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Top Quark Mass Measurement using the Dynamical Likelihood Method in the lepton plus jets channel at CDF Run II

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We present a new measurement of the top quark mass using $1.7 fb^{-1}$ of CDF data in the lepton+jets channel with DLM (Dynamical Likelihood Method). DLM treats each event as a quantum process and uses the differential cross section as a posterior probability for the observed quantities. One of the most important improvements since previous results with DLM is that we introduce 2D likelihood as a function of Mtop and JES instead of 1D likelihood as a function of Mtop, improving the JES uncertainty. With this method, we measured top quark mass to be 171.6 $\pm$ 2.0(stat.+JES) $\pm$ 1.3(syst.) GeV/c^2.

Preliminary Results

I. INTRODUCTION

The top quark mass is an important parameter of the Standard Model. The precise measurement of the top quark mass proviedes a constraint on the Higgs boson mass and other parameters in beyond-the-standard models.

This note describes a precise measurement of the top quark mass with $1.7fb^{-1}$ in $\bar{p}p$ collisions at $\sqrt{s}=1.96$ TeV with the CDF detector at the Fermilab Tevatron. To reconstruct top quark mass, we use top quark pair production events. Each of the top pair decays into a W boson and a bottom quark with probability approximately of 100% and then the two W bosons decay into one of three channels: '(a) 'dilepton channle", where both of W bosons decay leptonic (a charged lepton and a neutrino that is observed as missing E_T), (b)"lepton+jets channel", where one of the W bonsons decays leptonic and the other decays hadronic which is observed as two jets, (c) "all hadronic channel", in which both of the W bosons decay hadronic. In this analysis, we select the "lepton+jets channel".

The measurement uses the Dynamical Likelihood Method (DLM) [1], which reconstructs each event using likelihood defined by differential cross section and Transfer Function(TF) and the top quark mass is determined with maximum likelihood method. The most important improvement since previous DLM results using $318pb^{-1}$[?] is to improve the Jet Energy Scale(JES) uncertainty by integrating a variable of JES into the likelihood so that the lieklihood is a function of M_{top} and ΔJES. With this improvement of likelihood, most part of JES uncertainty is considered not as a systematic but as a statistical uncertainty.

II. DATA & MONTE CARLO SAMPLE

This analysis is based on an integrated luminosity of $1.7fb^{-1}$ collected with CDFII detector. We indentify "lepton+4jets" channel by selecting four high energy jets from four quarks and a charged lepton and a neutrino from one of the W bosons. For the charged lepton, an electron is reauired with E_T >20 GeV/c^2 or a muon with $p_T > 20\ GeV/c$ in the central region of the detector. The neutrino is required as missing $E_T > 20\ GeV/c^2$. For the jets, we require exactly four jets with $E_T > 20\ GeV/c^2$ and at least one jet tagged as a b-jet using a secondary tagging alghorithm. We finaly observed 343 events after event selection.

For the background estimation, five sources are considered: W+HF(heavy flavor) events which are associate production of W boson and heavy flavors($b,\bar{b},c,\bar{c}$), W+LF(light flavor) events which are W bosons associated with light quarks including b-mistagged jet, QCD events which do not contain W boson but a fake lepton. Three of the above background are considered as main sources of background. On the other hand, the single top and diboson events are estimated to be small number of backgrounds. TABLE I shows expected number of backgrounds

Model	Number
Diboson	3.19 ± 0.32
Single top	3.61 ± 0.34
W$b\bar{b}$	14.37 ± 5.82
W$c\bar{c}$/Wc	11.41 ± 4.66
Mistag	14.57 ± 3.29
Non-W	12.48 ± 10.97
Signal	232.29 ± 32.15
Total	293.69 ± 35.95
Data	343

TABLE I: The number of expected events for indivisual background source.

