Size Matters: Spacetime geometry in subatomic collisions*

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Since the commissioning of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven Lab in late 2000, a flood of new information about the physics of hot, dense matter has been produced. With the "interested layperson" in mind, I will describe the field of relativistic heavy ion physics and discuss its aims. I will focus particularly on the special role of space-time geometry in heavy ion collisions– why it is crucial, how we measure it and what it is telling us.

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I. INTRODUCTION AND OUTLINE

The field of relativistic heavy ion (RHI) physics is a large and growing area of research. It involves very significant financial and human resources, with large theoretical centers and experimental facilities on-line or being built around the world. Brookhaven National Lab has been a dominant force in RHI research, with its largely completed RHI program at its Alternating Gradient Synchrotron (AGS) facility and its new program at the recently constructed Relativistic Heavy Ion Collider (RHIC) complex. More than 3 km in circumference, RHIC is presently the only heavy ion collider in the world and generates collisions at the highest energies ever studied. The programs of RHI facilities typically last more than a decade and involve several hundred– and now thousands– of physicists. In the beginning of my lecture, here contained in Section II, I will try to briefly outline the main motivation driving this huge scientific undertaking.

Although RHI physics has its roots in nuclear physics, and many (or most) of the scientists in the field today have been trained in nuclear physics, increasingly we have had to learn and incorporate the tools and concepts of high energy or particle physics into our studies. The huge experimental collaborations, often composed of several hundred scientists employing detectors larger than a house, further reflect the convergence of the once quite distinct fields of nuclear and high energy physics. In Section III, I will discuss the main defining and crucial aspects of this new field distinguishing it from traditional nuclear or particle physics. In particular, I hope to illuminate exactly why "size matters" to us. I will also discuss that while we aim to study "big" systems, these are only big at the subatomic scale. I will try to give an idea of the relative lengths and time scales of the system– the tiniest and shortest-lived systems ever geometrically measured.

In Section V, I very will briefly discuss some of the experimental tools we use to study heavy ion collisions at RHIC, as well as the mechanism behind our amazing ability to measure length-scales on the order of 10^{-15} m with detectors of size scale 10 m. Finally, in Section VI, I will go through some examples of results on size measurements we have made in RHI collisions and what they may be telling us and summarize in Section VII.

While I get somewhat more technical towards the later part of this writeup, I will try to keep it general. Perhaps a good sign is that there are only three equations, and they sit in a section which I encourage you to skip!

II. THE FORCES IN NATURE

So far as we know, *all* of the interactions between objects in our Universe occur through only four forces. In our daily lives, we are familiar with two of them. These are the gravitational and electric (or electromagnetic) forces. These two forces dominate most of the structures with which we are familiar, and they share common characteristics.

Of course, the gravitational interaction is an attractive force between any two objects with mass. Mass is the "charge" upon which gravity acts– the more massive an object, the stronger the gravitational force it exerts (and experiences). We also know that the gravitational force decreases in strength rapidly as the separation between objects is increased. Doubling the separation between two objects decreases the attractive gravitational force between them by a factor of four.

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Quite similar is the electric force. Here, the "charge" upon which the force acts is the electrical charge. In contrast to gravity, there are two types of electrical charge, which we call "positive" and "negative" charge. The interaction between two objects having opposite-sign charge (i.e. between a negatively and a positively charged object) is attractive, similar to the gravitational force. On the other hand, two charges with the same sign experience a repulsive electric force. The dependence of the strength of the force on the separation between the objects is identical to that for gravity– the force vanishes at very large separations. Both of these forces are rather well-understood theoretically.[1] We can confidently predict the trajectory of a long fly ball in baseball[2] and understand the structure of the solar system, based on our understanding of gravity. Similarly, our understanding of the atom and materials like metals is based on our understanding of the electric force between atoms and atomic nuclei, as well as the theory of quantum mechanics which becomes important for very small systems.

With a good foundation for understanding systems at the atomic scale and larger, we turn our attention to the subatomic world of the nucleus. As we now know, the nucleus is composed of protons (with positive electrical charge) and neutrons (with no electrical charge), tightly packed together to form a system with radius of several femtometers (1 femtometer = 1 fm = 10^{-15} m).

But wait! We just said that one electrically-positive charge repels another, and that this repulsion grows as the separation decreases. And now we have a system with *several* positive charges at very *small* separation. And yet the protons and neutrons stick together. So, there much also be another, attractive, force which "overrides" the electrical repulsion. This force between protons and neutrons has been called the "nuclear" force.

Virtually **all** of the energy which makes our existence possible relies on the nuclear reactions taking place in our Sun.[3] Keep in mind that the energy stored in fossil fuels and the motion of water and air (hydro and wind energy) has, as its source, energy from the Sun; fossil fuel and motions of these fluids are only "storage devices" for solar energy. Our only other significant energy source are our still somewhat primitive nuclear reactors. On a darker note, we are all familiar with the destructive potential associated with the huge energy available through the nuclear force.

This obviously important force has been studied for nearly a century. We now understand that the quite complicated "nuclear" force between protons and neutrons is a remnant of a more fundamental force between the quarks and gluons making up the protons and neutrons. In addition to other types of charges (mass, electric charge), quarks have so-called "color," the charge of the Strong force. Just like electrically-charged objects combine to form electrically-neutral atoms, the colored quarks and gluons combine to form color-neutral ("white") protons and neutrons.[4]

However, the Strong force differs from the other forces in a crucial respect: the strength of the force between colored objects *grows* as the separation between them increases. Thus, if we try to separate the red, green and blue quarks[5] which make up a proton (say, by hitting the red quark with a high-energy electron), an attractive "force field" will be generated between the quarks. The energy in this field will grow with separation until there is sufficient energy to simply create brand-new quarks (remember– $E = mc^2$!) which then recombine with the original quarks to form white objects (called "hadrons;" the proton is an example). The upshot is that we can never isolate a single color charge (e.g. a quark) to study in the lab. This feature of the Strong force, called color confinement, greatly complicates the experimental and theoretical study of the force. One could imagine how much we would understand of the electric force if we were not able to remove electrically-charged electrons from electrically-neutral atoms!

So, can we ever study "free" quarks and gluons, not confined within a tight, colorless system? Quantum Chromodynamics (QCD), believed to be the correct theory of the Strong force[6], says that we can! If nuclear matter is extremely

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[1] This is not to imply that *all* is understood. In particular, active research is underway to address the questions of how the general relativistic theory of gravity can be "married" to quantum mechanics and the way that *all* of Nature's forces can be understood in one "unified" theory at a most fundamental level.

[2] Another force relevant in this case is the so-called "frictional" force due to the ball passing through air. However, friction in general is understood on a more basic level in terms of microscopic electrical forces.

[3] Actually, all four forces in Nature are involved in solar energy. The gravitational force compresses protons and neutrons together, allowing fusion through the nuclear force which releases much more energy than that "put in" by gravity. A useful analogy is that gravity initiates nuclear fusion much like the striking of a match initiates exothermic chemical reactions between the sulphur in the match tip and oxygen in the air. A third interaction– the "weak force," which I do not elaborate upon– is responsible for crucial details facilitating the burning of the nuclear fuel. Finally, the electromagnetic interaction is responsible for delivering the generated energy to us by way of light.

[4] In this context, the "nuclear" force between color-neutral protons is analogous to the complex "molecular" forces between electrically-neutral atoms; they are both residuals of more fundamental forces between their non-neutral constituents.

[5] Here, "red," "green" and "blue" are just fanciful labels for the three color charges of the Strong force. They do not respond to colors in the same way which we commonly use the term.

[6] With the theory of the Strong force in hand, one might fairly wonder why further experimental study is required. The first answer, generic to any theory, is that the theory needs to be tested in reality. The second, specific to the present case, is that it turns out that QCD calculations are theoretically intractable at length-scales of the order of the proton radius or larger, leaving the most crucial questions (such as the nature of confinement) unanswered in detail.

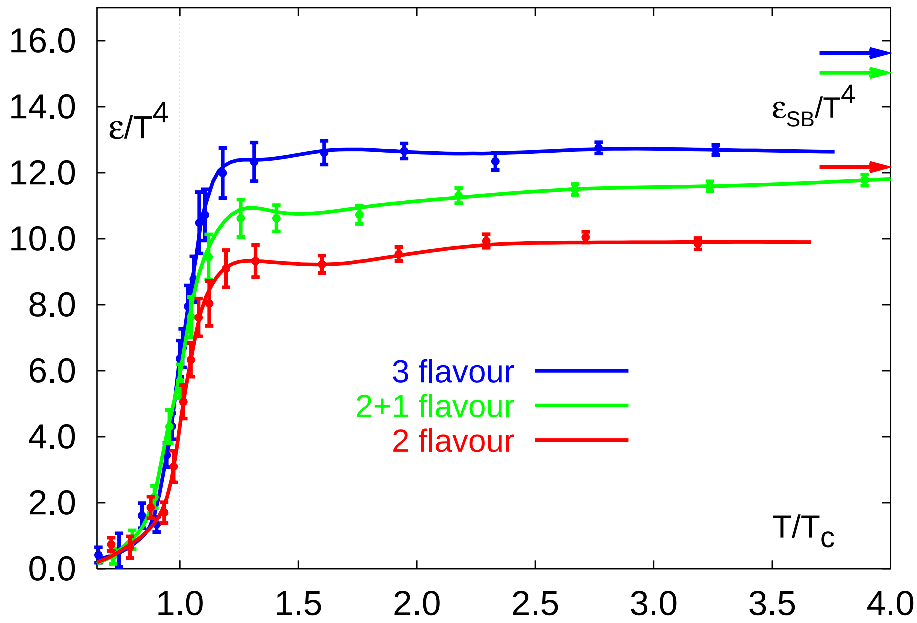

FIG. 1 The energy density divided by the fourth power of the temperature is related to the "number of degrees of freedom" of the system. The rapid rise in this quantity as the temperature is increased signals a fundamental change in the system. See text for details.

compressed and/or heated, quarks and gluons become "deconfined"– composite particles like protons "melt" into their constituents. This extreme state of color-deconfined matter is usually called the quark-gluon plasma (QGP). Figure 1 shows results of numerical calculations (1) of the energy density (ϵ) of colored matter, as the temperature (T) of the system is varied. The ratio ϵ/T^4 is related to the so-called number of degrees of freedom which describe a system. The rapid rise in this quantity signals the onset of deconfinement above some critical temperature T_c. Below this temperature, quarks and gluons are confined inside hadrons, so to describe the system, one need only discuss protons and neutrons, etc. At higher temperatures, however, each of the many colored objects themselves become relevant; the number of degrees of freedom has increased. The critical temperature is given in terms of energy units as about 170 MeV. Converted to more conventional units, this is about two trillion degrees centigrade– more than 100,000 times hotter than the Sun's core.

The left side of Figure 2 shows a schematic "phase diagram" of Strongly-interacting matter. Labels indicate the phase of matter which describes the system in given regions of temperature T and density ρ, and curves on the diagram indicate transitions between the various phases. Such phase diagrams are ubiquitous in so-called "condensed matter" physics– the study of bulk materials. An example for a familiar system is shown on the right in Figure 2. If we would like to understand water at a fundamental level, we need to focus on its phase diagram. In particular, one focuses especially on the transition lines between the different phases (melting/freezing in the liquid-solid transition, etc). From the study of water to superconductivity to semiconductors, the most essential insight into the physics of a system comes from a study of the transitions between phases. The goal of RHI physics is to vary the temperature and density of the Strongly-interacting matter (by colliding nuclei of various sizes with various collision energies) to probe the phase diagram and gain insight into fundamental nature of the Strong force.

In short, at a detailed level, what happens when two nuclei collide at high energy is itself interesting; especially from an experimental point of view, disentangling physical information from the extremely complex system we observe is fascinating and (personally speaking) a lot of fun. However, at a scientific level, we are not particularly interested in what happens when two nuclei collide. Instead, heavy ion collisions serve as the only available *tool* to generate a new state of matter– deconfined color matter or the QGP– and to study its properties. And even the QGP is only a tool for something more fundamental. At the end, we want to learn more about the Strong force, especially its most peculiar, important, and least-well-understood feature: the extremely strong interaction which hides color charges to make the composite particles (e.g. protons and neutrons) which we observe in "normal" matter. It's a big, non-trivial enterprise, but an important one.

