regulatory contexts where the harm from emissions can be mitigated if potentially impacted parties have better information about the nature and quantity of emissions. It also exists in other regulatory settings such as product safety regulation. Disclosure of product defects and hazards has direct social benefits, but it is desirable that firms face a cost (either liability or fines) when their products cause harm in order to induce care.

There are certainly many avenues for future work in this area. One could imagine two policymakers, one of whom chooses a tax and the other the audit probability (e.g., legislature and executive or regulatory agency) but who have different objective functions and interact strategically. A regulator may have other policy instruments at his discretion, including choosing the audit probability for a firm in a dynamic setting based on past behavior. One also might consider an endogenous audit process in which the probability of audit is a decreasing function of disclosed emissions. We have not modeled the choice between putting enforcement resources into more frequent audits or more effective audits. Clearly a regulator must achieve an optimal balance, and the model we've developed could provide a framework for exploring this issue. We have assumed that disclosure costs (tax) and penalties are constant per unit, and that audit effectiveness is independent of firm size or total emissions. Relaxing these assumptions significantly complicates the analysis, but could inform important issues regarding how regulation affects industry structure.

Figure 1. Firm's demand for emissions



Appendix

Proofs for Section II

Proof of Proposition 1:

Define $\upsilon \equiv 1-\frac{\alpha}{p\beta}$. The comparative static results for $z^* = F^{-1}\left(1-\frac{\alpha}{p\beta}\right) = F^{-1}(\upsilon)$ where $F(\cdot)$ is monotonically increasing in its augment and $\alpha < p\beta$ for an interior solution are given by:

$$\frac{\partial z^*}{\partial \alpha} = -\frac{1}{p\beta}\left(\frac{1}{f(z^*)}\right) < 0 \tag{A1}$$

$$\frac{\partial z^*}{\partial p} = \frac{\alpha}{p^2\beta}\left(\frac{1}{f(z^*)}\right) > 0 \tag{A2}$$

$$\frac{\partial z^*}{\partial \beta} = \frac{\alpha}{p\beta^2}\left(\frac{1}{f(z^*)}\right) > 0 \tag{A3}$$

Proof of Proposition 2:

The second order condition for minimization is satisfied: $-B''(e^*) > 0$. The comparative static results for *e* are derived implicitly.

$$\frac{\partial e^*}{\partial \alpha} = \frac{z^*}{B''(e^*)} < 0 \tag{A4}$$

$$\frac{\partial e^*}{\partial p} = \frac{\beta}{B''(e^*)}\int_{z^*}^{b}(u-z^*)f(u)du < 0 \tag{A5}$$

$$\frac{\partial e^*}{\partial \beta} = \frac{p}{B''(e^*)}\int_{z^*}^{b}(u-z^*)f(u)du < 0 \tag{A6}$$

Proof of Proposition 3:

The elements of the Hessian for the regulator's problem are:

$$\begin{aligned}\frac{\partial^2 V}{\partial \alpha^2} \equiv f_{11} &= e'(\mu^*)\left[(m-3sz^*-\mu^*)\frac{\partial z^*}{\partial \alpha} - (z^*)^2\right] \\ &\quad - se(\mu^*)\frac{\partial^2 z^*}{\partial \alpha^2} + e''(\mu^*)(m-sz^*-\mu^*)(z^*)^2\end{aligned} \tag{A7}$$

$$\begin{aligned}\frac{\partial^2 V}{\partial p^2} \equiv f_{22} &= e'(\mu^*)\left[(m-sz^*-\mu^*)\frac{\partial^2 \mu^*}{\partial p^2} - 2s\frac{\partial z^*}{\partial p}\frac{\partial \mu^*}{\partial p} - \left(\frac{\partial \mu^*}{\partial p}\right)^2\right] \\ &\quad - se(\mu^*)\frac{\partial^2 z^*}{\partial p^2} + e''(\mu^*)(m-sz^*-\mu^*)\left(\frac{\partial \mu^*}{\partial p}\right)^2 > 0\end{aligned} \tag{A8}$$

$$\frac{\partial^2 V}{\partial\alpha\partial p} \equiv f_{12} = e'(\mu^*)\left[(m - sz^* - \mu^*)\frac{\partial z^*}{\partial p} - s\frac{\partial z^*}{\partial\alpha}\frac{\partial\mu^*}{\partial p} - sz^*\frac{\partial z^*}{\partial p} - z^*\frac{\partial\mu^*}{\partial p}\right]$$
$$- se(\mu^*)\frac{\partial^2 z^*}{\partial\alpha\partial p} + e''(\mu^*)(m - sz^* - \mu^*)z\frac{\partial\mu^*}{\partial p} \qquad \text{(A9)}$$

