Coinductive Logic Programming and its Application to Planning and Model-Checking

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Outline

- Circular phenomena in computer science
- Coinduction: modeling circular phenonmenon
- Coinduction in logic programming (Co-LP)
- Applications of Co-LP:
  - Model checking
    - Timed model checking
  - Planning
    - Timed planning

Circular Phenomena in Comp. Sci.

- Circularity has dogged Mathematics and Computer Science ever since Set Theory was first developed:
  - The well known Russell's Paradox:
    - R = { x | x is a set that does not contain itself}

      Is R contained in R? Yes and No
  - Liar Paradox: I am a liar
  - Hypergame paradox (Zwicker)
- All these paradoxes involve self-reference (through some type of negation)
- Russell put the blame squarely on circularity and sought to ban it from scientific discourse:

  ``Whatever involves all of the collection must not be one of the collection" -- Russell 1908

Circular Phenomenon in Comp. Sci.

- All this changed with Kripke's paper in 1975 who argued that circular phenomenon are far more common and circularity can't simply be banned.
- Circularity has been banned from automated theorem proving and logic programming through the occurs check rule:

  An unbound variable cannot be unified with a term containing that variable: X = f(X) disallowed

- What if we allowed such unifications to proceed (as LP systems always did for efficiency reasons)?

Circularity in Computer Science

- If occurs check is removed, we will generate circular (infinite) structures:
  - X = [1,2,3 | X]
- Such structures, of course, arise in computing (circular linked lists), but banned in logic/LP.
- Subsequent LP systems did allow for such circular structures (rational terms), but they only exist as data-structures, there is no proof theory to go along with it.
  - One can hold the data-structure in memory within an LP execution, but one can't reason about it.

Coinduction

- Circular structures are infinite structures

  X = [1, 2 | X]   is logically speaking X = [1, 2, 1, 2, ….]
- Proofs about their properties are infinite-sized
- *Coinduction* is the technique for proving these properties [Aczel 1983; Moss & Barwise 1996]

  "Vicious Circles" by Moss and Barwise (1996)
- Our focus: inclusion of coinductive reasoning techniques in LP and its application to model checking and planning.

Induction vs Coinduction

- Induction is a mathematical technique for finitely reasoning about an infinite (countable) no. of things.
  - Naturals: 0, 1, 2, …
  - Lists: [ ],   [X],   [X, X],   [X, X, X], …
- 3 components of an inductive definition:

  (1) Initiality, (2) iteration, (3) minimality
  - for example, the set of lists is specified as follows:

    [ ] – an empty list is a list (**initiality**)   ……(i)

    [H | T] is a list if T is a list and H is an element (**iteration**) ..(ii)

    minimal set that satisfies (i) and (ii) (**minimality**)

Induction vs Coinduction

- Coinduction is a mathematical technique for (finitely) reasoning about infinite things.
  - Mathematical dual of induction
- 2 components of a coinductive definition:

  (1) iteration, (2) maximality
  - for example, for a list:

    [H | T] is a list if T is a list and H is an element (**iteration**).

    Maximal set that satisfies the specification of a list.
  - This coinductive defn. specifies all infinite sized lists

Example: Natural Numbers

- $\Gamma_N(S) = \{ 0 \} \cup \{ succ(x) \mid x \in S \}$
- Γ_N defines 2 sets.
  - $N = \mu\Gamma_N$ (least fixed-point)
  - $N' = \nu\Gamma_N = N \cup \{ \omega \}$ (greatest fixed-point)

Definition	Proof	Mapping
Least fixed point	Induction	Recursion
Greatest fixed point	Coinduction	Corecursion

- Co-recursion: recursive definition without a base case

Infinite Objects and Properties

- Traditional logic programming (based on LFP) is unable to reason about infinite objects and/or properties
- (The glass is only half-full)
- Example: perpetual binary streams
  - traditional logic programming cannot handle

```
bit(0).
bit(1).
bitstream( [ H | T ] ) :- bit( H ), bitstream( T ).
|?- X = [ 0, 1, 1, 0 | X ], bitstream( X ).
```

- Goal: Combine traditional LP with coinductive LP

Extending LP with Co-induction

- What is needed?: an operational semantics for incorporating coinduction into SLD resolution
- Declarative Semantics: across the board dual of traditional LP [Lloyd 87]:
  - greatest fixed-points
  - terms: co-Herbrand universe $U^{co}(P)$
  - atoms: co-Herbrand base $B^{co}(P)$
  - program semantics: maximal co-Herbrand model $M^{co}(P)$.