To validate our method, we use Monte Carlo(MC) samples for both signal and backgrounds. For the signal, we prepare PYTHIA [3] samples of five different top quark masses to check the Mtop dependence of the method. To examine the Monte Carlo generator dependece, HERWIG [4] samples are also used. For the study of the background effects, we adopt MC samples for W+HF, W+LF, single top and diboson background. All MC samples are simulated with CDFII simulation package.

On the other hand, QCD backgrond is derived from data using non-isolated leptons.

III. METHOD

A. Dynamical Likelihood Method

Differential corss section

The hard scattering parton process in a $p\bar{p}$ collision can generally be written as

$$a/p + b/\bar{p} \rightarrow \ldots \rightarrow C, \tag{1}$$

where a and b are the initial partons, a quark, anti-quark or gluon, in the proton and anti-proton, respectively, and $c_1, c_2, ...c_n$ are final state partons. States of partons are after the initial- and before the final-state radiation. In the case of the $t\bar{t}$ lepton+jets channel, the initial parton set (a, b) is $(q, \bar{q})$, $(\bar{q}, q)$ or (g, g), and the final partons are $l^+, \nu, q, b, \bar{q}', \bar{b}$ or their anti-particles, where $l = e$ or μ and q, q' are quarks from W. Hereafter, a particle symbol p represents its 4-momentum, and $\boldsymbol{p}$ its 3-momentum. The final partons are assumed to have their pole masses so that the 3-momenta define their states unambiguously.

The hadronic cross-section for the $t\bar{t}$ production is given by

$$d\sigma = dz_a dz_b d^2 p_T f^*_{a/p}(z_b) f_T(p_T) d\hat{\sigma}(a + b \rightarrow C; M_{top}), \tag{2}$$

where M_{top} is the mass of the top quark, and $d\hat{\sigma}$ is the parton level corss section,

$$d\hat{\sigma}(a + b \rightarrow C; M_{top}) = \frac{(2\pi)^4 \delta^4(a + b - C)}{4\sqrt{(a \cdot b)^2 - m_a^2 m_b^2}} |\mathcal{M}(a + b \rightarrow C; M_{top})|^2 d\Phi_n^{(f)}(a + b; C). \tag{3}$$

Variables z_a and z_b are the energy fraction of a and b in hadrons p and $\bar{p}$ respectively, and p_T is the total momentum ot the initial and finel systems in the plane transverse to the beam axis. Functions $f^*_{a/p}(z_a)$ and $f^*_{b/\bar{p}}(z_b)$ denote the effective Parton Distribution Function(PDF's), while $f_T(p_T)$ is the probability function for the total transverse momentum of the system acuired by initial state radiation. In Eq.(3), $\mathcal{M}$ is the matrix element of process (1), and $d\Phi_n^{(f)}$ is the Lorentz invariant phase space factor,

$$d\Phi_n^{(f)} \equiv \prod_{i=1}^{n} \frac{d^3 c_i}{(2\pi)^3 2E_i}. \tag{4}$$

The basic postulate of DLM is that final partons in a sigle event occupy an n-dimensional unit phase space in the neighborhood of $c(c_1, \ldots, c_n)$. The total probability for this final state to occur is obtained by integrating Eq.(2) with initial state variables, z_a, z_b and p_T, as

$$\frac{d\sigma}{d\Phi_n^{(f)}} = I(a, b)|\mathcal{M}(a + b \rightarrow C; M_{top})|^2, \tag{5}$$

where

$$I(a, b) = \frac{(2\pi)^4}{4\sqrt{(a \cdot b)^2 - m_a^2 m_b^2}} f^*_{a/p}(z_a) f^*_{b/\bar{p}}(z_b) f_T(p_T) \tag{6}$$

is the integration factor for the initial state. Because of the δ-function in Eq.(3), the initial parton momenta a and b are uniquely defined by that of C.

When the parton kinematics are given, we define the likelihood as

$$L_1(M_{top}|c) = l_0 \frac{d\sigma}{d\Phi_n^{(f)}}, \tag{7}$$

where l_0 stands for the integrated luminosity required to generate the event. But since the beam state is unknown, we set it to a constant, according to Bayes' postulate, "Equidistribution of Ignorance".