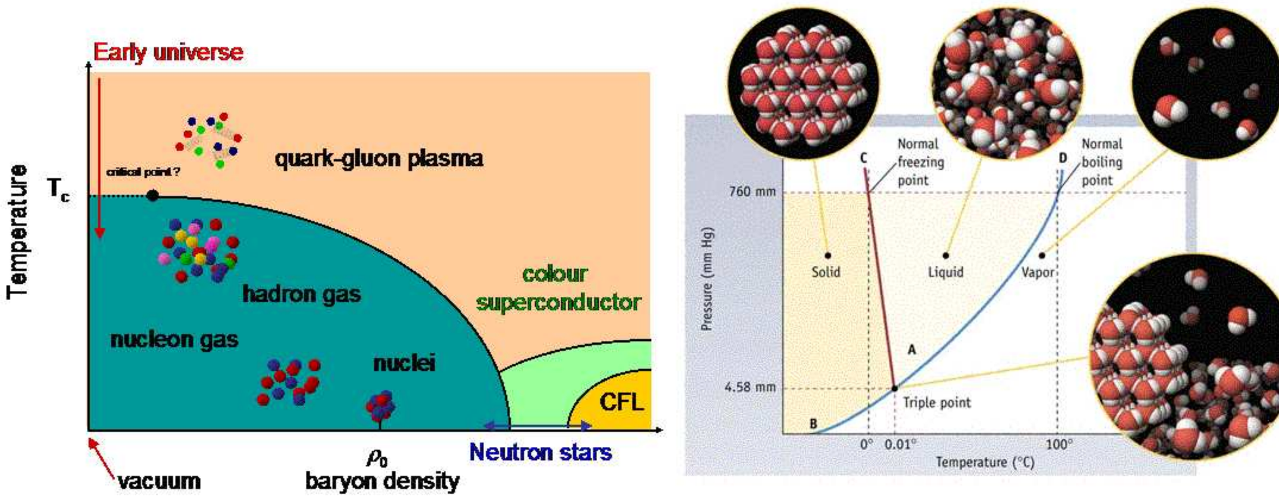

FIG. 2 Phase diagrams of strongly-interacting matter (left) and of water (right). As the pressure and density are varied, the matter passes into different phases (states) in which the fundamental degrees of freedom of the system vary. See text for details.

III. SIZE MATTERS

An experimental study of bulk quark matter requires creation of a color deconfined system in the laboratory, as well as experimental signatures which signal its creation and properties. According to our present understanding, this first condition requires huge collision energies of the magnitude generated at RHIC (1). This is the "relativistic" in relativistic heavy ion physics.

Here, I briefly discuss why size matters in achieving the goals we have outlined above; this stresses the "heavy" in relativistic heavy ion physics. In anticipation of the next Section, I also try to give an idea of the magnitude of the sizes I am talking about.

A. Size: the distinguishing feature of RHI physics

From the discussion of the previous Section, it is clear that any understanding of the phases of a system requires the creation and study of *bulk* matter. We would never discuss freezing a single water molecule!– one must have have a large system of molecules to discuss ice or steam.

The feature distinguishing RHI from high energy particle physics, then, is that the former aims, for the first time, to use the tools of condensed matter physics (phase diagrams etc.) to probe the Strong force. Particle physicists want to generate as "clean" a process as possible, and so usually collide the simplest possible particles (e.g. electrons); this is so that they can identify the elementary process governing the initial collision between the particles. RHI physicists focus more on the *system* created, and how it evolves. For us, size matters.

The feature distinguishing RHI from "traditional" nuclear physics is that the latter focuses on specific properties of the nuclei under investigation. This extremely interesting field studies, for example, why Uranium fissions spontaneously upon capture of a neutron, or how nuclear reactions take place in stars. In RHI physics, the nuclei are simply a tool to create a large system so hot that any distinguishing details of the colliding nuclei are irrelevant. In such a case, we hope to be able to use the tools of thermodynamics (e.g. to characterize the system with a temperature and density) to discuss a generic Strongly-interacting system, without reference to specific energy levels or so.

B. Big and Small

So, RHI physics aims to study "big" systems. But "big" relative to what? Well, big relative to the confinement scale– that is, the size of a proton whose radius is about 1 fm. To put things into perspective, let's look at the sizes of some systems around us:[7]

- Diameter of Andromeda galaxy: 10^{21} m = 1,000,000,000,000,000,000,000 m = 1 Zettameter
- Diameter of the Sun: 10^9 m = 1 gigameter
- Diameter of the Earth: 10^7 m = 10 megameters
- Diameter of the RHIC collider: 10^3 m = 1 kilometer
- Height of a person: 10^0 m = 1 meter
- Diameter of typical amoeba: 10^{-4} m = 100 micrometers
- Length of typical virus: 10^{-8} m = 10 nanometers
- Diameter of an atom: 10^{-10} m = 100 picometers (= 1 Angstrom)
- Size of hot system created at RHIC: 10^{-15} m = 0.000000000000001 m = 1 femtometer

The young scientists among you are increasingly comfortable with the SI (Système International) prefixes used above– mega = 10^6 etc. However, do not grow so comfortable as to grow numb to what they convey! They encode a huge diversity of scales which should not be taken casually. For example, according to the list above, we are much smaller than the Sun– 10^9 = 1 billion times smaller. However, this is *nothing* compared to how small is the source created at RHIC relative to us– it is 10^{15} = 1 million billion times smaller. To imagine that we can get an experimental handle on such a range of scales should amaze you. It does me.

C. Non-trivial Collision Geometries– Impact Parameter and Reaction Plane

Besides geometrical considerations in the *final* state (i.e. when the hot system is formed and evolves), we need to consider the geometry of the *initial* state. The necessity for such considerations is again a feature special to heavy ion collisions.

The two nuclei about to collide (let us call them A and B) move along the beam- or z-direction. The directions transverse to the beam we label x and y. The two nuclei may hit head-on, so that each nucleus overlaps entirely with the other in the collision. However, they may also experience a glancing collision, as illustrated in Figure 3. We define the *impact parameter* $\vec{b}$ as the vector connecting the centers of A and B and perpendicular to the beam direction.

The magnitude of the impact parameter $|\vec{b}|$ determines what fraction of A will overlap B; this overlap region is indicated in red on the right-hand side of Figure 3. For a head-on collision, $|\vec{b}| = 0$ and that fraction is 100%. At RHIC, the nuclei are moving at 99.995% the speed of light; at these energies, the non-overlapping portion of the nuclei are barely influenced by the collision, and continue down the beam direction. In the jargon of the field, the protons and nucleons who do not collide are called "spectators," while those in the overlap region are called "participants."

We are interested in what happens in the overlap region, as it is here where the energy density and temperature may be high enough to form color-deconfined matter. Clearly, the size and shape of this region depends on the impact parameter– a larger $|\vec{b}|$ means a smaller and less-round region in the $x - y$ plane.

The impact parameter also has a direction $\hat{b}$ associated with it, as shown in the Figure. The z-direction and $\hat{b}$ together define the so-called *reaction plane*. The deformed shape of the participant zone has mirror symmetry with respect to the reaction plane. In particular, the zone is deformed with a greater spatial extension out of the reaction plane than in it.

It is worthwhile to point out that we cannot "aim" the speeding nuclei at the femtometer level, to generate collisions at a given impact parameter. Instead, nuclear beams are directed towards each other, and the impact parameter varies at random from one collision to the next.[8] One of our jobs as experimentalists is to estimate $\vec{b}$ after the fact. This is discussed further in Section V.B.

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[7] Here, factors of 2-5 are ignored.

[8] To be precise, the *direction* varies randomly, while the magnitude $|\vec{b}|$ fluctuates in accordance with geometrical probabilities.

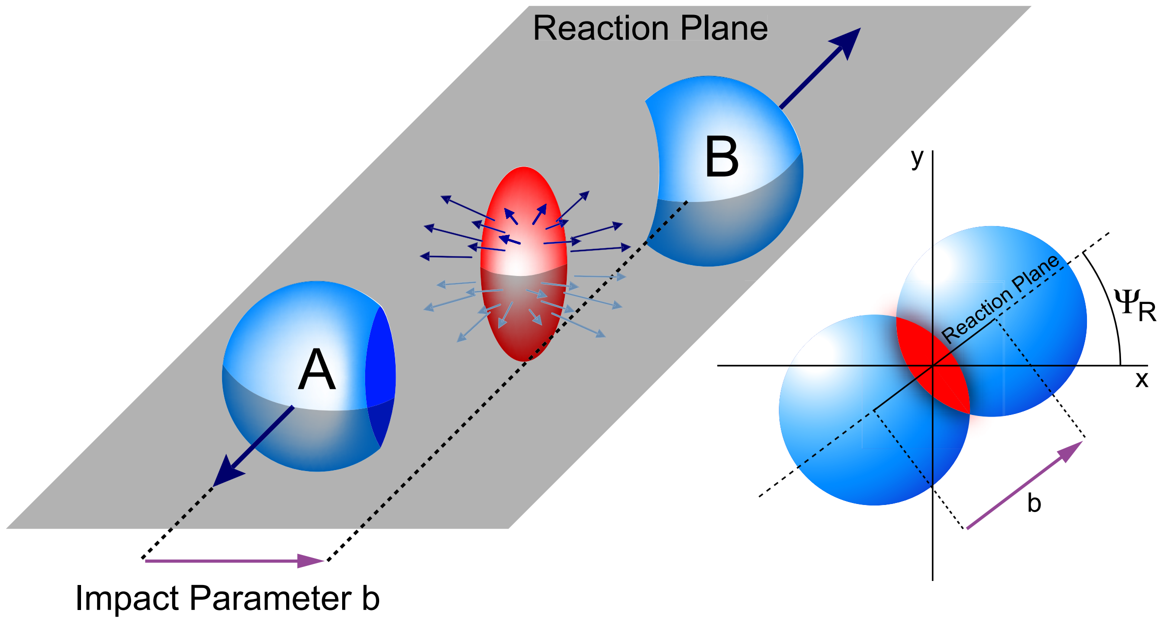

FIG. 3 The collision geometry of two colliding nuclei A and B. The nuclei only partially overlap in the collision leading to a small, deformed participant zone (red), and the spectators (blue) continue in the beam direction. The impact parameter vector $\vec{b}$ and the beam direction define the reaction plane of the collision. See text for details. Figure courtesy of Thomas Ullrich.

D. Slow and Fast

In addition to the extent of the hot system in *space*, we may also investigate its extent in *time*. In particular, as discussed later, we might expect that if the system forms a deconfined state, it will "live" longer before decaying. Similarly to the exercise in Section III.B, I compare the "long" times we measure to other physical processes:

- Age of the Universe: 15 billion years = 10^{17} s =100,000,000,000,000,000 s = 100 petaseconds
- Age of homosapien species: 10^{13} s = 10 teraseconds
- Typical human life: 10^9 s = 1 gigasecond
- Length of this lecture: 10^3 s = 1 kilosecond (though it may seem longer)
- Typical breath: 10^0 s = 1 second
- Very fast flash exposure: 10^{-7} s = 100 nanoseconds
- Pentium™ clock cycle: 10^{-9} s = 1 nanosecond
- Oscillation period of atomic clock : 10^{-10} s = 100 picoseconds
- Lifetime of source at RHIC: 10^{-23} s = 0.00000000000000000000001 s = 10 yoctoseconds[9]

Again, I am trying to impress you with the incredibly short times we claim to measure. Surely, the age of the Universe is much longer than the time for one breath– 10^{17} = a hundred thousand trillion times longer. However, that same breath is 10^{23} = a hundred *billion* trillion times longer than the the lifetime of the system we create at RHIC!

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[9] The prefix "yocto" is the smallest of the recognized SI prefixes. Neither I nor any of my colleagues to whom I spoke had even heard of this prefix before I prepared this talk. In the field, we typically use 1 fm/c = 3×10^{-24} s as a time unit.

