Top Quark Mass: Past, Present and Future

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Abstract. The top quark is the most massive elementary particle discovered thus far. Its large mass may help explain the mechanism by which fundamental particles gain mass - the Standard Model's greatest standing mystery. Today the top quark mass, together with the W boson mass, plays an important role in constraining the Higgs boson mass. The current status of the top quark mass measurement and a brief outline of the expectation at the Large Hadron Collider and the International Linear Collider will be covered.

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1. INTRODUCTION

The top quark is the heaviest of all elementary particles known today, with a mass roughly 35 times that of the next heaviest fermion, the b quark. As one of the fundamental fermions in the Standard Model (SM) measuring the top quark mass would be important by itself. But more importantly, the top quark's large mass produces sizable contributions to the electroweak radiative corrections. Therefore making an accurate measurement of the top quark mass is an important task for precision tests of the SM. Today the top quark mass, together with the W boson mass, provides the best constraint on the Higgs boson mass. In most extensions of the SM (e.g. MSSM) the top quark mass contributes (through loop corrections) to the Higgs boson mass. Therefore the consistency check of any future theory will make it likely that the business of making ever more precise measurements of the top quark mass will continue well into the future.

A general introduction to the properties of the $t\bar{t}$ final state used to measure the top quark mass will be given in Section 2. The mass measurements performed at the Tevatron will be described in Subsection 2.1. The top mass measurement expectation at the Large Hadron Collider (LHC) and the International Linear Collider (ILC) will be briefly described in Subsections 2.2 and 2.3. The question of why we should measure the top quark mass to better than 1% will be discussed in Subsection 2.4.

2. TOP QUARK MASS MEASUREMENTS

Top quarks at the Tevatron are produced in pairs through the strong interaction or singly by the weak interaction. Even though the single top quark cross section is only about a factor of two smaller than the $t\bar{t}$ cross section, all the top quark mass measurements have been done (and most likely will continue to be done) exclusively in the $t\bar{t}$ channel. The reason is that single top quark production offers fewer analysis constraints and since the final state has fewer jets the backgrounds are much larger.

In the SM the top quark decays almost 100% of the time to a b quark and a W boson. The W in turns decays equally to a pair of leptons or a pair of quarks (e.g. $W^+ \to \bar{e}\nu_e, \bar{\mu}\nu_\mu, \bar{\tau}\nu_\tau, u\bar{d}, c\bar{s}$). Since there are three colors for every quark the W boson decay to two jets is three times more probable than its decay to an electron or a muon. The $t\bar{t}$ final states can then be classified according to the W boson decay into all jets (both W's decay hadronically), lepton+jets (only one W decays hadronically), or dileptons (both W's decay leptonically).

In the W boson center of mass (CM) the decay products have a momentum of 40 GeV/c, and in the top quark CM the b and the W have a momentum of 66 GeV/c (for a top quark mass of 170 GeV/c^2). So the main objects in top quark analysis are a 40 GeV/c electron, muon, neutrino or light jet, and a 66 GeV/c b-jet. Lorentz boosts increase or decrease the particle's momenta depending on the direction of the boost relative to the particles. This has the effect of widening the momentum distributions, keeping the peaks at the CM momenta. Therefore most event selection criteria require that jets, leptons and missing (neutrino) transverse momentum be bigger than 15-20 GeV/c, that the pseudorapidity η of these objects be within the active part of the detector (typically $|\eta|<2$), and that the leptons are isolated from the jets to avoid confusion with leptons that can be generated from the decay of particles inside a jet. Jets coming from b quarks can be identified by reconstructing secondary vertices signaling the presence of a B hadron. If a secondary vertex is found inside a jet the jet is said to be b-tagged.

The 40 GeV/c electrons and muons are well understood objects and do not contribute much to the top quark mass uncertainty. Relying on jet universality, for the 40 GeV/c light jets we can look at the LEP W mass measurements in the all hadronic channel for guidance. All four LEP experiments have measured the W mass in $e^+e^- \to W^+W^-$ with $W \to q\bar{q}'$ [1]. The W mass error in this channel ranges from 80 MeV/c^2 (L3) to 137 MeV/c^2 (DELPHI). These errors are negligible in comparison to the errors currently achievable in the top quark mass ($\approx$ 2 GeV/c^2) and it will not be until the ILC that the top quark mass errors will reach the 100 MeV/c^2 level. So for top mass measurements at the Tevatron or the LHC we can assume that the light 40 GeV/c jets are fairly well understood. A word of caution is in order here. The $p\bar{p} \to t\bar{t}$ Monte Carlo programs currently used in the top mass measurement at the Tevatron do not include the effects of color reconnection (CR) included in the W mass measurement at LEP. Recent studies [2] show that CR could affect the top mass at the level of 0.5-1.0 GeV/c^2, suggesting that for the next round of top mass measurements it will be extremely useful to have MC generators that include CR effects.

