SUMMARY OF RESEARCH

On NASA Research Cooperative Agreement NCC-1-407

Entitled

"Research on Streamlines and Aerodynamic Heating for Unstructured Grids on High-Speed Vehicles"

with

NASA Langley Research Center
Hampton, VA 23681-2199

Technical Officer: Mr. H. Harris Hamilton
Mail Stop 408A

Covering the Period August 1, 2000 to June 30, 2001

Principal Investigator: Dr. Fred R. DeJarnette
Mechanical and Aerospace Engineering
N C State University
Raleigh, NC 27695-7910
Phone: 919-515-5243
Fax: 919-515-7968
E-mail: dejar@ncsu.edu

August 21, 2001

INTRODUCTION

Engineering codes are needed which can calculate convective heating rates accurately and expeditiously on the surfaces of high-speed vehicles. One code which has proven to meet these needs is the Langley Approximate Three-Dimensional Convective Heating (LATCH) code (ref. 1). It uses the axisymmetric analogue in an integral boundary-layer method to calculate laminar and turbulent heating rates along inviscid surface streamlines. It requires the solution of the inviscid flow field to provide the surface properties needed to calculate the streamlines and streamline metrics. The LATCH code has been used with inviscid codes which calculated the flow field on structured grids. Several more recent inviscid codes calculate flow field properties on unstructured grids. The present research develops a method to calculate inviscid surface streamlines, the streamline metrics, and heating rates using the properties calculated from inviscid flow fields on unstructured grids. Mr. Chris Riley, prior to his departure from NASA LaRC, developed a preliminary code in the C language, called "UNLATCH", to accomplish these goals. No publication was made on his research. The present research extends and improves on the code developed by Riley. Particular attention is devoted to the stagnation region, and the method is intended for programming in the FORTRAN 90 language.

INVISCID SURFACE STREAMLINES AND METRICS

Let $\vec{R}$ be the position vector

$$\vec{R} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k} \tag{1}$$

The body surface is described by

$$F(x, y, z) = 0 \tag{2}$$

Therefore, the Cartesian coordinates x, y, and z are not independent. We will consider y and z as independent and that x can be determined from eq. (2) as some function of y and z. The method developed here is similar to that in ref. 2 for the metric.

The unit normal to the body surface is given by

$$\hat{e}_n = \frac{\nabla F}{|\nabla F|} = \frac{F_x\hat{i} + F_y\hat{j} + F_z\hat{i}}{[F_x^2 + F_y^2 + F_z^2]^{1/2}} \tag{3}$$

The velocity is given by

$$\vec{V} = u\,\hat{i} + v\,\hat{j} + w\,\hat{k} \tag{4}$$

On the body surface, $\vec{V}$ and $\hat{e}_n$ are perpendicular. Therefore,

$$\vec{V} \circ \hat{e}_n = 0$$

and hence

$$uF_x + vF_y + wF_z = 0$$

or

$$u = -\frac{(vF_y + wF_z)}{F_x} \tag{5}$$

We will also use streamline coordinates t and β, where $S = \int V dt$ is distance along a streamline and β is a coordinate constant along a streamline. The differential arc lengths are given by

$$ds = V dt \tag{6}$$

and

$$h_\beta \, d\beta \tag{7}$$

where h_β is the metric along a t = constant line. The unit vector in the direction of a streamline is $\hat{e}_s$, where

$$\vec{V} = V\hat{e}_s \tag{8}$$

and the unit vector perpendicular to $\hat{e}_s$ and $\hat{e}_n$ is

$$\hat{e}_\perp = \hat{e}_s \times \hat{e}_n \tag{9}$$

$\hat{e}_\perp$ is therefore tangent to the surface and normal to $\hat{e}_s$. If we consider t and β as independent variables for surface streamlines, then

$$\frac{\partial \vec{R}}{\partial t} = \vec{V} \tag{10}$$

and

$$\frac{\partial \vec{R}}{\partial \beta} = h_\beta \, \hat{e}_\beta \tag{11}$$

$\hat{e}_\beta$ is the unit vector tangent to a t = constant line on the surface and it is not perpendicular to $\hat{e}_s$.

We set

$$\hat{e}_\beta = \cos\eta \, \hat{e}_\perp + \sin\eta \, \hat{e}_s \tag{12}$$

where η is to be determined. Let n be the straight line coordinate normal to the body. Then

$$\frac{\partial \vec{R}}{\partial n} = \hat{e}_n \tag{13}$$

and $\vec{R} = \vec{R}(t, \beta, n)$. Also $\vec{R} = x\,\hat{i} + y\,\hat{j} + y\,\hat{k}$ and the derivative of $\vec{R}$ gives

$$d\vec{R} = dx\hat{i} + dy\hat{j} + dz\hat{k} = Vdt\hat{e}_s + h_\beta d_\beta \hat{e}_\beta + dn\hat{e}_n \tag{14}$$

We note that t, β, and n are not orthogonal coordinates. Substitute eq. (12) for $\hat{e}_\beta$ in eq. (14) to get

$$dx\hat{i} + dy\hat{j} + dz\hat{k} = Vdt\hat{e}_s + h_\beta d_\beta(\cos\eta\,\hat{e}_\perp + \sin\eta\,\hat{e}_s) + dn\hat{e}_n \tag{15}$$