Resonance

When process (1) includes resonance r, i.e. the internal line of a Feynman graph, as

$$a/p + b/\bar{p} \rightarrow r + c_{j+1} + \ldots + c_j \tag{8}$$

$$r \to c_1 + \ldots + c_j, \tag{9}$$

the matrix element factorizes is factorized as

$$|\mathcal{M}(a+b \to C; M_{top})|^2 = |\mathcal{M}_{prod}|^2 \mathcal{P}(s_r) |M_{dec}|^2, \tag{10}$$

where $\mathcal{M}_{prod}$ and $\mathcal{M}_{dec}$ are matrix elements for process (8) and (9) respectively, and $\mathcal{P}(s_r)$ is the propagator factor of the resonance,

$$\mathcal{P} = \frac{1}{(s_r - M_r^2)^2 + M_r^2 \Gamma_r^2}. \tag{11}$$

In Eq.(11), s_r is the virtual mass squared of r, which satisfies

$$s_r = \left(\sum_{i=1}^{j} c_i \right)^2. \tag{12}$$

In $t\bar{t}$ production, the t, $\bar{t}$, W^+ and W^- are resonances. In the leptonic decay channel of the W, the z component of neutrino momentum ν_z is not observed. We infer s_W according to the posterior probability density for s_W, which is

$$\Pi(s_W) = \mathcal{P}(s_W) / \int_0^{\inf} \mathcal{P}(s_W) ds_W. \tag{13}$$

The z-component of the neutrino, ν_z, is obtained by solving Eq.(12). For other resonance in $l + jets$ process, s_r are given by Eq.(12) from momenta of their daughters.

Transfer function for observables

Final partons are not directly observed; they undergo parton-shower/hadronization and are observed by detectors with finite resolution. In analysis, quarks and gluons are reconstructed as jets. To describe the correlation between the final parton state and observed quantities (observables), we introduce the transfer function (TF) denoted by $w(y|x||M_{top})$, which reprents a M_{top} dependent probability density function for y when x is given. Symbol y is a set of observables and x is a parton variable set that corresponds to y. In the present analysis of the $l + jets$ process, we use TF for jets only. We assume $w(y|x||M_{top})$ can be factorized into the individual observable components,

$$w(y|x||M_{top}) = \prod_i w(y_i|x_i||M_{top}). \tag{14}$$

Abbreviating variable index i, TF for a variable set (x, y), $w(y|x||M_{top})$, is obtained from the (x, y) distribution in the $t\bar{t}$ Monte Carlo events by removing the cross section factor. We use TF's for b jets different from those for the W jets.

The parton kinematics reconstruction is made by the use of the posterior TF,

$$w(x|y||M_{top}) = w(y|x||M_{top}) / \int w(y|x||M_{top}) dx. \tag{15}$$

We also consider the effect from the Jet Energy Scale(JES). Obviously, y depends on JES, therefore we introduce TF depending on JES ($w(y|x||JES, M_{top})$). In other words, we treat the JES as a parmeter to be measured together with M_{top} and expect that JES uncertainty could be reduced to a statistical uncertainty. In this analysis, TF's are made from MC sample with top quark mass of 175 GeV/c^2.

Single path reconstruction and its likelihood

A set of parton kinematics is reconstructed in the following steps:

1. Jets are assigned to quarks. Each set of assignments is called a "topology", and is sorted by topology number $I_t (I_t = 1, \ldots, N_t)$.

2. A set of 3-momenta of observable partons b, $\bar{b}$, q, $\bar{q}'$ is generated from the TFs.

3. The transverse components of the neutrino momentum (ν_x, ν_y) are identified with the measured value of MET. One infers s_W according to $\Pi(s_W)$ with a fixed value of M_W, then the z-component of the neutrino momentum is obtained by solving Eq.(12). The two solutions(ν_{z1}, ν_{z2}) are sorted by integer $I_s (I_s = 1, 2)$.

