Title: Candy Factory Espionage

Brief Overview:

This unit focuses on patterns, functions, and algebraic thinking. Students are industrial spies who use their knowledge of patterns to break into a candy company and steal the secret formula of a new candy. They also will create and test their own pattern to safeguard their companies secrets to prevent enemy infiltration.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Number and Operations

- *Understand numbers, ways of representing numbers, relationships among numbers, and number systems.*
- *Understand meanings of operations and how they relate to one another.*
- *Compute fluently.*

Algebra

- *Understand patterns, relations, and functions.*
- *Represent and analyze mathematical situations and structures using algebraic symbols.*
- *Use mathematical models to represent and understand quantitative relationships.*
- *Analyze change in various contexts.*

Geometry

- *Use visualization, spatial reasoning, and geometric modeling to solve problems.*

- **Process Standards**

Problem Solving

- *Build new mathematical knowledge through problem solving.*
- *Solve problems that arise in mathematics and in other contexts.*
- *Apply and adapt a variety of appropriate strategies to solve problems.*
- *Monitor and reflect on the process of mathematical problem solving.*

Reasoning and Proof

- *Recognize reasoning and proof as fundamental aspects of mathematics.*
- *Make and investigate mathematical conjectures.*
- *Develop and evaluate mathematical arguments and proofs.*
- *Select and use various types of reasoning and methods of proof.*

Communication

- *Organize and consolidate their mathematical thinking through communication.*
- *Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*
- *Analyze and evaluate the mathematical thinking and strategies of others.*
- *Use the language of mathematics to express mathematical ideas precisely.*

Connections

- *Recognize and use connections among mathematical ideas.*
- *Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*
- *Recognize and apply mathematics in contexts outside of mathematics.*

Representation

- *Create and use representations to organize, record, and communicate mathematical ideas.*
- *Select, apply, and translate among mathematical representations to solve problems.*
- *Use representations to model and interpret physical, social, and mathematical phenomena.*

Grade/Level:

Grades 4/5 and Gifted & Talented (extensions)

Duration/Length:

Five days/ 45 minutes to one hour per day

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Problem solving strategies
- Use of calculators
- Mathematical language associated with patterns and functions
- Function tables
- Writing using a rubric - one is provided in Teacher Resource 10

Student Outcomes:

Students will:

- be able to use problem solving skills and strategies to analyze, solve, extend, and create patterns by using words, tables, and graphs to complete daily performance tasks and a summative assessment.
- have a working knowledge of various types of patterns, such as: growing patterns, repeating patterns, and recursive patterns as found in Fibonacci's sequence. Students also can be introduced to Pascal's triangle through an extension.

Materials/Resources/Printed Materials:

- All teacher answers are provided after each **Student Resource Sheet**
- Copy of all **Student Resource Sheets** for each student
- Pattern blocks
- Overhead of Hundred Chart
- Three Hundred Charts for each student
- Crayon or highlighter for each child
- Overhead of Honey Bees for Fibonacci's sequence
- http://www-groups.dcs.st-andrews.ac.uk/~history/Mathematicians/Fibonacci.html,
- http://www.schoolnet.ca/vp-pv/ECOS/e_fibol.htm
- http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/brickEXPLAIN.html
- Smoothey, Marion. *Let's Investigate- Number Patterns.* Marshall Cavendish. New York, 1943.
- Math Writing Rubric **Teacher Resource Sheet 10.** This is an optional math rubric. If there is one that you use continuously in your room or within your county, recommend that you post and review it daily.

