Cosmology Overview – Whence Cosmology?
Dynamics of the Universe
Fate of the Universe

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1 OVERVIEW

1.1 Vital Statistics of the Universe

$$R \approx 10^{28}\,\mathrm{cm} \qquad M \approx 10^{22}\,M_\odot$$
$$N_b \approx 10^{77} \qquad \rho \approx 10^{-29}\,\mathrm{g\ cm^{-3}}$$
$$T \approx 3\,\mathrm{K} \qquad t \approx 10^{10}\,\mathrm{y}$$
$$s \approx 10^{10}k \qquad p \approx 10^{-19}\,\mathrm{atm}$$

$$\text{Energy} \approx 0 \approx \text{Charge}$$

R = characteristic size of the universe

M = mass within the corresponding volume

N_b = number of baryons ("ordinary particles") in that volume

ρ = average mass/energy density in universe

T = characteristic background temperature of universe

t = age of universe

s = entropy per baryon of universe in units of Boltzmann's constant

p = average pressure of universe (in units of Earth's atmosphere)

Energy and charge, both conserved quantities, close to zero. Suggests possibility universe could have arisen from the null state – the **vacuum** – through **quantum fluctuations**. High entropy (s conserved).

Fairly simple. Is it fundamental (is there a choice)?

1.2 Whence Cosmology?

How can we approach a scientific description of the entire universe?

By using fundamental principles underlying physics – test.

Though the wheel has 30 spokes,

it is the space at the center that makes it useful,

A journey of a thousand li begins underneath one's feet...

– Lao Tse, *Dao De Qing*

2 EQUIVALENCE PRINCIPLE

How describe physics itself of the universe?

Einstein said laws of physics, i.e. *form* of physics, same everywhere in universe – **Principle of Equivalence**.

2.1 Principle of Equivalence

Theoretical form:

One can choose a coordinate system such that locally the form of the laws of physics is that in special relativity.

Thus only need to understand special relativity and coordinate transformations to write laws of physics in form valid anywhere in universe.

Experimental form:

$$m_i/m_g = \text{constant}$$

Acceleration caused by forces (inertial mass m_i) is equivalent to properties of gravitation (gravitational mass m_g) – independent of internal properties of objects. Verified to $\mathcal{O}(10^{-12})$.

Motto:

Acceleration = Gravitation = Curvature

Example: physics of bending of light independent of interpretation used; whether the local coordinate system is thought to be experiencing acceleration, gravitational force, or curved geometry.

Where does curvature come from?

Acceleration $\Rightarrow$ over time get $v = gh \Rightarrow z = v = gh$

But $t' \neq t_0 \Rightarrow$ parallel not parallel, i.e. curvature.

2.2 Metric

Mathematically, Principle of Equivalence implies that "distance" **interval** ds between events in spacetime must be given by a quadratic form (**metric** g_{ab}) of the coordinates dx^a:

$$ds^2 = g_{ab} dx^a dx^b$$

General relativity is math and physics of g_{ab} – need to know almost none for our universe. But here's 1 minute intro:

2.3 Curved Spacetime

Can develop gravitation theory 3 ways:

- Coordinate Invariance and Tensors
- Field Theory of Spin 0,1,2 Fields
  spin 0: no couple to light ; spin 1: repulsive (em)
  spin 2: $T^{ab}\langle h_{ab} h_{a'b'}\rangle T^{a'b'}$ propagator
- Differential Geometry

Each way leads to the same equations of General Relativity:

$$G_{ab}[g_{ab}] \equiv R_{ab} - (1/2)Rg_{ab} \qquad \text{(spacetime)}$$

$$R_{ab} - (1/2)Rg_{ab} = 8\pi T_{ab} \qquad \text{(Einstein Field Eqs)}$$

$$\text{(particle physics)} \qquad 8\pi T_{ab} \equiv (2/\sqrt{-g})[\delta(\sqrt{-g}\mathcal{L})/\delta g^{ab}]$$

where T_{ab} is the energy-momentum tensor and $\mathcal{L}$ is the Lagrangian.

3 COSMOLOGICAL PRINCIPLE

How interpret whole universe from local observations?

3.1 Isotropy

Observations indicate universe looks much the same in all directions – isotropy. E.g. counting galaxies on the sky; temperature of cosmic background radiation field – uniform in direction to $\mathcal{O}(10^{-5})$.

3.2 Homogeneity

Is universe much the same not just in all directions on the sky, but at all points in space – homogeneity.
Want 2-d sky information $\rightarrow$ 3-d map of the universe.