Operational Semantics: co-SLD

- nondeterministic state transition system
- states are pairs of
  - a finite list of goals [resolvent] (as in Prolog)
  - a set of syntactic term equations of the form x = f(x) or x = t
    - Given program p :- p. Then the query |?- p. will succeed.
    - Given p([1 | T]) :- p(T). |?- p(X) to succeed with X= [1 | X].
- transition rules
  - definite clause rule
  - "coinductive hypothesis rule"
    - if a coinductive goal Q is called,
      and Q unifies with an ancetor call made earlier
      then Q succeeds.

Example: Number Stream

```
:- coinductive stream/1.
stream( [ H | T ] ) :- num( H ), stream( T ).
num( 0 ).
num( s( N ) ) :- num( N ).

|?-  stream( [ 0, s( 0 ), s( s ( 0 ) )   |   T ] ).
```

1. MEMO: stream([0, s(0), s(s (0)) | T])
2. MEMO: stream([s(0), s(s (0)) | T])
3. MEMO: stream([s(s (0)) | T])
4. stream(T)

Answers:

T = [0, s(0), s(s(0)) | T]

T = [0, s(0), s(s(0)), s(0), s(s(0)) | T]

T = [0, s(0), s(s(0)) | T]   . . .

T = [0, s(0), s(s(0)) | X]   (where X is any rational list of numbers.)

Other Examples

- Append:
  :- coinductive append/3.
  append([ ], X, X).
  append([H | T], Y, [H | Z]) :- append(T, Y, Z).
  |?- X = [1, 2, 3 | X], Y = [3, 4 | Y], append(X, Y, Z).
  Answer: Z = [1, 2, 3 | Z].

  |?- Z = [1, 2 | Z], append(X, Y, Z).
  Answer: X = [ ], Y = [1, 2 | Z] ; X = [1, 2 | X], Y = _
  X = [1], Y = [2 | Z] ;
  X = [1, 2], Y = Z; …. ad infinitum
- Co-member(X, L): is X a member of infinite list L?
- Sieve of Eratosthenes (lazy evaluation)

Co-Logic Programming

- combines both halves of logic programming:
  - traditional logic programming
  - coinductive logic programming
- syntactically identical to traditional logic programming, except predicates are labeled:
  - Inductive, or
  - coinductive
- and stratification restriction enforced where:
  - inductive and coinductive predicates cannot be mutually recursive. e.g.,

    p :- q.

    q :- p.

  Program rejected, if p coinductive & q inductive
- Preliminary implementation on top of YAP available.

Applications of Co-LP

- With Co-LP one can perform LFP as well as GFP computations elegantly
- Declarative power of LP can be harnessed for more sophisticated applications
- Two major application domains:
  - Model checking: Need to compute LFPs & GFPs
  - Planning: rules describing domain may be circular
- Using LP brings other LP-specific techniques to bear on the problem:
  - Constraints (continuous time easily included)
  - Parallelism (verification/planning done in parallel)

Application: Model Checking

Finite Automata

```
automata([X|T], St):- trans(St, X, NewSt), automata(T, NewSt).
automata([ ], St) :- final(St).

trans(s0, a, s1).    trans(s1, b, s2).    trans(s2, c, s3).
trans(s3, d, s0).    trans(s2, 3, s0).    final(s2).

?- automata(X,s0).
   X=[ a, b];
   X=[ a, b, e, a, b];
   X=[ a, b, e, a, b, e, a, b];
   ……
   ……
   ……
```