Development/Procedures:

Day One:

Introduce the motivation to the students by distributing **Student Resource Sheet 1**. Each day the students will be working towards solving a problem that will open one of four doors to the Foreverlasting Gobstopper. During Day 1 students will be working on function tables involving pattern block manipulatives. Give students **Student Resource Sheet 2** and have pattern blocks available for their use. Students should be guided by the teacher through the process needed for creating a function table through the third pattern. Allow students to complete the pattern through the 7th term. Next, their explanations should be supported with a written explanation that includes a function table, number sentence and if necessary a picture following the rubric on **Teacher Resource Sheet 10.** Answers may be found on on **Teacher Resource Sheet 1**. Students will work on **Student Resource Sheet 3.** Students will be engaged in a similar activity involving hexagonal tables. Again, provide students with pattern blocks so that students are able to have the opportunity to create a visual if necessary. They will once again be asked to justify their rule by explaining using pictures, words, and tables following **Teacher Resource Sheet 10.** Answers may be found on **Teacher Resource Sheet 2.**

Day Two:

The focus of this lesson is finding patterns using the calculator. Each student should have his/her own calculator and a copy of **Student Resource Sheet 4.** If this is not possible, students should work in pairs, but each child should have the opportunity to enter the information. An overhead calculator is ideal for group instruction. Record answers on the board for a visual reference. Analyze the first five terms of the products with the students to illustrate the type of thinking that should lead them to solve the next five products without using the calculator. Answers may be found on **Teacher Resource Sheet 3**. Finally, the students will work on **Student Resource Sheet 5.** The students should be able to complete the pattern after the third term without using the calculator. They again will be asked to explain their answer using pictures, words, and tables. Students' writing can be graded on the rubric provided, **Teacher Resource 10**, your county's rubric, or your own class room rubric. Answers may be found on **Teacher Resource Sheet 4.**

Day Three:

Students will work with Fibonacci's sequence, **Student Resource Sheets 6 and 7,** by exploring the patterns within the numbers. Answers for these activities may be found on **Teacher Resource Sheets 5 and 6.** Additional resources may be obtained at the following web sites:

http://www-groups.dcs.st-andrews.ac.uk/~history/Mathematicians/Fibonacci.html,
http://www.schoolnet.ca/vp-pv/ECOS/e_fibol.htm
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/brickEXPLAIN.html

or through, Smoothey, Marion. *Let's Investigate- Number Patterns.* Marshall Cavendish. New York, 1943.
**Examples of Fibonacci's sequence also can be found in nature and can be easily displayed and examined. Examples may include, pine cones, pineapples, and conch shells.

The preparation activity challenges the students to extend the number pattern by finding the rule (add the two previous numbers together). The students again will be asked to explain their answer which can be graded using the rubric on **Teacher Resource Sheet 10**. Next, the students will be engaged in a family tree activity, **Student Resource Sheet 8**. To begin this activity, the teacher should model/review a family tree on the board. The children will then create a family tree for honeybees. Students must understand that only female (queen) bees can produce both a male and a female where as, male (drone) bee can only produce a female (queen) bee. This activity can be modeled for the first four generations by enlarging the bees and creating an overhead. Cut the overhead pieces out and create the family tree. Then, allow the students to continue to identify the pattern. Students may need assistance in understanding that when each generation is added, it equals the next number in the Fibonacci's sequence. It would be helpful to the students to place the number of bees in each generation along the side. Students will explain whether the honeybee family tree follows Fibonacci's sequence following the guidelines in **Teacher Resource Sheet 10**. Answers may be found on **Teacher Resource Sheet 7.**

Day Four:

Teacher Preparation - Distribute three Hundred Charts to each student, use Student Resource Sheets 9 and 10, provide a highlighter or crayon to each student, and make an overhead transparency of the Hundred Chart.

Today students will examine patterns found in a hundred chart by examining only part of the pattern and predicting, based on their observations of the pattern, whether certain numbers will be included in the sequence. They will use a highlighter or crayon to color in each of the given numbers to ascertain the pattern. A discussion about the given pattern should ensue which will lead the students to complete the next 3 terms. **Students are not to complete the pattern through 100.** Allow students to first examine the pattern to see if they are able to predict, based on their shaded numbers, if the additional terms will be part of their pattern. It is helpful to have an overhead copy that allows the students to compare their work to yours and provides for an overall class discussion. From the information provided, a frequency table can be made for students having difficulty seeing the relationship on the chart. If students are able to obtain the idea allow them to work on their own for numbers 4, 5, and 6 while you continue to assist others. Answers for this activity may be found on **Teacher Resource Sheet 8.** The students will then continue with the final door challenge **Student Resource Sheet 11,** which is a growing pattern. They should explain the pattern using words, pictures, and symbols to meet the rubric criterion on **Teacher Resource Sheet 10**. Answers for this activity may be found on **Teacher Resource Sheet 9.**