Only one local vantage point so isotropy about that one point doesn't guarantee homogeneity. But would if that one point were random location.

Copernican Principle says we not in preferred location at center of universe (Cosmological Principle or Principle of Cosmic Modesty). If accepted then our universe can be taken to be homogeneous.

Seems reasonable we not in special location, but want to test. Must build 3-d map through astronomical surveys.

3.3 Evolution

But finite speed of light says looking out in distance is looking back in time – things may change.

Can't directly compare distant objects to nearby ones without understanding **evolution**. To verify Cosmological Principle – to do cosmology – must also study astrophysics and understand galaxies and clusters.

Through combination of *assuming* Cosmological Principle and using partial and ever improving verifications of it from large scale surveys, we will adopt homogeneity as a property of our universe.

3.4 Robertson-Walker Metric

Isotropy and homogeneity immediately give specific form to the metric g_{ab} which determines the spacetime geometry and physics of the universe.

This is **Robertson-Walker** form. Distance interval ds is simple; in some common coordinate choices:

Table 3.1: Forms of the Robertson-Walker Metric

Isotropic:

$$ds^2 = -dt^2 + a^2(t)\,(1 + kr^2/4)^{-2}[dr^2 + r^2 d\omega^2]$$

Comoving:

$$ds^2 = -dt^2 + a^2(t)\,[dr^2 + \chi^2 d\omega^2]$$

$$\chi = (\sin r,\ r,\ \sinh r) \quad \text{for} \quad k(>, =, <)0$$

Standard:

$$ds^2 = -dt^2 + a^2(t)\,[(1 - kr^2)^{-1} dr^2 + r^2 d\omega^2]$$

Here $d\omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ is the usual interval on the 2-sphere.

Two important properties to note about RW metric:

1) stays isotropic and homogeneous over time (never mixes r and t and only depends on r as a whole, never x, y, z)

2) involves one variable dependent on time, $a(t)$, and one constant, k

3.5 Distances and Kinematics (curvature and expansion)

The constant k enters the spatial part of the metric and causes it to deviate from the Euclidean form; k is the **spatial curvature constant**.

- spatial curvature only constant because homogeneous, isotropic
- spatial curvature different from spacetime curvature;
  even if $k = 0$ there is spacetime curvature: 4-d not Euclidean

dr is the **coordinate distance** interval.
$(1 - kr^2)^{-1/2}dr$ is the **comoving distance** interval.
$a(t)\,(1 - kr^2)^{-1/2}dr$ is the **proper distance** interval

The parameter $a(t)$ multiplies the spatial metric and is called the **scale factor**. Note $a(t)$ just scales proper to comoving distance intervals: $dr_{\mathrm{prop}} = a\,dr_{\mathrm{com}}$ – evolution in time scaled out in comoving frame so coordinate positions constant – objects "move with" the spacetime evolution and appear fixed in that frame. Later call $a(t)$ **expansion parameter**.

Often normalize $a(t)$ so $a(\mathrm{today}) = 1$. Sometimes normalize $k = \pm 1$. Can't do both!

Can read off all kinematics of universe from metric – redshift, Hubble expansion. (Done by Weyl 1921 – preHubble!)

For dynamics (forces), need to know $a(t)$ – need to relate metric to energy content through theory of gravitation, e.g. general relativity.

4 DYNAMICS OF THE UNIVERSE

GR says all forms of energy have mass: energy, momentum, pressure. Homogeneity and isotropy say T_{ab} **perfect fluid**.

$$G_{ab}(a,k) = 8\pi T_{ab}(\rho, p)$$

Our universe is remarkably simple enough to treat quasiNewtonian.

4.1 Newtonian Approach

"Matter evolving in closed system" recalls thermodynamics.

- $a(t)$ – changing scales, i.e. volume, is like 1st law. Adiabatic $\Rightarrow$

$$dU = d(\rho V) = -p\,dV$$
$$\dot{\rho} = -(\rho + p)\frac{\dot{V}}{V}$$

- Motion of particle: 1st derivative gives energy conservation

$$\frac{1}{2}m\dot{R}^2 + G\frac{mM}{R} = \text{constant}$$
$$\left(\frac{\dot{R}}{R}\right)^2 = -\frac{8\pi}{3}\rho + \text{constant} \times R^{-2}$$

- 2nd derivative gives Newton's 2nd law

$$m\ddot{R} = \frac{GMm}{R^2}$$
$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}\rho$$

$R \sim a(t)$, $V \sim a^3$, and GR includes pressure in gravity:

FRIEDMANN EQUATIONS

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho - ka^{-2}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p)$$

4.2 General Relativity

GR well tested, gives rigorous derivation of Friedmann eqs.