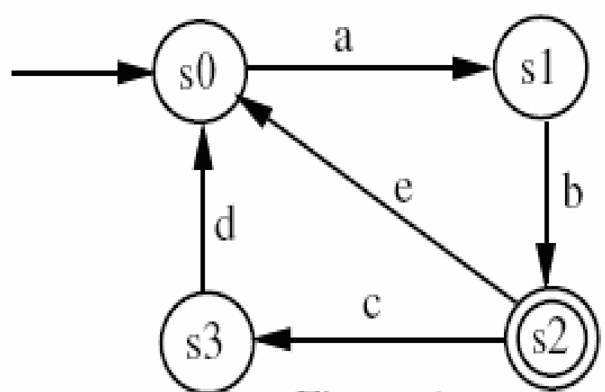


Figure A

Infinite Automata

automata([X|T], St):- trans(St, X, NewSt), automata(T, NewSt).

trans(s0,a,s1). trans(s1,b,s2). trans(s2,c,s3).
trans(s3,d,s0). trans(s2,3,s0). final(s2).

?- automata(X,s0).
X=[a, b, c, d | X];
X=[a, b, e | X];

When the same goal is seen
=> infinite cycle in the automata
=> HALT

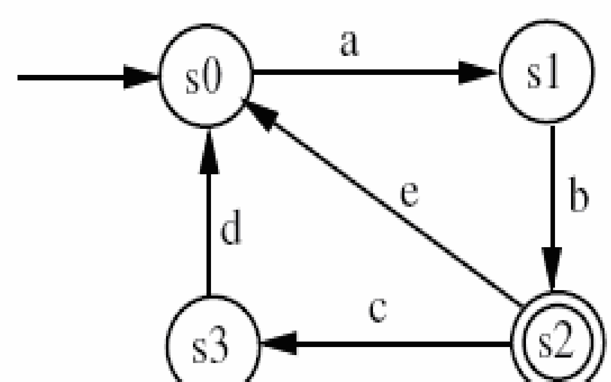


Figure A

Verifying Liveness Properties

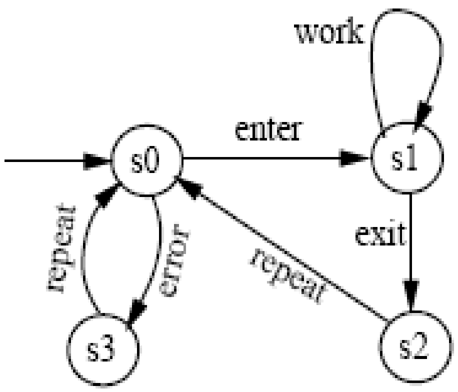
- Verifying safety properties in LP is relatively easy: safety modeled by reachability
- Accomplished via tabled logic programming (an efficient engine for computing LFP)
- Verifying liveness is much harder: a counterexample to liveness is an infinite trace
- Verifying liveness is transformed into a safety check via use of negations in model checking and tabled LP
  - Considerable overhead incurred
- Co-LP solves the problem more elegantly:
  - Infinite traces that serve as counter-examples produced as answers

Verifying Liveness Properties

- Consider Safety:
  - Is an unsafe state S_u (wrongly) considered safe?

