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Nonlocal Character of Quantum Theory *

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Abstract

According to the usual conception of causality, the truth of a statement that refers only to events confined to an earlier time cannot depend upon an unconstrained choice to be made by an experimenter at a later time. According to the usual ideas of relativity this causality condition should be valid in all Lorentz frames. It is shown here that this concept of relativistic causality is incompatible with certain simple predictions of quantum theory.

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Disclaimer

1. Introduction

Certain generalizations [1] of the works of Einstein, Rosen, and Podolsky [2] and John Bell [3] show that the predictions of quantum theory are incompatible with a property called "local realism". The word "local" refers here to a putative causality condition that claims that, in any Lorentz frame, the truth of a statement pertaining only to outcomes of experiments localized in one region at a given time cannot depend upon which experiment is freely chosen and performed in a spatially separated region at a later time. The word "realism" signifies that those demonstrations depend on the validity of an Einstein-type conception of physical reality, or perhaps on an assumption of determinism or of hidden variables. But if any of these three reality concepts is used then locality is not placed in jeopardy, for orthodox quantum philosophy claims the failure of all three reality concepts in the realm of quantum phenomena. I intend to show here, in a way perhaps simpler than before [4,5], that the locality property itself is incompatible with certain predictions of quantum theory, without assuming determinism, hidden variables, or Einstein reality.

2. Hardy-type Experiment

Use of a Hardy-type experiment [6,7,8] allows it to be shown that the putative locality condition itself is incompatible with the predictions of quantum theory. The essential features of the Hardy-type experimental set up are first that it involves two regions that are spatially separated from each other. Here they are called **R** and **L**. In **R** there are two alternative possible experiments, called $R1$ and $R2$, and in **L** there are two alternative possible experiments, called $L1$ and $L2$. In some Lorentz frame the experimenter's choice in **R** occurs *later* than the appearance and recording of the observable outcomes of the experiments in **L**. According to the usual causality concept it is reasonable to entertain the notion that the truth of a statement that refers only to observable properties in **L** at an earlier time cannot depend upon which measurement will be freely chosen and performed in **R** at a later time. According to the relativistic causality concept there is an analogous condition with **R** and **L** interchanged. But in this second case "earlier" and "later"are specified by using a different Lorentz frame.

What is under consideration here, then, is a set of just four possible experimental set ups that are specified to be identical except for the unconstrained bivalent choices of each of the two experimenters, and the various differences that can arise from differences in these two choices. If the defining conditions of two of these possibilities differ only by the choice to be made in one region at a later time then our putative locality condition claims that outcomes in the spatially separated region at the earlier time must be *the same* in these two cases: what appears earlier cannot be one outcome if the later free choice is Choice 1, but a different outcome if the later free choice is Choice 2.

The logical structure of the Hardy experiment is shown in Figure 1, along with the four pertinent predictions of quantum theory.