Performance Assessment:

Day Five:

A review of the motivation should take place before beginning the day's task. The students will be creating their own code today when they find out that they are actually Wompa security agents. Reflect with the students on the various types of patterns that were explored during the week. The task will engage students to work with a partner to create a pattern, based on the week's lessons and other patterns explored in class, that will help protect the Foreverlasting Gobstopper. The rubric for this activity is provided on their **Student Resource Sheet 12.** Once students have followed all of the given directions they will then switch with a fellow security group to "decode" their pattern. You may also want to create a **P**raise **Q**uestion **P**olish paper that will allow students to reflect and make changes if need be.

Extension/Follow Up:

- Read aloud *Willy Wonka and the Chocolate Factory* while working on this unit.
- Patterns involving decimals may be incorporated into the first and second day.
- Pascal's triangle can be modeled after the lesson on Fibonacci's sequence. Students can find Fibonacci's sequence within Pascal's triangle. Additional resources can be found on the Web which are located at the following sites,
  http://members.tripod.com/~absolutebow/ptri2.html
  http://www.math.umass.edu/~mconnors/fractal/generate/pascal.html
  http://www.orst.edu/instruct/phi302/philosophers/pascal.html
  http://www.maxmon.com/1640ad.htm
- After familiarizing students with Pascal's triangle, write Pascal's triangle so that the left most 1's of each row are aligned with each other. Next, either highlight or draw diagonal lines through this triangle and add the values on the diagonals. They will be Fibonacci numbers.
- A discussion about fractals is applicable to this unit.and information can be located at the same web site as Pascal's triangle. http://members.tripod.com/~absolutebow/ptri2.html
- Students may go more in depth into Fibonacci's sequence by examining the Golden Rule/Rectangle found in the Parthenon, Michelangelo's paintings, and the ratio of the hand (1 hand which has, 5 fingers, each of which has, 3 parts, separated by, 2 knuckles).

- Students familiar with exponents may try the following activity that correlates with **Student Resource Sheet 7**.

  Choose a series of three numbers: 5, 8, 13 Multiply the outside numbers; 5 * 13 = 65 and square the middle one; $8^2 = 64$ What is the difference? 65 - 64 = 1 Is it always the same no matter which group of three you choose?

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Spy Games!

You are an employee of the Jupiter Candy Company. Your mission, should you choose to accept it, is to break into the William Wampa Candy Factory and take the secret formula for the Foreverlasting Gobstopper. You will need to use your knowledge of patterns and functions to successfully infiltrate the Wampa security system. A Wampa insider informed us that you must unlock four doors before reaching the secret formula for the Foreverlasting Gobstopper. Each door can only be unlocked by successfully solving a pattern. Good luck!

Door One Preparation!

The Wampa insider has given you a clue to help solve the pattern to unlock the first door. Solve the following pattern (clue) to help prepare you to unlock Door Number One.

Ficcolos Restaurant has 10 triangular shaped tables. Each side of the tables seat one person. If the tables are put together in a straight line with one side touching another, how many people can sit at the table? Explain your answer.