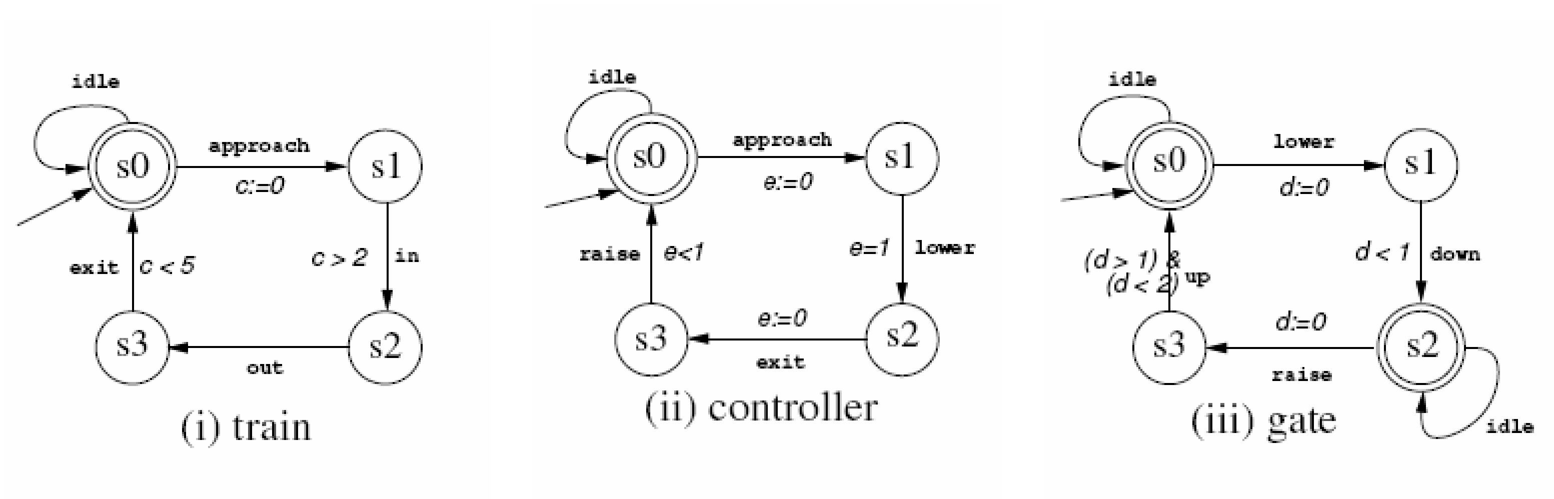
    i.e., is S_u reachable?
  - If answer is yes, the path to S_u is the counter-ex
    - Tabled LP will produce this path as an answer
- Consider Liveness, then dually
  - Is a dead state D (wrongly) considered live?
  - If answer is yes, the infinite path containing D is the counter example
    - Co-LP will produce this infinite path as an answer
- Liveness checking as easy as safety checking

Nested Finite and Infinite Automata




```
:- coinductive state/2.
state(s0, [s0,s1 | T]):- enter, work,
                          state(s1,T).
state(s1, [s1 | T]):- exit, state(s2,T).
state(s2, [s2 | T]):- repeat, state(s0,T).
state(s0, [s0 | T]):- error, state(s3,T).
state(s3, [s3 | T]):- repeat, state(s0,T).
work.     enter. repeat. exit. error.
work :- work.
|?- state(s0,X), absent(s2,X).
   X=[ s0, s3 | X ]
```

Verification of Real-Time Systems "Train, Controller, Gate"

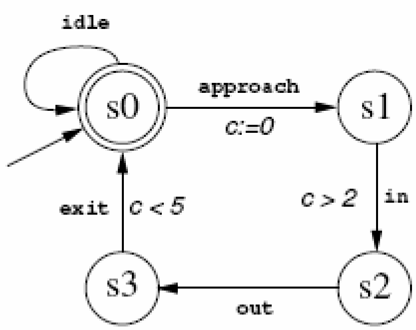


Timed Automata

- ω-automata w/ time constrained transitions & stopwatches
- straightforward encoding into CLP(*R*) + Co-LP

(clock to be reset in every cycle)

Verification of Real-Time Systems "Train, Controller, Gate"

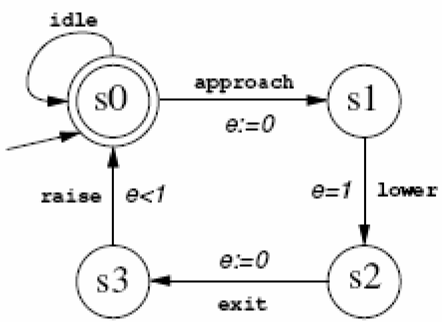


(i) train

```
:- use_module(library(clpr)).
:- coinductive driver/9.

train(X, up, X, T1,T2,T2).        % up=idle
train(s0,approach,s1,T1,T2,T3) :- {T3=T1}.
train(s1,in,s2,T1,T2,T3):-{T1-T2>2,T3=T2}
train(s2,out,s3,T1,T2,T3).
train(s3,exit,s0,T1,T2,T3):-{T3=T2,T1-T2<5}.
train(X,lower,X,T1,T2,T2).
train(X,down,X,T1,T2,T2).
train(X,raise,X,T1,T2,T2).
```

