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LOW-ENERGY IMPACT RESISTANCE OF GRAPHITE-EPOXY PLATES AND ALS HONEYCOMB SANDWICH PANELS

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ABSTRACT

Low energy impact may be potentially dangerous for many highly optimized "stiff" structures. Impact by foreign objects such as birds, ice and runways stones or dropping of tools occur frequently and the resulting damage and stress concentrations may be unacceptable from a designer's standpoint. The present work is concerned with the barely visible,yet potentially dangerous dents due to impact of foreign objects on the Advanced Launch System (ALS) structure. Of particular interest is the computation of the maximum peak impact force for a given impactor mass and initial velocity. The theoretical impact forces will be compared with the experimental dropweight results for the ALS face sheets alone as well as the ALS honeycomb sandwich panels.

1. INTRODUCTION.

One of the earliest work on the low velocity impact on composite sandwich structures was performed by Rhodes (1974,.1978) and Rhodes et al (1979). They showed that the honeycomb core considerably reduced the area of delamination damage as compared with the unsupported graphite-epoxy laminates. and the cripping of the core allows high bending stresses in the face sheets. Oplinger and Slepetz (1975) found from the dropweight experiments that the graphite sandwich panels exhibit marked damage due to nominal impact energy levels as low as 2 ft-lb. due to low strain to fracture of graphite material and low compressive crushing strength of the honeycomb core. They analysed the sandwich panel as a plate resting on an elastic foundation. Sharma (1981) measured the preload and impact energy combination necessary to cause catastrophic failure of graphite/epoxy sandwich structures and examined the residual

strength of the specimens. Further, 't Hart (1981) examined the effect of impact damage on the tension-compression fatigue properties of Carbon/Epoxy Sandwich Panels and found that significant damage may occur at low impact energy. Labor and Bhatia (1980) examined the impact resistance of hybrid and graphite layups and investigated the effects of core densities of the sandwich structure. Gottesman et al. (1987) and Bass (1986) presented experimental and analytical results on the strength of sandwich structures due to low velocity impact. Further studies on the impact resistance and damage tolerance of sandwich plates were reported by Bernard (1987) and Bernard and Lagace (1987). They found that damage in the impacted facesheets was primarily delaminations with the largest delamination occurring between the bottom two plies (5^{th} and 6^{th} plies) in the top facesheet; and debonding of the top facesheet from the adhesive layers was more pronounced in stiffer core. Recent work on the instrumented impact testing of composite sandwich panels was performed by Shih and Jang (1979). They found that the impact resistance was mainly controlled by the facesheets and relatively independent of the density of the poly-vinyl-chloride (PVC) foam core, provided the facesheet material is tough enough; however, for less tough facesheets, the impact failure becomes foam core dominated rather than facesheet dominated. In particular, the macroscopic and microscopic failure modes and energy absorbing characteristics of these sandwich panels were examined by Shih and Jang (1979).

The NASA Langley researchers have done extensive experimental and analytical studies on the low energy impact resistance of graphite/epoxy plates. Bostaph and Elber (1982) presented : static indentation tests on graphite epoxy plates and the results are compared with the theoretically predicted plate stiffness and maximum strain energy at failure. Bostaph (1984) from the US Army showed that toughened materials were able to delay insipient delamination, but not significantly, in quasi-static indentation tests. Many other experiments on impact resistance of composite plates have been performed by numerous investigators (see for example Ciarns and Lagace 1988, Sjoblom et al. 1988). For brievity, the remaining part of this introduction will focus on the"theoretical" studies on impact of composite plates.

One of the earliest theoretical investigations on the impact of isotropic-homogeneous plates was presented by Eringen (1953) and Timoshenko in 1913 as reported in Timoshenko and Goodier (1970). An excellent survey of the historical contributions of various authors was presented by Greszczuk in 1982. Based on the Hertzian contact parameters for transversely isotropic laminates (Conway 1956), Greszczuk presented theoretical and experimental "peak" maximum impact force for quasi-siotropic circular plates under a spherical impactor at the plate center. Further investigations on the theoretical prediction of low velocity impact force and the delamination growth analysis in quasi-isotropic laminates were reported by Shivakumar and Elber (1984) and Shivakumar et al. (1985a,b). The present work follows closely the method presented by Shivakumar (1985b) which includes the finite deflection effects.

2. Impact Analysis of Graphite-Epoxy Plates

The energy-balance method is used to obtain the peak impact force for circular or square graphite-epoxy plates under a concentrated at the plate center. This method was used by Greszczuk(1975, 1981,1982), Greszczuk and Chao (1977) and Shivakumar et al. (1985b) and the theoretical peak impact force was computed based on the equation,

$$(1/2)(M_I)(V_I)^2 = (1/2)(K_b)W^2 + (1/4)K_m W^4 + (2/5)\, P^{5/3}/n^{2/3} \qquad (1)$$

In the above expression, M_I and V_I are the mass and velocity of the impactor, K_b and K_m are the bending and membrane stiffness of the plate, W is the deflection at the center of the plate and the impact force P is proportional to the ralative displacement α raised 3/2 power according to Hertz law (Conway 1956, Willis 1966 and Timoshenko and Goodier 1970) such that,

$$P = n\,\alpha^{3/2} \qquad (2)$$

The various coefficients of Hertz law are computed for the present graphite-epoxy plate and they can be found in Appendix B. From finite deflection plate theory, the impact force can also be written as (Shivakumar et al. 1985b, Vol'mir 1967 and Timoshenko and Woinowsky-Krieger 1959),

$$P = K_b W + K_m W^3 \qquad (3)$$

Although the above formulae are intended for isotropic-homogeneous plates, they can also be applied to transversely isotropic material

(see Dahan and Zarka 1977) and quasi-isotropic materials (Agarwal and Broutman 1980 and Greszczuk 1982), provided one can obtain the average Young's modulus and average Poisson's ratio as described in Appendix A. The energy-balance can be written in a non-dimensional form by dividing through by $E_{av}h^3$ (the total thickness of the composite plate is h),

$$e_o = (1/2)\, k_b\, (W/h)^2 + (1/4)\, k_m\, (W/h)^4 + (2/5)\, p^{5/3}/(n^*)^{2/3} \tag{4}$$

where,

$$e_o = (1/2)M_I V_I^{\,2}/(E_{av}h^3)$$

$$k_b = K_b/(E_{av}h) \tag{5}$$

$$k_m = K_m h/E_{av}$$

$$p = P/(E_{av}h^2) = k_b(W/h) + k_m(W/h)^3$$

$$n^* = n/(E_{av}h^{1/2})$$

The value of n^* for the contact between a steel punch and a graphite-epoxy plate is given by eqn. B10 in Appendix B, using the average value of the composite plate,

$$E_{av} = 7.0 \times 10^6 \text{ psi}, \qquad \nu_{av} = 0.30\ , \qquad n^* = 0.567400142 \tag{6}$$

so that the energy balance equation becomes,

$$e_o = (1/2)\, k_b(W/h)^2 + (1/4)k_m\, (W/h)^4 + 0.583624746\, p^{5/3} \tag{7}$$

From Shivakumar et al. 1985b, there are four possible boundary conditions for the axisymmetric deflection of a circular plate under a concentrated load at the plate center. They are, using h = 0.081 in. , a = 1.50 in. , based on tee grverning equilibrium and compatibiltiy eqns described in Appendix D.