4. For a set of variables thus reconstructed, we scan the value of M_{top} and ΔJES in its search region, where ΔJES is the shift from default JES in the unit of its uncertainty σ_{def}

Steps from (1) through (4) constitute a unit of the reconstruction procedure, which we call a "path". A path is specified by integers (I_t, I_s, k), where $k(k=1,\dots,K)$ is the path number, representing a set of (s_W, x). When observable set $y^{(i)}$ is given for the i-th event, the single path likelihood for the k-th path is defined by

$$L_1^{(i,k)}(M_{top}, \Delta JES; I_t, I_s, k|x^{(k)}) = l_0 \frac{d\sigma}{d\Phi}(I_t, I_s, k; M_{top}|x^{(k)}) w(x^{(k)}|y^{(i)}||\Delta JES, M_{top}) \tag{16}$$

where subscript 1 stands for a single path. Note that the likelihood for a path is a function of M_{top} and ΔJES.

Single event likelihood
The likelihood of the i-th event is defined by the average (or the expectation value) of the single path likelihood over all reconstructed paths,

$$L^{(i)}(M_{top}, \Delta JES) = \frac{1}{2KN_t} \sum_{I_t}^{N_t} \sum_{k=1}^{K} \sum_{I_s}^{2} L_1^{(i,k)}(M_{top}, \Delta JES||I_t, I_s, k(x^{(k)})). \tag{17}$$

By this definition, the event likelihood for the true values of I_t and I_s is always included in $L^{(i)}(M_{top}, \Delta JES)$, if the event is signal. The scanning of k, i.e. kinematical reconstruction, is to be done until the value of the event likelihood converges.

Joint likelihood for multiple events and maximum likelihood estimate of M_{top}
The event likelihood given by Eq.(17) for individual events are mutually independent probabolity functions for common parameters M_{top} and ΔJES. Thus to obtain the top quark mass and the jet energy scales from a total of N_{ev} events, we multiply the event likelihood functions, and take -2ln(likelihood) of the product,

$$\Lambda(M_{top}, \Delta JES) = -2ln\left(\prod_{i=1}^{Nev} L^{(i)}(M_{top}, \Delta JES)\right). \tag{18}$$

The mass of top quark and the jet energy scale are determined from the maximum likelihood estimators of M_{top} and ΔJES,

$$\hat{M}_{top} = M_{top} \ at \ minimum \ of \ \Lambda(M_{top}, \Delta JES), \tag{19}$$

$$\hat{\Delta} JES = \Delta JES \ at \ minimum \ of \ \Lambda(M_{top}, \Delta JES). \tag{20}$$

B. Matrix Element Calculation in the $l + jets$ channel

In this channel, the reconstruction path from observed y to parton momenta $c(c_1, \dots, _n)$ are specified by the assignment of the 4 jets to quarks $(b, \bar{b}, q, \bar{q})$, the value of x, and the two solutions for the neutrino longitudinal momentum(ν_z). Since we require at least one b-tagged jet, we consider 12 or 4 combinations for the reconstruction depending on whether 1 or 2 jets are b-tagged. The parton distribution functions $f_{a/p}(z_1)$ and $f_{b/\bar{b}}(z_2)$ are obtained from PYTHIA using the leading order PDF, CTEQ5L. Other PDF sets are used as a systematic uncertainty. The transverse distribution function $f_T(p_T)$ is obtained by running the PHYTHIA generator.