For an interior solution, the determinant of the Hessian, denoted $|H|$, must be positive or $|H| = f_{11}f_{22} - (f_{12})^2 > 0$. Therefore for an interior solution to the regulator's problem, $f_{11} > 0$ since $f_{22} > 0$.

The following second order effects are necessary to compute the comparative static results of interest:

$$\frac{\partial^2 V}{\partial\alpha\partial m} \equiv f_{1m} = e'(\mu^*)z^* < 0 \qquad \text{(A10)}$$

$$\frac{\partial^2 V}{\partial p\partial m} \equiv f_{2m} = e'(\mu^*)\frac{\partial\mu^*}{\partial p} < 0 \qquad \text{(A11)}$$

$$\frac{\partial^2 V}{\partial\alpha\partial s} \equiv f_{1s} = -e(\mu^*)\frac{\partial z^*}{\partial\alpha} - e'(\mu^*)(z^*)^2 > 0 \qquad \text{(A12)}$$

$$\frac{\partial^2 V}{\partial p\partial s} \equiv f_{2s} = -e(\mu^*)\frac{\partial z^*}{\partial p} - e'(\mu^*)z^*\frac{\partial\mu^*}{\partial p} \qquad \text{(A13)}$$

$$\frac{\partial^2 V}{\partial\alpha\partial w} \equiv f_{1w} = 0 \qquad \text{(A14)}$$

$$\frac{\partial^2 V}{\partial p\partial w} \equiv f_{2w} = 1 \qquad \text{(A15)}$$

We begin with the comparative static results for the optimal tax on reported emissions, denoted α^*.

$$\frac{\partial\alpha^*}{\partial m} = \frac{1}{|H|}\begin{vmatrix} -f_{1m} & f_{12} \\ -f_{2m} & f_{22} \end{vmatrix} = \frac{1}{|H|}\left[-f_{1m}f_{22} + f_{2m}f_{12}\right] > 0 \text{ given } |H| > 0$$

since

$$-f_{1m}f_{22} + f_{2m}f_{12}$$
$$= se(\mu^*)e'(\mu^*)\left[\frac{\partial\mu^*}{\partial\alpha}\frac{\partial^2 z^*}{\partial\alpha^2} - \frac{\partial\mu^*}{\partial p}\frac{\partial^2 z^*}{\partial\alpha\partial p}\right] \qquad \text{(A16)}$$
$$+ [e(\mu^*)]^2\left[(m - sz^* - \mu^*)\left(\frac{\partial\mu^*}{\partial p}\frac{\partial z^*}{\partial p} - z^*\frac{\partial^2\mu^*}{\partial p^2}\right) + s\frac{\partial\mu^*}{\partial p}\left(\frac{\partial z^*}{\partial p} - \frac{\partial z^*}{\partial\alpha}\right)\right] > 0$$

$$\frac{\partial\alpha^*}{\partial s} = \frac{1}{SOC}\begin{vmatrix} -f_{1s} & f_{12} \\ -f_{2s} & f_{22} \end{vmatrix} = \frac{1}{SOC}\left[-f_{1s}f_{22} + f_{2s}f_{12}\right] < 0 \text{ given } SOC > 0$$