(i) Clamped,In-Plane Immovable, $P = (616.588\ lb)(W/h) + (274.273\ lb)(W/h)^3$

$$k_b = K_b/(E_{av}h) = 4.603066\ (h/a)^2 = 0.01342254$$

$$k_m = K_m h/E_{av} = 2.0479915\ (h/a)^2 = 0.005971943$$

(ii) Clamped,In-Plane Movable, $P = (616.588\ lb)(W/h) + (124.012\ lb)(W/h)^3$

$$k_b = K_b/(E_{av}h) = 0.01342254$$

$$k_m = K_m h/E_{av} = 0.92599413\ (h/a)^2 = 0.002700199 \qquad (8a,b,c,d)$$

(iii) Simply Supported, In-plane Immovable,
$P = (242.846\ lb)(W/h) + (347.017\ lb)(W/h)^3$

$$k_b = K_b/(E_{av}h) = 1.813329093\ (h/a)^2 = 0.005287668$$

$$k_m = K_m h/E_{av} = 2.59117215\ (h/a)^2 = 0.007555858$$

(iv) Simply Supported, In-plane Movable,
$P = (242.846\ lb)(W/h) + (66.09859\ lb)(W/h)^3$

$$k_b = 0.005287668$$

$$k_m = K_m h/E_{av} = 0.493558614\ (h/a)^2 = 0.001439217$$

The above k_b and k_m values for all four types of boundary conditions for a circular plate agree with those reported by Vol'mir (1967 section 43).

The bending stiffness K_b should be replaced by $K_b c_s$ using thick plate theory which takes into account transverse shear effect (Lukasiewicz 1976, 1979 and Shivakumar et al. 1985b) where,

$$c_s = \left[1 + (K_b/K_s)\right]^{-1} \tag{9}$$

$$K_b/K_s = \left[3/(4\pi)\right]\left[K_b/(G_{zr}h)\right]\left[1 - 4\nu_{rz}(G_{zr}/E_{av})\right]\left[(4/3) + \ln(a/a_{contact})\right]$$

Using

$$\begin{aligned} \nu_{rz} &= 0.28, \\ G_{zr} &= 0.5959 \times 10^6 \text{ psi} \\ E_{av} &= 7.0 \times 10^6 \text{ psi} \\ \ln(a/a_{contact}) &= \ln(2a/h) = 3.6119184 \end{aligned} \tag{10}$$

one obtains,

$$K_b/K_s = 1.0680294\, k_b\, (E_{av}/G_{zr}) = 12.54607\, k_b \tag{11}$$

Thus, we have, for each of the four boundary conditions,

(i) Clamped Immovable, or Movable, $c_s = 0.855871$, $k_b c_s = 0.01148796$
(ii) (12)

(iii) or (iv) Simply Supported, Immovable or Movable, $c_s = 0.9377876$

$$k_b c_s = 0.0049587 \tag{13}$$

For a simply supported square plate under a concentrated load P at the plate center, the load deflection relation is (see Timoshenko and Woinowsky-Krieger 1959,section 34 and Ugural 1981 section 3.3),

$$W = 0.01160\ Pb^2/D \tag{14}$$

where b is the length of the side of the square plate and D is the flexural rigidity and W is the deflection at the plate center. This equation can be re-arranged as, letting ν_{av}=0.30,

$$P = 86.20689(hD/b^2)\ (W/h) = 7.894403(E_{av}h^4/b^2)\ (W/h) \tag{15}$$

or (letting b=diameter of the circular plate = 3.00 in.),

$$k_b = K_b/(E_{av}h) = 7.894403\ (h/b)^2 = 0.00575502$$

$$P = (264.3107 \text{ pounds})\ (W/h) \tag{16}$$

Finally, for a clamped square plate under a concentrated load at the center of the plate, one obtains (see Ugural 1981, section 3.12),

$$W = 0.005592\ Pb^2/D \quad , \quad \text{let } b=2a = 3.00 \text{ in.},$$

$$P = 184.327\ (hD/b^2)(W/h) = 16.87984\ (E_{av}h^4/b^2)\ (W/h)$$

$$k_b = K_b/(E_{av}h) = 16.87984\ (h/b)^2 = 0.012305 \tag{17}$$

$$P = (565.150 \text{ pounds})\ (W/h)$$

Comparing the impact force on a simply supported circular plate with radius a and a simply supported square plate with side b=2a such that the circular plate is just inscribed inside the square plate, the impact force on the circular plate is smaller than that of the square plate. That is, the deflection of the circular plate is larger than that of the square plate for the same impact concentrated force.

$$P = (242.846\ \text{lb})(W/h) + \ldots \qquad \text{s.s. circular plate, radius a}$$

$$P = (264.310\ \text{lb})(W/h) + \ldots \qquad \text{s.s. square plate , side =2a} \quad (264.31/242.846= 1.08838) \tag{18}$$

This somewhat unusual result arises from the fact that the deflection mode shape for a square plate is quite different from that of the circular plate. Further, the reaction forces at the four corners of the plate tend to produce a convax upward deflections under the applied downward concentrated force P. On the other hand, for clamped plates,

$$P = (616.588\ \text{lb})(W/h) + \ldots \qquad \text{clamped circular plate, radius a}$$

$$P = (565.150\ \text{lb})(W/h) + \ldots \qquad \text{clamped square plate, side =2a} \quad (616.588/565.15=1.09101) \tag{19}$$

the effects of the reaction forces at the four corners are much less apparent due to the reaction moments along the four edges of the square plate. Since the area of the circular plate is πa^2 and the area of the square plate with side =2a is $4a^2$, the ratio of the area is 1.27323. The impact force of the circular plate is larger than that of the square plate. Since the experimental data lies somewhere between simply supported and clamped conditions, it is expacted that the impact forces for the circular plates are approximately the same as the square plate.

3. Impact Analysis of ALS Honeycomb Sandwich Plates

The ALS honeycomb sandwich structure is composed of a top and bottom graphite/epoxy facesheets and a honeycomb core. The core is made of HFT-3/16-2.0 glass-phenolic material and the core material was manufactured by Hexcel Inc. The facesheets are identical to those examined in section two and each sheet is composed of 16-layer quasi-isotropic laminate. The material property of the honeycomb core is,

$$E_{core} = 17000 \text{ psi},$$

$$G_{xz}(\text{L-straight direction}) = 15000 \text{ psi} \qquad (20)$$

$$G_{yz}(\text{W-non-straight direction}) = 5000 \text{ psi}$$

The above core material properties are higher than those for the Nomex honeycomb core reported by Bernard and Lagace (1987). The sandwich structure is being analysed as a plate resting on an elastic foundation. The flexural rigidity of the top sheet is,

$$D = E_{av}h^3/\left[12(1-\nu_{av}^2)\right] = 340.6673 \text{ lb-in} \qquad (21)$$

The elastic foundation is assumed to be linear elastic so that the force is proportional to the displacement and the core stiffness is,

$$k = E_{core}/h_{core} = 17\text{-}00 \text{ psi}/\ 1.370 \text{ in} = 12408 \text{ lb/in}^3 \qquad (22)$$

where the core thickness is 1.370 and the total thickness is

$$h_{total} = h_{core} + 2h = 1.370 + (2)(0.081\ in) = 1.532\ in \quad (23)$$

The characteristic length is

$$\ell = (D/k)^{1/4} = 0.40705\ in \quad (24)$$

Thus, from Timoshenko and Woinowsky-Krieger (1959) and Hui (1986), the concentrated force is related to the deflection (directly at the location of the application of the force)

$$P = (8Dh/\ell^2)\ (W/h) = (1332.324\ lb)\ (W/h) \quad (25)$$

In the above, the top facesheet is assumed to be infinitely large and this assumption is reasonable since the deflection is highly localized near the concentrated load location (within about 1/2 inch radius as seen from experiments). Recall that $P = K_b W$ so that,

$$K_b = 16448.\ lb/in \quad (26)$$

$$k_b = K_b/(E_{av}h) = 0.029009 \quad (27)$$

4. Discussions of Results

The graphite-epoxy plates and the ALS honeycomb sandwich panels were subjected to a dropweight impactor loading with a mass of either 2.66 lb or 3.9 lb. By varing the initial height of the impactor, the velocity of the impactor just before hitting the plate was recorded by the machine. The "Dynatup" IBM/PC Impact Testing System was manufactured by General Research Corporation, 5383 Hollister Ave., Santa Barbara, Calif. 93111 Tel (805)-964-7724., dated Sept. 12, 1985.