Verification of Real-Time Systems "Train, Controller, Gate"



(ii) controller

contr(s0,approach,s1,T1,T2,T1).

contr(s1,lower,s2,T1,T2,T3):- {T3=T2, T1-T2=1}.

contr(s2,exit,s3,T1,T2,T1).

contr(s3,raise,s0,T1,T2,T2):-{T1-T2<1}.

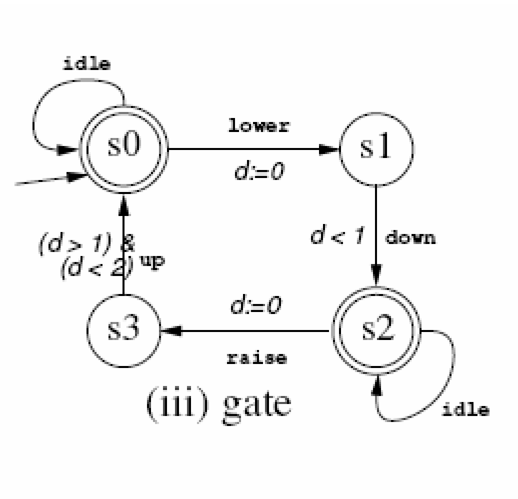
contr(X,in,X,T1,T2,T2).

contr(X,up,X,T1,T2,T2).

contr(X,out,X,T1,T2,T2).

contr(X,down,X,T1,T2,T2).

Verification of Real-Time Systems
“Train, Controller, Gate”



(iii) gate

gate(s0,lower,s1,T1,T2,T3):- {T3=T1}.

gate(s1,down,s2,T1,T2,T3):- {T3=T2,T1-T2<1}.

gate(s2,raise,s3,T1,T2,T3):- {T3=T1}.

gate(s3,up,s0,T1,T2,T3):- {T3=T2,T1-T2>1,T1-T2<2 }.

gate(X,approach,X,T1,T2,T2).

gate(X,in,X,T1,T2,T2).

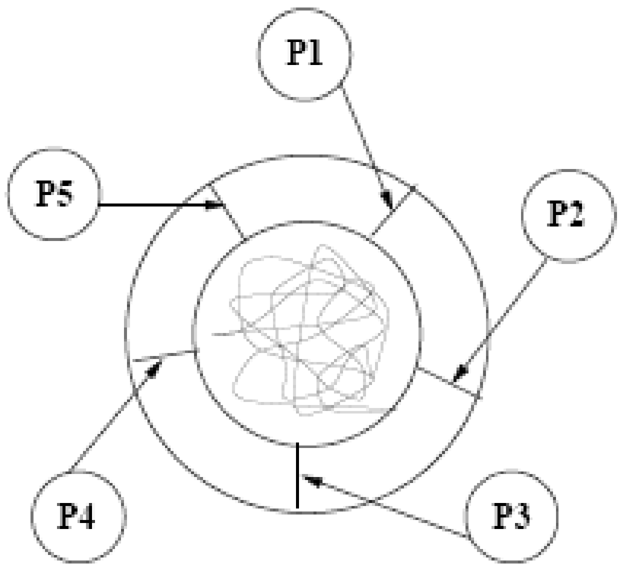
gate(X,out,X,T1,T2,T2).