Since y and z are functions of t, β, and n, we can write

$$dy = \frac{\partial y}{\partial t}dt + \frac{\partial y}{\partial \beta}d\beta + \frac{\partial y}{\partial n}dn \tag{16}$$

and

$$dz = \frac{\partial z}{\partial t}dt + \frac{\partial z}{\partial \beta}d\beta + \frac{\partial z}{\partial n}dn \tag{17}$$

where t, β, and n are independent coordinates and $n = 0$ on the body surface. Substitute eq. (16) into eq. (15) and take the dot product of both sides of the resulting equation with $\hat{j}$. Equate the coefficient of dβ on both sides to get

$$\frac{\partial y}{\partial \beta} = h_\beta[\cos\eta\,\hat{e}_\perp \circ \hat{j} + \sin\eta\,\hat{e}_s \circ \hat{j}] \tag{18}$$

Similarly, substitute eq. (17) into eq. (15), take the dot product of both sides with $\hat{k}$, and equate the coefficients of dβ to get

$$\frac{\partial z}{\partial \beta} = h_\beta[\cos\eta\,\hat{e}_\perp \circ \hat{k} + \sin\eta\,\hat{e}_s \circ \hat{k}] \tag{19}$$

Since $\hat{e}_\perp = \hat{e}_s \times \hat{e}_n$, then use eqs. (3) and (4) to perform the cross product. The result is

$$\hat{e}_\perp = \frac{[\hat{i}(vF_z - wF_y) + \hat{j}(wF_x - uF_z) + \hat{k}(uF_y - vF_x)]}{V\,|\nabla F|} \tag{20}$$

Using eq. (20) in eqs. (18) and (19) we get

$$\frac{\partial y}{\partial \beta} = h_\beta \left[\cos \eta \, \frac{(wF_x - uF_z)}{V \, |\nabla F|} + \sin \eta \, \frac{v}{V} \right] \tag{21}$$

and

$$\frac{\partial z}{\partial \beta} = h_\beta \left[\cos \eta \, \frac{(uF_y - vF_x)}{V \, |\nabla F|} + \sin \eta \, \frac{w}{V} \right] \tag{22}$$

We can eliminate sin η from eqs. (21) and (22) to get

$$h_\beta \cos \eta = \frac{\left[w \frac{\partial y}{\partial \beta} - v \frac{\partial z}{\partial \beta} \right] |\nabla F|}{VF_x} \tag{23}$$

In obtaining eq. (23), u was found from eq. (5). The left side of eq. (23) is the metric $h = h_\beta \cos \eta$ needed for the axisymmetric analogue to replace the axisymmetric body radius. Note that h$d\beta$ is the perpendicular distance between adjacent streamlines as shown in Fig. 1.

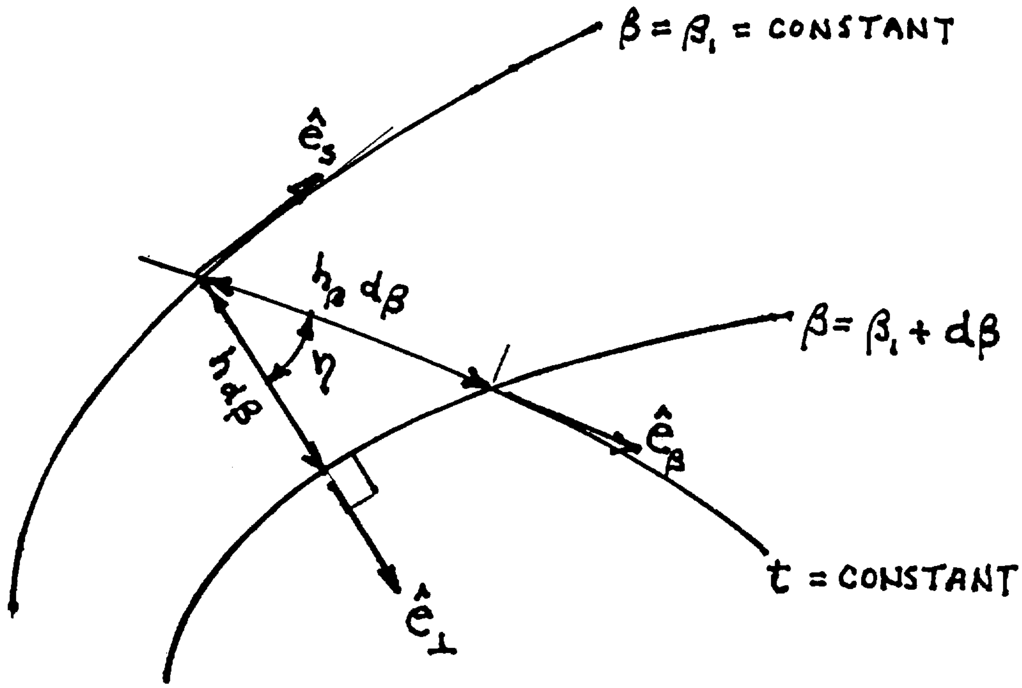

Figure 1. Streamlines and metrics

We can also eliminate cos η from eqs. (21) and (22) to get

$$h_\beta \sin \eta = \frac{(vF_x - uF_y) \frac{\partial y}{\partial \beta} + (wF_x - uF_z) \frac{\partial z}{\partial \beta}}{F_x V} \tag{24}$$

and again u can be obtained from eq. (5). Equations (23) and (24) allow us to determine η if needed. Equation (23), however, is the relation needed to obtain the metric $h = h_\beta \cos\eta$ for the axisymmetric analogue. An alternate method is given in the appendix which uses orthogonal coordinates.