Table 1 shows the maximum peak impact force for graphite/epoxy circular plates and the initial kinetic energy, assuming no energy loss. Both the clamped in-plane immovable boundary condition and the simply supported in-plane movable condition are considered and the results are tabulated in this table. It appears that the experimental results show that the plates are closer to being clamped rather than simply supported at the edge.. The majority of the energy was loss due to the vibration of the impactor system and it ranges from 70 to 75% energy loss. Assuming a 75% energy loss, the predicted maximum peak impact forces are plotted in Figure 1 along with the experimental data. Three different boundary conditions are used in the experiments (i) the circular plates are glued to the circular blocks to avoid in-plane slipping (ii) no glue is applied and in-plane slipping may not be fully prevented (iii) the circular plate is resting on a three inches diameter hole with no supporting system on top of the plate. The peak impact forces for the circular plates with these three boundary conditions differ from 5 to 12% and the theoretical values agree with the experiments.

Figure 2 shows the peak impact force versus the initial kinetic energy for graphite/epoxy square plate with side being 3 inches. It can be seen that the measured impact forces for square plates are almost identical to the circular plates. This experimental observation is consistent with the theoretical predictions described in section 2 for square plates. Again, the theoretical impact forces (shown by the solid curves) agree with the experimental values.

Finally, the maximum peak impact force versus the initial kinetic energy for honeycomb sandwich panels are shown in Figure 3. In order to test the validity of the theoretical model of a plate resting on an elastic foundation, the "loose" honeycomb sandwich panels are used for comparison purposes. The "loose" honeycomb sandwich panel consists of top and bottom face sheets and a honeycomb core but the face sheets are not glued to the core. Experimental data show that the "loose" and "bonded" honeycomb sandwich panels have approximately the same impact force. The theoretical impact force are tabulated in Table 2. Based on a 70% energy loss, the theoretical impact forces are plotted in solid line in Figure 3 and they agree with the the experimental data. The theoretical impact forces are higher than the experimental data since the effects of "buckling" or "crushing" of the core is neglected in the plate on elastic foundation model.

In all these three figures, it can be seen that the peak impact force is proportional to the initial kinetic energy, at least for low initial kinetic energy.

W/h	P Clamped Immovable (pounds)	$(1/2)M_I V_I^2$ (ft-lb)	P SS Movable (pounds)	$(1/2)M_I V_I^2$ (ft-lb)
0.10	53.0350	0.2418323	22.8399	0.099145
0.20	107.7156	0.9534993	46.07643	0.392906
0.30	165.6876	2.1523598	70.01061	0.8848849
0.40	228.59651	3.876113	95.32561	1.5833349
0.50	298.08794	6.179474	122.1314	2.500326
0.60	375.80758	9.133731	150.92029	3.651588
0.70	463.40106	12.826800	182.0886	5.056470
0.80	562.51402	17.363477	216.0332	6.737934
0.90	674.79211	22.865793	253.1505	8.72257
1.00	801.88096	29.473412	293.8371	11.04062
1.10			338.4896	13.72602
1.20			387.50479	16.81643
1.30			441.27900	20.35326
1.40			500.2089	24.38172
1.50			564.69117	28.95090
1.60			635.12231	34.11373
1.70			711.89895	39.927134
1.80			795.4176	46.45196
1.90			886.0750	53.753146
2.00			984.2677	61.89964

Table 1 Maximum peak impact forces and the initial kinetic Energy

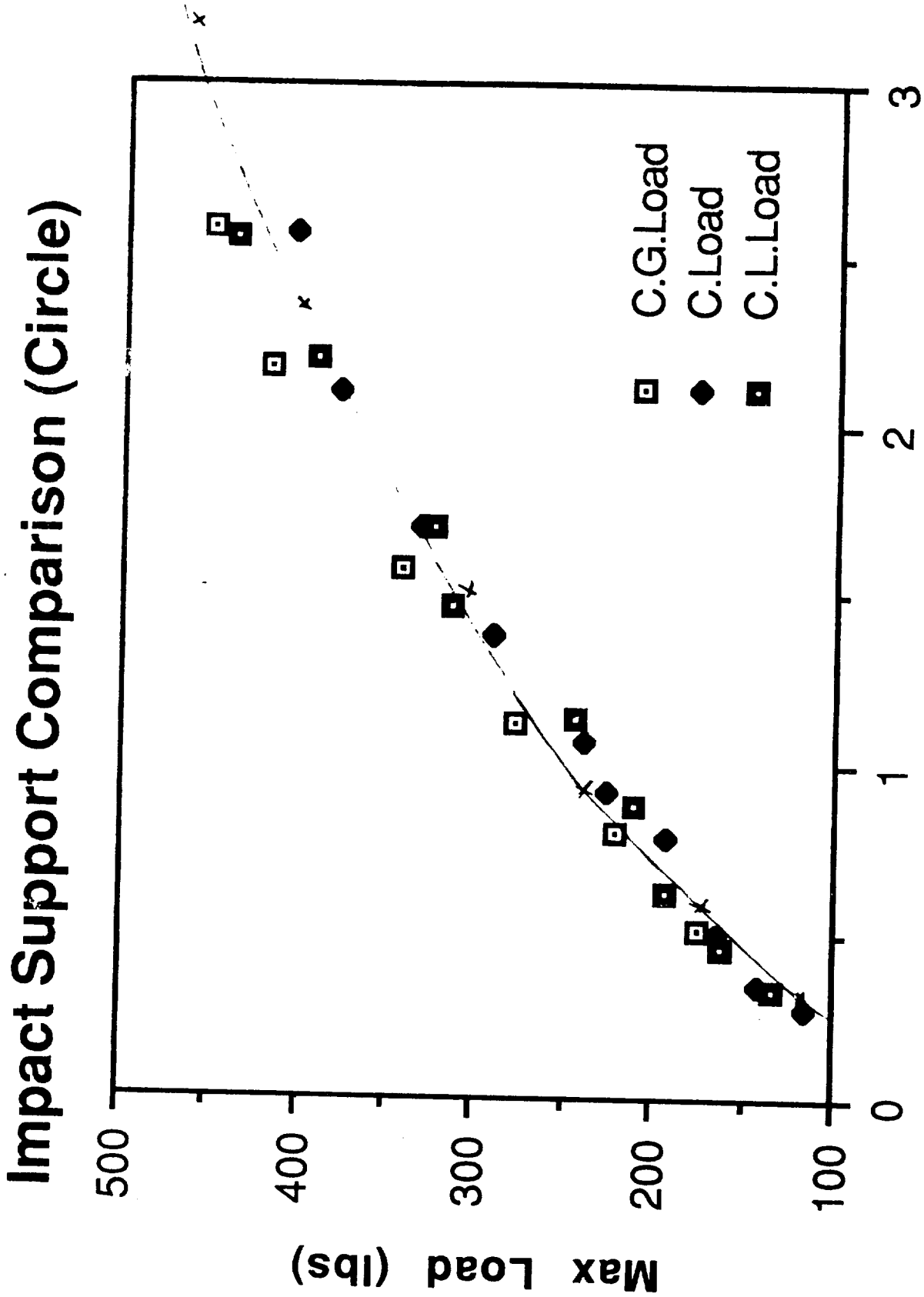

Figure 1 Maximum Peak Impact Force versus the Initial Kinetic Energy for circular graphite-epoxy plates (diameter = 3 inches)