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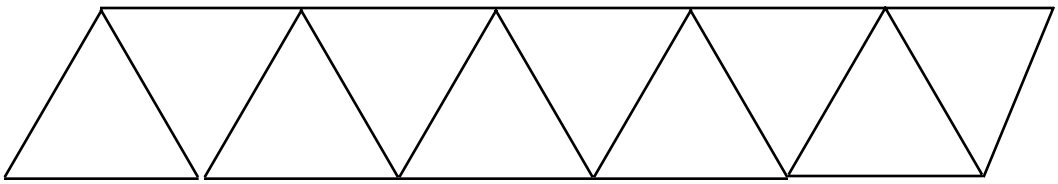
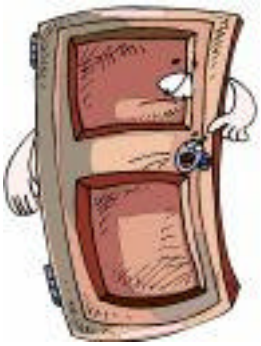
# tables	# chairs
1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10
9	11
10	12

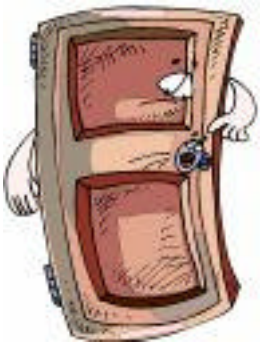
The number of tables increases by one and the number of chairs increases by one for each additional table added on. The relationship between tables and chairs is the number of tables plus two.





Door One

The Wampa Company workers are having a taste testing banquet for the new fall candies. Wampa has ten hexagon tables. Each side of the table seats one person. The tables need to be arranged in a straight line with one side touching another. How many Wampa workers can sit at the table? Explain your answer using words, pictures, and tables.



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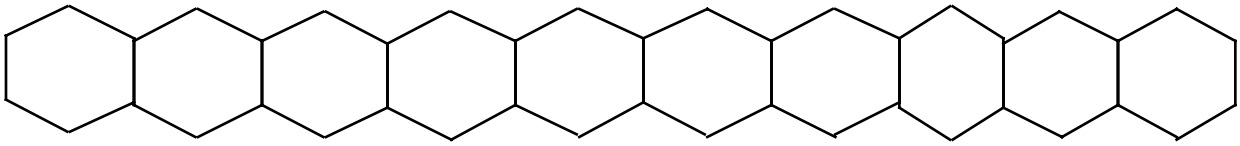
# tables	***# chairs***
1	**6**
2	**10**
3	**14**
4	**18**
5	**22**
6	**26**
7	**30**
8	**34**
9	**38**
10	**42**

The first table seats six and each additional table adds four seats. The relationship between tables and chairs is the number of tables times 4 plus 2. So, if you multiply the number of tables by four, you have to add two to take into account the extra two chairs from the first table.



Door Two Preparation!

Directions: Our insider has informed us that the second door requires the use of a calculator. This activity is designed to prepare you to use a calculator to solve the pattern. Record your answers in the blanks provided.

1. Multiply 999 by 1. _____
2. Multiply 999 by 2. _____
3. Multiply 999 by 3. _____
4. Multiply 999 by 4. _____
5. Multiply 999 by 5. _____

Without using the calculator continue the pattern for the next five terms. Explain how you continued the pattern.

Describe at least three other relationships you see within the patterns.

Door Two Preparation!

Directions: Our insider has informed us that the second door requires the use of a calculator. This activity is designed to prepare you to use a calculator to solve the pattern. Record your answers in the blanks provided.

1. Multiply 999 by 1. _____**999**____

2. Multiply 999 by 2. _____**1998**____

3. Multiply 999 by 3. _____**2997**____

4. Multiply 999 by 4. _____**3996**____

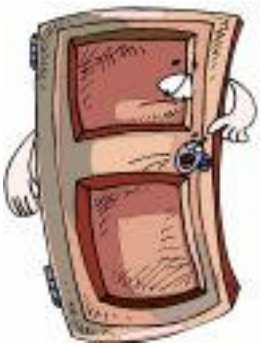
5. Multiply 999 by 5. _____**4995**____

Without using the calculator continue the pattern for the next five terms. Explain how you continued the pattern.

5994, 6993, 7992, 8991, 9990

Describe at least three other relationships you see within the patterns.

The ones decrease by one while the thousands increase by one. The tens and the hundreds remain the same. The ones plus the thousands equal 9. The difference between the sum of the first two digits and the last two digits is one.



Door Two

Use a calculator to solve the following pattern. Record your answers in the blanks provided.