Stagnation Region

We will consider first the stagnation region for a body which is symmetric about the x-y plane. The stagnation point is located at $y = y_0$ and $z = 0$. In the region near the stagnation point we can approximate v and w by the relations.

$$v \approx v_y (y - y_0) \tag{25}$$

and

$$w \approx w_z z \tag{26}$$

where $$v_y = \left(\frac{\partial v}{\partial y}\right)_0 \text{ and } w_z = \left(\frac{\partial w}{\partial z}\right)_0 .$$

Since $$\frac{\partial y}{\partial t} = v \text{ and } \frac{\partial z}{\partial t} = w ,$$

eqs. (25) and (26) can be written as

$$\frac{\partial y}{\partial t} = v_y (y - y_0) \tag{27}$$

and

$$\frac{\partial z}{\partial t} = w_z z \tag{28}$$

Equations (27) and (28) can be integrated, holding β constant, to give

$$y - y_0 = C_1(\beta) e^{v_y t} \tag{29}$$

and

$$z = C_2(\beta) e^{w_z t} \tag{30}$$

where $C_1(\beta)$ and $C_2(\beta)$ are functions of β and thus constant along a streamline. The stagnation point $y = y_0$ and $z = 0$ requires $t = -\infty$ since $v_y > 0$ and $w_y > 0$ in general. We have some liberty in defining $C_1(\beta)$ and $C_2(\beta)$; therefore, we set

$$C_1(\beta) = \varepsilon \cos\beta \tag{31}$$

and $$C_2(\beta) = \varepsilon\sqrt{\frac{v_y}{w_z}}\sin\beta \tag{32}$$

where ε is a small parameter with the dimension of length. The factor $\sqrt{\frac{v_y}{w_z}}$ in eq. (32) is inserted for convenience to make the curve $t = 0$ perpendicular to the streamlines which cross that line. However, all the other lines of t = constant are not perpendicular to the streamlines.

Using eqs. (31) and (32) in (29) and (30) give

$$y - y_0 = \varepsilon\cos\beta\, e^{v_y t} \tag{33}$$

and $$z = \varepsilon\sqrt{\frac{v_y}{w_z}}\sin\beta\, e^{w_z t} \tag{34}$$

Eliminating t from eqs. (33) and (34) gives another form of the streamline equation as

$$y - y_0 = \varepsilon\cos\beta\left[\frac{z}{\varepsilon\sqrt{\frac{v_y}{w_z}}\sin\beta}\right]^{\left(\frac{v_y}{w_z}\right)} \tag{35}$$

where, again, β is constant along a streamline. If we eliminate β from eqs. (33) and (34) we get

$$\left[\frac{y - y_0}{\varepsilon\, e^{v_y t}}\right]^2 + \left[\frac{z}{\varepsilon\sqrt{\frac{v_y}{w_z}}\, e^{w_z t}}\right]^2 = 1 \tag{36}$$

which is the equation for an ellipse for t = constant lines. Equations (35) and (36) show that streamlines (β = constant) are perpendicular to t = constant lines only for $t = 0$, unless $v_y = w_z$. For eq. (23) we can use eqs. (25), (26), (33) and (34) to determine the factor

$$\left[w\frac{\partial y}{\partial\beta} - v\frac{\partial z}{\partial\beta}\right] = \left[-w_z\,\varepsilon^2\sqrt{\frac{v_y}{w_z}}\sin^2\beta - v_y\,\varepsilon^2\sqrt{\frac{v_y}{w_z}}\cos^2\beta\right]e^{(v_y + v_z)t}$$

$$= -\varepsilon^2 \sqrt{v_y w_z} \left[\sin^2 \beta + \frac{v_y}{w_z} \cos^2 \beta \right] e^{(v_y + w_z)t} \tag{37}$$

Then eq. (23) becomes

$$h = h_\beta \cos \eta = \frac{|\nabla F|_0}{(-F_x)_0} \frac{\varepsilon^2 \sqrt{v_y w_z}}{V} \left[\sin^2 \beta + \frac{v_y}{w_z} \cos^2 \beta \right] e^{(v_y + w_z)t} \tag{38}$$

Note that $F_x < 0$ for the nose region of a blunt nosed body in order to make $\hat{e}_n$ the unit outer normal to the body. By using eq. (5) for u and eqs. (25), (26), (33) and (34) the magnitude of the velocity becomes

$$V = \sqrt{u^2 + v^2 + w^2} = \left[\frac{(F_y v + F_z w)^2}{F_x^2} + v^2 + w^2 \right]^{1/2}$$

$$= \frac{1}{|F_x|} \left[(F_x^2 + F_y^2) v^2 + (F_x^2 + F_z^2) w^2 + 2 F_y F_z v w \right]^{1/2}$$

$$= \frac{\varepsilon \sqrt{v_y w_z}}{|F_x|_0} \left[(F_x^2 + F_y^2)_0 \frac{v_y}{w_z} \cos^2 \beta \, e^{2 v_y t} + (F_x^2 + F_z^2)_0 \sin^2 \beta \, e^{2 w_z t} \right.$$

$$\left. + 2 (F_y F_z)_0 \sqrt{\frac{v_y}{w_z}} \sin \beta \cos \beta \, e^{(v_y + w_z)t} \right]^{1/2} \tag{39}$$

For bodies symmetric about the *x-y* plane, $F_z = 0$ in the plane of symmetry.