since

$$
\begin{aligned}
& - f_{1s} f_{22} + f_{2s} f_{12} \\
& = s[e(\mu *)]^2 \left(\frac{\alpha *}{p *} \psi \frac{\partial z *}{\partial \alpha} \right) + sz * e(\mu *) e'(\mu *) \left(\frac{\partial \mu *}{\partial p} \frac{\partial^2 z *}{\partial \alpha \partial p} - z * \frac{\partial^2 z *}{\partial p^2} \right) \\
& + e(\mu *) e''(\mu *)(m - sz * - \mu *) \frac{\partial \mu *}{\partial p} \left[\frac{\partial z *}{\partial \alpha} \frac{\partial \mu *}{\partial p} - z * \frac{\partial z *}{\partial p} \right] \\
& + [e'(\mu *)]^2 z * (m - sz * - \mu *) \left(z * \frac{\partial^2 \mu *}{\partial p^2} - \frac{\partial z *}{\partial p} \frac{\partial \mu *}{\partial p} \right) \\
& + sz * [e'(\mu *)]^2 \frac{\partial \mu *}{\partial p} \left[\frac{\partial z *}{\partial \alpha} \frac{\partial \mu *}{\partial p} - z * \frac{\partial z *}{\partial p} \right] \\
& + e(\mu *) e'(\mu *) \left(s \frac{\partial z *}{\partial p} + \frac{\partial \mu *}{\partial p} \right) \left(z * \frac{\partial z *}{\partial p} - \frac{\partial z *}{\partial \alpha} \frac{\partial \mu *}{\partial p} \right) < 0
\end{aligned}
\qquad \text{(A17)}
$$

where $\psi \equiv \left[\frac{\partial F(\upsilon)}{\partial \upsilon} \right]^{-1} \frac{1}{(p *)^2 \beta} > 0$ with $\upsilon \equiv 1 - \frac{\alpha *}{p * \beta}$.

The comparative static result for *w* on $\alpha *$ is generally ambiguous:

$$
\frac{\partial \alpha *}{\partial w} = \frac{1}{SOC} \begin{vmatrix} - f_{1w} & f_{12} \\ - f_{2w} & f_{22} \end{vmatrix} = \frac{1}{SOC} [- f_{1w} f_{22} + f_{2w} f_{12}] = \frac{1}{SOC} [f_{12}]
$$

Given $SOC > 0$, the sign of $\frac{\partial \alpha *}{\partial w}$ equals the sign of f_{12}, which is given as equation (A9) above.

We now derive the comparative static results for the optimal audit probability, $p *$. The comparative static result for *w* on $p *$ is given by:

$$
\frac{\partial p *}{\partial w} = \frac{1}{SOC} \begin{vmatrix} - f_{11} & f_{1w} \\ - f_{12} & f_{2w} \end{vmatrix} = \frac{1}{SOC} [- f_{11} f_{2w} + f_{12} f_{1w}] = \frac{1}{SOC} [- f_{11}] < 0 \text{ since } f_{11} > 0 .
$$

The signs of $\frac{\partial p *}{\partial m}$ and $\frac{\partial p *}{\partial s}$ are generally ambiguous. The respective expressions follow:

$$
\begin{aligned}
\frac{\partial p *}{\partial m} &= \frac{1}{SOC} \begin{vmatrix} - f_{11} & f_{1m} \\ - f_{12} & f_{2m} \end{vmatrix} = \frac{1}{SOC} [- f_{11} f_{2m} + f_{12} f_{1m}] \\
&= \frac{1}{SOC} \left\{ se(\mu *) e'(\mu *) \left[\frac{\partial \mu *}{\partial p} \frac{\partial^2 z *}{\partial \alpha^2} - z * \frac{\partial^2 z *}{\partial \alpha \partial p} \right] + [e(\mu *)]^2 (m - 2sz * - \mu *) \left(z * \frac{\partial z *}{\partial p} - \frac{\partial z *}{\partial \alpha} \frac{\partial \mu *}{\partial p} \right) \right\}
\end{aligned}
$$

$$\frac{\partial p^*}{\partial s} = \frac{1}{SOC}\begin{vmatrix} -f_{11} & f_{1s} \\ -f_{12} & f_{2s} \end{vmatrix} = \frac{1}{SOC}\left[-f_{11}f_{2s} + f_{12}f_{1s}\right]$$

$$= \frac{1}{SOC}\left\{ s[e(\mu^*)]^2\left(\psi\frac{\partial z^*}{\partial \alpha}\right) + sz^* e(\mu^*)e'(\mu^*)\left(\frac{\partial \mu^*}{\partial \alpha}\frac{\partial^2 z^*}{\partial \alpha \partial p} - \frac{\partial \mu^*}{\partial p}\frac{\partial^2 z^*}{\partial \alpha^2}\right)\right.$$

