VEHICLE MASS, STIFFNESS AND THEIR RELATIONSHIP

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ABSTRACT

Vehicle stiffness is a commonly used parameter in the field of vehicle safety. But a single-valued "stiffness", although well defined for the linear case, is not well defined for non-linear systems, such as vehicle crashes. Moreover, the relationship between vehicle stiffness and mass remains confusing. One previous work [1] addresses this issue. Multiple definitions of stiffness were used to address the lack of a clear definition of stiffness. The R^2 values for the correlation between mass and each stiffness measure were presented. The results showed that no clear relationship existed between mass and any of the stiffness measures. The results from a statistical analysis indicated that there were differences in stiffness between different types of vehicles.

This paper extends the same research by including a significant amount of new data samples as well as some different analysis procedures. Results show that mass is poorly correlated to stiffness and for some vehicle types mass correlates better to vehicle crush than to stiffness. In addition, it is shown that even without a well-defined definition of stiffness different levels of stiffness can be defined and differences in stiffness between different vehicle types can be quantitatively and qualitatively established.

INTRODUCTION

The most influential vehicle parameters in frontal crash and compatibility are the mass [2, 3, 4], the stiffness as well as the geometry; the latter two are not well understood and appear to be less significant. The relationship between mass and stiffness (a single-valued parameter) has been a subject of study for some time. One proposition [5] is that the mass ratio of two colliding vehicles has historically been incorrectly identified as the cause of compatibility problems because stiffness is the actual parameter at play, except it is not available in accidents statistics, but it is related to mass. However, stiffness, which is a description of the crash response, is a complicated concept, and warrants a systematic discussion.

The crash response of each vehicle, used to derive stiffness, depends on its detailed force displacement history. A complete characterization would involve details at the micro level. While this is necessary for an individual vehicle analysis, it results in a computationally intractable problem when general analysis across a vehicle fleet is needed. A characterization at a higher level is more appropriate. One such characterization of the crash response is to use barrier (rigid or deformable) test data. Indeed, this approach has been used lately in several studies [6, 7]. The term "stiffness", without being clearly defined, has been used loosely as a measure that describes the force-crush behavior. This has resulted in some difficulty, confusion and misinterpretation of results and conclusions.

A recent study [1] attempted to address the topic of stiffness and its relationship with mass. Vehicle stiffness, either static or dynamic, is a vague concept. It has precise meaning only in linear elastic deformation which is not the case in a vehicle crash. In an attempt to address the lack of a well-formed stiffness definition, multiple stiffness definitions were used in that study [1]. The use of several physically meaningful stiffness measures allows a relationship between mass and stiffness to be studied without having to have one specific stiffness definition. This assumes that there are enough stiffness measures that are different enough and they span the space of "reasonable" stiffness definitions; therefore, if there is a relationship between mass and stiffness for the majority of these stiffness definitions, then there is a relationship between mass and stiffness in general.

Eight different stiffness measures were defined in the previous study [1]. Then the mass-stiffness correlation analysis and stiffness characteristic study among different type of vehicles were performed by using the NHTSA NCAP tests, mostly from model year 1999 to 2003. It was conclude that only a weak relationship between mass and stiffness appeared to exist. The stiffness of different vehicle types and their relative ranking system were found to depend on the definition of the stiffness.

The objective of this study is to advance the previous study by including a larger sample size with a more extensive model year coverage. The previous study included 175 vehicles from 1999 to 2003, while the current study includes 585 vehicles and spans the model years 1979-2004. The analysis procedures used in the previous study are followed here. The only difference is the addition of a new parameter, the maximum crush, and its relationship with mass.

METHODOLOGY

The NCAP test data used in this paper were obtained from NHTSA database for the model years 1979 to 2004. The vehicles were grouped into four different types, Car, Van, SUV, and Truck, according to the U.S. Environmental Protection Agency (EPA) classifications. Among the vehicles used, about 66% were Cars, 14% were SUVs, 11% were Trucks, and 9% were Vans.

Based on barrier force and vehicle displacement, 4 different definitions of stiffness for a total of 9 measures were considered. Test data were excluded from the analysis if the linear impulse and momentum were not consistent (the integral of force-time was not close to the vehicle momentum), load cell or accelerometer data were missing, or there was instrumentation errors. The SAE CFC60 filter was used to filter the barrier forces.