The spin averaged matrix element squared $|\mathcal{M}|^2$ is factorized into 3 parts: (1) $t\bar{t}$ production ($|\mathcal{M}_{t\bar{t}}|^2$), (2) the propagators of the top and anti-top ($|\Pi_{tl}|^2$ and $|\Pi_{th}|^2$), and (3) the top decay matrices $|\mathcal{D}_{tl}|^2$ and $|\mathcal{D}_{th}|^2$, where tl and th indicate leptonic and hadronic decay channels ot top respectively. Namely,

$$|\mathcal{M}|^2 = |\mathcal{M}_{t\bar{t}}|^2 |\Pi_{tl}|^2 |\Pi_{th}|^2 |\mathcal{D}_{tl}|^2 |\mathcal{D}_{th}|^2. \tag{21}$$

The production matrix element for the $q\bar{q}$ process is

$$|\mathcal{M}_{t\bar{t}}(q\bar{q} \to t\bar{t})|^2 = \frac{g_s^4}{9}(2 - \beta^2 sin^2\theta^*), \tag{22}$$

where θ^* is the angle between the top quark and the incident quark in the proton in the $t\bar{t}$ center of mass system, β is the velocity of the top and g_s is the strong coupling constant.
For the gg process, the matrix element can be expressed as

$$|\mathcal{M}_{t\bar{t}}(gg \to t\bar{t})|^2 = g_s^4 \left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8}\right)\left(\tau_1^2\tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2}\right), \tag{23}$$

where

$$\tau_1 = \frac{2(q\cdot t)}{\hat{s}},\ \tau_2 = \frac{2(\bar{q}\cdot t)}{\hat{s}},\ \rho = \frac{4M^2}{\hat{s}}. \tag{24}$$

In these equations, the $t\bar{t}$ spin correlations for both the $q\bar{q}$ and gg processes are averaged out, since correlation do not affect the mass measurement. Likewise, whether the $t\bar{t}$ is produced via $q\bar{q}$ or gg has no effect on the mass measurement, hence the likelihoods for the two processes are summed up in the event likelihood with the appropriate PDF weight.

The decay matrix elements for the leptonic and hadronic channel are expressed as

$$|\mathcal{D}_{tl}|^2 = N_W \frac{g_w^4}{4}\frac{(t\cdot l)(b\cdot \nu)}{(S_{l\nu} - M_W^2)^2 + M_W^2\Gamma_W^2}, \tag{25}$$

$$|\mathcal{D}_{th}|^2 = N_W \frac{g_w^4}{4}\frac{1}{2}\sum_{i\leftrightarrow j}\frac{(\bar{t}\cdot j_i)(\bar{b}\cdot j_j)}{(S_{2j} - M_W^2)^2 + M_W^2\Gamma_W^2}, \tag{26}$$

where $S_{l\nu}$ and S_{2j} represent the invariant masses squared of the lepton+neutrino and the 2 jets from W respectively. As for the mass and the decay width of W, we assume the world average values, $M_W = 80.4\,GeV/c^2$ and $\Gamma_W = 2.1\,GeV/c^2$. N_W in (25) and (26) is the normalization constant to convert $\mathcal{P}(s_W)$ to $\Pi(s_W)$, which is a constant for the fixed values of M_W and Γ_W. Since the z-component of the neutrino momentum is not measured, $S_{l\nu}$ is infered with $\Pi(s_W)$. The generated value of $S_{l\nu}$ gives the z-component of the neutrino momentum via Eq.(12). In Eq.(26), we make both possible assignments of the 2 jets to q and $\bar{q}'$ from the W, and the likelihoods corresponding to the two possibilities are averaged.

C. Mapping Function

Since the maiximum point of the 2D likelihood $L(M_{top}, \Delta JES)$ to determine M_{top} and ΔJES is affected by wrong jet assignments and background contamination, we need to correct the offset from true value. To determine the offset from the true values, a large number of pseudo-experiments are done for five MC samples produced with different top quark masses and different JES so that we make a 2D function $f(M_{top}, \Delta JES)$ called the mapping function. The mapping function gives the true value when the measured values M_{top} and ΔJES is given. Figure 1 shows the two mapping functions for measured M_{top} and ΔJES as a function of input M_{top} and input ΔJES.
Since the errors for measured M_{top} and ΔJES are also shifted by the mapping functions, they should be corrected further so that the pull width is 1.