For the laminar heating rate, we use $dS = Vdt$ to determine the integral

$$I \equiv \int_0^s h^2 V ds = \int_{-\infty}^t h^2 V^2 dt \tag{40}$$

Using eq. (38), eq. (40) can be integreted to yield

$$I = \frac{|\nabla F|_0^2}{(F_x)_0^2} \varepsilon^4 v_y w_z \left[\sin^2 \beta + \frac{v_y}{w_z} \cos^2 \beta \right]^2 \frac{e^{2(v_y + w_z)t}}{2(v_y + w_z)} \tag{41}$$

For the laminar momentum thickness, we need the ratio $I^{1/2} / hV$ which becomes

$$\frac{I^{1/2}}{hV} = \frac{1}{\sqrt{2(v_y + w_z)}} \tag{42}$$

Note that this result is independent of β, t, and ε in the stagnation region.

Region Beyond the Stagnation Region

It is convenient to let $t = 0$ at the edge of the stagnation region. Then $-\infty < t \leq 0$ for the stagnation region, where the equations given above are valid.

For $t > 0$, we can determine the metric $h = h_\beta \cos \eta$ from eq. (23) provided we can calculatae $\frac{\partial y}{\partial \beta}$ and $\frac{\partial z}{\partial \beta}$. On the body surface y and z are independent variables and also t and β are independent variables. The velocity components are given at specific positions x, y and z from the inviscid solver. We note that

$$\frac{\partial y}{\partial t} = v \tag{43}$$

and

$$\frac{\partial^2 y}{\partial \beta \partial t} = \frac{\partial v}{\partial \beta} \tag{44}$$

But $v = v(y, z)$ and $\frac{\partial^2 y}{\partial \beta \partial t} = \frac{\partial^2 y}{\partial t \partial \beta}$. Therefore eq. (44) can be written as

$$\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial \beta}\right) = \left(\frac{\partial v}{\partial y}\right)_z \frac{\partial y}{\partial \beta} + \left(\frac{\partial v}{\partial z}\right)_y \frac{\partial z}{\partial \beta} \tag{45}$$

In a similar manner

$$\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial \beta}\right) = \left(\frac{\partial w}{\partial y}\right)_z \frac{\partial y}{\partial \beta} + \left(\frac{\partial w}{\partial z}\right)_y \frac{\partial z}{\partial \beta} \tag{46}$$

Starting at the ege of the stagnation region, where $t = 0$ and $\frac{\partial y}{\partial \beta}$ and $\frac{\partial z}{\partial \beta}$ are known, we can determine y, z, $\frac{\partial y}{\partial \beta}$ and $\frac{\partial z}{\partial \beta}$ along a streamline by integrating eqs. (43), (45), (46) and $\frac{\partial z}{\partial t} = w$ numerically with t or $S = \int V dt$ as the independent variable. In order to do that, we must be able to interpolate the inviscid solution on the surface to determine v, w, $\frac{\partial v}{\partial y}$, $\frac{\partial v}{\partial z}$, $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial z}$ for the equations given above.

Surface Fitting Geometry

Consider the inviscid solution on an unstructured grid. For the body surface, we would know

$$x_i, u_i, y_i, v_i, z_i, \text{ and } w_i$$

at points on the surface for Cartesian coordinates (x, y, z) with corresponding inviscid velocity components (u, v, w). The points do not, in general, correspond to any regular pattern on the surface. The equation for the body surface is generally not known, but can be represented by eq. (2). Then the unit normal, $\hat{e}_n$, to the surface, although not known, could be determined from eq. (3) where $(\hat{i}, \hat{j}, \hat{k})$ are unit vectors in the (x, y, z) directions, respectively. Also, $F_x = \frac{\partial F}{\partial x}$, $F_y = \frac{\partial F}{\partial y}$, and $F_z = \frac{\partial F}{\partial z}$. As shown earlier, $\vec{V} \circ \hat{e}_n = 0$ implies

$$uF_x + vF_y + wF_z = 0 \tag{5}$$

We will consider 3 adjacent points on the body surface as shown in Fig. 2.