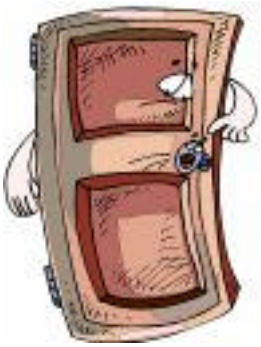
1. Multiply 37 by 3. _____

2. Multiply 37 by 6. _____

3. Multiply 37 by 9. _____

Without using the calculator continue the pattern for the next three products. Using pictures, words, and tables, explain your answer.

Describe another relationship or pattern you found in this problem.



Door Two

Use a calculator to solve the following pattern. Record your answers in the blanks provided.

1. Multiply 37 by 3. _____**111**_____

2. Multiply 37 by 6. _____**222**_____

3. Multiply 37 by 9. _____**333**_____

Without using the calculator continue the pattern for the next three products. Using pictures, words, and tables, explain your answer.

444, 555, 666, ...

Describe another relationship or pattern you found in this problem.

The multipliers are multiples of three.

Door Three Preparation

The Wampa insider provided us with the following activity to prepare you for the next door.

Complete the next four terms in the following pattern.

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , ___, ___, ___, _____, ...

What is the rule? Explain your answer using words, symbols, and pictures.

This pattern is called the Fibonacci sequence. It is named after an Italian mathematician, Leonardo of Pisa or Leonard Pisano who was born in Pisa, Italy (the city with the famous leaning tower). Fibonacci discovered this famous math pattern in the thirteenth century.

Door Three Preparation

The Wampa insider provided us with the following activity to prepare you for the next door.

Complete the next four terms in the following pattern.

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , **34**, **55, 89, 144, …..**

What is the rule? Explain your answer using words, symbols, and pictures.

To get the next digit in the sequence you need to add the previous two terms. For example, The first term plus the second term equals the third term (1+1=2). The second term plus the third term equals the fourth term (1+2=3).

This pattern is called the Fibonacci sequence. It is named after an Italian mathematician, Leonardo of Pisa or Leonard Pisano who was born in Pisa, Italy (the city with the famous leaning tower). Fibonacci discovered this famous math pattern in the thirteenth century.

Fibonacci Fun

There are many different patterns and relationships that can be found in the Fibonacci Sequence. The following activity explores one of the many relationships.

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , 144 , . . .

1. Choose a series of four consecutive terms from the Fibonacci Sequence and write them below.

2. Multiply the first and last terms. Record your answer _____.

3. Multiply the two middle terms. Record your answer _____.

4. Find the difference between the two numbers. _____

5. Does this relationship always work? Repeat this for two or more groups of four terms to test your prediction.

Fibonacci Fun

There are many different patterns and relationships that can be found in the Fibonacci Sequence. The following activity explores one of the many relationships.

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , 144 , . . .

1. Choose a series of four consecutive terms from the Fibonacci Sequence and write them below.

Example: ***2 , 3 , 5 , 8 ,***

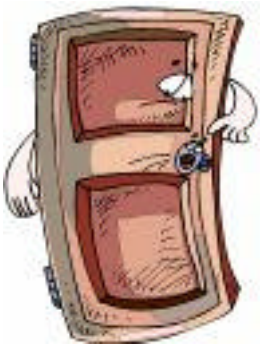
2. Multiply the first and last terms. Record your answer ____**16**___.

3. Multiply the two middle terms. Record your answer ____**15**___.

4. Find the difference between the two numbers. _____**1**____

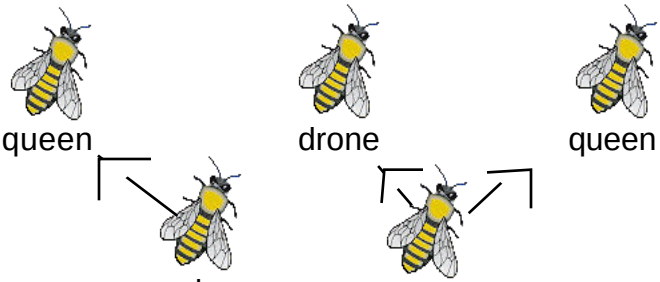
5. Does this relationship always work? Repeat this for two or more groups of four terms to test your prediction.