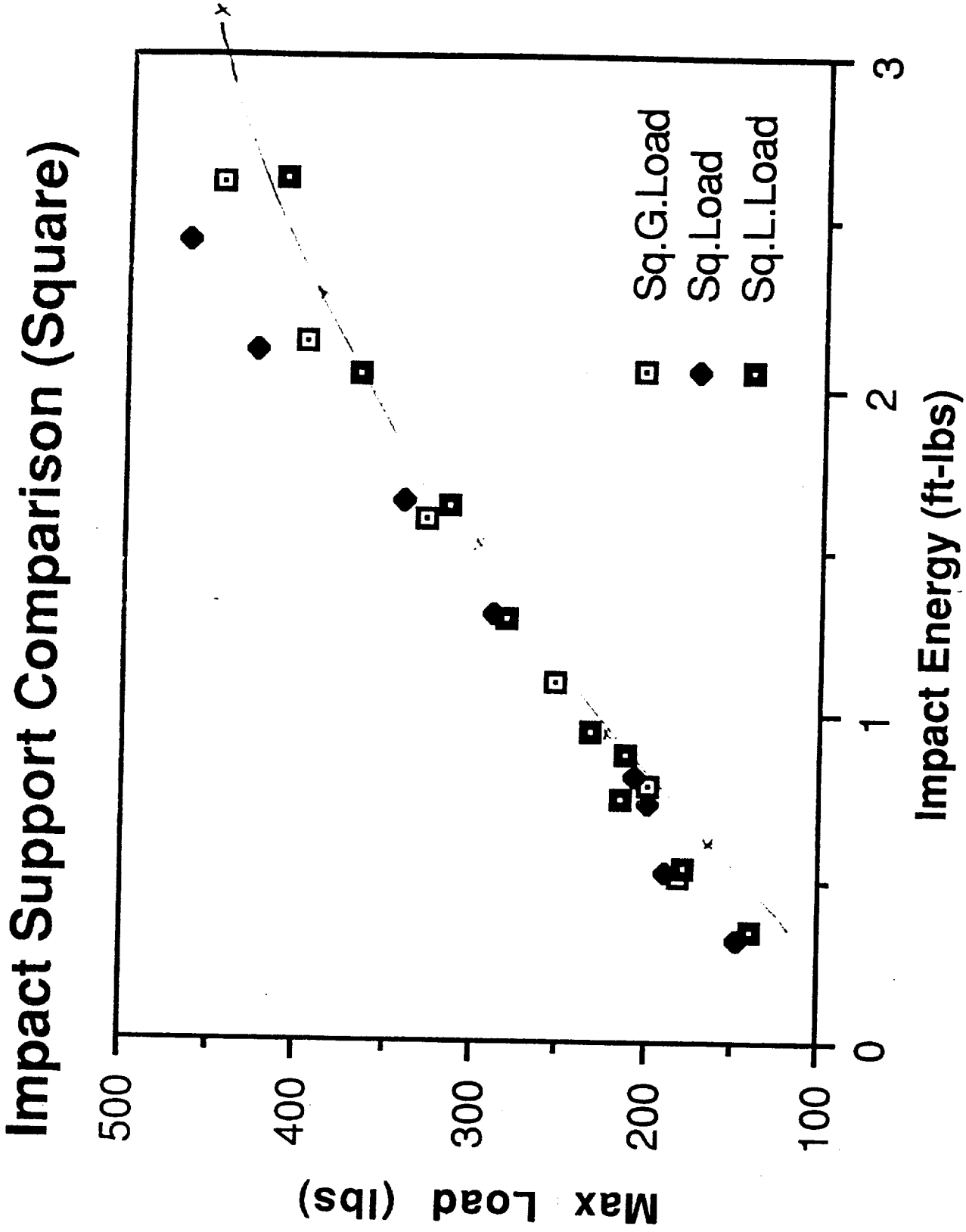

Figure 2 Maximum Peak Impact Force versus Initial Kinetic Energy for Graphite Epoxy Square Plates with Side being 3 inches

ALS Honeycomb Sandwich Panel

W/h	P (lbs)	$(1/2)M_I V_I^2$ (ft-lbs)
0.10	133.2323	0.667702
0.20	266.464	2.56509
0.30	399.6970	5.65577
0.40	532.929	9.924744
0.50	666.1618	15.3627
0.60	799.3941	21.96337
0.70	932.626	29.7215
0.80	1065.85	38.6334

Table 2 Predicted Peak Impact Forces using a Plate on a Linear Elastic Foundation Model