$$+ e(\mu^*)e''(\mu^*)(m - sz^* - \mu^*)z^*\left[z^*\frac{\partial z^*}{\partial p} - \frac{\partial z^*}{\partial \alpha}\frac{\partial \mu^*}{\partial p}\right]$$

$$+ [e'(\mu^*)]^2 z^*(m - sz^* - \mu^*)\left(\frac{\partial z^*}{\partial \alpha}\frac{\partial \mu^*}{\partial p} - \frac{\partial z^*}{\partial p}\frac{\partial \mu^*}{\partial \alpha}\right)$$

$$\left. + s(z^*)^2[e'(\mu^*)]^2\left[z^*\frac{\partial z^*}{\partial p} - \frac{\partial z^*}{\partial \alpha}\frac{\partial \mu^*}{\partial p}\right] + e(\mu^*)e'(\mu^*)\left(s\frac{\partial z^*}{\partial \alpha} + z^*\right)\left(\frac{\partial z^*}{\partial \alpha}\frac{\partial \mu^*}{\partial p} - z^*\frac{\partial z^*}{\partial p}\right)\right\}$$

Proofs for Section III

Below, we reexamine the model presented in Section II relaxing the homogeneous firms and perfect information assumptions. Consider first the problem facing a *k*-type firm, among many heterogeneous firms. The firm's reported emissions are denoted by ez. The emissions revealed by audit are $x = e \cdot (u + k)$, where k represents the firm's individual characteristic that is unknown to the regulator. k is defined on the support $[-\varepsilon, \varepsilon]$ with mean zero. *u* is a random variable with probability density function $f(u)$ on the interval $[1-d, 1+d]$. $f(u)$ is unimodal and symmetric around 1. The firm is found underreporting if $x > ez$. The expected level of underreporting for a *k*-type firm is

$$\int_{z-k}^{d}[e(u+k) - ez]f(u)du = e\int_{z-k}^{d}(u+k-z)f(u)du$$

The firm's objective function is then

$$\underset{e,z}{Min}\ \ e\cdot\left[\alpha z + p\beta\int_{z-k}^{d}(u+k-z)f(u)du\right] - B(e)$$

The first order conditions for an interior solution on *e* and *z* are given respectively by:

$$\alpha z + p\beta\int_{z-k}^{d}(u+k-z)f(u)du - B'(e^*) = 0 \tag{A18}$$

$$e\left[\alpha - p\beta\int_{z^*-k}^{d}f(u)du\right] = 0 \tag{A19}$$

Solving (A19) for z^* yields

$$z^* = F^{-1}\left(1 - \frac{\alpha}{p\beta}\right) + k\,.$$

Because *k* is a constant, the comparative static results on *z** are the same as in the homogenous firm model (see equations (A1) through (A3) above).
The comparative static results on e^* are given as follows:

$$\frac{\partial e^*}{\partial \alpha} = \frac{z^*}{B''(e^*)} < 0 \tag{A20}$$

$$\frac{\partial e^*}{\partial p} = \frac{\beta}{B''(e^*)} \int_{z^*-k}^{d} (u + k - z^*) f(u) du < 0 \tag{A21}$$

$$\frac{\partial e^*}{\partial \beta} = \frac{p}{B''(e^*)} \int_{z^*-k}^{d} (u + k - z^*) f(u) du < 0 \tag{A22}$$

Now consider the regulator's problem when firms' demands for emissions are linear and given by $e(\mu) = a - c\mu$. Given incomplete information on *k*, the regulator now minimizes, $E(V)$, with respect to his choices of α and *p* where

$$E(V) = E\left\{ e(\mu^*)(m - sz^* - \mu^*) - \frac{1}{2} e(\mu^*)\left(\frac{a}{c} - \frac{1}{c}\mu^*\right) + pw \right\}$$

$$= pw + am - \frac{1}{2}\frac{a^2}{c} - asE(z^*) - cmE(\mu^*) + csE(\mu^* \cdot z^*) + \frac{1}{2} cE\left((\mu^*)^2\right)$$

with $\mu^* = \alpha z^* + p\beta \int_{z^*-k}^{d} (u + k - z^*) f(u) du$ denoting the marginal cost of emitting given the optimal report, *z**.