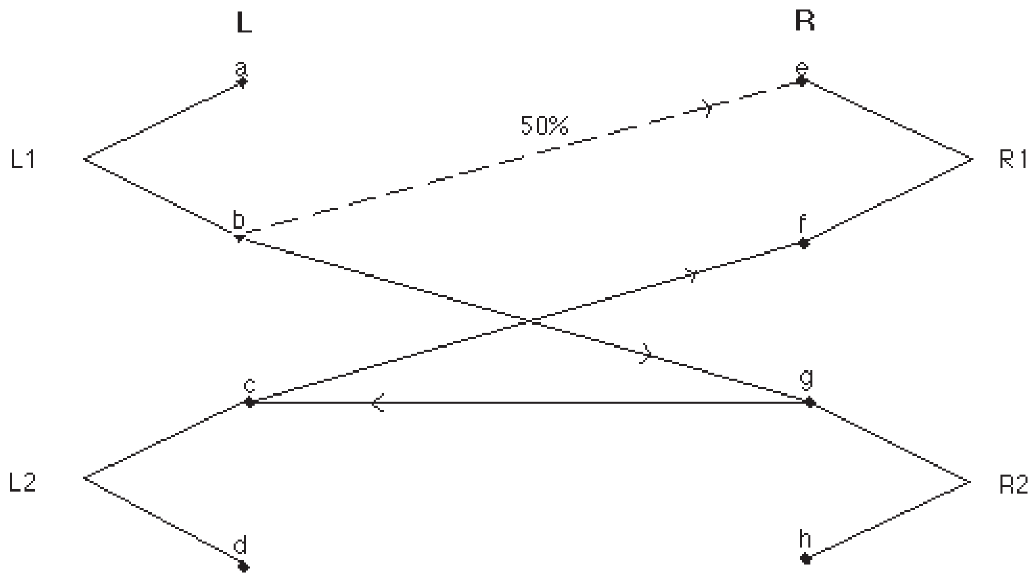

Figure 1: The logical structure of the Hardy experiment is represented, together with the four pertinent predictions of quantum theory. The three solid paths between pairs of labelled points represent predictions that have conditional probability equal to unity. The dotted line represents a connection that has conditional probabilty equal to 50%. (The 50% comes from the Eberhard/Rosselet version: Hardy's optical case gives 67%)

For example, the solid line from b to g represents the prediction that:

if $L1$ is performed in **L** and the outcome there is b, then if $R2$ is performed in **R** the outcome there will be g.

This prediction can be written symbolically as:

$$(L1 \wedge b) \Longrightarrow (R2 \Longrightarrow g),$$

where $\Longrightarrow$ stands for *implies*, the strict conditional, and $\wedge$ stands for conjunction. Equivalently, this prediction can be written in the form of line 9 below.

Informally, the general line of argument goes as follows. Suppose we consider a frame in which the measurement in region **L** is chosen and performed earlier than the measurement in region **R**. Suppose experiment L2 is chosen in **L**. Then the LOC condition that the outcome at the earlier time cannot be one outcome if the later choice in **R** is to perform experiment R2, but a different outcome if the later choice is to perform experiment R1, combined with the two QM predictions $g \rightarrow c$ and $c \rightarrow f$, allows one to conclude, under this condition that the earlier measurement is L2, that $g \rightarrow f$: if under the condition that R2 is performed the outcome g appears in **R** then we know, on theoretical grounds, that if the later choice had been to perform measurement R1, instead of measurement R2, then the outcome in **R** would necessarily have been f. This is because the outcome c at the earlier time entailed by QM and the premise g cannot be altered by changing the choice of which measurement is performed in **R** at the later time.

Following the symbolic conventions of modal logic we express this conclusion in the following way:

$$(L2 \wedge R2 \wedge g) \Longrightarrow [R1 \Box\!\!\rightarrow (L2 \wedge R1 \wedge f)],$$

which is equation 5 of the proof. If says that:
If L2 and R2 are performed and the result g appears in **R** then if R1 had been performed there, instead of R2, then the outcome f would have appeared.

Statements of this general kind are commonplace in physics: theory often allows one to deduce from the outcome of certain measurements on a system what the outcome of some alternative possible measurement would necessarily be. That is why modal logic really had to be developed: to cope with the fact that statements of this general kind are endemic in science.

The conclusion obtained above can be formulated as a statement that if measurement L2 is performed then $g \rightarrow f$, where the consequence is a statement referring only observables in region **R**. [See equation 6 below for the complete statement.]

Now the truth of this statement cannot depend on how we select the frame of reference, which is, according to the ideas of relativity theory, merely a notational convention. So let us shift to the other frame, where the measurements in region **L** are chosen and performed later. The statement $g \rightarrow f$ should still be true. But then the truth of this statement, which refers only to observables in **R**, cannot, according to the putative locality principle LOC, depend on whether measurement L2 or L1 is chosen to be performed at the later time. Yet if this implication $g \rightarrow f$ holds also under the condition that L1 is performed in **L** then we can combine that implication with $b \rightarrow g$ to get $b \rightarrow f$, which contradicts the prediction of QM that b entails e some nonzero fraction of the time.

3. The Proof

I state the steps of the proof in logical symbols, and also in words for the first few steps in order to make clear the meanings of the symbols. Later, I justify each step on the basis of the predictions of quantum mechanics (QM) or the locality condition described above (LOC), or simple logic (LOGIC).