3 unknowns (a, ρ, p) so need 3 eqs, but conservation eq redundant (used energy conservation twice/only 2 eq from hom and iso).

To close system, need **equation of state** from particle physics.

All components of interest follow EOS

$$p = w\,\rho$$

for example,

matter ($w = 0$: "dust")

radiation ($w = 1/3$: photons, neutrinos, GW, relativistic matter)

cosmological constant ($w = -1$)

REVIEW OF PRINCIPLES

Equivalence Principle

$\Rightarrow$ Metric Description

Homogeneity and Isotropy

$\Rightarrow$ Metric is Robertson-Walker

$\Rightarrow$ Energy-Momentum Tensor is in perfect fluid form

Gravitational Field Equations (general relativity)

& Homogeneity and Isotropy

$\Rightarrow$ Friedmann Equations for evolution of spacetime

Equation of State

& Friedmann Equations

$\Rightarrow$ Evolution of matter

4.3 Density Evolution

Solution to conservation Friedmann eq ($\dot{\rho} = -3(\dot{a}/a)(\rho + p)$), each component separately,

$$\rho_w \sim a^{-3(1+w)}$$

EOS intuitive: $\rho_w \sim E_w N_w / V \sim E_w N_w a^{-3}$

At any one time, one component tends to dominate. But each evolves at different rate so can be overtaken.

Components' EOS and Evolution

General	$\rho_w \sim a^{-3(1+w)}$	$a \sim t^{2/[3(1+w)]}$	$t_0 = \frac{2}{3(1+w)} H_0^{-1}$
Radiation	$\rho_{1/3} \sim a^{-4}$	$a \sim t^{1/2}$	$t_0 = \frac{1}{2} H_0^{-1}$
Nonrelativistic matter	$\rho_0 \sim a^{-3}$	$a \sim t^{2/3}$	$t_0 = \frac{2}{3} H_0^{-1}$
Curvature	$\rho_{-1/3} \sim a^{-2}$	$a \sim t$	$t_0 = H_0^{-1}$
Cosmological constant	$\rho_{-1} \sim a^0$	$a \sim e^{H_0 t}$	$t_0 = \infty$

We define

$$
\begin{aligned}
H &\equiv \dot{a}/a && \text{(Hubble parameter)} \\
\rho_c &\equiv \frac{3H^2}{8\pi} && \text{(critical density)} \\
\Omega &\equiv \rho/\rho_c && \text{(dimensionless density)}
\end{aligned}
$$

with $H_{\rm today} \equiv H_0$ – **Hubble constant**. Note $H_0^{-1} = 10^{10} h^{-1} yr \approx 14$ Gyr.

5 EVOLUTION OF THE UNIVERSE

5.1 Expansion

Scale factor $a(t)$ multiplies all free distances, so wavelengths should expand (*cf.* adiabatic box in quantum theory or organ pipe).

$$z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} \equiv \Delta\lambda/\lambda = \frac{a(t_o)}{a(t_e)} - 1$$

$$\lambda \sim a \qquad\qquad \rho_\gamma \sim a^{-4} \qquad\qquad T_\gamma \sim a^{-1}$$

Expansion leads to lower density, lower temperature (analogy to air escaping thru lips).

> *Observation of expansion leads to picture of hot dense early universe:* **Big Bang model** *with age of order* H_0^{-1}

5.2 Curvature

Velocity Friedmann eq relates expansion rate H to energy content

$$H(z) \equiv (\dot{a}/a) = H_0 \left[\sum \Omega_w (1+z)^{3(1+w)} + (1-\Omega)(1+z)^2 \right]^{1/2}$$

Can rewrite in terms of curvature constant

$$k = [\Omega(z) - 1] a^2 H^2(z) = (\Omega - 1) a_0^2 H_0^2$$

Different components cause different expansion evolution (acceleration/deceleration) and curvature density $1 - \Omega$ to evolve differently:

- if $w > -1/3$ (normal) then $|1 - \Omega|$ increases with time,
- if $w < -1/3$ (exotic) then $|1 - \Omega|$ decreases with time.

Fate of universe controlled by acceleration.

Today, after many characteristic Hubble expansion times, universe has $1 - \Omega \approx 0 - 0.75$, close to unstable critical density $\Omega = 1$.