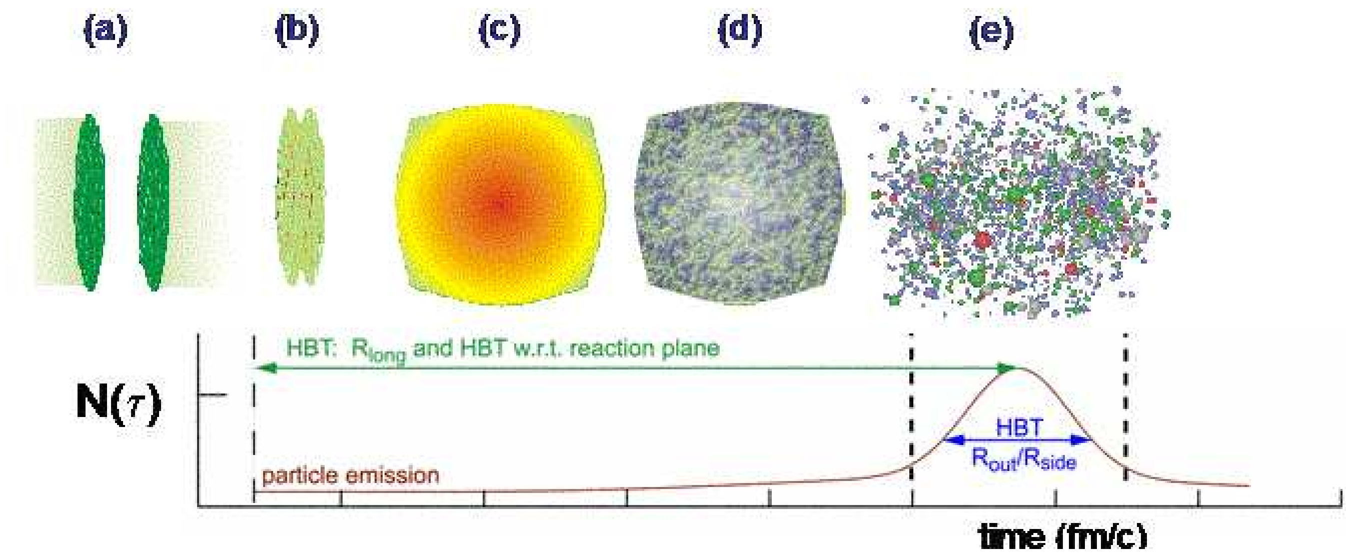

FIG. 4 Schematic sketch stages of a heavy ion collision as a function of time (top) and the time distribution of hadron emission (bottom). Graphics courtesy of Stephen Bass.

IV. GEOMETRICAL CONNECTIONS TO THE UNDERLYING DYNAMICS

I have discussed how space-time geometry is a special consideration of heavy ion collisions– that one requires "large" systems, which have non-trivial geometric configurations. By estimating the impact parameter of the collision (see Section V.B), we can calculate this *initial* geometry. But we are interested in the system formed *after* the initial overlap of the colliding nuclei: *is* it in fact a "system" which may be described by bulk properties (temperature, pressure, etc)?; what *are* those bulk properties?; how does the system evolve back to confined matter?; are there any remnant signatures in this final confined state which tell us that a deconfined state was indeed achieved during the collision?

Here, I briefly discuss some aspects of these questions.

A. Evolution of a RHI Collision

In the top of Figure 4 is plotted a schematic sketch of the time evolution of a RHI collision. Panel (a) shows two nuclei approaching each other near the speed of light. (They look like pancakes instead of spheres because of the so-called length contraction along the direction of velocity due to special relativity.) The collision starts when they begin to interpenetrate (panel (b), $t = 0$) and collisions between the protons and quarks within them deposits energy into the participant zone. The hope is that a thermalized system (panel (c)) is formed as the scattered and created particles subsequently collide with many *other* particles in the participant zone, so that energy is distributed statistically; this perhaps is a deconfined state of matter. Even if it is deconfined matter, the system will expand and cool and "hadronize"– that is, form white hadrons like protons and pions; this not-well-understood process is indicated in panel (d). Finally, these hadrons may continue to collide with each other. After a hadron's final collision, we consider it to be "emitted," and it proceeds undisturbed to our detector. The collection of points in space and time at which particles are emitted is often called the "freeze-out space-time distribution."

In the bottom of Figure 4 is sketched the number of hadrons emitted as the system evolves. In the early phases of the collision (panels (a)-(c)), one expects relatively few particles to escape the dense medium. After hadronization, particles will begin to be emitted. We crudely characterize the time distribution of this emission by some wide peak in time. The time until the peak (indicated by the green arrow in Figure 4 may be called the system "lifetime" or evolution duration, while the width of the peak (blue arrow) is the disintegration duration. I should point out that while this emission time distribution may be calculated in theoretical models, it is not possible to measure directly with any type of very fast stopwatch or so. As discussed below, however, we *can* get a handle on timescales characterizing the distribution. That is, we can put approximate numbers on the x-axis at the bottom of Figure 4.

Due to the complex nature of RHI physics, a truly rigorous "theory" of the entire collision is as yet not at hand. Instead, one constructs "models" of the collision, which hopefully describe the important physics associated with a given measurement. There are many such models in the field; here, I just mention on one of the most popular and successful which concentrates on the evolution of the bulk medium– hydrodynamics. A nice recent overview hydrodynamic models at RHIC can be found at (2).

As the name suggests, hydrodynamical models treat the hot system created in a collision as a fluid. This implicitly assumes that we have a "thermalized" system which can be discussed in terms of *bulk* (thermodynamic) quantities such as density, temperature, etc– i.e. that we do not need to follow the motion of each particle to characterize the driving physics. Hydrodynamic models actually reproduce a large range of experimental data at RHIC; while this does not *prove* the validity of the thermalization assumption, it does provide strong support of it. Unambiguously proving thermalization turns out to be a challenging (probably impossible) proposition. While this topic remains a hot (so to speak) issue in the field, most evidence points in the direction of thermalization, and I will not discuss it further here. Importantly, hydrodynamical calculations need to assume a relationship between quantities like temperature and pressure. This so-called "Equation of State" is a fundamental property of any material, describing, for example, the amount of pressure required to compress a water droplet by 10%. It also carries information about what degrees of freedom describe the system and about the nature of the interaction between parts of the system.

B. System Size and Lifetime

As discussed in Section II, if the system goes from a confined to a deconfined state, the number of degrees of freedom increases. In a thermalized system, this implies an increase in "entropy." Entropy is a quantitative representation of the degree of disorder in a system; alternatively, one may think of it as the length of a written list of all the possible configurations of the system. A law of thermodynamics is that in any physical process, the entropy may not decrease. Thus, even though we do not much understand hadronization, we can be sure that the increased entropy generated in the transition to the deconfined state survives as the system returns to the confined state. One way that this entropy had been expected to show up in RHIC data was in the observation of a "large" emission source (3). Here, "large" means large compared to RHI collisions at lower energies, where a transition to QGP– and the generation of increased entropy– had not taken place.

Other consequences of the generation of a deconfined state were expected in the characteristic timescales indicated in Figure 4. If energy is deposited into a droplet of hadrons, much of it goes into generating outward pressure; this pressure causes the system to expand and disintegrate quickly. On the other hand, if there is just sufficient energy to cause a transition to a deconfined state, any additional energy goes into the melting of the hadrons; at this point, the pressure drops sharply, the system does not expand quickly, and thus "lives" longer before disintegrating (i.e. freezing-out). Hydrodynamical calculations of the system disintegration time with and without a transition confirm this picture, though the magnitude of the effect depends on the details of the calculation (4).

C. Signs of Collectivity - Space-Momentum Correlations

As we have said, it is pointless to discuss pressure, or a "state of matter" or the size of "the system" if there *is* no system. Figure 5 shows two possibilities for the participant zone in a non-head-on heavy ion collision. In one (a), the protons (or the quarks within the protons) from one nucleus collide with the protons (or quarks) of the other, and the particles emitted from these collisions make their way undisturbed out of the participant zone. The other (b) is the result of a hydrodynamical calculation (2), which assumes that the energy deposited by the initial collisions is somehow distributed throughout the participant zone, and the zone becomes really a *system*. One way to see the distinction is that, in (b), particles emitted from a position on the right-hand size of the participant zone "know" that they are emitted from that region– it is reflected in their velocity direction (which is indicated by the small arrows). In (a), this is not the case: the velocity of a particle emitted from a given proton-proton collision does not depend on where (or if) the other proton-proton collisions occur.

The connection between a particle's emission point and its velocity which we observe in scenario (b) is called a "space-momentum correlation."

If scenario (a) holds, the game is up: we have no "system," no large length-scale deconfinement, no state of matter to discuss. Indeed, there was some fear of this before initial data was taken at RHIC. Fortunately, reality appears to favor scenario (b). In the following, I mention two reasons– intimately connected to geometry– why we think this.

1. Elliptic Flow

If we believe we have created a thermalized system (by statistical distribution of collision energy via rescattering or whatever), then basic understanding of the initial collision geometry and hydrodynamics tells us that the pressure at

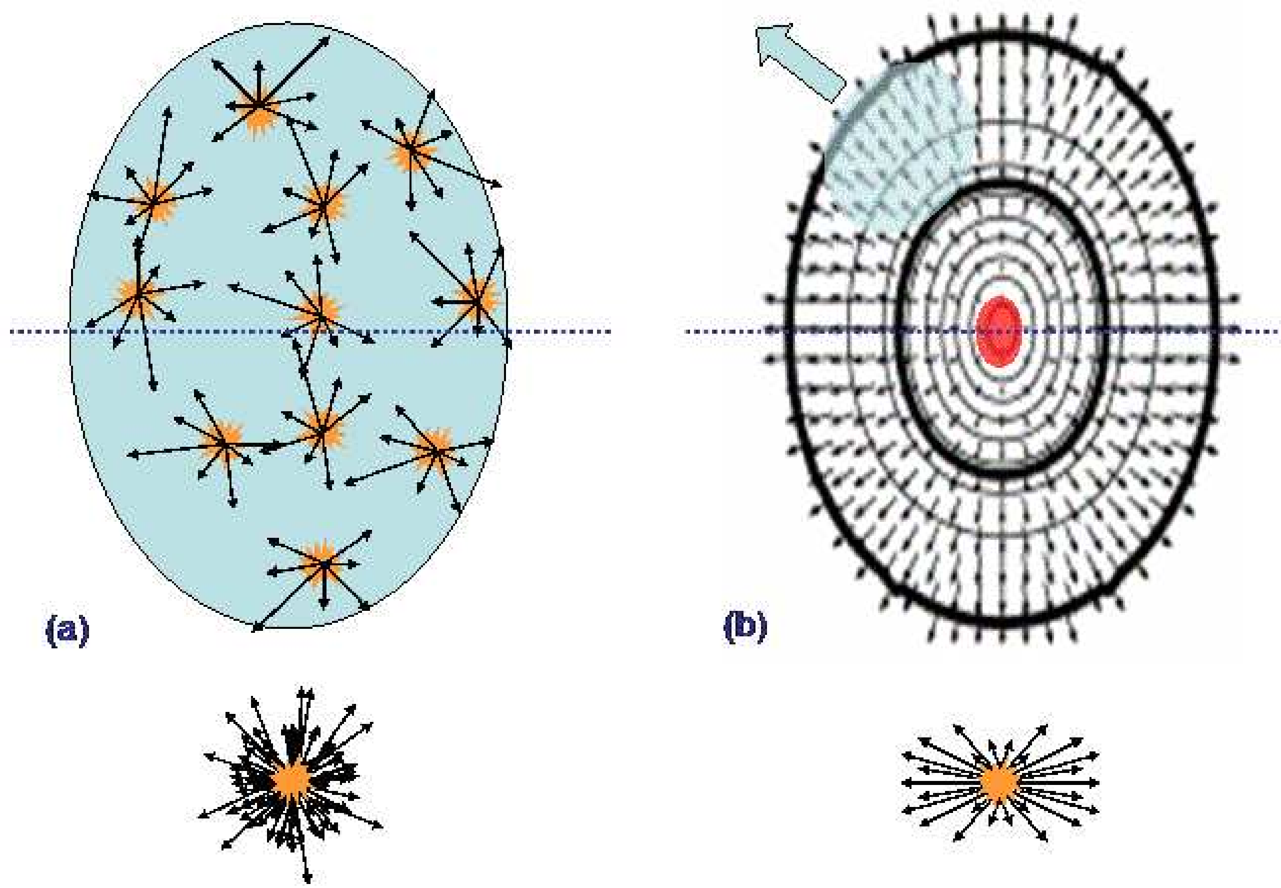

FIG. 5 Two possible pictures of the participant zone of a heavy ion collision, as seen along the beam axis. On the left, the zone is comprised of individual proton-proton collisions, each of which emit particles, which to not interact further, in all directions from all space points. On the right is the result of a hydrodynamical calculation (2). Here, a bulk system is formed, leading to an ordered correlation (suggested by the small arrows) between spatial emission point and the direction of emission. Shaded blue regions indicate those areas which emit particles in the "upper-left" direction. (For the left-hand figure, the entire participant zone is shaded.) In both cases, the reaction plane is indicated by the blue dotted line. Bottom figures indicate momentum distributions, with spatial information removed. See text for details.

the center of the participant zone is very high (suggested by the red region in panel (b) of Figure 5) and the pressure is zero outsize the participant zone. In between, the pressure drops steadily as one moves outward from the center.