A Z boson mass measurement in the channel $Z \to b\bar{b}$ with a precision similar to that achieved for the W boson was not performed at LEP. Recently CDF presented a measurement of the energy scale of b-jets using the $Z \to b\bar{b}$ decay [3]. The energy scale was determined with a 2% error, which translates into $\approx$1.5 GeV/c^2 error in the top quark mass. This will help to greatly reduce the systematic errors in the dilepton channel. Current measurements in the lepton+jets channel use the W mass to determine an overall scale factor that is applied to both light and heavy jets, and rely on MC programs to correctly model the difference between heavy and light jets. The uncertainty on that MC ratio produces a systematic error of about $\approx$0.5 GeV/c^2.

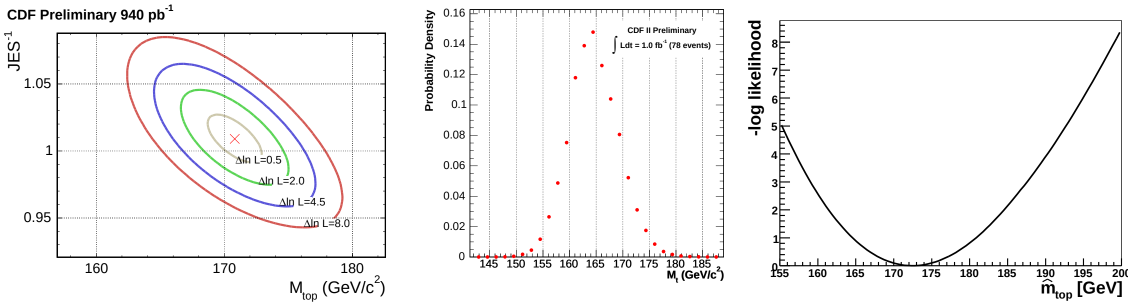

FIGURE 1. Left: 2D likelihood $L(M_{top}, JES)$ for the l+jets channel in CDF. Center: 1D likelihood $L(M_{top})$ for the dilepton channel in CDF. Right: 1D $-ln[L(M_{top})]$ for the dilepton channel in D0.

2.1. Top quark mass at the Tevatron

The top mass measurements at the Tevatron can be classified as measurements in which a number (category one) or a probability density function or pdf (category two) is calculated for every event. In the first category the number is entered in a histogram, which is then compared with a template obtained from MC. The templates are constructed by building high statistics histograms (in the same way as done for data) for different values of the top quark mass M_{top}. The top quark mass is then extracted comparing the data histogram with templates for different values of M_{top}. Since only one number per event is calculated, this way of extracting M_{top} is equivalent to calculating averages as $\overline{M} = \sum_i W(M_i)M_i$ where the weight $W(M)$ is determined by the shape of the template. In the second category the pdfs for each event are multiplied together to form a likelihood $L(M_{top})$ and the top quark mass is extracted by maximizing L with respect to M_{top}. Events that are better measured (narrower pdf's) contribute more to the measurement and this way of extracting the mass is equivalent to calculating averages as $\overline{M} = (\sum_i M_i/\sigma_i^2)/(\sum_i 1/\sigma_i^2)$, where M_i and σ_i are the average and rms of the pdf for event i. It can be shown that this is the most accurate way of calculating averages. Both categories have been extended to more dimensions, by calculating more than one number per event in the first category and by calculating pdfs as a function of more variables in the second category.