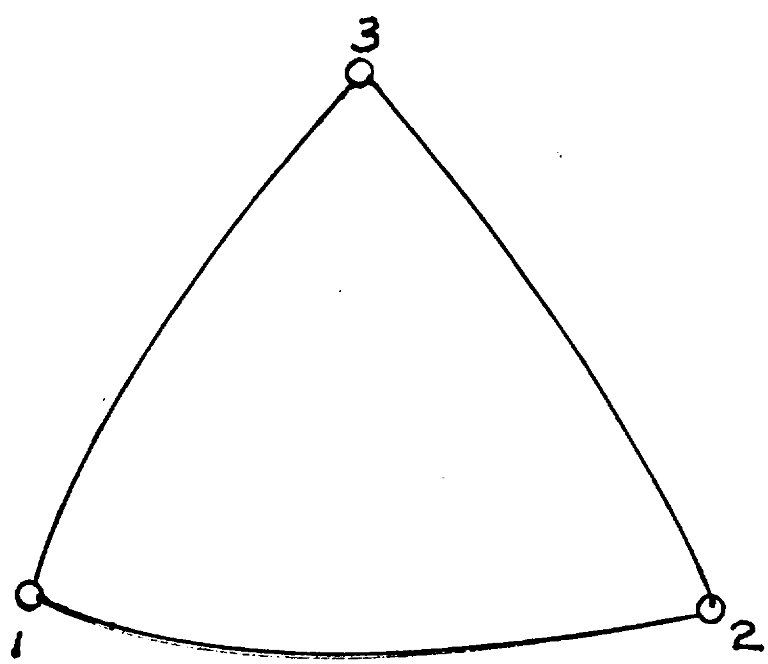

Figure 2. Adjacent points on body surface

We wish to surface fit the conic equation

$$F(x, y, z) = y^2 + a_1 y + a_2 z^2 + a_3 z + a_4 x^2 + a_5 x + a_6 = 0 \tag{47}$$

to the region inside the triangle formed by the 3 points. The coefficient of y^2 is chosen to be unity, which means this term cannot be zero. We could have chosen some other term to have unity as its coefficient, as long as we know that term is not zero. With x the coordinate in the longitudinal direction, then the y^2 term would not be zero except for some flat surfaces.

For the surface fit to the triangular region, we need to determine the coefficients a_1, a_2, a_3, a_4, a_5, and a_6. Therefore we need 6 equations. These equations follow by applying equtions (5) and (47) to the points 1, 2, and 3 on the surface. From eq. (47) we get

$$\left.\begin{aligned} F_x &= 2a_4 x + a_5 \\ F_y &= 2y + a_1 \\ F_z &= 2a_2 z + a_3 \end{aligned}\right\} \tag{48}$$

then eq. (5) becomes

$$u(2a_4 x + a_5) + v(2y + a_1) + w(2a_2 z + a_3) = 0 \tag{49}$$

Rearranging, we get

$$a_1 v + a_2(2wz) + a_3 w + a_4(2ux) + a_5 u = -2vy \tag{50}$$

Now apply eqs. (47) and (50) at the 3 points on the surface. This gives 6 equations for the 6 unknowns $a_1, ..., a_6$. After determining these 6 coefficients, we can calculate the surface normal, $\hat{e}_n$, from eq. (3) and x as a function of y and z from eq. (47). However, there is no guarantee that the surface equation is continuous with adjacent triangles to the 3 points. Note that $F(x, y, z) = 0$ being the body surface, there are only two independent variables. It is convenient to use y and z as these two indepdendent variables since the streamines near a blunt nose ($x = 0$, $y = 0$, $z = 0$) has the distance along the streamline varying as y and z but $x^{1/2}$ for x-direction. This can be seen more clearly when looking at the stagnation region. As an example, consider a blunt-nosed body described by the simplier conic shape

$$y^2 + Kz^2 = 2ax + Bx^2 \tag{51}$$

where the cross section for x = constant is an ellipse and K, a, and B are constants. K determines the ellipticity for the cross sectional shape, "a" is the nose radius of curvture in the $(y - x)$ plane,

(a/K) is the nose radius of curvature in the $(z-x)$ plane, and B determines the conic section in the x-direction, i.e.,

$B < 0$ for elliptic, $B = 0$ for parabolic and $B > 0$ for hyperbolic longitudinal shapes.

Here we write eq. (51) as

$$F(x, y, z) = y^2 + kz^2 - 2ax - Bx^2 = 0 \tag{52}$$

We can solve for x from the quadratic formula, with $B \neq 0$, using $x = 0$ at $y = 0$ and $z = 0$, to determine the proper sign. Thus eq. (52) becomes

$$x = \frac{-a + [a^2 + B(y^2 + Kz^2)]^{1/2}}{B} \tag{53}$$

which also can be written as

$$a + Bx = [a^2 + B(y^2 + kz^2)]^{1/2} \tag{54}$$

Equation (54) is convenient because eq. (52) gives

$$F_x \equiv \frac{\partial F}{\partial x} = -2a - 2Bx = -2(a + Bx) = -2[a^2 + B(y^2 + kz^2)]^{1/2} \tag{55}$$

Also
$$F_y \equiv \frac{\partial F}{\partial y} = 2y \quad \text{and} \quad F_z = 2kz \tag{56}$$

For zero angle of attack, the nose point is the stagnation point, and for the region near the stagnation point

$$v \approx v_y \, y \quad \text{and} \quad w \approx w_z \, z \tag{57}$$

where $v_y = \left(\dfrac{\partial v}{\partial y}\right)_0$ and $w_z = \left(\dfrac{\partial w}{\partial z}\right)_0$

u is determined from eq. (5) as

$$u = -\frac{1}{F_x}(vF_y + wF_z)$$

and using eqs. (55), (56), and (57) we get

$$u \approx \frac{[v_y y^2 + w_z K z^2]}{[a^2 + B(y^2 + K z^2)]^{1/2}} \tag{58}$$

and near the stagnation point,

$$u \approx \frac{v_y y^2 + w_z K z^2}{a} \tag{59}$$

Thus $u \sim y^2$ and z^2 whereas $v \sim y$ and $w \sim z$. Therefore, we will use y and z as independent variables on the surface and x can be determined from $F(x, y, z) = 0$. Also, we will work with v and w since u can be determined from eq. (5).