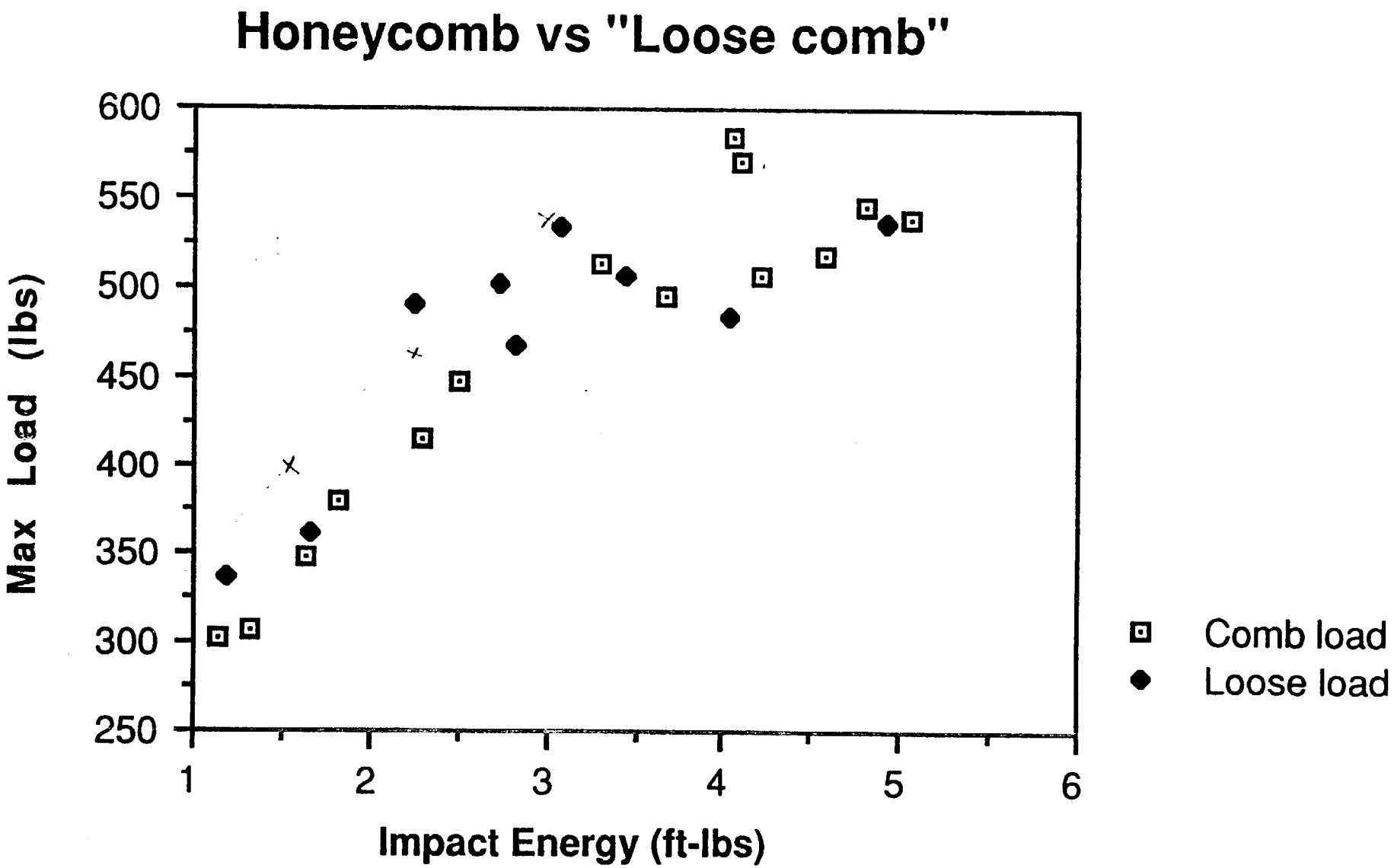

Figure 3 Maximum Peak Impact Forces versus the Initial Kinetic Energy for for "loose" or "bonded" honeycomb Sandwich Panels

5. Conclusions

The theoretical energy balance model is used to predict the maximum impact forces based on a given initial kinetic energy of the impactor. The low energy impact resistance of graphite/epoxy circular and square plates subjected to concentrated forces at the plate center is examined. A theoretical model of a plate resting on a linear elastic foundation is proposed for the ALS honeycomb sandwich sructure and the theoretical impact forces agree with the experimental data.

Further work is being planned for the four point bending of the ALS sandwich panel. The sandwich panels were previously damaged and it is important to study the residual strength of the structure due to various loads including the shear loads.

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The author would like to thank Alan Nettles for his many helpful advices, especially for his guidance in the experimental portion of the work. Sincere thanks are due to David Lance (a summer student from Georgia Institute of Technology) for his experimental skill in obtaining the data. Finally, Dr. Jerry Patterson provided an excellent opportunity to conduct research in a well-equipped laboratory. It was my pleasure to work for the Composite Branch in a friendly environment.

APPENDIX A Constitutive Equations for Laminated Plates

For a symmetrically laminated plate, there is no bending-stretching coupling and the membrane stress resultants(N_x, N_y, N_{xy})are related to the in-plane strains of the middle surface(ε_x, ε_y , γ_{xy}) by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{A1}$$

Further, the bending stress resultants (M_x, M_y, M_{xy}) are related to the curvatures (κ_x, κ_y, κ_{xy}) by

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{A2}$$

In the above expressions, the extensional stiffness A_{ij} and the bending stiffenss D_{ij} can be obtained from,

$$A_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij}(k^{th} \text{ layer}) \, (Z_k - Z_{k-1}) \tag{A3}$$

$$3D_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij}(k^{th} \text{ layer}) \, (Z_k^3 - Z_{k-1}^3) \tag{A4}$$

Each layer is assumed to be orthotropic and the angle between the fiber direction and the x axis is θ. Denoting $S = \sin\theta$ and $C = \cos\theta$, the stress strain relations are,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{A5}$$

where,

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}C^4 + (2Q_{12} + 4Q_{66})\, S^2C^2 + Q_{22}S^4 \\
\bar{Q}_{22} &= Q_{11}S^4 + (2Q_{12} + 4Q_{66})\, S^2C^2 + Q_{22}C^4 \\
\bar{Q}_{12} &= (Q_{11} + A_{22} - 4Q_{66})\, S^2C^2 + A_{12}\,(S^4 + C^4) \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\, S^2C^2 + Q_{66}\,(S^4 + C^4) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\, SC^3 + (Q_{12} - Q_{22} + 2Q_{66})\, S^3C \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\, S^3C + (Q_{12} - Q_{22} + 2Q_{66})\, SC^3
\end{aligned} \tag{A6}$$

Note that the stress strain relations in the material coordinate are,

$$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} \tag{A7}$$

The face sheet of the Advanced Launch System (ALS) structure is made of graphite/epoxy T300/934 material where the material parameters are,

$$E_L = 20.1 \times 10^6 \text{ psi} \quad \text{tension}, \qquad E_L = 19.4 \times 10^6 \text{ psi compression}$$

$$E_T = 1.5 \times 10^6 \text{ psi tension}, \qquad E_T = 2.4 \times 10^6 \text{ psi compression}$$

$$G_{LT} = 0.66 \times 10^6 \text{ psi}$$

$$\nu_{LT} = 0.294 \,, \tag{A8}$$

$$\text{density} = \rho = 0.057 \text{ lb-mass/in}^3$$

In the present analysis, the tensile values of E_L and E_T are used so that,

$$E_L/E_T = 13.400$$

$$G_{LT}/E_T = 0.4400 \tag{A9}$$

$$\nu_{TL} = (\nu_{LT})(E_T/E_L) = 0.021940299$$

From Jones (1975), eqn. 2.61, one obtains,

$$(Q_{11}, Q_{22}, Q_{12}) = (1/c_o)(E_L, E_T, \nu_{LT}E_T)$$

$$Q_{66} = G_{LT} \,, \qquad c_o = 1 - \nu_{LT}\nu_{TL} \tag{A10}$$

Thus, in non-dimensional form,

$$(q_{11}, q_{22}, q_{12}, q_{66}) = (1/E_T)(Q_{11}, Q_{22}, Q_{12}, Q_{66})$$

$$= (13.48699717, \ 1.006492326, \ 0.295908744, \ 0.440000) \tag{A11}$$

The ALS composite plate consists of 16 layers with a total thickness of 0.081 inch where the fiber angles are,

$$(0,\ 45^0,\ 90^0,\ -45^0,\ -45^0,\ 90^0,\ 45^0,\ 0^0)_s \tag{A12}$$

Since the laminate consists of 16 equal-thickness layers, the extensional stiffness can be written as (h = total thickness = 0.081 inch),

$$A_{ij} = (h/4)\left[\bar{Q}_{ij}(\theta=0^0) + \bar{Q}_{ij}(\theta=45^0) + \bar{Q}_{ij}(\theta=-45^0) + \bar{Q}_{ij}(\theta=90^0)\right] \tag{A13}$$