Define $\phi \equiv F^{-1}\left(1 - \frac{\alpha}{p\beta}\right)$ and $\gamma \equiv p\beta \int_{\phi}^{d} (u - \phi) f(u) du$. Given our notation,

$$E(z^*) = F^{-1}\left(1 - \frac{\alpha}{p\beta}\right) = \phi$$

$$E(\mu^*) = \alpha F^{-1}\left(1 - \frac{\alpha}{p\beta}\right) + p\beta \int_{\phi}^{d} (u - \phi) f(u) du = \alpha\phi + \gamma$$

$$E(z^* \cdot \mu^*) = E[(\phi + k) \cdot (\alpha(\phi + k) + \gamma)] = \alpha\phi^2 + \gamma\phi + \alpha Var(k)$$

$$E\left[(\mu^*)^2\right] = E\left[(\alpha(\phi + k) + \gamma)^2\right] = \gamma^2 + \alpha^2\phi^2 + \alpha^2 Var(k) + 2\alpha\gamma\phi$$

where $Var(k)$ denotes the variance of the random variable *k*.

After substituting the above expressions into $E(V)$, we can write the first order conditions for an interior solution as:

$$\frac{\partial E(V)}{\partial \alpha} \equiv g_1 = -s\frac{\partial \phi}{\partial \alpha}[a - c(\alpha\phi + \gamma)] - c\phi[m - s\phi - (\alpha\phi + \gamma)] + (cs + c\alpha)Var(k) = 0$$

$$\frac{\partial E(V)}{\partial p} \equiv g_2 = w - s\frac{\partial \phi}{\partial p}[a - c(\alpha\phi + \gamma)] - c\frac{\gamma}{p}[m - s\phi - (\alpha\phi + \gamma)] = 0$$

The second order effects follow. Each expression includes a comparison between the second order effect in the heterogeneous firm model (denoted by g's), and the associated second order effect that would obtain in the homogeneous firm model assuming linear demand for emissions (denoted by $\bar{f}$'s). The latter model is a special case of the more general model in Section II of the paper (see equations (A10) through (A15) for the second order effects with a more general demand function).

$$\frac{\partial^2 V}{\partial \alpha^2} \equiv g_{11} = -s\frac{\partial^2 \phi}{\partial \alpha^2}[a - c(\alpha\phi + \gamma)]$$
$$+ c\left[\phi^2 + 2s\phi\frac{\partial \phi}{\partial \alpha} - (m - s\phi - (\alpha\phi + \gamma))\frac{\partial \phi}{\partial \alpha}\right] + cVar(k) \quad \text{(A23)}$$
$$= \bar{f}_{11} + cVar(k)$$

$$\frac{\partial^2 V}{\partial p^2} \equiv g_{22} = -s\frac{\partial^2 \phi}{\partial p^2}[a - c(\alpha\phi + \gamma)]$$
$$+ c\left[\left(\frac{\gamma}{p}\right)^2 + 2\frac{s\gamma}{p}\frac{\partial \phi}{\partial p} + (m - s\phi - (\alpha\phi + \gamma))\frac{\alpha}{p}\frac{\partial \phi}{\partial p}\right] = \bar{f}_{22} > 0 \quad \text{(A24)}$$

$$\frac{\partial^2 V}{\partial \alpha \partial p} \equiv g_{12} = -s\frac{\partial^2 \phi}{\partial \alpha \partial p}[a - c(\alpha\phi + \gamma)]$$
$$+ c\left[\phi\frac{\gamma}{p} + s\frac{\gamma}{p}\frac{\partial \phi}{\partial \alpha} + s\phi\frac{\partial \phi}{\partial p} - (m - s\phi - (\alpha\phi + \gamma))\frac{\partial \phi}{\partial p}\right] = \bar{f}_{12} \quad \text{(A25)}$$

$$\frac{\partial^2 V}{\partial \alpha \partial m} \equiv g_{1m} = -c\phi = \bar{f}_{1m} < 0 \quad \text{(A26)}$$

$$\frac{\partial^2 V}{\partial p \partial m} \equiv g_{2m} = -c\frac{\gamma}{p} = \bar{f}_{2m} < 0 \quad \text{(A27)}$$

$$\frac{\partial^2 V}{\partial \alpha \partial s} \equiv g_{1s} = -\frac{\partial \phi}{\partial \alpha}[a - c(\alpha\phi + \gamma)] + c\phi^2 + cVar(k) = \bar{f}_{1s} + cVar(k) \quad \text{(A28)}$$