Answers will vary based on the four terms chosen.

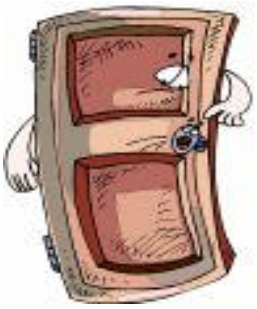


Door Three

Every honeybee colony has a special female bee called a queen. The queen bee has two parents a queen and a male drone. Male bees are called drones and do not work. Drones have only one parent -- the queen. The family tree below traces the parents and grandparents of a drone. Continue the tree back for another three generations.

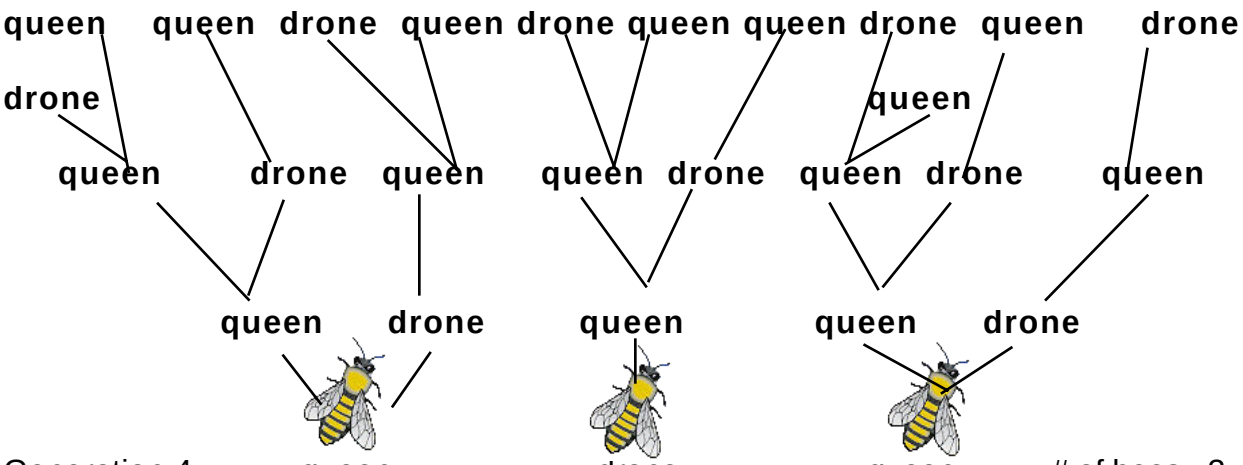


Does this bee family tree follow Fibonacci's pattern? Explain your answer using words, tables, and pictures on the back of this sheet.

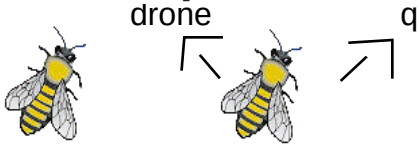


Door Three

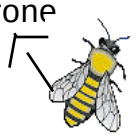
Every honeybee colony has a special female bee called a queen. The queen bee has two parents a queen and a male drone. Male bees are called drones and do not work. Drones have only one parent -- the queen. The family tree below traces the parents and grandparents of a drone. Continue the tree back for another three generations.



Generation 4 queen drone queen # of bees _3



Generation 3 drone queen # of bees _2



Generation 2 queen # of bees _1

Generation 1 drone # of bees _1

Does this bee family tree follow Fibonacci's pattern? Explain your answer using words, tables, and pictures on the back of this sheet. Yes!

Door Four Preparation

The Wampa insider has provided you with the following activity to prepare you for the final door.

1. Using your hundred chart and a highlighter or crayon, shade in the following numbers on your chart:

   3 , 6 , 9 , 12 , 15 , 18 , 21 , 24 , 27 , ...