Analogous to pencil standing on point.

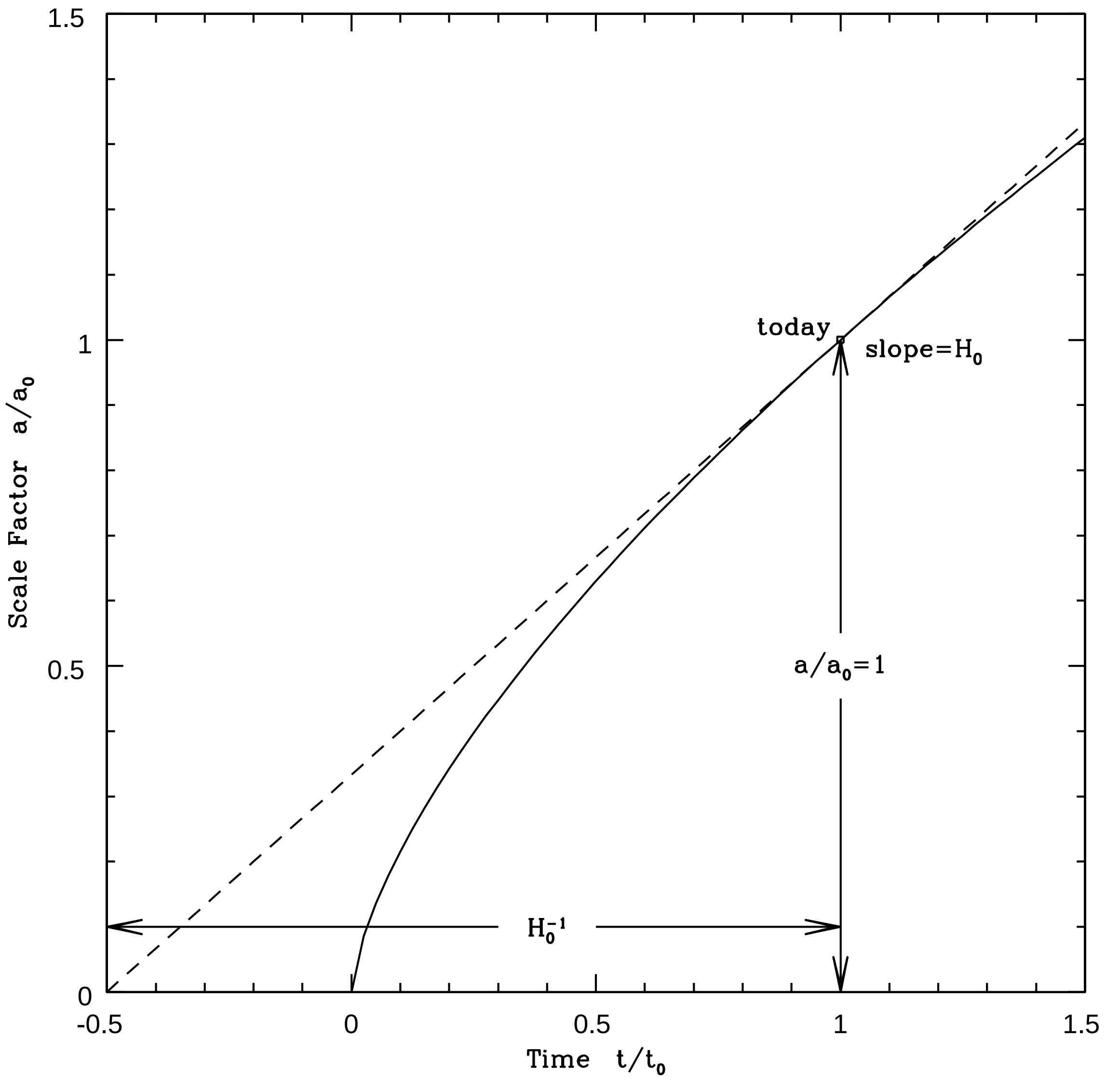
Suggestive of exotic matter episode driving $|1 - \Omega| \to 0$ (**inflation**).

5.3 Fate of the Universe

Fate of universe controlled by **deceleration parameter**

$$q_0 = -\frac{\ddot{a}a}{\dot{a}^2}$$

Gives *spacetime* curvature.



Age t_0 > or < H_0^{-1}, depending on energy type and magnitude.

To determine fate, need to measure densities and EOS of components of universe (matter, cosmological constant, etc.). Search for Ω, H_0, Ω_w.