Computational Differentiation in BVPs in Ordinary Differential Equations

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Extended Abstract

We will describe the use of Computational Differentiation as the basis of a fast, reliable integrator based upon the use of the Taylor series. The Frobenius recurrence relations is the means of representing ordinary differential equations.

We will also discuss an analysis of routines that perform basic operations on power series to determine their efficiency. We examined the number of (1) floating point additions (and subtractions), (2) floating point multiplications, (3) floating point divisions (in case they are more expensive than multiplications,) and (4) function evaluations, *e.g.* exponential, sine, cosine, square root, etc.

Thus, the analysis of some common power series operations is given in terms of the length of the truncated power series. It is shown that functions on power series can be efficiently computed. This can be the basis of accurate, efficient integrators for ordinary differential, integral, and integro-differential equations.

This analysis led to careful construction of these functions and the run times were surprising. Previous tests had indicated run time reductions (from the IMSL library routines) of approximately 80% based on the test suite in the classic Hull, Enright, ... paper. We now get more than 90% reduction in run time. We feel this improvement is due to improved algorithms and improved compilers that can take better advantage of computer architectures.

We will also report on computational differentiation activities that have been done to relieve the programmers burden in linearizing the set of nonlinear differential equations that are common in these models.

The primary application that was planned for these integrators is in a code for the solution of multipoint boundary-value problems in ordinary differential equations. The term boundary-value is wrong for the intended applications as the values are observed and should generally be met in a best-fit sense. Early versions of this code have been instrumented and the run time is over 90% in integration. Thus, improvements in run time for this part are significant.

This code was created using Krommes' `FWEB` system for literate programming. The source is available for anonymous ftp.

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