2. Continue the pattern on your hundred chart for 3 more terms. Explain how you solved the pattern using words, symbols, and pictures.

3. Will the numbers 64, 76, 90, or 93 be shaded in this pattern? Explain your answer.

4. Use another hundred chart to extend this pattern for three more terms:

   4 , 8 , 12 , 16 , 20 , 24 , 28 , 32 , ...

5. Explain how you solved the pattern using words, symbols, and pictures.

6. Will the numbers 64, 75, 88, or 97 be shaded in this pattern? Explain your answer.

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2. Continue the pattern on your hundred chart for 3 more terms. Explain how you solved the pattern using words, symbols, and pictures.

   The pattern is multiples of three.

3. Will the numbers 64, 76, 90, or 93 be shaded in this pattern? Explain your answer.

   Only 90 and 93 will be shaded because they are multiples of three.

4. Use another hundred chart to extend this pattern for three more terms:
   4 , 8 , 12 , 16 , 20 , 24 , 28 , 32 , ...

5. Explain how you solved the pattern using words, symbols, and pictures.

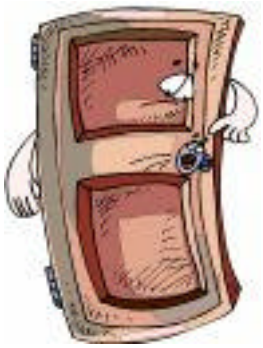
   The pattern is multiples of four.

6. Will the numbers 64, 75, 88, or 97 be shaded in this pattern? Explain your answer.

   Only 64 and 88 will be shaded because they are multiples of four.

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



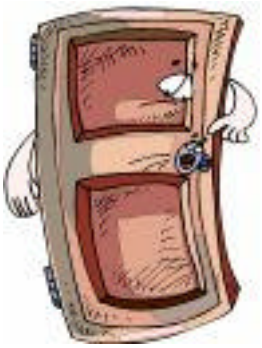
Door Four

1. Using a hundred chart and a highlighter or crayon, shade in the following numbers on your chart.

   1, 3, 6, 10, 15, 21, 28, ……..

2. Continue the pattern on your hundred chart for three more terms. Explain how you solved the pattern using words, symbols, and pictures.

3. Will the numbers 64, 76, 90, or 93 be shaded in this pattern? Explain your answer.



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   1 , 3 , 6 , 10 , 15 , 21 , 28 , ……..

2. Continue the pattern on your hundred chart for three more terms. Explain how you solved the pattern using words, symbols, and pictures.

   This is a growing pattern (add one, add two, add three, etc.) the rest of the pattern is as follows:

   36 , 45 , 55 , 66 , 78 , 91 , 105 , …

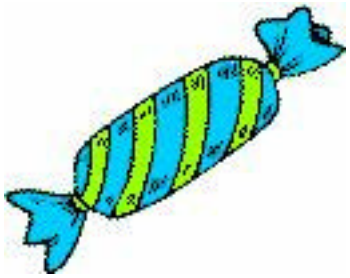
3. Will the numbers 64, 76, 90, or 93 be shaded in this pattern? Explain your answer.

Congratulations!

Surprise! You are actually an employee in the security department of the William Wampa Candy Company and now that you have discovered how easy it is to break-in, you have been asked to create a new pattern to protect the secret formula for the Foreverlasting Gobstopper.

Based on your knowledge and experience with patterns, create a pattern with a partner that will be tested by other security agents. **Your pattern must include the following to be accepted by the William Wampa Candy Company (check box when finished):**

- ☐ A pattern showing at least 5 terms that can be easily extended for three more terms.
- ☐ A relationship that works for every term.
- ☐ An explanation of the pattern that includes math vocabulary.



Math Writing Rubric

3 Accomplishes purpose of question.
Math communication is clear.

2 Partially accomplishes purpose of question.
Math communication lacks total clarity.

1 Shows limited understanding of question.
Math communication is not clear.