Essentially two statistical analyses were performed: a linear regression analysis, and a multiple comparison analysis. The linear regression was applied to study the correlation of vehicle mass and various stiffness measures. The coefficient of linear determination, R^2, has been used to describe the degree of linear association [8].

The next step was to perform a multiple comparison analysis of vehicle-type stiffness values by means of the Tukey procedure [8], using a family confidence coefficient of 90%. The family confidence coefficient pertaining to the multiple pairwise comparisons refers to the proportions of correct families, each consisting of all pairwise comparisons, when repeated sets of samples are selected and all pairwise confidence intervals are calculated each time. A family of pairwise comparisons is considered to be correct if every pairwise comparison in the family is correct. Thus, a family confidence coefficient of 90% indicates that all pairwise comparisons in the family will be correct in 90 percent of the repetitions.

STIFFNESS DEFINITIONS

Definition 1: "Linear" Stiffness

For a given barrier force versus vehicle displacement curve F(d), a line is obtained by a least square fit over a given displacement range. The slope of this line is defined as the "linear" stiffness over the displacement range considered. Two "linear" stiffness measures were used in this paper, depending on the range of the displacement considered. The first, noted as K1, has a displacement range from 25 to 250 mm, as shown in Figure 1. The second, K2, has a displacement range from 25 to 400 mm.

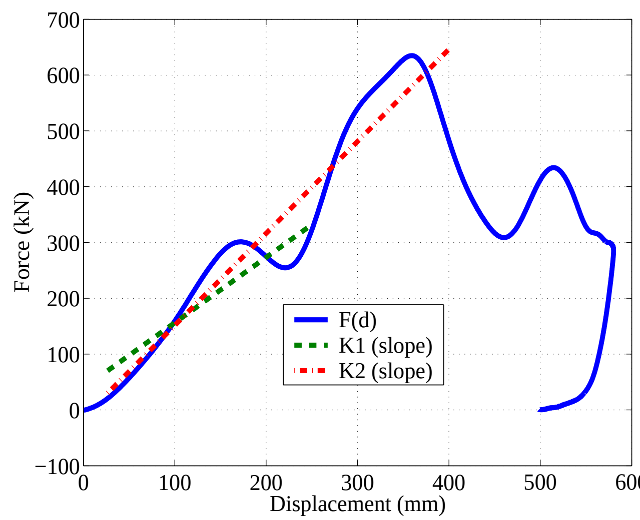

Figure 1. Illustration of "Linear" Stiffness (K1 and K2)

Definition 2: Energy-Equivalent Stiffness

The energy-equivalent stiffness (Ke) is defined as Ke=F*2/d, where F is the average force over the displacement range [25 mm, d]. Two energy-equivalent stiffness measures were used, one with d equal to 250 mm, and the other with d equal to 400 mm, i.e., Ke1=F1*2/(250-25) (unit: force/mm) and Ke2=F2*2/(400-25) (unit: force/mm), as shown in Figure 2.

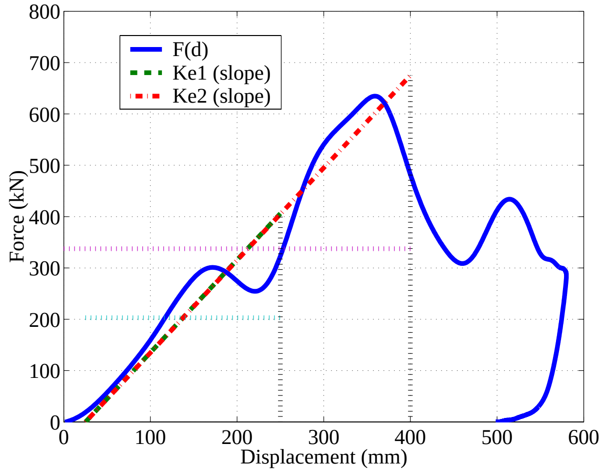

Figure 2. Illustration of Energy-Equivalent Stiffness (Ke1 and Ke2)

Definition 3: Global Linear Energy-Equivalent Stiffness

The global linear energy-equivalent stiffness is defined as $Ke3=M*V^2/Xm^2$, where M is the vehicle mass, V is impact speed and Xm is the maximum displacement of the vehicle. This is obtained by an approximation of the conservation of total energy and by assuming the force is F=Ke3*d, where Ke3 is a stiffness and d is the vehicle displacement.