If the pressure is greater in one region than another, there is a force pushing towards the low-pressure region. We may think of a tank containing compressed gas; there is an outward force on the walls of the tank. The larger this change in pressure, the larger the push. Furthermore, the net push is greater when the pressure change occurs over a shorter distance.

With this in mind and looking at panel (b) of Figure 5, we immediately realize that there is an overall outward "push" on elements of the system. Further, we see that the push is greater on those elements *in* the reaction plane (left/right of center in the Figure) than on those *out* of the reaction plane (above/below the center in the Figure).

So, if a system is created, we will measure a very "non-round" momentum distribution – more particles will be emitted in some particular direction (and opposite that direction) than at right angles to that direction, as suggested in the bottom part of the Figure.[10] From this distribution, we estimate the reaction plane for each event (see below).

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[10] For a detailed understanding of this anisotropic distribution, it turns out that there are *two* important geometric effects producing the anisotropy: Firstly, the geometric anisotropy in pressure leads to a greater push in the reaction plane, so that *faster* particles are emitted in-plane. Secondly, due to the geometric anisotropy and the fact that the particles are pushed "outward," *more* particles are emitted in-plane. See e.g. (5)

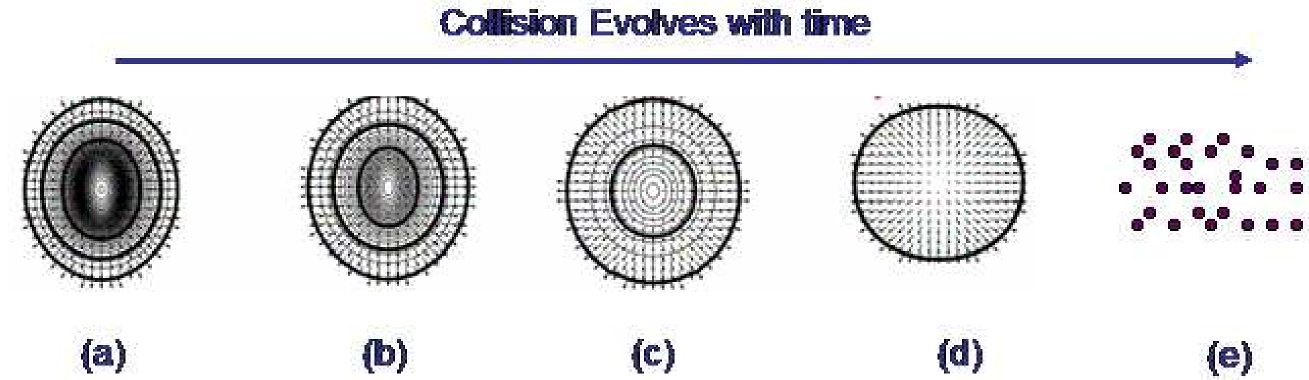

FIG. 6 Panels (a)-(d) show a hydrodynamic calculation of the evolution of the participant zone as a function of time (2). Panel (e) is a cartoon, motivated by reference (6), of the possible geometry after a non-hydrodynamic hadronic rescattering phase. See text for details.

The collective motion generating these space-momentum correlations and non-round momentum distributions is called "elliptic flow," which we discuss further below. The magnitude of the elliptic flow is an indication of the pressure generated in the collision.

2. Size, Shape, and Homogeneity Regions

Clearly, elliptic flow is a measure of collectivity intimately related to geometry. However, we do not actually *measure* the geometry of the system with elliptic flow. Instead, the collision looks like the lower panels of Figure 5. We only measure non-round *momentum* distributions, not space-time geometry. The system is far too tiny to "trace back" the particles' velocities to different regions of the emitting region.

What would be some more direct probes of the geometric consequences of collectivity?

Well, the first would clearly be an increase in the overall size of the system. If there is indeed an outward flow of the system, and if the system "lives" for a while, then the system will grow during the stages (b) to (e) in Figure 4. So, in addition to the entropy discussion of Section IV.B, we have another reason to look for "large" system size at freeze-out, especially given the lifetime arguments of the same Section.

Another consequence would be a change in shape of the system. Depending on the magnitude of the impact parameter, the participant zone is initially (at overlap of the colliding nuclei) quite geometrically deformed, with the small axis of the ellipse in the reaction plane (c.f. Figure 5). According to the previous Section, the pressure gradients cause the system to expand more in the in-plane direction, so, as the collision evolves, the system will get more round, and may even become deformed such that the in-plane axis is longer than the out-of-plane one. See Figure 6 for a hydrodynamical calculation of the system evolution as a function of time. It is reasonable to assume that after the hydrodynamical stage of the collision has ended and emitted hadrons, these hadrons will continue to interact for a while. In this case, the system may evolve from an initially out-of-plane extended shape (panel (a)) to an in-plane one (panel (e)) (6).

Both the overall expansion and shape change mentioned above are indirect geometric measures of collective flow. They may be generated either by large flow or long lifetimes (or both). A more direct geometric measure of collective flow is by measuring the size of so-called "homogeneity regions" (7), discussed below.

Imagine that we only look at high-momentum particles which are emitted from the system "up and to the left" as indicated by the blue-shaded regions in Figure 5. In scenario (a), such particles will come from many of the individual proton-proton collisions– the "up-left" emitting region is in fact the entire participant zone. In scenario (b), however, the space-momentum correlations mean that up-left-going particles are emitted mostly by regions geometrically up-left. Furthermore, slow particles will be emitted from a larger region than faster ones, which come mostly from small regions near the edge of the source. Regions which emit particles with a chosen momentum are called homogeneity regions (7). If we could measure the size (and shape) of these homogeneity regions for many momenta, we would have a good measure of collectivity of the system.

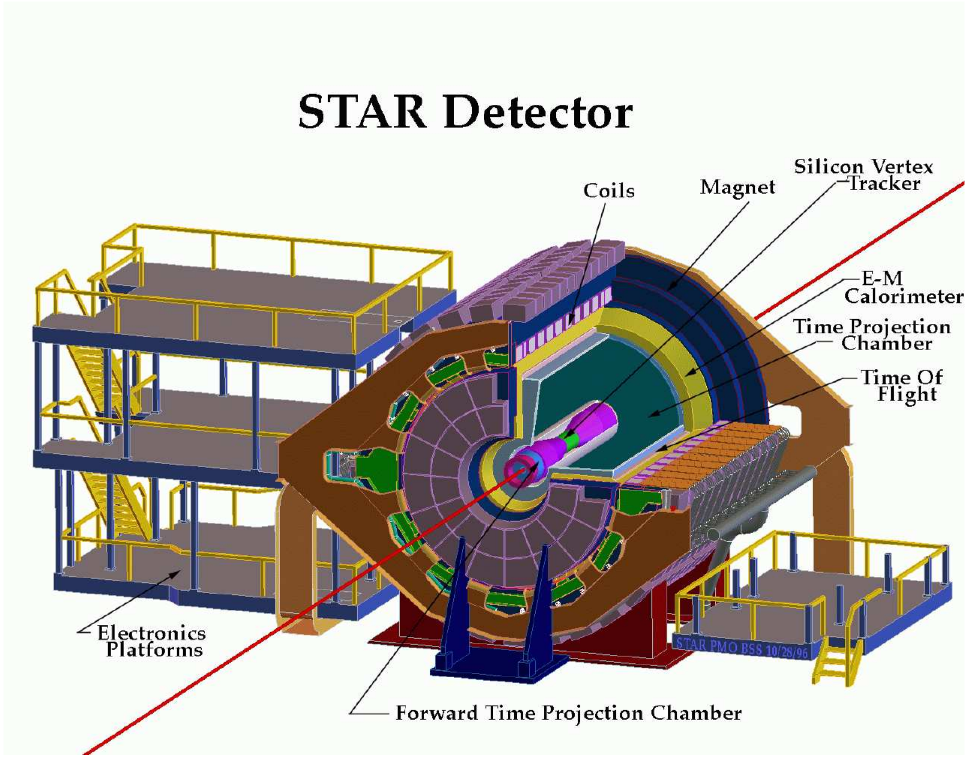

FIG. 7 A cartoon of the STAR detector at RHIC. Various detector subsystems surround the collision in a cylindrical geometry. The colliding beams run along the axis of the cylinder.

V. EXPERIMENTAL METHODS

In one collision at RHIC, thousands of particles are produced. Two large (about 500 people each) experimental collaborations and two smaller ones operate massive detector systems as big as a house to catch many of these particles. Reconstructing the particle momentum and identity (i.e. is a measured particle a proton or a pion?) from the signal pattern the particles generate is a huge job. Equally challenging is the analysis effort to sift through all of this information and extract physical information about what is happening in the collision. Here, I just touch upon a few of the experimental issues directly relevant to our discussion.

A. Very Brief Description of the STAR Experiment

Due to time constraints, I here give only the briefest taste of one part of one experiment at RHIC. The details of each of the experiments and their physical set-up are simply fascinating, and I encourage everyone to take advantage of the many tours offered by the collaborations on-site at Brookhaven. For much more information on the detectors, see reference (8).

The STAR (Solenoidal Tracker At RHIC) collaboration is comprised of about 500 scientists and engineers from 13 countries on four continents. The detector systems, built over the past decade and still being upgraded, are shown schematically in Figure 7. There, the solenoidal (cylindrical) geometry of the 3-story-high experiment is clear, with the two colliding nuclear beams running along the axis of the cylinder and colliding in the center. Many sub-detectors surround this collision point, measuring different aspects of the emitted particles.

At the heart of STAR is the Time Projection Chamber (TPC) which measures, in three dimensions, the paths of particles emitted from the collision as they fly at close to the speed of light through the detector. The TPC itself is a gas-filled cylinder. The particles flying through it plow through the gas, separating electrons from the gas atoms and thus leaving a 3D tracks much like the contrail left by an airplane. With an electric field, a contrail is slowly drifted towards the ends of the TPC, where it is sensed, digitized, and recorded to tape. This information is later analyzed to reconstruct a full 3D picture of each collision. An example is shown in Figure 8. In the "side view" on the left, we see most of the several hundred tracks pointing towards the collision point along the beam axis. Careful examination of the "end-on view" on the right shows that the tracks are curving somewhat. This is due to the presence of a large