In the proof I shall use the symbol $R1\Box\rightarrow$. It is taken from modal logic, and is to be read: "If $R1$ is performed, instead of $R2$, then...". It is to be used in situations where there is an unconstrained (free) choice between the two alternative possible measurements $R1$ and $R2$. The form

$$(X \wedge R2) \Longrightarrow [R1\Box \rightarrow Y]$$

means that the truth of statement X under the condition that $R2$ is chosen entails that Y would be true if $R1$ were to be measured instead of $R2$.

In modal logic the truth of such a statement is generally supposed to be justified by appeal to the notion of "closeness of worlds", which is not part of physics. Here such statements are justified by appeal to our putative physics-based causality condition LOC, which brings in the condition of non-dependence directly in terms of the idea of non-dependence of earlier results upon which of two alternative possible measurement is freely chosen later.

Proof

1. LOC: "If $L2$ is chosen and performed (at some earlier time) and the outcome of $L2$ is c and $R2$ is chosen and performed (at a later time), then under the alternative possible condition that $R1$ is chosen and performed (at the later time), instead of $R2$, the outcome of $L2$ (at the earlier time) would still be c":
$(L2 \wedge c \wedge R2) \Longrightarrow [R1 \Box\!\!\rightarrow c]$.

2. LOGIC: "If $L2$ and $R2$ are chosen and performed and the outcome of $L2$ is c then under the alternative possible condition that $R1$ is chosen and performed (at the later time), instead of $R2$, then $L2$ and $R1$ would be performed and the outcome of $L2$ would be c":
$(L2 \wedge R2 \wedge c) \Longrightarrow [R1 \Box\!\!\rightarrow (L2 \wedge R1 \wedge c)]$.

3. QM: "If $L2$ is chosen and performed (at the earlier time) and $R2$ is chosen and performed (at the later time) and the outcome of $R2$ is g, then $L2$ and $R2$ are performed and the outcome of $L2$ is c":
$(L2 \wedge R2 \wedge g) \Longrightarrow (L2 \wedge R2 \wedge c)$.

4. QM: "If $L2$ and $R1$ are performed and the outcome of $L2$ is c then $L2$ and $R1$ are performed and the outcome of $R1$ is f":
$(L2 \wedge R1 \wedge c) \Longrightarrow (L2 \wedge R1 \wedge f)$.

5. LOGIC: $(L2 \wedge R2 \wedge g) \Longrightarrow [R1 \Box\!\!\rightarrow (L2 \wedge R1 \wedge f)]$.

6. LOGIC: $(L2) \Longrightarrow [(R2 \wedge g) \Longrightarrow (R1 \Box\!\!\rightarrow f)]$.

7. LOC: $(L1) \Longrightarrow [(R2 \wedge g) \Longrightarrow (R1 \Box\!\!\rightarrow f)]$.

8. LOGIC: $(L1 \wedge R2) \Longrightarrow [g \Longrightarrow (R1 \Box\!\!\rightarrow f)]$.

9. QM: $(L1 \wedge R2) \Longrightarrow [b \Longrightarrow g]$.

10. LOGIC: $(L1 \wedge R2) \Longrightarrow [b \Longrightarrow (R1 \Box\!\!\rightarrow f)]$.

11. LOC: $(L1 \wedge R2) \Longrightarrow [R1 \Box\!\!\rightarrow (b \Longrightarrow f)]$.

12. LOGIC: $R2 \Longrightarrow [L1 \Longrightarrow [R1 \Box\!\!\rightarrow (b \Longrightarrow f)]]$.

13. QM: $L1 \Longrightarrow [R1 \Longrightarrow \neg(b \Longrightarrow f)]$.

14. LOGIC: $L1 \Longrightarrow [R1 \Box\!\!\rightarrow \neg(b \Longrightarrow f)]$.

15. LOGIC: $R2 \Longrightarrow [L1 \Longrightarrow [R1 \Box\!\!\rightarrow \neg(b \Longrightarrow f)]]$.

The symbol $\neg$ is negates the proposition that follows it.

The conjunction of 12 and 15 contradicts the assumption that the experimenters in regions **R** and **L** are free to choose which experiments they will perform. Thus the incompatibility of the assumptions of the proof is established.