Note that,

$$\bar{Q}_{16}\ (\theta=0) = 0, \quad \bar{Q}_{16}(\theta=90^0) = 0,$$
$$\bar{Q}_{16}(\theta=45^0) = -\bar{Q}_{16}(\theta=-45^0) \tag{A14}$$

so that by inspection, $A_{16}=0$ and similarly, $A_{26}=0$. Further, $\bar{q}_{ij} = \bar{Q}_{ij}/E_T$

$$(a_{11},\ a_{22},\ a_{12},\ a_{66}) = (A_{11},\ A_{22},\ A_{12},\ A_{66})\left[1/(E_T h)\right]$$
$$= (1/4)\left[\bar{q}_{ij}(\theta=0^0) + 2\bar{q}_{ij}(\theta=45^0) + \bar{q}_{ij}(\theta=90^0)\right] \tag{A15}$$

Thus, we have,

$$a_{11} = (1/4)\left[\bar{q}_{11}(\theta=0^0) + 2\,\bar{q}_{11}(\theta=45^0) + \bar{q}_{11}(\theta=90^0)\right]$$

$$a_{11} = (1/4)\left[q_{11} + (2/4)(q_{11} + 2q_{12} + 4q_{66} + q_{22}) + q_{22}\right] = 5.729035748$$

$$a_{22} = (1/4)\left[\bar{q}_{22}(\theta=0^o) + 2\,\bar{q}_{22}(\theta=45^o) + \bar{q}_{22}(\theta=90^o)\right]$$

$$a_{22} = (1/4)\left[q_{22} + (2/4)\,(q_{11} + 2q_{12} + 4q_{66} + q_{22}) + q_{11}\right] = a_{11}$$

$$a_{12} = (1/4)\left[\bar{q}_{12}(\theta=0^o) + 2\bar{q}_{12}(\theta=45^o) + \bar{q}_{12}(\theta=90^o)\right]$$

$$a_{12} = (1/4)\left[q_{12} + (2/4)\,(q_{11} + q_{22} - 4q_{66} + 2q_{12}) + q_{12}\right] = 1.813617745$$

$$a_{66} = (1/4)\left[q_{66} + (2/4)(q_{11} + q_{22} - 2q_{12} - 2q_{66} + 2q_{66}) + q_{66}\right] = 1.957709001$$

(A16a,b,c)

From Jones (1975), the extensional stiffness for an isotropic homogeneous plate is,

$$A_{ij} = \left(\frac{E_{av}h}{1-\nu_{av}^2}\right)\begin{bmatrix} 1 & \nu_{av} & 0 \\ \nu_{av} & 1 & 0 \\ 0 & 0 & (1-\nu_{av})/2 \end{bmatrix} \qquad \text{(A17)}$$

Thus, the in-plane "average" Young's modulus and "average" Poisson's ratio can be obtained from,

$$5.729035784 \quad E_T = E_{av}/(1-\nu_{av}^2)$$

$$1.813617745 \quad E_T = \nu_{av}E_{av}/(1-\nu_{av}^2) \qquad \text{(A18)}$$

$$1.957709001 \quad E_T = E_{av}/2(1+\nu_{av})$$

which implies,

$$\nu_{av} = 1.813617745/5.729035784 = 0.316565966$$

$$E_{av} = 7.732359197 \times 10^{6} \text{ psi} \tag{A19}$$

For a 16-layer laminate, the bending stiffness D_{ij} can be computed using the appropriate weighting factor for each layer (Tsai and Hahn 1980, Table 6.6, page 234),

$$3d_{ij} = 2(1/16)^3 \{169\, \bar{q}_{ij}(\theta=0^o) + 127\bar{q}_{ij}(\theta=45^o) + 91\bar{q}_{ij}(\theta=90^o) + 61\bar{q}_{ij}(\theta=-45^o) + 37\bar{q}_{ij}(\theta=-45^o) + 19\bar{q}_{ij}(\theta=90^o) + 7\bar{q}_{ij}(\theta=45^o) + \bar{q}_{ij}(\theta=0^o)\}$$

$$d_{ij} = D_{ij}/(E_T h^3) \tag{A20}$$

Since $\bar{q}_{16}(\theta=0^o)=0$, $\bar{q}_{16}(\theta=90^o)=0$, one obtains,

$$3d_{16} = 2(1/16)^3 [134\bar{q}_{16}(\theta=45^o) + 108\, \bar{q}_{16}(\theta=-45^o)] = 2(1/16)^3\, 26q_{16}(\theta=45^o)$$

$$d_{16} = (2/3)(1/16)^3 (26/4)(q_{11}-q_{22}) = 0.013203659$$

$$d_{26} = d_{16} = 0.013203659 \tag{A21}$$

The remaining bending stiffness coefficients (d_{11}, d_{22}, d_{12}, d_{66}) can be computed from,

$$d_{ij} = (2/3)(1/16)^3 \{170\bar{q}_{ij}(\theta=0^o) + 232\, \bar{q}_{ij}(\theta=45^o) + 110\bar{q}_{ij}(\theta=90^o)\} \tag{A22}$$

so that, $(d_{22}/d_{11} = 0.778487408)$,

$$d_{11} = (2/3)(1/16)^3 \left[170q_{11} + (232/4)(q_{11}+2q_{12}+4q_{66}+ q_{22})+ 110q_{22}\right]$$

$$d_{11} = 0.550216712 = D_{11}/(E_T h^3)$$

$$d_{22} = (2/3)(1/16)^3 \left[170q_{22} + (232/4)(q_{11}+2q_{12}+4q_{66}+ q_{22}) + 110q_{11}\right]$$

$$d_{22} = 0.428336782 = D_{22}/(E_T h^3)$$

$$d_{12} = (2/3)(1/16)^3\left[170q_{12} + (232/4)(q_{11}+q_{22}- 4q_{66}+2q_{12}) +110q_{12}\right]$$

$$d_{12} = 0.139277710 \qquad \text{(A23)}$$

$$d_{66} = (2/3)(1/16)^3 \left[170q_{66} + (232/4)(q_{11}+q_{22}- 2q_{12}) + 110q_{66}\right]$$

$$d_{66} = 0.151285315$$

From Jones (1975), the bending stiffness for an isotropic homogeneous laminate is,

$$D_{ij} = \left(\frac{\bar{E}h^3}{12(1-\bar{\nu}^2)}\right)\begin{bmatrix} 1 & \bar{\nu} & 0 \\ \bar{\nu} & 1 & 0 \\ 0 & 0 & (1-\bar{\nu})/2 \end{bmatrix} \qquad \text{(A24)}$$

Therefore, by comparison,

$$0.550216712\ E_T = \bar{E}/\left[12(1-\bar{\nu}^2)\right]$$

$$0.428336782\ E_T = \bar{E}/\left[12(1-\bar{\nu}^2)\right]$$

$$0.139277710\ E_T = \bar{\nu}\,\bar{E}/\left[12(1-\bar{\nu}^2)\right] \qquad \text{(A25)}$$

$$0.151285315\ E = \bar{E}/\ 24(1+\bar{\nu})$$

Thus, by assuming that $\bar{\nu}$=0.3, one obtains,

$$\bar{E} = 9.0125497x1w^{6} \text{ psi}$$

$$\bar{E} = 7.016156489x10^{6} \text{ psi} \qquad \text{(A26)}$$

$$\bar{E} = 7.6045629x10^{6} \text{ psi}$$

$$\bar{E} = 7.08015274x10^{6} \text{ psi}$$

Since D_{22}/D_{11}= 77.848%, it is obvious that the bending stiffness in the 1-direction is not the same as that in the 2-direction. This causes a non-axisymmetric behavior in the deflection of a circular plate under a lateral concentrated force at its center. For a conservative design, one should use the average Young's modulus of $7.01615x10^{6}$ psi.