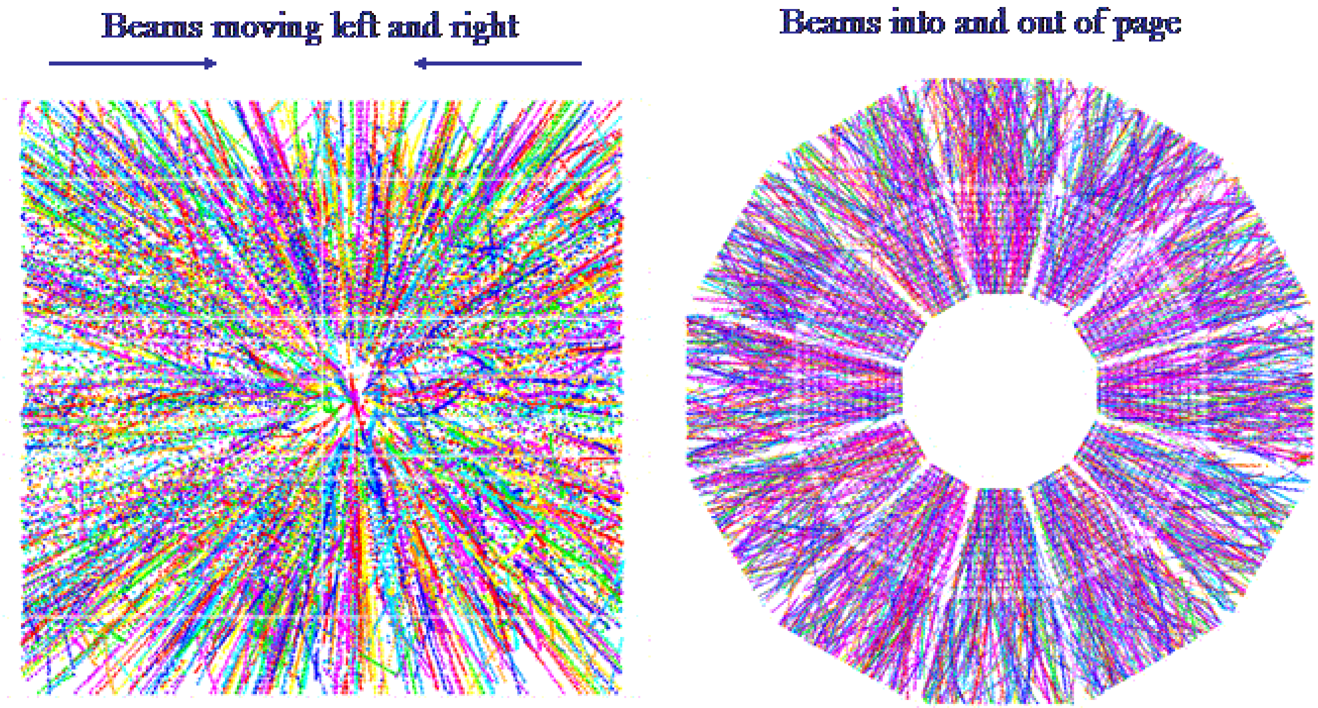

FIG. 8 Particle tracks from one gold+gold collision reconstructed in three dimensions in the STAR TPC.

magnet surrounding the TPC and helps us identify the particle leaving a track– particles with negative electrical charge bend one way and with positive charge bend the other way.

This is a criminally brief description of a complex experiment. See (9) for more information on the STAR TPC and (8) for details of all RHIC experiments. From now on, I will focus on how we extract physics from the particles which we reconstruct.

B. Estimating the Impact Parameter Vector

As discussed in Section III.C, the impact parameter vector determines the geometric size, shape and orientation of the participant zone of a collision. Since we cannot control $\vec{b}$, we must estimate it for each measured collision "after the fact."

To estimate the magnitude of the impact parameter, $|\vec{b}|$, we use the fact that head-on collisions result in a greater number of emitted particles than glancing ones. The number of particles emitted from a collision is also referred to as the *multiplicity* of the collision. Figure 9 shows a model calculation (which looks very much like the experimental distribution) of the multiplicity on the x-axis versus the likelihood of an event on the y-axis, as well as cartoons indicating the fraction of the colliding nuclei which overlap. Thus, simply by counting the number of tracks in the TPC, we estimate $|\vec{b}|$. Figure 8 is a picture of a small-$|\vec{b}|$ ("central") collision; a large-$|\vec{b}|$ ("peripheral") collision would have just a few tracks.

Once we know $|\vec{b}|$, we also know the initial *shape* of the participant zone; a central collision has a rounder geometry than a peripheral one. However, simply counting tracks does not tell us the *orientation* of the zone (i.e. where does the "long" direction of the participant zone in Figure 3 point in the STAR TPC– up? sideways?); for this, we need the direction $\hat{b}$. We estimate this direction by using the fact that elliptic flow (c.f. Section IV.C.1) gives us more particles in the reaction plane than out of it. The azimuthal direction with the most particles is approximately $\hat{b}$.

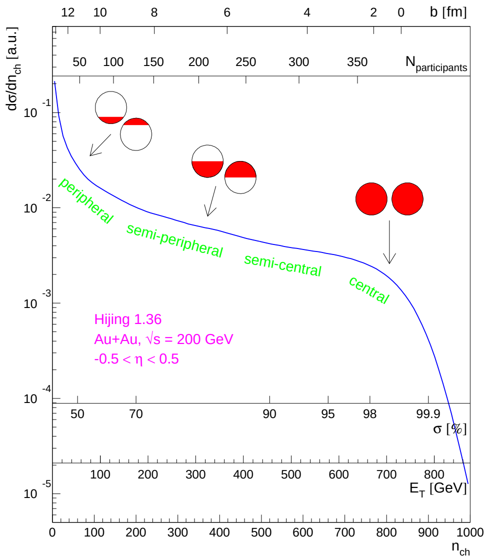

FIG. 9 Model calculation of the event multiplicity distribution for collisions of various centrality. See text for details. Figure courtesy of Thomas Ullrich.

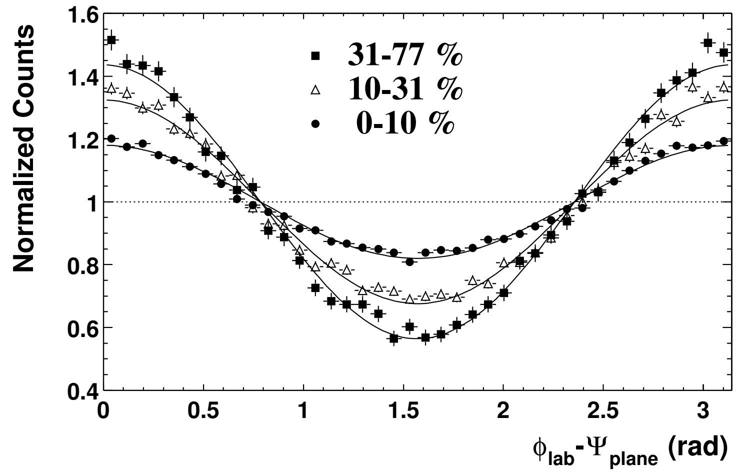

FIG. 10 Distribution of particle momentum relative to the reaction plane for events of three different centrality measured by STAR (10).

C. Measuring Elliptic Flow

Well, once we have identified the direction of $\hat{b}$, we can simply look at how *many* more particles are emitted in the direction of the reaction plane than out of it. This gives a quantitative measure of elliptic flow.[11]

Figure 10 shows a measurement by the STAR collaboration of the azimuthal angle of emitted particles relative to the reaction plane of the event. Results for three values of $|\vec{b}|$ are shown. In all cases, we observe more particles emitted in the direction of $\hat{b}$ (i.e. at 0° =0 radians and 180° = 3.14 radians) than at right angles to it (i.e. at 90° = 1.5 radians). As we would expect, central collisions (labeled "0-10%"), which have a rounder initial geometry, show a smaller effect than the most peripheral collisions (labeled "31-77%").

Numerically, we quantify the magnitude of the anisotropy with a quantity called v_2. Essentially, it is the difference between the number of particles measured at 0° and the number at 90°.[12]

D. Extracting the Source Size

We have said that we want to measure homogeneity regions as small as a few femtometers (10^{-15} m) with detector systems of the size of several meters. Clearly this involves techniques quite different from, e.g electron microscopy which images surfaces with Angstrom (10^{-10} m) resolution using tiny tips on the same Angstrom scale.

The technique we use to extract these tiny source sizes is called intensity interferometry, a method invented for use in radioastronomy by Robert Hanburry-Brown and Richard Twiss in the 1950's. Their goal, measuring the diameter of large objects (stars), may seem quite different from our goal of measuring small ones. However, since stars are so far away, the *angular* size, from our standpoint, is incredibly tiny. And it is really this angular size which Hanburry-Brown and Twiss measured. Even in particle or RHI physics, intensity interferometry analyses are often called "HBT" analyses, in honor of these pioneering scientists.

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[11] The astute reader may realize that there is some danger in using an anisotropy in the momentum distribution to identify the reaction plane, and then measuring the momenta of particles relative to that very reaction plane. Issues of this sort are often called "autocorrelations" and may indeed skew measurements. Sophisticated methods to avoid such problems have been developed in the field, and indeed the short description of elliptic flow measurements in the text above is a simplified version of actual practice.

[12] To be precise, v_2 is the second-order Fourier component of the azimuthal distribution. The distributions in Figure 10 are approximately described by $\frac{dN}{d(\phi_{\rm lab}-\Psi_{\rm plane})} = (1 + 2v_2\cos(\phi_{\rm lab} - \Psi_{\rm plane})$.

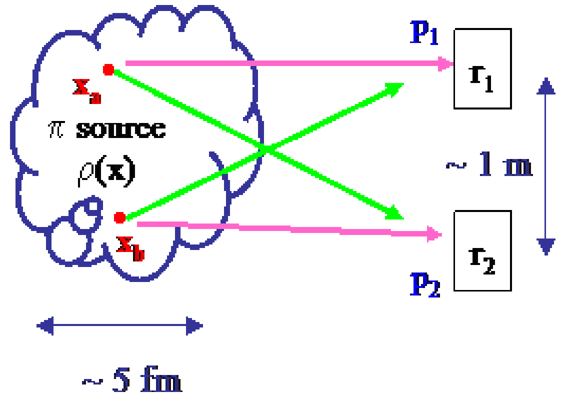

FIG. 11 Schematic illustration of two pions emitted with momenta p_1 and p_2 measured at detector positions r_1 and r_2. They are emitted from spacetime points x_b and x_a. Pink lines correspond to the process in which particle 1 is emitted from position x_a, green lines indicate it was emitted from position x_b. See text for details.

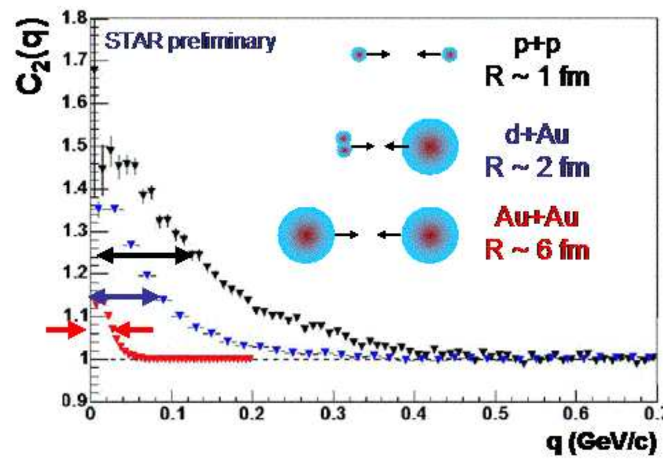

FIG. 12 Preliminary correlation functions measured by STAR at RHIC for the colliding systems of proton+proton (black), deuteron+gold (blue) and gold+gold (red). The width of the correlation function (indicated by the colored arrows) is big for proton+proton collisions, indicating a small source, while the gold+gold collisions yield a narrow correlation, hence a larger source. From reference (11). See text for details.

I give next a basic mathematical motivation of the technique we use, involving simple quantum mechanical concepts. In the spirit of this lecture, however, the reader may feel free to jump to Section V.D.2 for the "bottom line."

1. HBT Motivation

Imagine some region emitting two particles; particle 1 is emitted from spacetime point x_a with momentum p_1, and particle 2 from x_b with momentum p_2. Once emitted, we imagine that they travel undisturbed (i.e. p_1 and p_2 don't change) to some device which detects the particles at r_1 and r_2. This is suggested by the pink lines in Figure 11. Since the particles travel undisturbed (an assumption which is not usually true, but we account for this in practice) the wave function of each particle is a plane wave, and the 2-particle wave function for the whole process is

$$\psi \sim U(x_a, p_1) \cdot e^{i(r_1 - x_a)\cdot p_1} \cdot U(x_b, p_2) \cdot e^{i(r_2 - x_b)\cdot p_2} \tag{1}$$

However, we do not directly measure x_a and x_b. Furthermore, the particles we are detecting are both photons (for Hanburry-Brown and Twiss) or same-charge pions (in our case). We are thus measuring bosons which are identical in the quantum mechanical sense, and we must symmetrize the wave function for the two-particle process with respect to "a" and "b". In this case:

$$\psi_{\text{identical}} \sim \frac{1}{\sqrt{2}} \left[U(x_a, p_1) \cdot e^{i(r_1 - x_a)\cdot p_1} \cdot U(x_b, p_2) \cdot e^{i(r_2 - x_b)\cdot p_2} + U(x_b, p_1) \cdot e^{i(r_1 - x_b)\cdot p_1} \cdot U(x_a, p_2) \cdot e^{i(r_2 - x_a)\cdot p_2} \right]. \tag{2}$$

The second term in Equation 2 corresponds to the process suggested by the green arrows in Figure 11, in which the particle detected at r_1 originates at x_b.