Definition 4: Peak Force as a Stiffness Metric

In addition to the three stiffness definitions above, three peak barrier forces were also used. It is recognized that the use of the peak force is the least representative of the stiffness. They are defined as following:

Fp1 = max (F(d)), 25 mm<d<250 mm;
Fp2 = max (F(d)), 25 mm<d<400 mm;
Fp = max (F(d)), 25 mm<d<Xm,
where Xm is the maximum displacement.

RESULTS AND OBSERVATIONS

Linear Regression Procedure

The R^2 values which describe the degree of linear association between mass and the various stiffness measures are presented in Table1. The results for five different vehicle groups are listed in the Table. The first group in row two includes all the vehicles, and the other four are subgroups of different vehicle types.

Table 1. Mass-Stiffness R^2 Value (1979-2004)

	K1	Ke1	K2	Ke2	Ke3	Fp1	Fp2	Fp
All	0.27	0.24	0.11	0.28	0.35	0.32	0.19	0.48
Car	0.02	0.02	0.08	0.08	0.13	0.04	0.09	0.37
SUV	0.05	0.03	0.00	0.02	0.00	0.03	0.00	0.13
Truck	0.02	0.05	0.02	0.01	0.01	0.02	0.02	0.06
Van	0.13	0.06	0.10	0.10	0.12	0.12	0.10	0.36

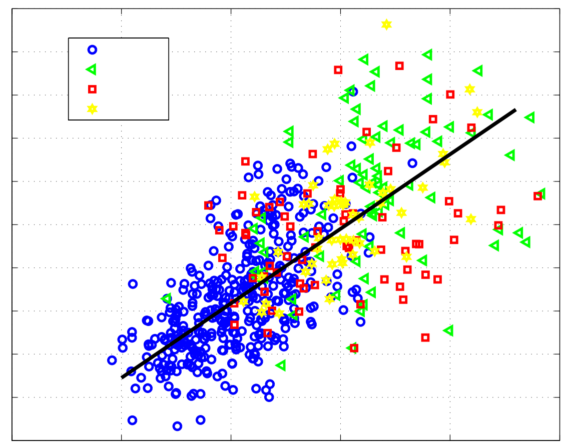

Figure 3. Mass and Peak Force

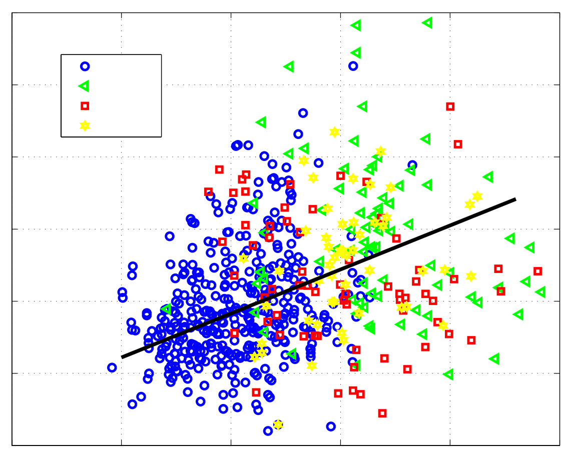

Figure 4. Mass and Stiffness Measure K2

Many values in Table 1 are close to zero with the highest value below 0.5, which indicates a very weak correlation between the two. The highest correlation exists between mass and peak force, with the R^2 value of 0.48, for the all vehicle group, as shown in Figure 3. The lowest correlation exists between mass and K2 with the R^2 value of 0.11, for the all vehicle group, which is shown in Figure 4. Moreover, the correlation, for most stiffness measures, in the all vehicle group is, in general, higher than that for each individual subgroup.