Both D0 and CDF performed top quark mass measurements of the first category. In the l+jets final state during Run I CDF used the fitted or reconstructed mass M_{reco} as the one parameter that is measured per event [4]. M_{reco} is obtained by performing a constrained fit to the $t\bar{t}$ events and then of all possible solutions selecting the one with the smallest χ^2. D0 expanded the technique to two dimensions using M_{reco} and a discriminant $D = S/(S+B)$ to separate signal (S) from background (B) [5]. In Run II CDF used 1) M_{reco} and the two jet (or W) invariant mass as the two numbers measured per event to obtain the most precise template mass measurement so far [6, 7], and 2) the B meson transverse decay length in b-jets to obtain a mass measurement with large errors, but independent of jet energies [8]. Due to the two missing (un-measured) neutrinos the $t\bar{t}$ events in the dilepton final state are under-constrained. In this case a "weight" distribution as a function of the top quark mass is calculated and some parameters of that distribution (e.g. the mean or peak) are used as the measured numbers for each

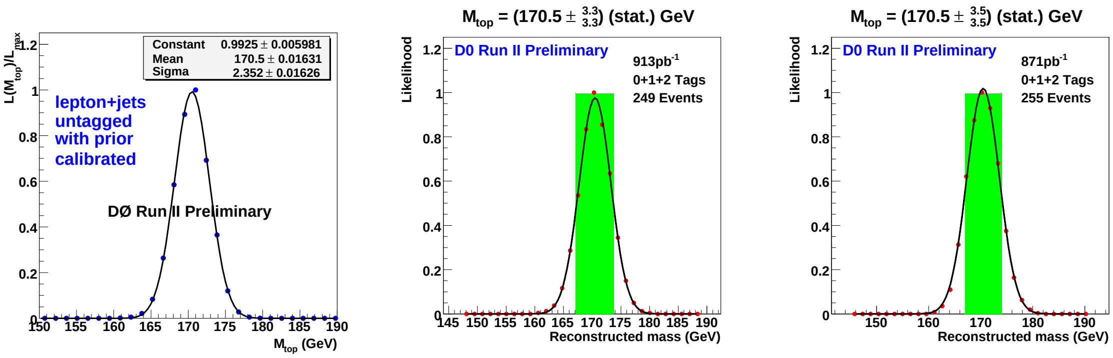

FIGURE 2. $L(M_{top})/L_{max}$ plots for the l+jets channel in the D0 experiment, for the un-tagged analysis (left), and the b-tagged analysis in the e+jets channel (middle) and the μ+jets channel (right).

event and compared to MC templates to extract the true top quark mass [9, 10, 11, 12]. Other "weight" distributions have also been used, like the invariant mass of a lepton and a b-jet, or the b-jets' energy [10].

Several D0 and CDF top quark mass measurements belong in the second category. In the l+jets final state D0 measured the top mass using Run I data by calculating a pdf as a function of M_{top} and the fraction of signal events [13]. A similar measurement was performed by CDF using Run II data [7, 14]. As the statistics increased both D0 and CDF extended the pdfs to include a Jet Energy Scale parameter JES [15, 16, 17] to reduce the largest systematic error by taking advantage of the W mass constraint. Category two measurements were also performed by CDF in dileptons [18] and all jets [19]. Only the most precise measurement in each channel will be covered in the rest of this section.

The most precise CDF and D0 top quark mass measurements are performed in the l+jets channel where a pdf $P(x|\alpha)$ is calculated for every event. The variable x labels the leptons and jets four momenta and $\alpha = (M_{top}, JES, f)$ labels the top quark mass M_{top}, Jet Energy Scale JES and fraction of signal events f. Given that all events are independent, the probability (or likelihood) for a sample of N events is the product of the probabilities for each event $L(\alpha) = \prod_{i=1}^{N} P(x_i|\alpha)$. If α is known then L will be very close to maximal, otherwise a different set of events would have been observed. Or if α is unknown it can be determined by maximizing L. A given event in the final sample has a certain probability $P_{t\bar{t}}(x|\alpha)$ of being signal and a probability $P_B(x)$ of being background. Therefore the probability for an individual event is $P(x|\alpha) = fP_{t\bar{t}}(x|\alpha) + (1-f)P_B(x)$. A two dimensional likelihood $L(M_{top}, JES)$ is obtained by projecting or maximizing with respect to f. The left plot in Figure 1 shows the two dimensional likelihood for the l+jets channels in the CDF experiment [17]. A fit to this likelihood yields the top mass given below in Eq. 2.