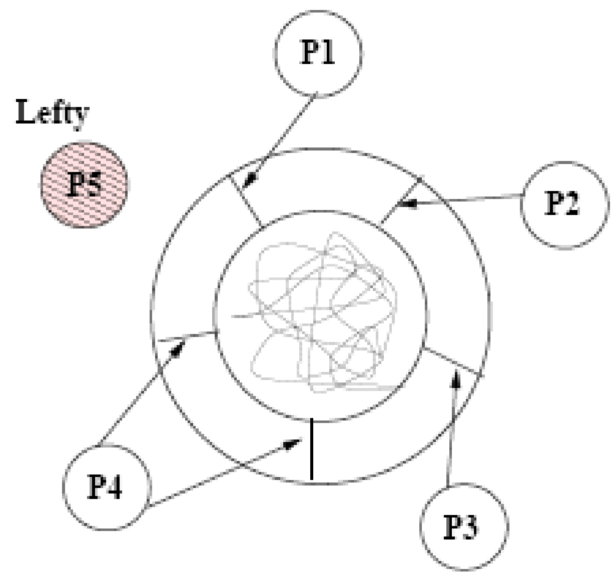
gate(X,exit,X,T1,T2,T2).

Verification of Real-Time Systems

```
:- coinductive driver/9.
driver(S0,S1,S2, T,T0,T1,T2, [ X | Rest ], [ (X,T) | R ]) :-
        train(S0,X,S00,T,T0,T00),  contr(S1,X,S10,T,T1,T10),
        gate(S2,X,S20,T,T2,T20), {TA > T},
        driver(S00,S10,S20,TA,T00,T10,T20,Rest,R).
|?- driver(s0,s0,s0,T,Ta,Tb,Tc,X,R).
  R=[(approach,A), (lower,B), (down,C), (in,D), (out,E), (exit,F),
      (raise,G), (up,H) | R ],
  X=[approach, lower, down, in, out, exit, raise, up | X] ;
  R=[(approach,A),(lower,B),(down,C),(in,D),(out,E),(exit,F),(raise,G),
      (approach,H),(up,I)|R],
  X=[approach,lower,down,in,out,exit,raise,approach,up | X] ;
  %  where A, B, C, ... H, I are the corresponding wall clock time of events generated.
```

DPP – Safety: Deadlock Free



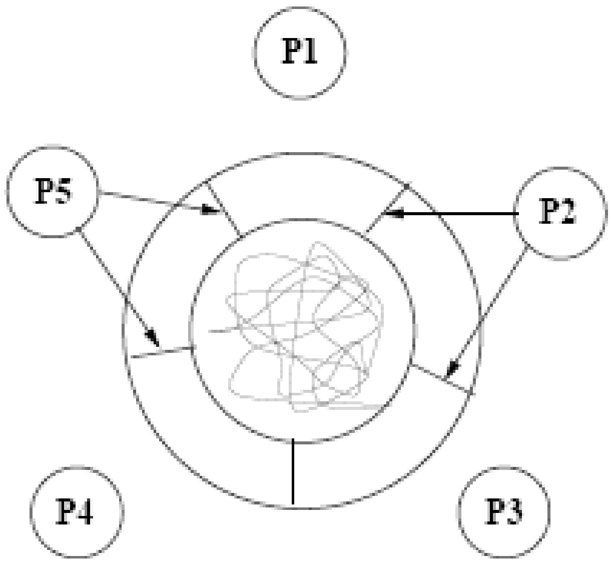


- A solution
  - Force one philosopher to pick forks in different order than others
- Checking for deadlock
  - Bad state is not reachable
  - Implemented using Tabled LP

```
:- table reach/2.
   reach(Si, Sf) :- trans(_,Si,Sf).
   reach(Si, Sf) :- trans(_,Si,Sfi),
                              reach(Sfi,Sf).

?- reach([s,s,s,s,s],
     [w,w,w,w,w]).
   no
```

DPP – Liveness: Starvation Free



- Phil. waits forever on a fork
- One potential solution
  - phil. waiting longest gets the access
  - implemented using stop watches
- Checking for starvation
  - once in bad state, is it possible to remain there forever?
  - implemented using co-LP

```
starved(X) :-X = 1, driver([s, s, s, s, s], [w, _, _, _,_]);
             X = 2, driver([s, s, s, s, s], [_, w, _, _,_]);
             X = 3, driver([s, s, s, s, s], [_, _, w, _,_]);
             X = 4, driver([s, s, s, s, s], [_, _, _, w,_]);
             X = 5, driver([s, s, s, s, s], [_, _, _, _,w]).
```

```
?- starved(X).
   no
```