APPENDIX B Coefficients of Hertz Law

According to Hertz Law, the energy due to the caontact between two elastic bodies is,

$$E_c = (2/5)\, P^{5/3} / n^{2/3} \tag{B1}$$

where P is the contact force and (see Shivakumar et al. 1985b)

$$n = 4\, R_I^{1/2} / \left[3\pi (K_1+K_2)\right]$$

$$K_1 = (1-\nu_I^2)/(\pi E_I) \tag{B2}$$

$$K_2 = \frac{(C_{22})^{1/2}\left\{\left[(C_{11}C_{22})^{1/2} + G_{zr}\right]^2 - (C_{12}+G_{zr})^2\right\}^{1/2}}{(2\pi)(G_{zr})^{1/2}\,(C_{11}C_{22} - C_{12}^2)}$$

In the above, R_I is the radius of the impactor, E_I and ν_I are the Young's modulus and Poisson's ratio of the impactor and

$$c_{11} = C_{11}/E_{av} = (E_z/E_{av})(1-\nu_{av})\beta$$

$$c_{22} = C_{22}/E_{av} = (\beta)(1-\nu_{zr}^2\delta)/(1+\nu_{av})$$

$$c_{12} = C_{12}/E_{av} = \nu_{zr}\beta, \qquad g_{zr} = G_{zr}/E_{av} \tag{B3}$$

$$\beta = 1/\left[1-\nu_{av} - 2\nu_{zr}^2\delta\right]$$

$$\delta = E_{av}/E_z$$

Thus, equations B2 can be written in the non-dimensional form,

$$n^* = n/(E_{av}h^{1/2}) = 4\,(R_I/h)^{1/2}/\left[3\pi(E_{av}K_1+E_{av}K_2)\right]$$

$$E_{av}K_1 = (1-\nu_I^2)(E_{av}/E_I)/\pi$$

$$E_{av}K_2 = \frac{(c_{22})^{1/2}\left\{\left[(c_{11}c_{22})^{1/2}+g_{zr}\right]^2 - (c_{12}+g_{zr})^2\right\}^{1/2}}{(2\pi)(g_{zr})^{1/2}\,(c_{11}c_{22}-c_{12}^2)} \qquad \text{(B4)}$$

In the present analysis, we have,

$$E_{av} = 7.0000 \times 10^6 \text{ psi}, \qquad \nu_{av} = 0.30 \qquad \text{(B5)}$$

From Shivakumar et al 1985b, one can estimate the following material parameters,

$$\nu_{zr} = 0.060, \qquad G_{zr} = 0.59\times10^6 \text{ psi}, \qquad E_z = 1.70\times10^6 \text{ psi} \qquad \text{(B6)}$$

so that,

$$\delta = E_{av}/E_z = 4.117647059,$$

$$\beta = 1.491751492 \qquad \text{(B7)}$$

$$(c_{11}, c_{22}, c_{12}, g_{zr}) = (0.253597754,\ 1.130491131,\ 0.089505090,\ 0.084285714)$$

Based on the computed values of c_{11}, c_{22}, c_{12}, g_{zr}, one obtains,

$$E_{av}K_2 = 1.244177058 \tag{B8}$$

For the impactor,

$$\nu_I = 0.30, \qquad E_I = 29.0\text{x}10^6 \text{ psi}, \qquad R_I = 0.250 \text{ in}$$

$$E_{av}K_1 = 0.069918413 \tag{B9}$$

so that,

$$n^* = 0.567400142 \tag{B10}$$

APPENDIX C Design Guideline for Stacking Sequence of Quasi-Isotropic Laminated Plates

From Appendix A, it was demonstrated that the bending stiffness in the 2-direction is only 77.848% of the bending stiffness in the 1-direction. The material parameters of the ALS graphite-epoxy T300/934 laminate are,

$$E_L/E_T = 13.400, \qquad G_{LT}/E_T = 0.44, \qquad \nu_{LT} = 0.294$$

and the stacking sequence is,

$$(0^o, 45^o, 90^o, -45^o, -45^o, 90^o, 45^o, 0^o)_s$$

There are sixteen layers and the total thickness is 0.081 in. From Tsai and Hahn (1980 table 5.4 and table 6.6) and Jones (1975), if the laminate consists of "equal-thickness" lamina, the extensional stiffnesses (A_{11}, A_{22}, A_{12}, A_{66}, A_{16}, A_{26}) are independent of the stacking sequence That is, an interchange of say any two of the above layers would not change the "in-plane" A_{ij} stiffnesses and each layer carries the weighted factor of unity. On the other hand, the "out-of-plane" bending stiffnesses (D_{11}, D_{22}, D_{12}, D_{66}, D_{16}, D_{26}) depends on the stacking sequence such that outer layers carry a larger weighted factor than the layers near the middle surface (halfway between the top and bottom fibers). For example in the small deflection bending of a beam subjected to a three point bending load, the top half of the beam is under compression and the bottom half is under tension so that the middle surface has zero in-plane stress and the layers near the middle surface should carry a smaller weighted factor than those near the outer fibers. The weighted factors

for a 32 layers laminate is (Tsai and Hahn 1980),

$$(721, 631, 547, 469, 397, 331, 271, 217, 169, 127, 91, 61, 37, 19, 7, 1)_s$$

in the present ALS panel which consists of 16 layers, the weighted factors are (there are "four" sets of the 0, 90, 45,-45 layup),

$$(169, 127, 91, 61, 37, 19, 7, 1)_s$$

It was demonstrated from the Utah-laminate computer program that as one increases the number of "sets" to 16 for a 64 layer laminate, the bending stiffness D_{11} and D_{22} are essentially the same (within 1%). If the number of layers is only 16, one can interchange say two layers so that the reatio of D_{22} to D_{11} would be closer to unity. For example, in the above stacking sequence, one obtains,

for the 0^o, the weighted factor is 169+1 = 170
for the 45^o, the weighted factor is 127+7 = 134
for the 90^o, the weighted factor is 91+19 = 110
for the -45^o, the weighted factor is 61+37 = 98

Consider the following stacking sequence,

$$(45, -45, 0, 90, 90, -45, 0, 45)_s$$

one can easily show that D_{11} is identical to D_{22} since 91+7 = 61+37. However, such stacking sequence has the drawback that bending stiffness in the +45 or -45 directions are not the same (169+1=170 and 127+19=146). A compromise stacking sequence may be,

$(A, B, C, D, B, D, A, C)_s$

where the weighted factors of A, B, C, D are (169+7=176, 127+37=164, 91+1 =92, 61+19=80) and A, B, C and D refer to 0^o, 90^o, 45^o, -45^o respectively or A,B,C,D, may refer to 90^o, 0^o, 45^o, -45^o, respectively, etc. This stacking sequence represents the "best" design since the composite plate behaves like an isotropic-homogeneous plate.

Halpin (1984, section 4.6.2) suggested that the interlaminar shear stresses σ_{zx} will be "significantly" lower if the $+45^o$ and -45^o layers are separated by a 0^o or 90^o layer.for a quasi-isotropic layup involving (45, -45, 90, 0). He showed that there are only 12 distinct stacking possibilities. Among these 12 possibilities, six of them involve adjacent $\pm 45^o$ layers and six with $\pm 45^o$ interspered between 0^o or 90^o layers.(see tables B1 and B2 from Halpin 1984).