$$\frac{\partial^2 V}{\partial p \partial s} \equiv g_{2s} = -\frac{\partial \phi}{\partial p}[a - c(\alpha\phi + \gamma)] + c\frac{\gamma}{p}\phi = \bar{f}_{2s} \quad \text{(A29)}$$

$$\frac{\partial^2 V}{\partial \alpha \partial w} \equiv g_{1w} = 0 = \bar{f}_{1w} \quad \text{(A30)}$$

$$\frac{\partial^2 V}{\partial p \partial w} \equiv g_{2w} = 1 = \bar{f}_{2w} \quad \text{(A31)}$$

We now state the comparative static results for the regulator's choice variables in the heterogeneous firms, incomplete information model.

$$\frac{\partial \alpha^*}{\partial m} = \frac{1}{|H|}\begin{vmatrix} -g_{1m} & g_{12} \\ -g_{2m} & g_{22} \end{vmatrix} = \frac{1}{|H|}\left[-g_{1m}g_{22} + g_{2m}g_{12}\right] = \frac{1}{|H|}\left[-\bar{f}_{1m}\bar{f}_{22} + \bar{f}_{2m}\bar{f}_{12}\right] > 0$$

by equation (A16). As above $|H|$ denotes the determinant of the Hessian. For an interior solution, $|H| > 0$ which implies $g_{11} = \bar{f}_{11} + cVar(k) > 0$.

$$\frac{\partial \alpha^*}{\partial s} = \frac{1}{|H|}\begin{vmatrix} -g_{1s} & g_{12} \\ -g_{2s} & g_{22} \end{vmatrix} = \frac{1}{|H|}\left[-g_{1s}g_{22} + g_{2s}g_{12}\right] = \frac{1}{|H|}\left[-\bar{f}_{1s}\bar{f}_{22} + \bar{f}_{2s}\bar{f}_{12}\right] - \frac{1}{|H|}cVar(k)g_{22} < 0$$

since the term in brackets is negative by (A17) and $g_{22} > 0$.

$$\frac{\partial p^*}{\partial w} = \frac{1}{|H|}\begin{vmatrix} -g_{11} & g_{1w} \\ -g_{12} & g_{2w} \end{vmatrix} = \frac{1}{|H|}\left[-g_{11}g_{2w} + g_{12}g_{1w}\right] = \frac{1}{|H|}\left[-g_{11}\right] = \frac{1}{|H|}\left[-\left(\bar{f}_{11} + cVar(k)\right)\right] < 0.$$

The signs of $\frac{\partial \alpha^*}{\partial w}$, $\frac{\partial p^*}{\partial m}$, and $\frac{\partial p^*}{\partial s}$ are generally ambiguous. The respective expressions follow:

$$\frac{\partial \alpha^*}{\partial w} = \frac{1}{|H|}\begin{vmatrix} -g_{1w} & g_{12} \\ -g_{2w} & g_{22} \end{vmatrix} = \frac{1}{|H|}\left[-g_{1w}g_{22} + g_{2w}g_{12}\right] = \frac{1}{|H|}\left[-\bar{f}_{1w}\bar{f}_{22} + \bar{f}_{2w}\bar{f}_{12}\right] = \frac{1}{|H|}\left[\bar{f}_{12}\right]$$

$$\frac{\partial p^*}{\partial m} = \frac{1}{|H|}\begin{vmatrix} -g_{11} & g_{1m} \\ -g_{12} & g_{2m} \end{vmatrix} = \frac{1}{|H|}\left[-g_{11}g_{2m} + g_{12}g_{1m}\right] = \frac{1}{|H|}\left[-\bar{f}_{11}\bar{f}_{2m} + \bar{f}_{12}\bar{f}_{1m}\right] - cVar(k)g_{2m}$$

$$\frac{\partial p^*}{\partial s} = \frac{1}{|H|}\begin{vmatrix} -g_{11} & g_{1s} \\ -g_{12} & g_{2s} \end{vmatrix} = \frac{1}{|H|}\left[-\bar{f}_{11}\bar{f}_{2s} + \bar{f}_{12}\bar{f}_{1s}\right] + \frac{1}{|H|}cVar(k)\left(\bar{f}_{12} - \bar{f}_{2s}\right)$$

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