D. Sanity Check

After applying the mapping functions, M_{top} and ΔJES should be measured correctly. We apply the mapping functions to the measured values M_{top} and ΔJES for each pseudo-experiments with a variety of input top quark mass and ΔJES. Fig.2 shows that the linearity of our method with the mapping functions and Fig.3 shows that the residuals between the measured value and true value as a function of top quark mass or ΔJES.
We also varify if the errors of measured values are estimated correctly. To do this varification, we introduce the pull which is defined by

$$\sigma = \sigma^+(M_{rec} < M_{input}) \tag{27}$$
$$= \sigma^-(M_{rec} > M_{input}) \tag{28}$$

The pull is obtained for each pseudo-experiment. If our estimation of an error is correct, the width of pull distribution with a large number of pseudo-experiments should be 1. Fig.4 shows that the pull width as function of input top quark mass or input ΔJES.

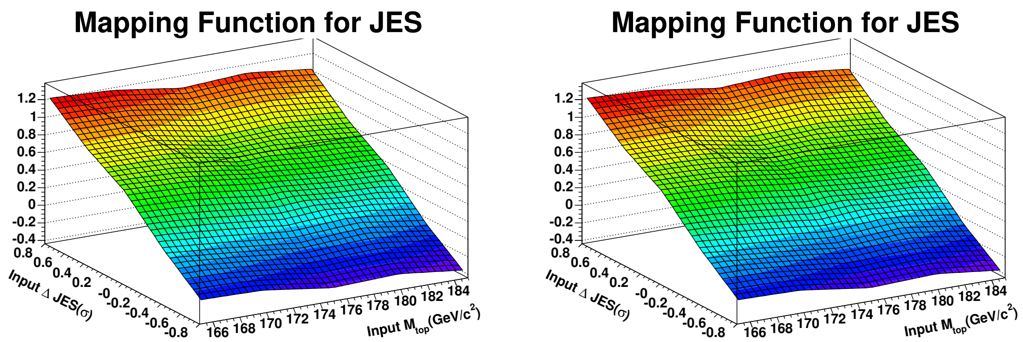

FIG. 1: The two mapping functions for measured M_{top} and ΔJES as a function of input M_{top} and input ΔJES.

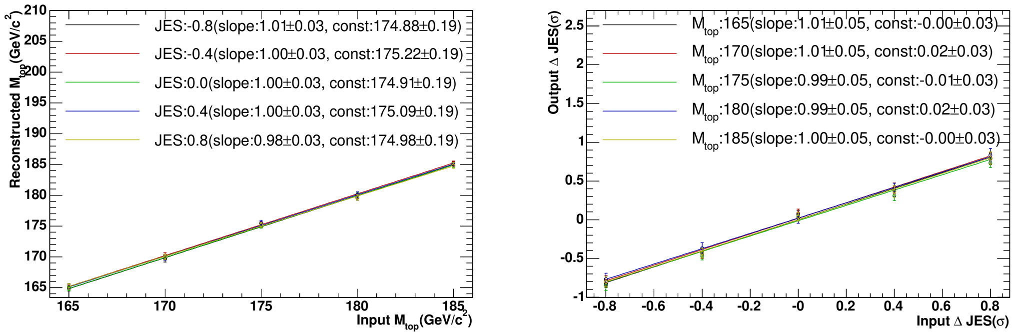

FIG. 2: The left plot shows that measured M_{top} as a function of input M_{top} for a variety of input ΔJES. The right plot shows that measured ΔJES as a function of input ΔJES for a variety of input M_{top}. Each point in the plots are obtained a large number of pseudo-experiments with the mapping functions.