Surface Fitting Velocity Components

If we only use the 3 points on the triangle 1, 2, 3, then the velocity components v and w can only be fit as linear variations in y an z, i.e.,

$$\left.\begin{aligned} v &= b_1 y + b_2 z + b_3 \\ w &= c_1 y + c_2 z + c_3 \end{aligned}\right\} \tag{60}$$

where the constants can be determined by applying the equations above for v and w to the 3 points where v and w are known. If we wish more than linear variations in y and z, then we must consider more than 3 points.

One possibility is to add 3 adjacent points as shown in Fig. 3.

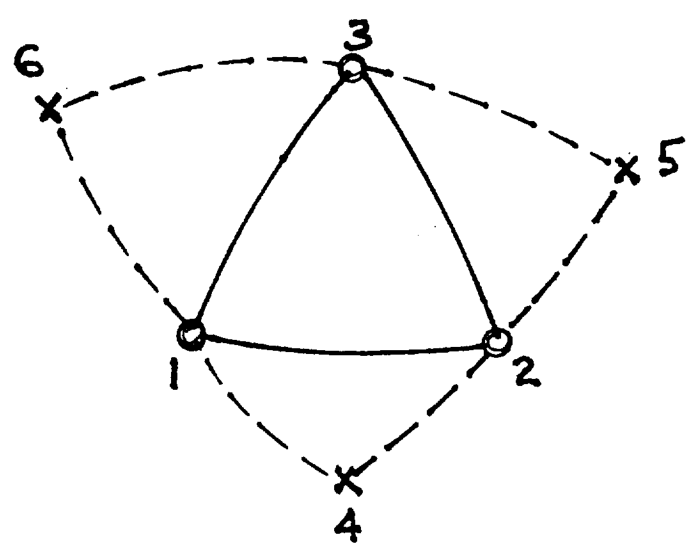

Figure 3. Adding three adjacent points

With 6 points, we could write v and w as

$$v = b_1 y + b_2 z + b_3 + b_4 y^2 + b_5 z^2 + b_6 yz$$

$$w = c_1 y + c_2 z + c_3 + c_4 y^2 + c_5 z^2 + c_6 yz \tag{61}$$

The 6 coefficients for v and for w could be determined by applying the equations above to the 6 points where y, z, v, and w are known. Although 6 points are used to determine the coefficients, the equations above are applied only to the region inside the triangle formed by points 1, 2, and 3.

If we go outside the triangle 1, 2, 3 then other points would be used for that region. For example, suppose we go beyond (1, 2, 3) into the triangle (2, 5, 3). Then we would add 2 points (7 and 8) to the points (1, 2, 3, 5) to make 6 points to determine the coefficients for the region (2, 5, 3). See Fig. 4.

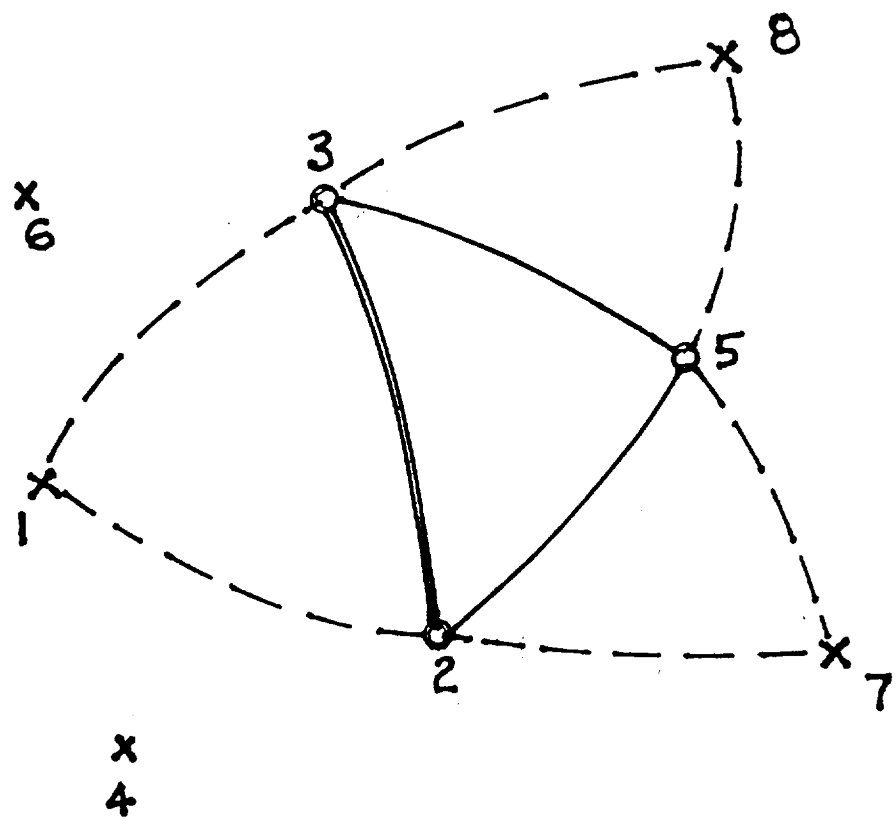

Figure 4. Adjacent triangle (2, 5, 3)