D0 used a Gaussian likelihood prior $G(JES)$ with a mean $JES = 1$ and an rms of 0.037 derived from a large sample of γ+jets events to further improve the measurement. The γ+jets events were used to perform an independent study of the jet energy scale and since the $t\bar{t}$ and γ+jets samples are independent, the likelihood for the combination of both samples is just the product of L and G. The top quark mass is then extracted by

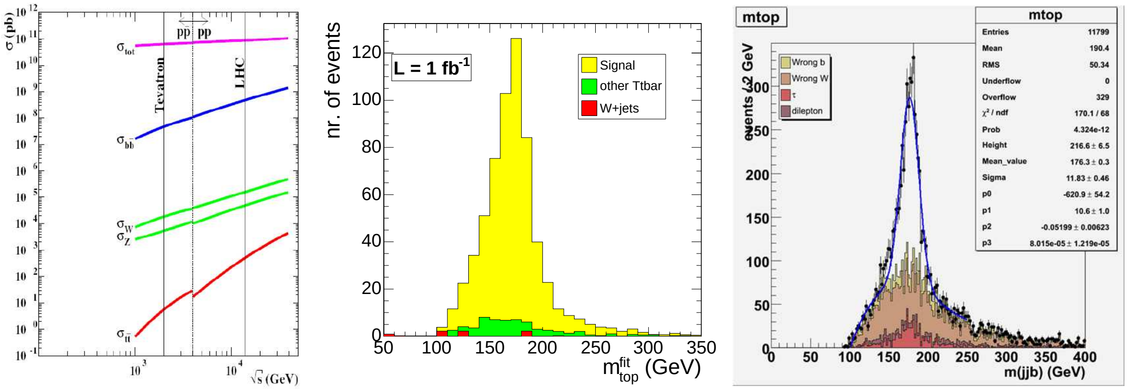

FIGURE 3. Left: Total cross sections as a function of energy for several different final states. Center: CMS simulation of the three jet invariant mass in the l+jets channel after applying a constraint kinematic fit. Right: ATLAS simulation of the three jet invariant mass in the l+jets channel.

projecting this product onto the M_{top} axis:

$$L(M_{top}) = \int df\, dJES\;\; L(M_{top}, JES, f)\; G(JES) \tag{1}$$

The normalized likelihoods $L(M_{top})/L_{max}$ are shown in Figure 2. The figure on the left shows the likelihood for the combined e+jets and μ+jets samples for the un-tagged analysis. The center (right) figure shows the likelihoods for the e+jets (μ+jets) sample in the b-tagged analysis. The final top quark mass derived from these plots [20] is given in Eq. 3 below.

$$\text{CDF } l\text{+jets: } M_{top} = 170.9 \pm 2.2(\text{stat+JES}) \pm 1.4(\text{syst}) = 170.9 \pm 2.6\ \text{GeV}/c^2 \tag{2}$$

$$\text{D0 } l\text{+jets: } M_{top} = 170.5 \pm 2.4(\text{stat+JES}) \pm 1.2(\text{syst}) = 170.5 \pm 2.7\ \text{GeV}/c^2 \tag{3}$$

With a combined weight of about 80%, the above two measurements dominate the world top quark mass combination [22].

In the dilepton channel CDF's best measurement with 1 fb^{-1} of data was obtained calculating pdfs for the signal and many different backgrounds [18]. The final likelihood as a function of M_{top} is shown in the center plot in Figure 1, and the final result is given in Eq. 4 below. The best D0 measurement in the dilepton channel with 1 fb^{-1} of data was obtained using the mean and rms of the "weight" distribution and comparing the resulting 2D histogram with a series of signal and background templates for the ee, $e\mu$ and $\mu\mu$ channels [20]. The right plot in Figure 1 shows the -ln(L) of the data to template comparison. The final result is given in Eq. 5.

$$\text{CDF dileptons: } M_{top} = 164.5 \pm 3.9(\text{stat}) \pm 3.9(\text{syst}) = 164.9 \pm 5.5\ \text{GeV}/c^2 \tag{4}$$

$$\text{D0 dileptons: } M_{top} = 172.5 \pm 5.8(\text{stat}) \pm 5.5(\text{syst}) = 172.5 \pm 8.0\ \text{GeV}/c^2 \tag{5}$$

$$\text{CDF all jets: } M_{top} = 171.1 \pm 3.7(\text{stat}) \pm 2.1(\text{syst}) = 171.1 \pm 4.3\ \text{GeV}/c^2 \tag{6}$$