IV. TOP MASS EXTRACTION

We have 343 $t\bar{t}$ candidate events passing our event selection criteria. The joint likelihood of these events is fitted with quadratic surface as

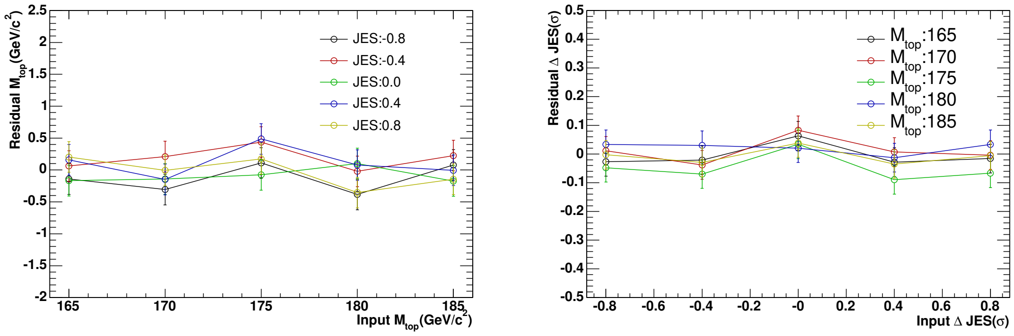

FIG. 3: The left plot shows that residuals between measured M_{top} and true M_{top} as a function of input M_{top} for a variety of input ΔJES. The right plot shows that residuals between measured ΔJES and true ΔJES as a function of input ΔJES for a variety of input M_{top}. Each point in the plots are obtained a lot of pseudo-experiments with the mapping functions.

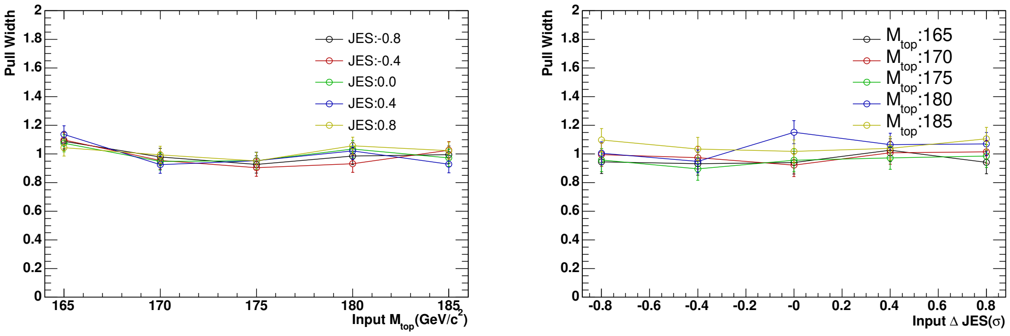

FIG. 4: The pull width as a function of input M_{top} or ΔJES.

$$f(x,y) = ax^2 + by^2 + cxy + dx + ey + f; \tag{29}$$

From the fit, we obtain $M_{top} = 171.6 \pm 2.0$(stat.+JES) GeV/c^2 after applying mapping function. Fig.5 shows the joing likelihood as a function of M_{top} and ΔJES.

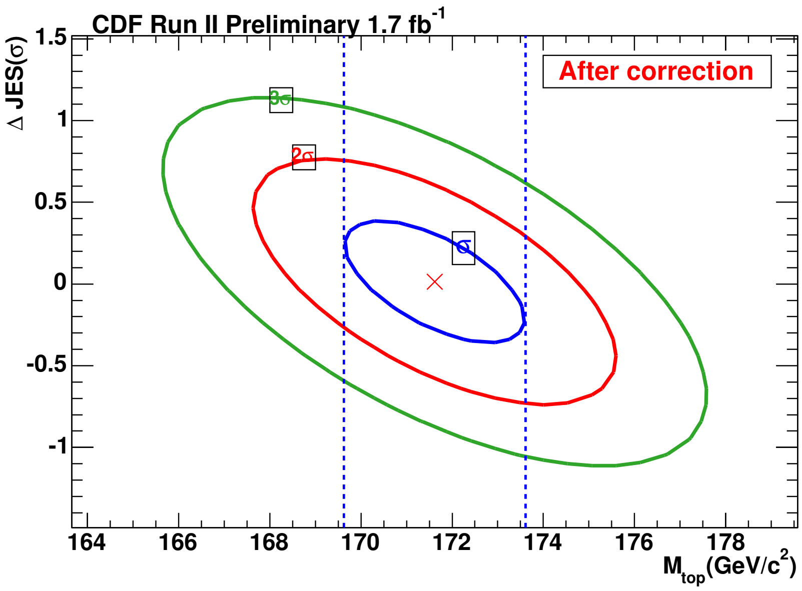

FIG. 5: The joing likelihood as a function of M_{top} and ΔJES. The cross mark in the center of figure is the maximum point of likelihood and blue curve is one sigma curve which is extracted to axis of M_{top}.