4. Justification of each step

1. The statement $(R2 \wedge X) \Longrightarrow [R1 \Box\!\!\rightarrow Y]$ asserts: "If $R2$ is performed and X is true then [if $R1$ is performed, instead of $R2$, then Y is true.]" The validity of line 1 thus follows from LOC, which claims that the truth of statements referring only to measurements performed and outcomes appearing in **L** at the earlier time cannot be affected by changing in **R**, at the later time, the freely chosen $R2$ to the freely chosen $R1$.

2. This line is just a rewriting of line 2.

3. This is the prediction of QM corresponding to the path from g to c in Fig. 1.

4. This is the prediction of QM corresponding to the path from c to f in Fig. 1.

5. This follows from lines 2, 3, and 4 by two syllogisms.

6. This line follows from line 5 by elementary logic.

7. This follows from line 6 and the LOC claim that (also in the second Lorentz frame, in which the experiments in **L** occur later) a true statement referring only to experiments and observables that can appear only at an earlier time in **R** cannot be made false by changing the free choice made at a later time in **L** from $L2$ to $L1$.

8. This is just a restatement of line 7.

9. This is the prediction of QM corresponding to the path from b to g in Fig. 1.

10. This follows from lines 8 and 9 by syllogism.

11. Note that in line 10 the statement b is made under the condition that $R2$ is performed whereas in line 11 statement b is made under the condition

that $R1$ instead of $R2$ is performed. But then line 11 follows from line 10 and LOC, for LOC implies that the truth of b, which is fixed in **L** at the earlier time, cannot be altered by changing the choice in **R** at the later time from $R2$ or $R1$. It follows from this that if 10 is true then so is 11.

12. This is just a re-writing of 11.

13. This line is entailed by the dotted line from b to e in Fig. 1. Under the condition that $L1$ and $R1$ are performed it is not true that that if b appears in **L** then f must appear in **R**: 50% of the time the outcome e appears instead of f.

14. If something is true under a condition $R1$ then it is true if $R1$ is performed instead if the alternative.

15. This is implied by 14.

As mentioned previously, I have used a symbol, $\Box\rightarrow$, that is similar to one used in modal logic, and which has a verbal translation identical to the one used in modal logic. But the meaning of this symbol in modal logic is usually tied to a notion of "closeness of possible worlds" that is not part of physical theory, and for which various definitions can be given. Consequently, there are many modal logics, and appeal to "modal logic" is, by itself, not sufficient to determine the correctness of arguments [5].

The present proof, although structurally more complex than the proof given in [4] is logically simpler in that all of the steps that follow from LOGIC are true in the *general* theory of counterfactuals [9], without appeal to the special rules that define closeness of worlds.

The present proof, although dealing with "instead of" conditionals, is self contained and uses, in addition to the general logical principles, only ordinary ideas from quantum physics. These are, first of all, the idea that the choices to be made by experimenters can be considered to be free (i.e., unconstrained) variables, and then the idea that an outcome that appears "now", although not completely fixed by past events, does become fixed when the experiment is performed, and hence cannot depend upon what an experimenter will choose to do at a later time. These assumptions, together with the assumed validity of the predictions of quantum theory, lead to line 6.

The conclusion asserted in line 6 refers only to observables appearing in region R. It is derived under the supposition that $L2$ is performed earlier. Then the ideas of special relativity are used to assert that the truth of statements cannot depend upon which frame of reference is used, and hence, in particular, that the established validity of the conclusion in line 6 must continue to be true in a frame in which the choice L2 is made later rather than earlier.

Then a normal idea of causality is used to claim that if a statement S pertaining only to possible outcomes of possible experiments confined to a spacetime region R is known to be true under the supposition that the experimenter in region L freely chooses *at a later time* to perform the experiment L2, then S must be true also if that experimenter were, at the later time, to make the other possible choice: the validity of a statement that relates possible events occurring in the system under study at an earlier time cannot depend upon an unconstrained choice to be made by an experimenter at a later time.

There is, however, no apparent problem with the assumption that these causality conditions hold in some single preferred frame, such as the cosmic black-body radiation rest frame.

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