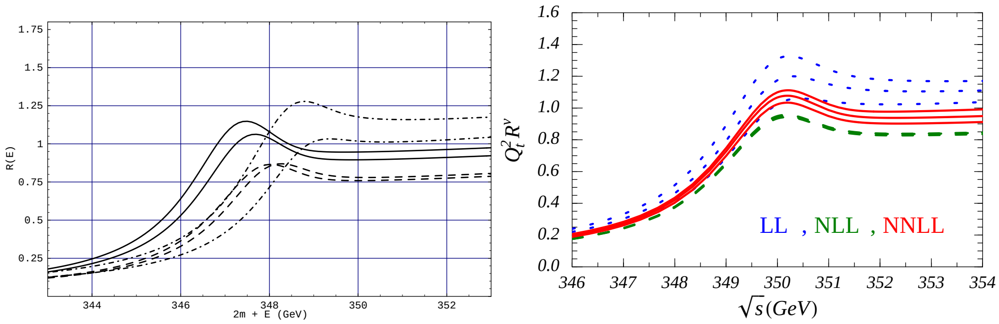

FIGURE 4. Normalized cross section $R_t = \sigma(e^+e^- \to t\bar{t})/\sigma(e^+e^- \to \mu^+\mu^-)$ as a function of $\sqrt{s}$ calculated using the pole mass (left) or the 1S mass (right). See text for details.

In the all jets channel CDF produced with $\approx$1 fb^{-1} of data a template measurement as a function of the top quark mass and the Jet Energy Scale [21]. The result is given in Eq. 6.

The left plot in Figure 5 displays a summary of all the top quark mass measurements that enter the world combination [22] . As the plot also shows the world average, 170.9 ± 1.8 GeV/c^2, has reached the extraordinary precision of 1%.

2.2. Top quark mass at the LHC

The left plot in Figure 3 shows how the cross section increases with CM energy for several different channels. The $t\bar{t}$ cross section increases by a factor of $\approx$160 between the Tevatron and the LHC. At the same time the main backgrounds, W+jets and multi-jets, only increase about 6 times. So while at the Tevatron the signal to background ratio is of order $S/B \approx 1$ at the LHC it will be about $S/B \approx 20$. This enormous increase in statistics and signal to background ratio will allow for a large reduction in the top quark mass statistical error. But perhaps more importantly it will allow for 1) the important cross check provided by the ability to accurately measure the mass in three different decay channels (dilepton, l+jets and all jets) and 2) for the possibility of extracting the systematic errors from the data.

Assuming an integrated luminosity of about 8 fb^{-1} by the end of the Tevatron run the stat+JES error in the l+jets channel (see Eqs 2-3) will reach the 600 MeV/c^2 level when both the D0 and CDF measurements are combined. This clearly indicates that the top quark mass error will be completely dominated by systematic errors. Today the systematic errors are extracted from MC studies, and if nothing changes the LHC top quark mass measurements will most likely not improve much beyond the Tevatron ones. It is the author's hopes that by measuring the mass as a function of jets rapidity, or b-jets momentum, or the b-jets angle with the beam, etc. the LHC experiments will be able to reduce the systematic errors substantially.

The center plot in Figure 3 shows a CMS full detector simulation of the three jet invariant mass corresponding to the hadronic decay of a top quark in the l+jets final state with two b-tagged jets [23]. Two b-jet to b-quark assignments are possible when

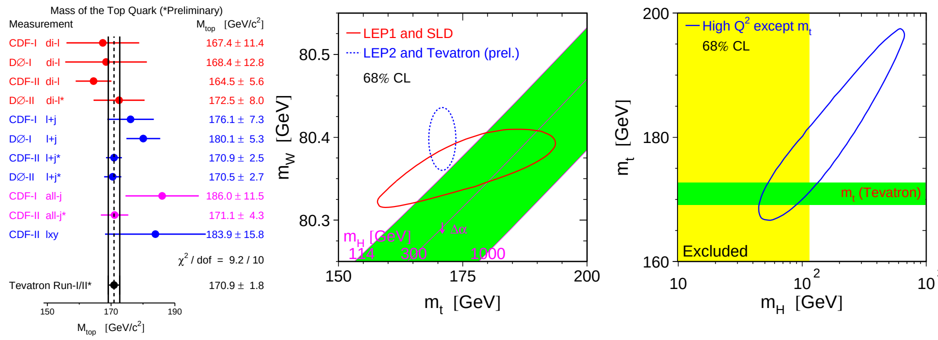

FIGURE 5. Top quark masses included in the world average (left). Electroweak fit plots of the W boson mass vs. the top quark mass (center) and of the top quark mass vs. the Higgs boson mass (right).

two jets are tagged. The permutation with the best constraint kinematic fit to the event was selected to calculate the three jet invariant mass.