We compare measured statistical uncertainty with expected uncertainty obtained by many pseudo experiments with MC events whose input top mass is 170 GeV/c^2. 68% of pseudo experiments had smaller uncertainty than our measured statistical uncertainty.

V. SYSTEMATIC UNCERTAINTY

The systematic uncertainties are summarized in Table II. The largest one still comes from the uncertainty in the jet energy measurement, although it is considerably improved since the previous analysis[**?**].

The initial and final state radiation uncertainties are estimated using PYTHIA samples, where the rage of QCD parameters are determined by studying Drell-Yan data. For the parton distribution functions, we added uncertainties

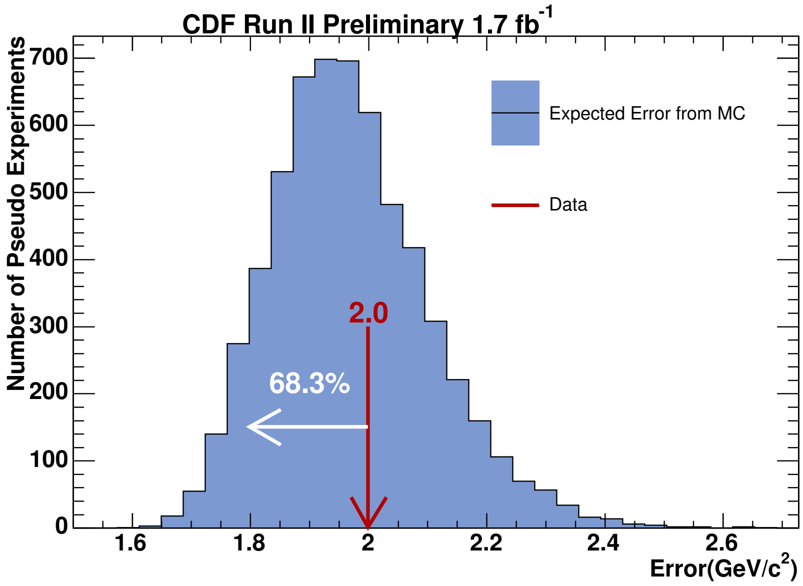

FIG. 6: The measured statistical uncertainty and expected uncertainty obtained by many pseudo experiments with MC events whose input top mass is 170 GeV/c^2. 68% of pseudo experiments had smaller uncertainty than our measured statistical uncertainty.

from two sources in quadrature: the differences from the 20 pairs of CTEQ6M uncertainty eigenvectors($\pm\sigma$) and MRST using two different α_s values. The sum in quadrature of all the systematic uncertainties is 1.3 GeV/c^2.

Systematic	2-D systematic (GeV/c^2)
Residual JES	0.62
PDF	0.23
ISR	0.35
FSR	0.54
Generator	0.54
lepton p_T	0.28
Background shape	0.12
Backgronnd composition	0.31
Backgronnd fraction	0.14
b-jet energy scale	0.40
b-tagging efficiency	0.01
NLO	0.41
Calibration	0.05
Multiple initeraction	0.05
Total	1.32

TABLE II: The total systematic uncertainty.

VI. CONCLUSION

Using the dynamical likelihood method, we have measured the top quark mass to be 171.6±2.0±1.3 GeV/c^2 from 343 events corresponding to an integrated liminosity of 1.7fb^{-1} accumulated by the CDF in Run II.

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