Observations

- Finite and infinite automata nested within each other:
  - Not possible in standard methods for model checking?
- Continous quantities such as time can be introduced
  - Hybrid systems can be modeled elegantly, as long as a solver/consistency-checker can be built for that domain
- Liveness checking as easy as safety checking
- Parallelism can be implicitly exploited from logic programs; (timed) model checking can be readily performed in parallel

Application: (Timed) Planning

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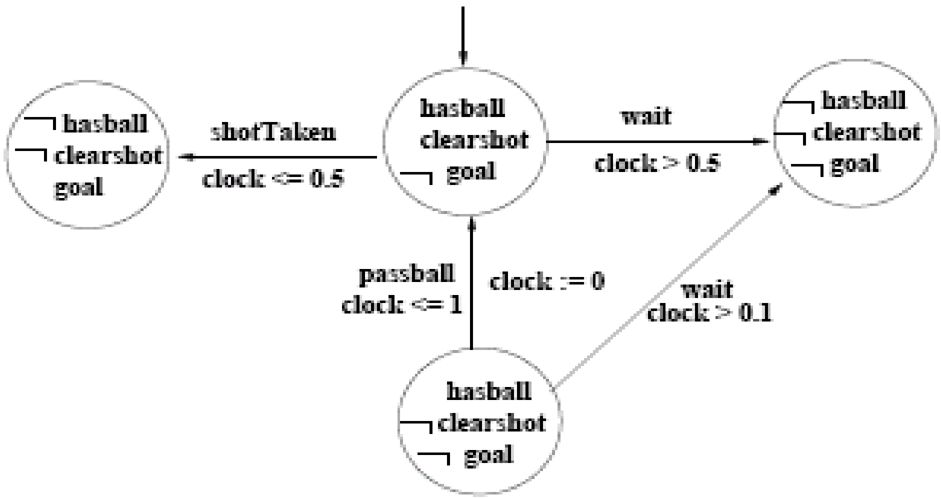
- *Planning:* Given (i) domain D, (ii) observations about initial state O, (iii) a set of *fluents* $g_1, \ldots, g_n$, find a set of actions $a_1, \ldots, a_m$, such that D will entail $g_1, \ldots, g_n$.
  - *Action description languages* (like *A*) describe domains (with actions and change) used for planning problems
- Planning may involve self referential rules:

  hasball *if* receivedpass & ~hasball
- Planning & verification: two sides of the same coin

Real-Time Soccer Playing Domain



¬

$$
\begin{aligned}
&ShotTaken \textbf{ causes } \neg HasBall,\ \neg ClearShot,\ Goal \textbf{ when } Clock \leq 0.5\\
&\qquad\qquad \textbf{if } HasBall,\ ClearShot,\ \neg Goal\\
&PassBall \textbf{ causes } ClearShot \textbf{ resets Clock when } Clock \leq 1\\
&\qquad\qquad \textbf{if } \neg ClearShot\\
&\textbf{wait causes } \neg HasBall,\ \neg ClearShot \textbf{ when } Clock > 0.5\\
&\qquad\qquad \textbf{if } HasBall,\ ClearShot\\
&\textbf{wait causes } \neg HasBall, \textbf{ when } Clock > 1 \textbf{ if } HasBall
\end{aligned}
$$

Real-Time Soccer Playing Domain

$PassBall$ **causes** $ClearShot$ **resets Clock when** $Clock \leq 1$
if $\neg ClearShot$

- holds(clearShot, res(passBall,S)) :-
  not_holds(clearShot, S), b_getval(clock, Clock), {Clock =< 1},
  {NewClock > 0}, b_setval(clock, NewClock).

wait causes $\neg HasBall$, **when** $Clock > 1$ **if** $HasBall$

- not_holds(hasBall, res(wait,S)) :-
  holds(hasBall,S), b_getval(clock,Clock), {Clock > 1},
  {NewClock > Clock}, b_setval(clock,NewClock).