Experimentally, we do not measure wavefunctions, but probabilities, which we calculate by squaring the wave function. Thus, after some rearrangement, the probability to measure the two particles is

$$P_2(x_1, x_2, p_1, p_2) \sim \psi^\dagger \psi \sim U_1^\dagger U_1 \cdot U_2^\dagger U_2 \cdot [1 + \cos(q \cdot (x_a - x_b))]\,. \tag{3}$$

This probability contains three parts. The first two, $U_1^\dagger U_1$ and $U_2^\dagger U_2$ represent the probability to emit particles 1 and 2 individually. The last part is an interference term containing $q \equiv p_1 - p_2$, which we measure, and $x_a - x_b$, which is what we want to extract. So, by dividing the two-particle probability by the single-particle probabilities, we end up with an experimentally-determined quantity as a function of relative momentum between the particles, whose only unknown is the two-particle *spatial* separation upon emission!

Using a little more formalism and some assumptions, one discovers that if one constructs, as a function of relative momentum q, a "correlation function" equal to the two-particle probability divided by the single-particle probabilities, there will be a "bump" at low q. The width of this bump is inversely proportional to the size of the homogeneity region.

I have given only a only a simplified description between the correlation function and the underlying spacetime geometry, but you have seen the essential point. For much more detail, see (12).

2. The Bottom Line

The bottom line is that, due to quantum mechanical interference between identical pions, if I observe one pion with momentum p_1 (this is one of the tracks in Figure 8), there will be an increased probability to observe *another* pion with momentum p_2 very close to p_1 in the *same* collision.

We measure the increased probability in a "correlation function" which is plotted versus the momentum difference $q = p_1 - p_2$. The correlation has a bump (corresponding to the increased probability) at $q = 0$. If the bump is wide (i.e. extends out to large q), then the homogeneity region is small; if it is narrow, then the homogeneity region is large.

Correlation functions have been measured for many types of collisions, and, in general, the results make sense. The source size is small when we expect it to be small (like when we look at peripheral collisions or collisions between small objects like protons) and large when we expect it to be large. Furthermore, the sizes come out about right– several femtometers. Figure 12 shows preliminary correlation functions measured by the STAR Collaboration for three different types of collisions: proton+proton; deuteron+gold (a deuteron is a type of hydrogen nucleus, about twice as big as a proton); and gold+gold. The sizes that one obtains by looking at the width of the bump are mostly as one expects– collisions between protons (who have a radius about 1 femtometer) give a source of about one femtometer size; deuteron+gold collisions produce a source about twice as big, and central gold+gold collisions have a size on order of 5 fm.

I want to point out again that this is simply amazing. Even ignoring the physics information we can obtain from these measurements, the fact that we can access these length scales in a way that seems to make sense is itself breath-taking. Above I briefly motivated what we think is the physics producing the correlation function. With a system and technique as complicated as the one at hand, a battle-hardened physicist would not be at all surprised if these measurements simply didn't work: he might reasonably fear/expect that there are complicating factors in the game, and that one would erroneously extract crazy sizes like centimeters or so.

3. More than "just" size – shape, lifetime and dynamics

As if the ability to measure the overall size of homogeneity regions at the femtometer scale were not amazing enough, we can actually get considerably more detailed information. This is essentially because we can measure the relative momentum between particles in all three directions. This means we have the length of the regions in each of the three directions.

So, which three directions should we choose? We can select them any way we want; it's just a matter of convenience and custom. One might choose to measure the homogeneity lengths in the z (beam), x (impact parameter), and y directions. This would work, but it is customary (and most useful, actually) to use the so-called "long," "out" and "side" directions. These are indicated in Figure 13. Since all of the energy is initially directed along the beam direction, it could well be that this direction is "special," so we single out "long" as its own direction. This leaves the two directions transverse to the beam. The "out" direction is taken to be along the direction of (transverse) motion of the particular pion pair, and "side" is then perpendicular to "long" and "out."

One nice feature of this parameterization is the fact that, if the pion-emitting source spits out pions over a long time (i.e. it has a long disintegration timescale), the effective homogeneity region gets larger in the "out" direction, but not in the "side" direction. This is sketched in Figure 13: panel (a) shows emission over a very short timescale, while in panel (b) the system disintegrates over a long time. So, by measuring some sort of difference or ratio between R_{out} and R_{side}, one gets a handle on this timescale. This is one of the important timescales mentioned in Section IV.B. In fact, one way the QGP had been expected to "show" itself was through an increase in the ratio R_{out}/R_{side} as the threshold energy for QGP production was passed. In Figure 14 are predictions of a hydrodynamical model for R_{out}/R_{side} in the case that a QGP is produced (thick lines) and in the case that it is not (thin lines). In the former case, the ratio is expected to rise as the energy threshold for QGP production is surpassed. There are several linestyles in the Figure, as well as two panels. These represent changing some of the various assumptions that go into the calculation. Thus, the *magnitude* of the expected increase is uncertain, but the *existence* of the increase is always seen in these calculations (4).

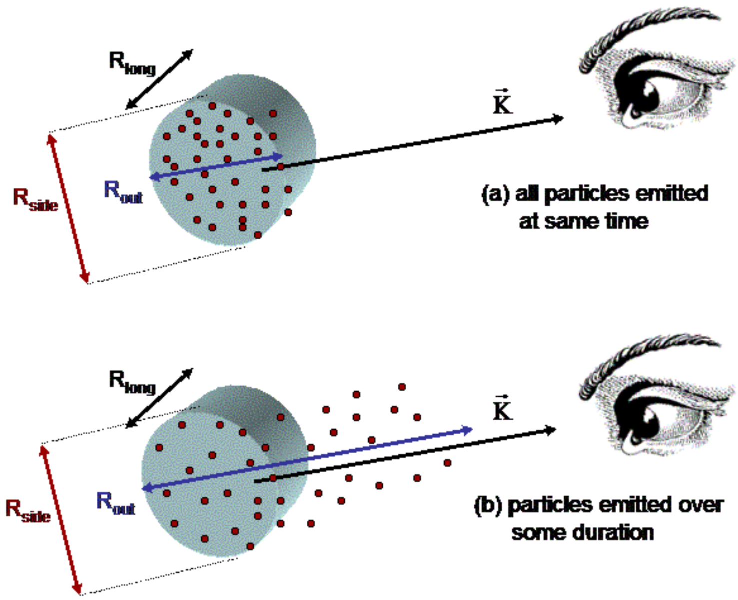

FIG. 13 **out-side-long caption here** The homogeneity lengths can be measured in three independent directions. By convention and convenience, these are chosen along the beam axis ("long"), along the transverse motion of the pions ("out") and perpendicular to these two directions ("side"). See text for details.

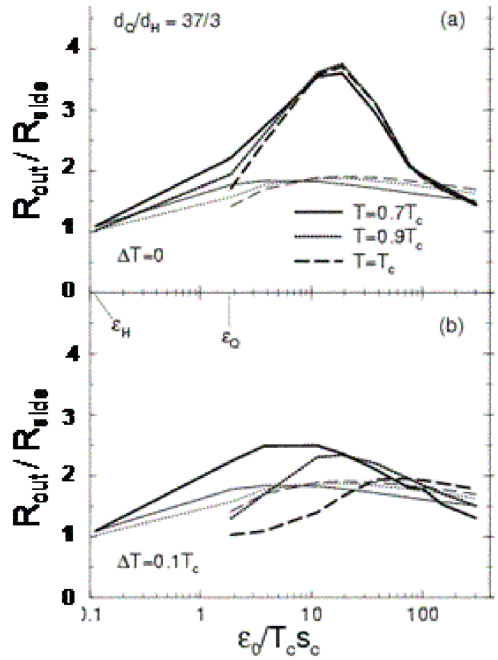

FIG. 14 Hydrodynamic predictions (4) of the ratio of homogeneity lengths R_{out}/R_{side} as a function of energy. Thick lines are for transition into QGP during the collision, while thin lines are for no transition. The various linestyles and the two panels represent various assumptions within the model.

Although I will not go into detail here, it is expected that R_{long} is connected to the other interesting timescale– the total evolution time of the collision from "bang!" (panel (b) in Figure 4) to disintegration (panel (e)). A long evolution time, expected if the system undergoes a transition to/from QGP, would lead to a large measured R_{long}.

In addition to size and time scales, with a three-dimensional analysis, clearly we can probe the *shape* of the homogeneity regions. For example, the homogeneity region in light blue in panel (b) of Figure 5 has a distinctly non-round shape– it is "squished" slightly against the wall of the source. As I mentioned, this effect is due to the nature of the collective flow, so getting a handle on the details of homogeneity region shapes reveals important information about collective dynamics of the system.

Looking further at the Figure, we also notice that the homogeneity region has a distinct "tilt"– the long axis of its ellipse makes an angle with respect to the reaction plane. By measuring these "tilts," we gain geometrical information about the *overall shape* of the source when it begins to emit particles. The relevance of this shape information to understanding the collision evolution was discussed in Section IV.C.2.

Finally, we know that collective dynamical effects ("flow") is a crucial feature which we'd like to study. As discussed in Section IV.C.2, we can do this by measuring homogeneity lengths as a function of particle momentum. In fact, by measuring the ways in which the size *and shape* of the homogeneity region vary with momentum, we extract details on the magnitude and microscopic nature of the collective flow which dominates the collision evolution.

Here, I have simply sketched out how our technique of measuring geometry can be extended to obtain this more detailed information, without going through all the math to prove it to you. For those who want more details, please see (12).

VI. GEOMETRICAL OBSERVATIONS AT RHIC

Now that we have discussed the ways in which we measure geometry and some of the physics information we can extract from it, I will only very briefly discuss a few of the geometrical results we have obtained at RHIC.

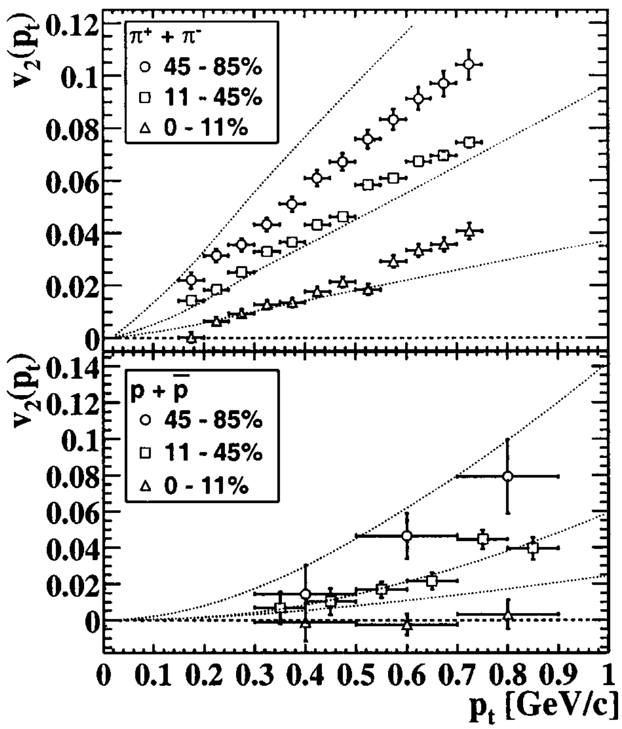

FIG. 15 The parameter v_2, which quantifies elliptic flow, as a function of the transverse momentum for pions (upper panel) and protons (lower panel), as measured by the STAR experiment. △: central collisions; □: midcentral collisions; ○: peripheral collisions. Lines show hydrodynamical.predictions for the elliptic flow.