What is the continuity of the v and w along the common boundary line 2-3? Along the line 2-3,

$$\frac{y - y_2}{y_3 - y_2} = \frac{z - z_2}{z_3 - z_2} \tag{62}$$

Thus $$v_{2-3} = b_1\left[y_2 + \frac{(y_3 - y_2)}{(z_3 - z_2)}(z - z_2)\right] + b_2 z + b_3 + b_4\left[y_2 + \frac{(y_3 - y_2)(z - z_2)}{(z_3 - z_2)}\right]^2$$

$$+ b_5 z^2 + b_6 z\left[y_2 + \left(\frac{y_3 - y_2}{bz_3 - z_2}\right)(z - z_2)\right] \tag{63}$$

Let $$v_{2-3} = B_1 z^2 + B_2 z + B_3 \text{ for } z_2 \le z \le z_3 \tag{64}$$

The coefficients B_1, B_2, and B_3 must be the same for triangle (1, 2, 3) as those for triangle (2, 5, 3). These conditions would replace pt. 1 for (2, 5, 3). Note that the v_{2-3} equation above already satisfies $v = v_2$ and $v = v_3$. Thus the relation given by eq. (63) or (64) along with v at points 5, 7, and 8 allow us to determine the 6 coefficients in eq. (61). Similarily for the w component of velocity.

CONCLUDING REMARKS

The method developed in this report can be used to calculate the heating rates on high speed vehicles. The improved solution for the stagnation region should provide more accurte heating rates and the technique can be used for regions downstream on unstructured as well as structured grids.

ACKNOWLEDGMENTS

The principal investigator acknowledges the assistance of Mr. H. Harris Hamilton, Dr. Chris Riley, and Mr. Artem Dynoknov, graduate assistance. Also, special thanks go to Mrs. Joyce Sorensen for word processing the manuscript.

REFERENCES

1. Hamilton, H. H., Greene, F. A., and DeJarnette, F. R.; "Approximate Method for Calculating Heating Rates on Three-Dimensional Vehicles", *Journal of Spacecraft and Rockets*, Vol. 31, No. 3, 1994, pp. 345-354.

2. Hassan, B., DeJarnette, F. R., and Zoby, E. V.; "Effect of Nose Shape on Three-Dimensional Streamlines and Heating Rates", *Journal of Spacecraft and Rockets*, Vol. 30, No. 1, 1993, pp. 69-78.

APPENDIX

Alternate Surface Coordinates

The use of t and β as independent coordinates on the body surface did not allow t = constant lines to be perpendicular to surface streamlines, along which β = constant. An alternate method is to use θ and β as independent coordinates where

$$dS = Vdt = h_s d\theta \tag{A-1}$$

and θ = constant lines are perpendicular to the surface streamlines, where β is still constant along a streamline. The metric associated with $d\theta$ is h_s and it is generaly not needed to calculate heating rates. The coordinate θ is defined such that the unit vector in the β direction is $\hat{e}_\perp$. Thus eq. (14) becomes

$$d\vec{R} = dx\hat{i} + dy\hat{j} + dz\hat{k} = h_s d\theta \hat{e}_s + hd\beta\, \hat{e}_\perp + dn\hat{e}_n \tag{A-2}$$

Now we are dealing with orthogonal coordinates θ, β, and n, and the metric needed for the axisymmetric analogue is h. Considering y and z as functions of θ, β, and n then eq. (A-2) dotted with $\hat{j}$ gives

$$\frac{\partial y}{\partial \beta} = he_\perp \circ \hat{j} = \frac{h(wF_x - uF_z)}{V\,|\nabla F\,|} \tag{A-3}$$

where eq. (20) was used for $\hat{e}_\perp$. Dotting Eq. (A-2) with $\hat{k}$ gives

$$\frac{\partial z}{\partial \beta} = h\hat{e}_\perp \circ \hat{k} = \frac{h(uF_y - vF_x)}{V\,|\nabla F\,|} \tag{A-4}$$

Note that $\dfrac{\partial}{\partial \beta}$ is now a partial derivative holding θ and n constant, whereas the $\dfrac{\partial}{\partial \beta}$ in eqs. (21) and (22) held t and n constant. Multiply eq. (A-3) by w and subtract eq. (A-4) multiplied by v to get

$$h = \frac{\left[w\dfrac{\partial y}{\partial \beta} - v\dfrac{\partial z}{\partial \beta} \right] |\nabla F\,|}{VF_x} \tag{A-5}$$

Eq. (A-5) is similar to eq. (23) except here $\dfrac{\partial}{\partial \beta}$ holds θ and n constant.