The right plot in Figure 3 shows an ATLAS full detector simulation of the three jet invariant mass in the l+jets channel [24]. The backgrounds correspond to the case in which a b-jet (Wrong b) or a light jet (Wrong W) are mis-assigned, or to events with a τ decay or two leptons in the final state. It is clear that at the LHC most of the backgrounds will come from mis-reconstructed top events.

The statistical error in the top quark mass will be mostly given by the rms of the previous plots divided by $\sqrt{N}$. With N in the tens of thousands it is clear that as mentioned before the top quark mass measurement at the LHC will be completely dominated by systematic errors.

2.3. Top quark mass at the ILC

The high accuracy with which the e^+e^- CM energy will be known at the ILC makes it possible to perform a less than 0.1% (or about 100 MeV/c^2) error measurement of the top quark mass by scanning the beam energy at the $t\bar{t}$ threshold. The left plot in Figure 4 shows the normalized cross section $R_t = \sigma(e^+e^- \to t\bar{t})/\sigma(e^+e^- \to \mu^+\mu^-)$ as a function of $\sqrt{s}$ computed with a pole mass of 175 GeV/c^2 at LO, NLO and NNLO (dashed-dotted, dashed and solid lines respectively). The pairs of curves correspond to different soft normalization scales [25]. The horizontal position and rate of rise of the curves in Figure 4 are determined by the values of the top quark mass and width respectively. But as we can see these curves are not stable and depend on the order of perturbation theory at which they are calculated. Top quarks decay too fast to be able to form bound states, but live long enough to produce a cross section enhancement with 1S quantum numbers (see bump in Figure 4). The curves become more stable when the top quark mass is defined as half of the mass of this enhancement. The right plot in Figure 4 shows the normalized cross section computed with a 1S mass of 175 GeV/c^2 at LL, NLL and NNLL (dotted, dashed and solid lines respectively). Each of the three curves correspond to different renormalization scaling parameters [26]. As we can see the curves are now more stable, which of course brings up the question of how to define the mass of a colored object like the top quark.

Currently the top quark mass is experimentally defined by the position of the peak of the invariant mass of a b-jet and a W, and closely corresponds to the pole mass of the top quark. Of course this is only approximately defined because the b-quark will cancel color with other partons in the event, making it impossible to identify all particles that come from the top quark decay. This ambiguity in the pole mass is of the order of Λ_{QCD} [27], and as the top quark mass error crosses the 1 GeV/c^2 line the question of what we mean by the mass of a top quark will become ever more relevant.

2.4. Why reduce the top mass error beyond 1%?

The right plot in Figure 5 shows the top quark mass vs. the Higgs boson mass [1]. The yellow area shows the Higgs boson masses excluded by LEP, the blue curve shows the 68% confidence area allowed by the electroweak fit (excluding M_{top}), and the green band shows the world average of the top quark mass. As the error in the top quark mass shrinks the green band will become narrower. If, as the error shrinks, the band moves down it could eventually exclude a SM Higgs, on the other hand if it moves up it will leave a window open. So currently the top quark mass plays an essential role in constraining the Higgs boson mass [1]. The center plot in Figure 5 contains similar information but now the W boson mass is also displayed. The blue dotted ellipse contains the world averages of the top and W masses and the green band displays the Higgs allowed region. Again we can see that as the top quark mass error is reduced the blue ellipse will shrink horizontally and it will move to the right or to the left making the SM more or less compatible with the allowed values of the Higgs boson mass.

Through loop corrections, in most extensions of the SM the top quark mass contributes to the Higgs boson mass. If (or once) the Higgs is seen at the LHC the top quark mass will become an important parameter to check the consistency of any future theory. Given the small error in the Higgs boson mass expected at the LHC ($\approx$ 0.2 GeV/c^2) the top quark mass error will become dominant in any future precision fit unless it reaches ILC precision levels [28]. It is therefore likely that the business of making ever more precise measurements of the top quark mass will continue well into the future.

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