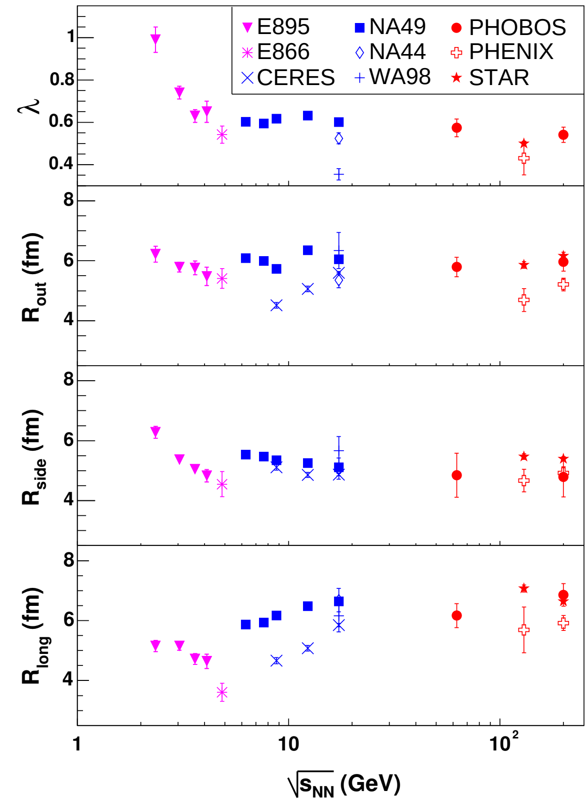

FIG. 16 Homogeneity lengths in "out," "side" and "long" directions extracted from three dimensional pion correlation functions from central Au+Au collisions, plotted as a function of collision energy. Data from RHIC experiments are in red. Blue and pink datapoints are results from other experiments at lower energy. From (14).

A. Elliptic flow

As discussed in Section IV.C, this is an important diagnostic tool to determine (a) whether we have indeed created a *system* in the collision, and (b) the magnitude of the pressure generated in this system (this gives insight into the Equation of State discussed in Section IV.A).

Thus, when one of the very first analyses of RHIC data showed clear evidence of very strong elliptic flow (13), it generated huge community interest. Elliptic flow at RHIC continues to be studied in ever-increasing detail, both theoretically and experimentally. Fully one quarter of STAR's 36 journal publications (as of Fall 2004) have been about elliptic flow!

We have already seen in Figure 10 a clear signal of elliptic flow in Au+Au collisions at RHIC, and as we discussed, the dependence on impact parameter makes intuitive sense. As mentioned in Section V.C, the effect is quantified by a parameter called v_2. This parameter is shown in Figure 15 for two different types of particles (pions and protons) for collisions with three different impact parameters. We see that v_2 increases with momentum, and is different for the different particle types; those are important details, but let's not worry about that. The main thing is that the elliptic flow is large. If $v_2 = 0.1$, this means that 50% more particles are emitted in the reaction plane than out of it; this is a huge effect!

Such a huge elliptic flow can only be explained if there is extremely strong self-interaction of the *system* in the early stages of the collision. Maximum self-interaction is one of the main components of hydrodynamics. Hydrodynamic calculations are shown in Figure 15 as lines, and are seen to agree fairly well with the data, except for the most peripheral collisions, for which too much elliptic flow is predicted. Failure to describe peripheral collisions, however, is to be expected, as the small reaction zone generated in those collisions do not allow collective behaviour to build up (size matters!).

The overall success of hydrodynamics to describe elliptic flow (and other measurements which I do not discuss) has generated hope that the system we produce *can* be understood in a theoretically consistent model. This was not the case when we had studied heavy ion studies at lower energies; thus, the success of hydro is seen by many as a welcome victory allowing us to peer through the collision evolution to the earliest times.

Finally, continued analysis confirms that not only can hydrodynamics explain the observed elliptic flow, but that it does so much better when it includes a transition to QGP in its Equation of State. This represents a good beginning to the determination of the Equation of State of hot dense matter– a main goal of the field.

B. Intensity Interferometry

We have already seen in Figure 12 correlation functions for three different colliding systems. The width of these one-dimensional correlations allow us to extract the overall size of the source. As discussed in Section V.D.3, we can gain much more information by studying the width of the correlations in *three* dimensions. Here, I discuss the homogeneity lengths R_{out}, R_{side} and R_{long} coming from such an analysis.

1. Size and Lifetime as a Function of Collision Energy

Well, the first things we were looking for in these measurements was evidence of a very big final source and/or long timescales. As discussed in Section IV.B, these observations would signal an expected increase in entropy and a QGP transition in the Equation of State. "Big" and "long" here mean relative to observations at lower collision energies where (presumably) QGP formation is impossible.

Figure 16 shows the world's data set of homogeneity lengths extracted from central Au+Au (or Pb+Pb) collisions as a function of the collision energy. Note that the x-axis is a logarithmic scale– we are looking at collision energies from 2 to 200 GeV, a huge range. However, we do not see the hoped-for big change in any of the homogeneity lengths as the energy crosses some QGP-creation threshold. The ratio R_{out}/R_{side} stays relatively constant at about 1.2, and R_{long} grows only slowly.

Figure 17 shows the story more clearly. The same hydrodynamic calculations (2) which explained the large elliptic flow at RHIC, and which gave us hope of achieving a detailed understanding of the collision evolution, fails to reproduce the homogeneity lengths. The predicted R_{long} is larger than observed, suggesting that the system "lives" more briefly in reality than it does in the calculation. The prediction for R_{out}/R_{side} is likewise too large, suggesting that the system falls apart at the end of its evolution more rapidly in reality than it does according to this otherwise-successful theory.

This discrepancy between theory and experiment has been termed the "RHIC HBT puzzle." Several attempts to fix this discrepancy have been performed. Some have been partially successful, but for most of those, once the homogeneity lengths match better with data, the elliptic flow is no longer reproduced. Some other approaches actually reproduce both of the observations relatively well (e.g. (15)), but are generally believed to rest on unphysical assumptions. It is fair to say that a consistent picture in an accepted physical theory has not yet been achieved.

Some in the field view the HBT puzzle as a minor annoyance. They believe that the connection between the calculated homogeneity lengths and the measured ones is not so simple as is currently believed. In other words, it is not the *theory* which is wrong, but the way in which we calculate homogeneity lengths in the theory. Furthermore, these folks point out, the experimental analysis and extraction of the homogeneity lengths is a tricky business, and it is easy to make several small systematic errors which might add up to a large error.

Since I think quite a bit about these issues, especially from the experimental side, I will weigh in with my opinion. I cannot disagree with the fact that, both from the theoretical and the experimental sides, extraction of homogeneity lengths is not so straightforward.

My discussion here has focused on the main points in an experimental interferometry analysis, and I have kept it rather simple. There are several somewhat complicated steps along the way, the majority of which *can* change the results. It is the experimentalist's job to estimate the importance of these effects and quantify them in terms of what are called systematic errors. To the best of our knowledge, the systematic errors in the experimental data do not explain the puzzle. However, I cannot deny the possibility that even experts in these analyses have overlooked something.

On the theory side, there are likewise several theoretical assumptions and simplifications that usually go into predicting homogeneity lengths. Many of these have been examined carefully and have not been found to create an effect sufficiently large to explain the puzzle. There are many promising ideas being explored now (e.g. (16; 17)), but the jury is, I believe, still out.

It may even be that correction of small errors on both sides will bring the experimental results on R_{out}/R_{side} and R_{long} up and the theoretical values down, so that they meet in the middle.

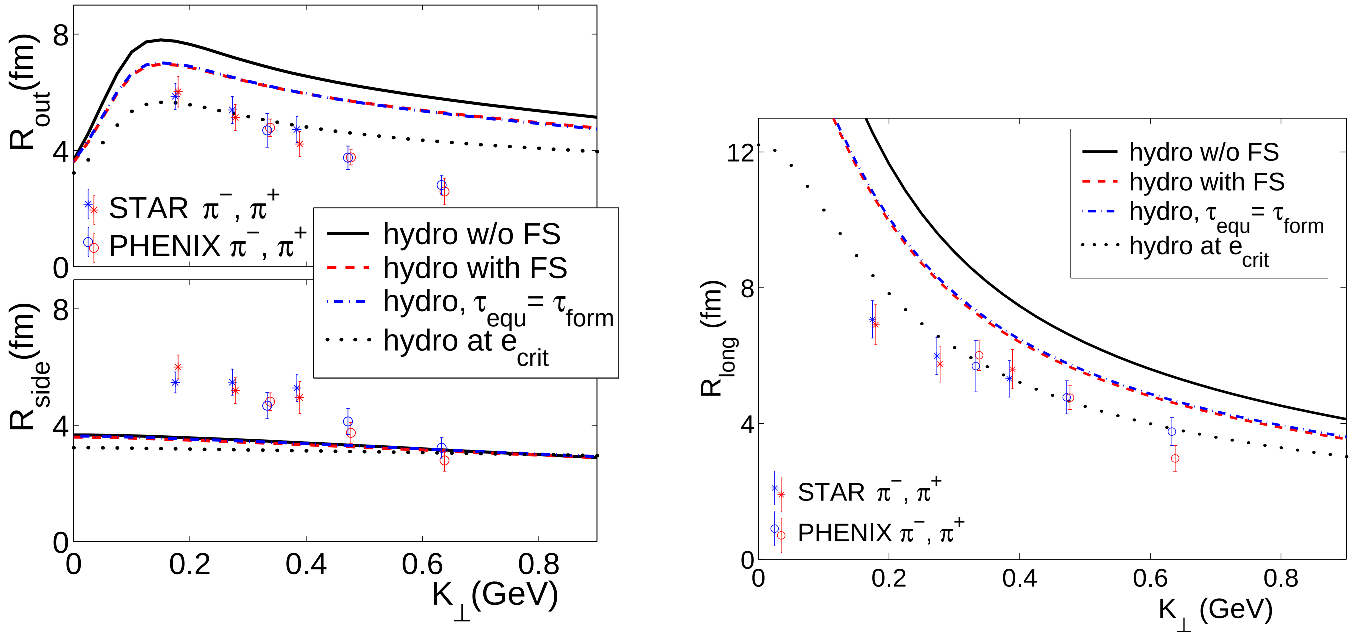

FIG. 17 Measured homogeneity lengths R_{out} and R_{side} (left) and R_{long} (right) are plotted as a function of momentum as colored datapoints. Superimposed as lines are the results of hydrodynamic calculations (2); the solid line is the "default" calculation which reproduces elliptic flow observations. Data and calculations are for central Au+Au collisions at RHIC.

My semi-educated guess is that further theoretical and experimental refinement of the method will help alleviate part of the discrepancy, but that there is still a problem at the heart. One reason is that it looks like the "usual" effects explored thus far are too small to explain the discrepancy. Another is that, once the experimental and theoretical numbers possibly change, the theory will still predict an increase in, e.g. R_{out}/R_{side} beginning at some energy as a consequence of QGP formation and decay, and the data will not show it.

Finally, the discrepancy already exists when we compare hydrodynamical predictions to the data. Hydrodynamics describes only the very dense stage of the collision. It is in this stage that elliptic flow is generated. However, after the very dense stage, there is naturally expected to be a more dilute stage in which the emitted protons, pions and other particles scatter off of each other. This is the stage indicated in panel (e) in both Figures 4 and 6. This is expected to considerably *increase* both the evolution and the disintegration timescales (6; 18), thus exacerbating the puzzle. It seems unlikely that the system would instantly go from a highly dense state to vacuum emission.

2. Geometric Evidence of Collectivity I - Momentum Dependence

As mentioned in Section IV.C.2, a collective system sends a clear geometric signal in the momentum dependence of the homogeneity lengths. Already in Figure 17 we see this signal clearly. The transverse[13] homogeneity lengths R_{out} and R_{side} become smaller as the particles become faster, both in the theory and in the experiment.

So, we know that hydrodynamical evolution leads naturally to collective (flow) behaviour, and describes well the v_2 signals measured in experiment. Further, we observe flow-like signals resembling those from hydro in the HBT analyses. However, as we have said, hydrodynamics does not describe the HBT data quantitatively. What to do? Can we, in fact, reconcile the two experimental results (v_2 and homogeneity lengths) in a single picture characterized by geometry and flow? If not, can we really claim that we understand the system we are creating?