Consider the stagnation region again, but using θ, β, and n as independent variables. θ is related to t by eq. (A-1) and t must be considered a function of θ and β on the surface. Eqs. (33), (34), (35), and (36), are valid here also. However, when determining $\frac{\partial}{\partial\beta}$ and $\frac{\partial z}{\partial\beta}$ from eqs. (33) and (34), we get

$$\left(\frac{\partial y}{\partial\beta}\right)_\theta = \varepsilon e^{v_y t}\left[-\sin\beta + v_y \cos\beta \frac{\partial t}{\partial\beta}\right] \tag{A-6}$$

and

$$\left(\frac{\partial z}{\partial\beta}\right)_\theta = \varepsilon\sqrt{\frac{v_y}{w_z}}\, e^{w_z t}\left[\cos\beta + w_z \sin\beta \frac{\partial t}{\partial\beta}\right] \tag{A-7}$$

Interestingly, when we form $\left[w\frac{\partial y}{\partial\beta} - v\frac{\partial z}{\partial\beta}\right]$ the terms involving $\frac{\partial t}{\partial\beta}$ cancel and eqs. (37) and (38) are also valid here. Although t is not an independent variable, we can still use it and eqs. (41) and (42) are also valid.

For the region beyond the stagnation region, we could use eq. (A-5) to determine h if we can determine $\frac{\partial y}{\partial\beta}$ and $\frac{\partial z}{\partial\beta}$ from adjacent streamlines. On the other hand, if we are tracing only one streamline, then we must integrate the differential equation derived below to determine h. It follows from eq. (A-2) that

$$\frac{\partial\vec{R}}{\partial\theta} = h_s\,\hat{e}_s \tag{A-8}$$

and

$$\frac{\partial\vec{R}}{\partial\beta} = h\,\hat{e}_\perp \tag{A-9}$$

We note that the equality of cross derivatives gives

$$\frac{\partial^2\vec{R}}{\partial\beta\partial\theta} = \frac{\partial^2\vec{R}}{\partial\theta\partial\beta} \tag{A-10}$$

Differentiate eq. (A-8) with respect to β and eq. (A-9) with respect to θ and then substitute into eq. (A-10) to get

$$h_s\frac{\partial\hat{e}_s}{\partial\beta} + \frac{\partial h_s}{\partial\beta}\hat{e}_s = h\frac{\partial\hat{e}_\perp}{\partial\theta} + \frac{\partial h}{\partial\theta}\hat{e}_\perp \tag{A-11}$$

Take the dot product of eq. (A-11) with $\hat{e}_\perp$ and divide both sides by $h_s h$ to get

$$\frac{1}{h}\frac{\partial h}{h_s \partial\theta} = \frac{1}{h}\frac{\partial \hat{e}_s}{\partial\beta} \circ \hat{e}_\perp \tag{A-12}$$

The partial derivative on the right side can be evaluted by using the transformation

$$\frac{1}{h}\frac{\partial \hat{e}_s}{\partial\beta} = \left(\frac{\partial \hat{e}_s}{\partial y}\right)_{z,n}\frac{1}{h}\frac{\partial y}{\partial\beta} + \left(\frac{\partial \hat{e}_s}{\partial z}\right)_{y,n}\frac{1}{h}\frac{\partial z}{\partial\beta} \tag{A-13}$$

Eqs. (A-3) and (A-4) give relations to determine $\frac{1}{h}\frac{\partial y}{\partial\beta}$ and $\frac{1}{h}\frac{\partial z}{\partial\beta}$. Then eq. (A-12) can be integrated along a streamline using

$$dS = h_s\, d\theta$$

as the variable of integration or with $dt = \frac{h_s\, d\theta}{V}$ as the variable of integration.

If we dot eq. (A-11) with $\hat{e}_s$ we get

$$\frac{\partial h_s}{\partial\beta} = h\frac{\partial \hat{e}_\perp}{\partial\theta} \circ \hat{e}_s \tag{A-14}$$

Divide both sides by $h_s h$ and use the vector relationship $\frac{\partial \hat{e}_\perp}{\partial\theta} \circ \hat{e}_s = -\,\hat{e}_\perp \circ \frac{\partial \hat{e}_s}{\partial\theta}$ to write eq. (A-14) as

$$\frac{1}{h_s}\frac{\partial h_s}{h\partial\beta} = -\frac{1}{h_s}\frac{\partial \hat{e}_s}{\partial\theta} \circ \hat{e}_\perp \tag{A-15}$$

The partial derivative on the right side can be transformed using

$$\frac{1}{h_s}\frac{\partial \hat{e}_s}{\partial\theta} = \left(\frac{\partial \hat{e}_s}{\partial y}\right)_{z,n}\frac{1}{h_s}\frac{\partial y}{\partial\theta} + \left(\frac{\partial \hat{e}_s}{\partial z}\right)_{y,n}\frac{1}{h_s}\frac{\partial z}{\partial\theta} \tag{A-16}$$

and from eq (A-2) we obtain

$$\frac{1}{h_s}\frac{\partial y}{\partial\theta} = \hat{e}_s \circ \hat{j} = \frac{v}{V} \tag{A-17}$$

and

$$\frac{1}{h_s}\frac{\partial z}{\partial \theta} = \hat{e}_s \circ \hat{k} = \frac{w}{V} \tag{A-18}$$

Eq. (A-15) gives the curvature term

$$\frac{1}{h_s}\frac{1}{h}\frac{\partial h_s}{\partial \beta}$$

which is needed when solving the cross-flow boundary layer equation. For heating rates, however, the cross-flow boundary layer equation is not needed.