Thus, there is no simple solution in order to satisfy the two criteria (i) the laminate behaves like an isotropic-homogeneous plate (ii) the interlaminar stresses are minimized. It appears that increasing the number of "sets" to say 6 for a total of 24 layers, making sure that the $+45^o$ and -45^o layers are separated by 0^o or 90^o layers, would be the best stacking sequence design.

Strength Heirarchy	Laminate Stress σ_y (KSI)	Laminate	Interface Moment in-/b in	Finite Element Maximum Stresses (KSI)
7	-26.5 →	90	-0.33	
	-0.5 →	0	-1.00	σ_z = -8.2
	13.5 •	45	1.50	τ_{zx} = 9.0
	13.5 ←	45	-1.67	
8	-26.5 →	90	-0.33	
	13.5 ←	45	0.82	σ_z = 7.4
	13.5 ←	-45	0.98	τ_{zx} = -9.2
	-0.5 →	0	-0.98	
9	-0.5 →	0	-0.01	
	-26.5 →	90	0.34	σ_z = -7.6
	13.5 ←	45	-0.85	τ_{zx} = -9.2
	13.5 ←	-45	-1.02	
10	-0.5 →	0	-0.01	
	13.5 •	45	0.15	σ_z = 10.0
	13.5 •	-45	0.65	τ_{zx} 8.3
	26.5 •	90	0.98	
11	13.5 ←	45	0.17	
	13.5 ←	-45	0.67	σ_z = 9.0
	-26.5 →	90	1.02	τ_{zx} = -7.7
	-0.5 →	0	1.02	
12	13.5 ←	45	0.17	
	13.5 •	45	0.67	σ_z = 10.9
	0.5 •	0	1.34	τ_{zx} 7.2
	26.5 →	90	1.67	

○ maximum τ_{zx}
□ maximum σ_z
T/300 5208 Graphite Epoxy
ϵ_x = 0.5%

Table C1 Normal Stress and Interlaminar Stress for adjacent $\pm 45^0$ Quasi-isotropic laminate

Strength Heirarchy	Laminate Stress σ_y (KSI)	Laminate	Interface Moment in-lb in	Finite Element Maximum Stresses (KSI)
1	-26.5 → 13.5 ← -0.5 → 13.5 ←	90 45 0 -45	-0.33 -0.83 -1.16 -1.33	σ_z = -6.8 τ_{zx} = -6.9
2	-0.5 → 13.5 ← 26.5 → 13.5 ←	0 -45 90 45	-0.01 0.15 0.15 0.02	σ_z = 6.2 τ_{zx} = 6.6
3	13.5 ← -26.5 → -0.5 → 13.5 ←	45 90 0 -45	0.17 0.17 -0.16 -0.33	σ_z = 6.6 τ_{zx} = 5.9
4	13.5 ← -26.5 → 13.5 ← 0.5 →	45 90 45 0	0.17 0.17 0.02 0.02	σ_z = 6.9 τ_{zx} = -6.5
5	13.5 ← 0.5 → 13.5 → -26.5 ←	45 0 90 -45	0.17 0.50 0.99 1.33	σ_z = 7.6 τ_{zx} = -5.8
6	13.5 ← -0.5 → 13.5 ← -26.5 →	45 0 90 -45	0.17 0.50 0.99 1.33	σ_z = 10.4 τ_{zx} = -6.0

○ - maximum $|\tau_{zx}|$
□ - maximum $|\sigma_z|$
T/300/5208 Graphite-Epoxy
ϵ_x = 0.5%

Table C2 Normal Stress and Interlaminar Stress for Interspersed $\pm 45^0$ with either a 0^0 or 90^0 Quasi-isotropic Laminate

Appendix D Equilibrium and Compaitbility equation for Circular Plates and Rectangular Plates

From Chia(1980,section 3.2), the nonlinear equilibrium and compatibility equations (written in terms of the out-of-plane displacement W and a stress function F) are, respectively, assuming axi-symmetric deflection,

$$(D)(R)\left[(1/R)(RW_{,R})_{,R}\right]_{,R} = \int_{s=0}^{s=R} sq\ ds + F_{,R}W_{,R} \tag{D1}$$

$$R\left[(1/R)(RF_{,R})_{,R}\right]_{,R} = (-Eh/2)\ (W_{,R})^2 \tag{D2}$$

where

$$N_r = (1/R)F_{,R}$$

$$N_\theta = F_{,RR} \tag{D3}$$

$$N_{r\theta} = 0$$

In the above expression, R is the radial coordinate, E is Young's modulus, h is the thickness, q is the applied stress, and N_r, N_θ are the membrane stress resultants. Assuming that the concentrated load can be represented by an applied stress over the contact area of radius $a_{contact}$, and dropping the nonlinear term $F_{,R}$ and $W_{,R}$ in the equilibrium equation for small deflection, one can show that the exact deflection is of the form,

$$W = W_1\ (1 + c_1 r^2 + c_2 r^{2+\varepsilon}) \tag{D4}$$

where ε is arbitrarily close to zero

The governing nonlinear equilibrium and compatibility equations for symmetrically laminated rectangular plates are, respectively, (Hui 1985a,b)

$$L_{D*}(W) = F_{,YY}W_{,XX} + F_{,XX}W_{,YY} - 2F_{,XY}W_{,XY} \tag{D5}$$

$$L_{A*}(F) = (W_{,XY})^2 - W_{,XX}W_{,YY} \tag{D6}$$

where W is the out-of-plane deflection, F is the stress function, X and Y are the in-plane coordinates and $L_{D*}(\ )$ and $L_{A*}(\ )$ are the differential operators defined by,

$$L_{A*}(\ ) = A^*_{22}(\ )_{,XXXX} + (2A^*_{12}+A^*_{66})(\ )_{,XXYY} + A^*_{11}(\ )_{,YYYY}$$

$$L_{D*}(\ ) = D_{11}(\ )_{,XXXX} + (2D_{12}+4D_{66})(\ )_{,XXYY} + D_{22}(\ )_{,YYYY}$$

The A^*_{ij} and D_{ij} are the material parameters defined by Jones (1975).

Finally, an extension of the work by Eringen(1953) to laminated plates was presented by Sun and Chattopadhyay (1975) based on the summation of the various deflection modes corresponding to different frequencies of an anisotropic rectangular palte. This method was used by Chou and Mortimer (1976) who presented a computer code to predict the contact force, deflection and bending strains of an anisotropic plate. Using this code, the predicated strains were in good agreement with the experimental data as reported by Dobyns and Porter (1981).

The present work deals with the prediction of the peak impact force due to low energy impact on graphite/epoxy circular and square plates and ALS boneycomb sandwich panels. Using the energy balance method, the initial kinetic energy is dissipated in terms of the bending energy of the plate, the membrane energy(due to the stretching of the mid-surface of the plate in finite deflection), the Hertzian contact energy (due to the imbeddment of the impact in the plate) and the energy loss (due to the vibration of the impactor punching system). The "average" Young's modulus and "average" Poisson's ratio as well as the estimated out-of-plane material parameters (see Shivakumar and Crews 1982 and Kriz and Stinchcomb 1979) are reported based on the ALS 16-layer quasi-isotropic layup. The Hertzian parameters are then computed in Appendix B. The theoretical contact forces are in good agreement with the experiments. Some design guidlines for various stacking sequence are presented in Appendix C.

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