If clearShot is false in the initial state, then the query

?- holds(goal, S). % produces the solution

S = res(shotTaken, res(passBall, s0))

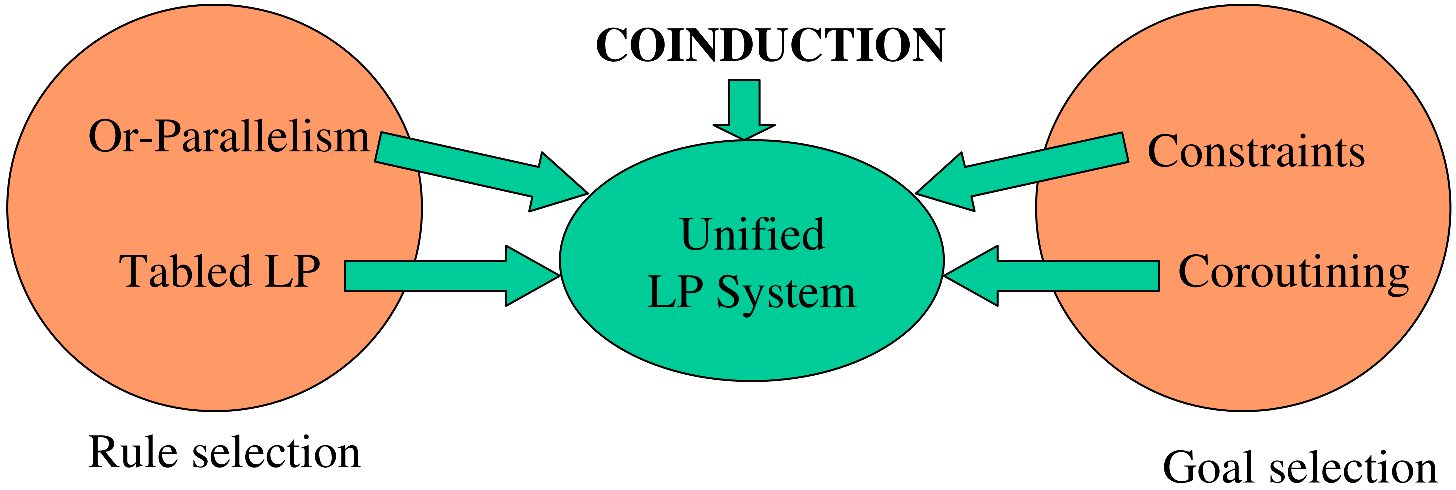
Other Applications & Status

- Other Applications:
  - Non monotonic reasoning: Co-LP allows goal-directed execution of Answer Set Programs
  - SAT Solvers: Goal-directed SAT solvers can be built
- Current Status:
  - A high level implementation of Co-LP on top of YAP Prolog completed.
  - Work in progress to build a low level implementation of Co-LP in an existing Prolog/CLP engine

Parallel Unified Reasoning Engine

- Lots of research in LP resulting in advances:
  - Constraints, Tabled LP, Parallelism, ASP, co-LP, coroutining
- Goal: build a system that combines them all

  build a system that run very large apps.



Conclusion

- Introducing coinduction into logic prog. allows one to compute both LFP & GFP
- GFP/LFP computations can be combined with other advanced features of LP allowing highly complex problems to be solved elegantly
  - Applications to (timed) model checking
  - Applications to (timed) planning
- Large instances of these applications can be run in parallel on a parallel logic programming system running on multicores

Related Publications

1. L. Simon, A. Mallya, A. Bansal, and G. Gupta. Coinductive logic programming. In *ICLP'06* .
2. L. Simon, A. Bansal, A. Mallya, and G. Gupta. Co-Logic programming: Extending logic programming with coinduction. In *ICALP'07*.
3. Coinductive Logic Programming and its Applications, ICLP'07 tutorial
4. A. Bansal, R. Min, G. Gupta. Goal-directed Execution of ASP. Internal Report, UT Dallas
5. R. Min, et al. Negation in coinductive Logic Programming. Forthcoming.