Many groups have tried to understand, comprehensively, several of the dynamical results coming from the experiments, using simplistic models of how the end of the collision looks like (as opposed to doing a full calculation of the

[13] In the simple picture I have given, collective flow in the *transverse* direction leads naturally to smaller *transverse* homogeneity lengths (R_{out} and R_{side}) for particles moving faster in the *transverse* direction. (As usual, "transverse" here means perpendicular to the beam direction.) See the discussion of Figure 5.
We also see that the *longitudinal* homogeneity length R_{long} is smaller for particles with high transverse velocity. Although it may not be obvious, this is due, in fact, to *longitudinal* flow, which I do not discuss here.

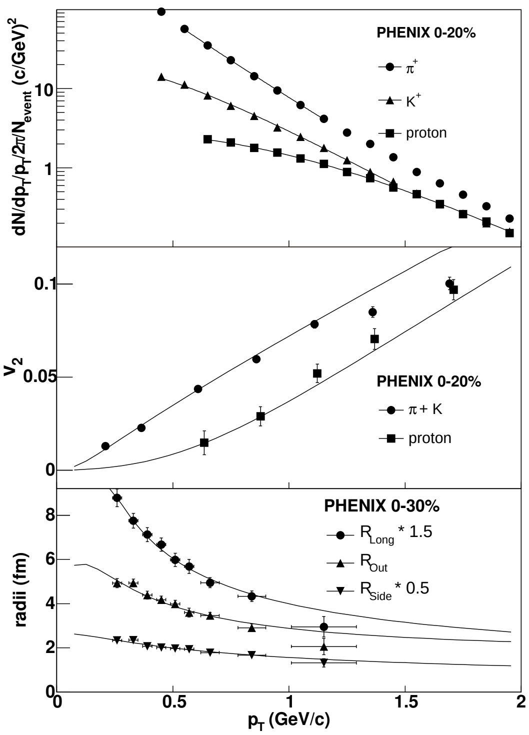

FIG. 18 Number of particles detected (top), elliptic flow (middle), and homogeneity lengths (bottom) are plotted as a function of transverse momentum. Measured data for different particle types are shown as symbols, while lines come from a simple calculation. See text for details. Figure from (19).

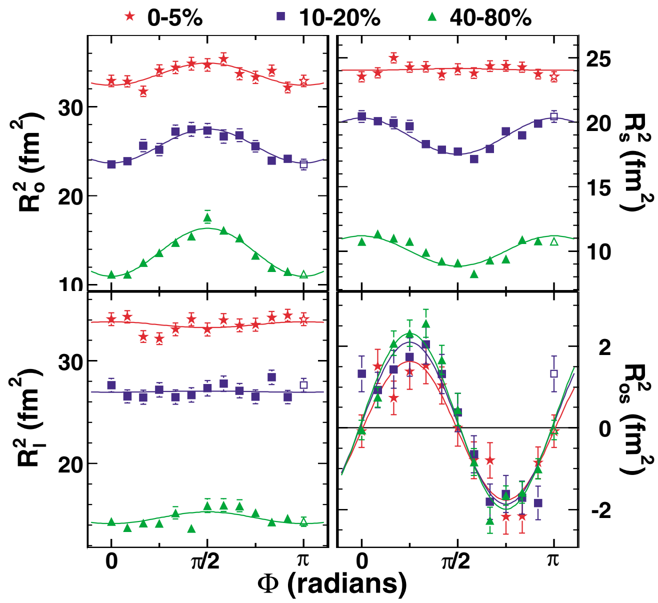

FIG. 19 Homogeneity lengths measured in Au+Au collisions at RHIC, plotted as a function of "viewing angle" with respect to the reaction plane. (This direction is indicated with the big blue arrow in Figure 5 panel (b).) $\phi = 0$ corresponds to the source as viewed from a detector sitting in the reaction plane, while $\phi = \pi/2$ corresponds to viewing perpendicular to the reaction plane. Central collisions are shown as red stars, mid-central as blue squares, and peripheral collisions as green triangles. The lines are not the result of any calculation or model, and I do not discuss them here. From (20).

system evolution). I show one of them in Figure 18.

Clearly, this simple model (5) can describe several aspects of the data (including others not shown) quite well. It does so by adjusting the amount of flow, system size, lifetime, shape, and other parameters, in order to reproduce the data. It should be pointed out however that, even with all the adjustable numbers, it is a priori far from clear that a given simple model will be able to reproduce the data. The main aspects of this model are (5):

- treatment of the system *as* a system (i.e. as bulk matter)
- a very strong flow (but we already expected that)
- very short evolution and emission timescales (of which we've already had a hint)

The good agreement with data suggests (though does not prove) that these elements of the model are important driving aspects of the physics of the final state of the system, and further investigation of the third bullet above might open the door to new insights about the system which we do not currently include in our theoretical calculations.

3. Geometric Evidence of Collectivity II - Overall Size and Shape

We have measured the homogeneity lengths in great detail. In addition to the energy and transverse momentum dependence discussed in the previous two Sections, we have looked at collisions of varying centrality and at particle emitted at all angles relative to the reaction plane. A detailed example (20) of the data is shown in Figure 19. There, homogeneity lengths are plotted as a function of "viewing angle" relative to the reaction plane (e.g. the blue arrow in Figure 5 represents a viewing angle of about 45°) for collisions of different impact parameter. (I will not be discussing the "length" R^2_{os} in the lower right panel of Figure 19.)

From these measurements, one can determine, e.g. using the simple model mentioned in the previous section, the *overall* [14] size and shape of the system. These are precisely the quantities we hoped to pin down in the discussions of Sections IV.B and IV.C.2 to prove important physics.

In Figure 19, it is clear that central collisions produce an overall bigger source than do peripheral collisions– the homogeneity lengths are always bigger in central collisions. Were it otherwise, we would start to doubt that we were measuring geometry at all! In central collisions, the transverse size is about 13 fm, and in peripheral it is about 7 fm. In all cases, the final size (which we measure with HBT) is about twice the initial size (which we know by knowing the impact parameter), again indicating collective explosive expansion of the system.

As I've said, larger sources and collective expansion in general are naturally understood in a hydrodynamic model. The problem is in the details. Hydrodynamics apparently produces a source which is too small (signaled by underprediction of R_{side} in Figure 17) and too long-lived (underprediction of R_{out} and R_{long} in Figure 17). The data and simplistic models seem to be making the impossible demand that the model expand the source to a *bigger* size, but to do it in a *shorter* time!! Naïvely, this would seem to suggest that we must generate *more* flow in the calculation. However: a) the flow generated by hydrodynamics is believed to be the maximum possible (2), and b) the strength of the flow coming from the hydrodynamical calculations already agrees with the simplistic models and with flow-sensitive data. Clearly, there is something we do not yet understand.

Going beyond the overall size, we now look at the shape, which we extract through the dependence of the homogeneity lengths on the viewing angle. If the particle-emitting source was shaped as shown in Figure 5, what would we expect? Well, if the source were viewed by a detector positioned in the reaction plane (in the Figure, this means a detector "to the right"), we would expect to have a larger R_{side} than we would if the detector was "above" the reaction plane.[15] We would also expect R_{out} to follow the opposite behaviour– being smaller when measured by an in-plane detector. To see this, you may want to look again at Figure 13 to recall how R_{side} and R_{out} are defined.

The data in Figure 19 in fact show this very behaviour, and we conclude that the system at the end of its evolution (which is the time when HBT signals are formed) is "stretched" out of the reaction plane. Recalling Figure 6 and the discussion of Section IV.C.2, we see that this is further evidence of a short evolution time. In fact, realistic calculations (6) produce a final scenario like that shown in panel (e) of Figure 6. Again, it appears that there is something which is still not understood.

VII. SUMMARY

I have tried to give some flavor of the field studying relativistic heavy ion collisions, and the motivation which drives large numbers of people to build huge facilities and detectors in this endeavor. The study of heavy ion collisions in themselves is indeed fascinating– it involves designing and understanding complex detector systems, performing challenging analysis of giant datasets, and extracting the underlying dynamics from a huge variety of phenomena exhibited in these collisions. However, understanding these collisions in itself is not the point; instead, we hope to use the collisions as a tool to create a new state of matter at the highest achievable temperatures and densities– color deconfined matter. Indeed, even creation and study of the QGP is itself only a bridge to the ultimate goal– gaining crucial insight into the most poorly understood aspect of the strongest force in Nature, the Strong force.

In order to construct this bridge we need to cross, we need *bulk matter*. As we have seen, large systems provide the best hope of generating this matter at scales larger than the deconfinement scale of ~ 1 fm. If the bulk system is indeed deconfined, it had been expected to live for a "long" time before decaying into the white (uncolored) objects which we finally measure. I tried to put this "large" and "long-lived" system into the context of other physical systems with which we are familiar. We saw that the scales with which we are dealing, and which we claim the ability to *measure* are unimaginably small in this broader context.

Small though they are, geometrical scales define the entire field of RHI physics, distinguishing it from its close cousin, particle physics. The crucial role of geometry comes not only in the *creation* of the matter under study at the initial stage, but in measurements to extract the properties of that matter in the final stage. I discussed the ways in which we determine the initial size and shape in space, and how we get a handle on the evolution of the system through momentum and space-time sensitive measurements.

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[14] I stress that "overall" here refers to the *whole* system, not just homogeneity regions which emit particles of particular momentum. Even the concept of the "whole system" implicitly assumes bulk matter. As I've mentioned, we believe this assumption justified.

[15] At this point, the alert reader may remember that we measure homogeneity regions, and not the "whole" source. Since the connection between the whole source and the homogeniety regions is nontrivial in a flowing system, she might then ask whether it is so clear how R_{side} will vary. She is right, in principle, and these effects are taken into account in the models. In fact, for sources created at RHIC, we believe that the argument made in the text holds even when these complicating factors are taken into account (5).

Through these measurements, we extracted strong evidence that in fact, a bulk system *is* indeed formed in collisions at RHIC, allowing us to apply the concepts of thermodynamics and states of matter. Momentum anisotropy observations indicate very high pressure generated in the high energy density and temperature regions of the system. Quantifying the relationship between these thermodynamical quantities is the Equation of State of Strongly-interacting systems. Hydrodynamical calculations can be employed to describe the evolution a bulk strongly self-interacting system, and the Equation of State is the defining physics which determines this evolution. We saw that using an Equation of State which includes a transition from confined to deconfined matter at the expected temperature can describe the data well, unlike the case in which there is no transition.

Moving beyond purely momentum measurements, we found that the geometric information extracted from particle interferometry made rough intuitive sense, a fact amazing in itself. Further, this information confirmed the picture of collective motion generated by a self-interacting system. Both the dynamical geometrical space-time information and the momentum measurements could be described by a simplistic model of the final stage of the collision.

The problem we saw is that this picture of the final stage does not quantitatively agree with results of the same hydrodynamical calculations which tie the momentum observations to the Equation of State. The system size is larger than expected, and, surprisingly, shorter-lived. As I said, this discrepancy is expected to be even worse if non-hydrodynamical effects– generally expected to be present– are included in the final stage of the model calculations. There are several possible sources of this "puzzle" at RHIC, including incomplete understanding of all systematic errors associated with the experimental analysis, incorrect procedures to model experimental results from model calculations, or even a fundamental misunderstanding of exactly particle interferometry measures at all. The most interesting possibility is that the underlying physics driving the system evolution is incompletely understood. Of course, several or all of these possibilities may be at work. My own opinion is that the final possibility is at least one of the factors at work, and that important insight may be gained through this geometrical information.

The second part of my title, "spacetime geometry in subatomic collisions," was probably a bit too grandiose. Due to time constraints, I discussed only some of the geometry-driven observations at RHIC. Others include correlations between non-identical particles (21), elliptic flow for higher momentum particles (10), and measurement of the number of "jets" (remnants of the very first quarks and gluons scattered in the collision) as a function emission angle relative to the reaction plane (22). Each of these probes different and truly exciting aspects of the system created at RHIC.

To reiterate the global theme: In addition to dominating many of the most important measurements in heavy ion collisions, a non-trivial geometry is crucial in creating what we really want to study– a thermalized bulk system of deconfined colored matter. Only then can we understand the Equation of State of Strongly interacting matter and gain insight into an issue of fundamental scientific interest, the poorly-understood mechanism of confinement. Thus, even if I reached a bit far in the second part of my title, the first part remains appropriate– clearly Size Matters!

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