Soft-Gluon Resummation for Heavy Quark Production in Hadronic Collisions*

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Heavy quark production has been a topic of intense interest since the discovery of the top quark at the Fermilab Tevatron. Complete calculations of heavy quark production have been carried out up to next-to-leading order (NLO) in the strong coupling constant α_s and the NLO corrections were found to be significant. Full results to higher orders do not exist beyond NLO. However, near the final-state production threshold, $z \equiv Q^2/s = 1$, where Q is the invariant mass of the heavy quark pair and s is the square of the center-of-mass energy of the partonic collision, there are large logarithms originating from soft gluon emission which can be resummed to all orders in pQCD. At nth order in perturbative QCD one encounters terms as singular as $(-\alpha_s^n/n!)[\ln^{2n-1}((1-z)^{-1})/(1-z)]_+$, which, when folded with parton distributions, give large and positive corrections. These Sudakov logarithms increase the NLO cross section.

The leading logarithms (LL) for heavy quark production arise from soft gluon emission from the quarks and gluons in the incoming hadrons and are therefore universal.

Beyond LL, the color exchange in the hard scattering must be taken into account. The resummation of the cross section is achieved by refactorizing the partonic cross section into hard components describing the short-distance hard-scattering, distributions associated with gluons collinear to the incoming partons, and a soft function associated with non-collinear soft gluons. The soft function is a matrix in color space which satisfies a renormalization group equation whose solution provides a matrix evolution equation, in terms of soft anomalous dimension matrices, that controls threshold logarithms. At NLL the matrix equation can be diagonalized and the eigenvalues and eigenvectors of the anomalous dimension matrix calculated.

The resummed $q\overline{q}$ partonic cross section in DIS scheme and in the diagonal basis is

$$\sigma_{q\overline{q}}^{\rm NLL}(s) = \sum_{i,j=1}^{2} \int_{-1}^{1} d\cos\theta \int_{s_{\rm cut}}^{s-2ms^{1/2}} ds_4$$
$$\frac{-1}{|\lambda_1-\lambda_2|^2}\frac{d\overline{\sigma}_{q\overline{q}}^{(0)}}{ds_4}\left[\left(\frac{4N^2\Gamma_{12}^2}{N^2-1}+|\lambda_1-\Gamma_{11}|^2\right)e^{E_{11}}\right.$$
$$+\left(\frac{4N^2\Gamma_{12}^2}{N^2-1}+|\lambda_2-\Gamma_{11}|^2\right)e^{E_{22}}$$
$$-\frac{8N^2\Gamma_{12}^2}{N^2-1}{\rm Re}\left(e^{E_{12}}\right)$$
$$\left.-2{\rm Re}\left((\lambda_1-\Gamma_{11})(\lambda_2-\Gamma_{11})^* e^{E_{12}}\right)\right], \qquad (1)$$

where Γ_{ij} are the components of the soft anomalous dimension matrix, $\lambda_{1,2}$ are the eigenvalues of the diagonalized anomalous dimension matrix, and $s_{\rm cut}$ is a nonperturbative cutoff. The exponents E_{ij} include both LL and NLL contributions.

The NLL resummed $q\overline{q}$ hadronic cross section, dominant for $t\overline{t}$ production at the Tevatron, is given by the convolution of parton distributions ϕ_i with the partonic cross section

$$\sigma_{q\overline{q}}^{\rm NLL}(S) = \sum_{q=u}^{b}\int_{\tau_0}^{1} d\tau \int_{\tau}^{1}\frac{dx}{x}\phi_q(x)\phi_{\overline{q}}(\frac{\tau}{x})\sigma_{q\overline{q}}^{\rm NLL}(\tau S)$$

where $\sigma_{q\overline{q}}^{\rm NLL}(\tau S)$ is defined in Eq. (1).

To match our results to the exact NLO cross section we define the NLL improved cross section

$$\sigma_{q\overline{q}}^{\rm imp} = \sigma_{q\overline{q}}^{\rm NLL} - \sigma_{q\overline{q}}^{\rm NLO,approx} + \sigma_{q\overline{q}}^{\rm NLO,exact}\ .$$

At $m = 175$ GeV$/c^2$ and $\sqrt{S} = 1.8$ TeV with $s_{\rm cut}/(2m^2) = 0.04$ is $5.7 < \sigma_{q\overline{q}}^{\rm imp} < 6.1$ pb compared to a NLO cross section of 4.5 pb.

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