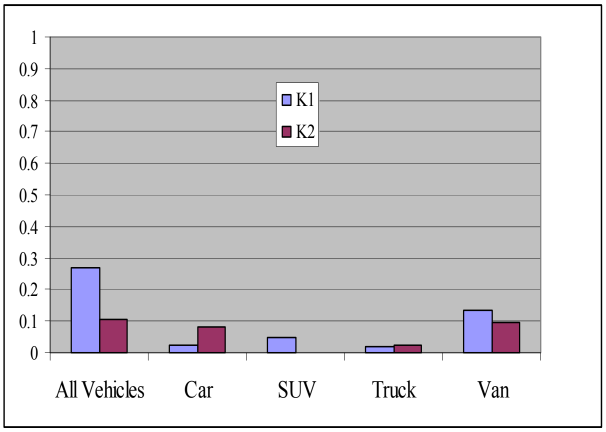

Figure 5. R^2 Values of Mass and "Linear" Stiffness

The results for the R^2 values between the two "linear" stiffness measures K1 or K2 and mass are shown in Figure 5. In general, there is very weak correlation between mass and either K1 or K2 for the subgroups.

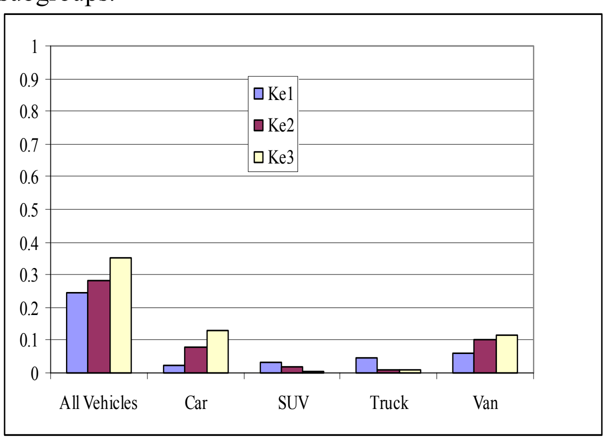

Figure 6. R^2 Values of Mass and Energy-Equivalent Stiffness

The results for the R^2 values between the three energy-equivalent stiffness measures, Ke1, Ke2, and Ke3 and mass are shown in Figure 6. All the values are below 0.35. It seems that there is almost no correlation between mass and any of these three stiffness measures for the SUV and Truck subgroups.

In general, Ke3 has a relatively higher correlation with mass, while Ke1 has a relatively lower correlation with mass for the Car and Van subgroups, even though the correlations are weak.

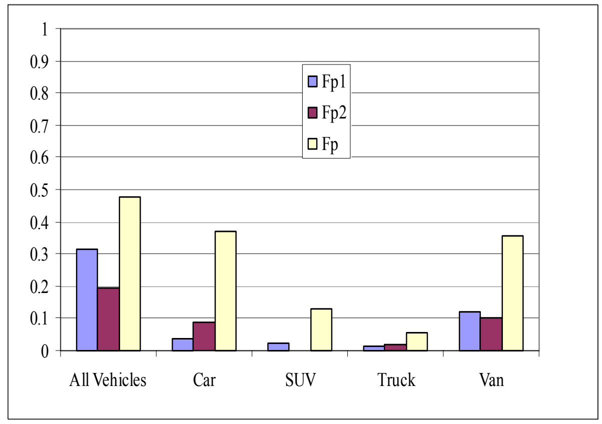

Figure 7. R^2 Values of Mass and Peak Barrier Force

The results for the R^2 values between peak forces and mass are shown in Figure 7. In general, Fp shows a relatively higher correlation across all the groups, especially for the Car and Van subgroups.

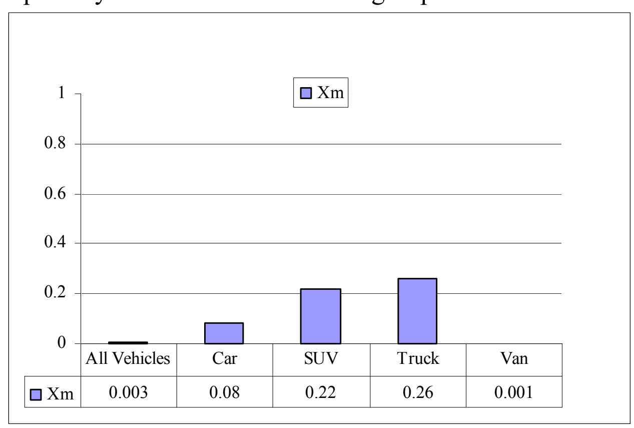

Figure 8. R^2 Values of Mass and Maximum Crush Xm

The maximum crush Xm, is the maximum vehicle displacement over the duration of the vehicle crash, obtained from double integration of the vehicle acceleration with the initial velocity of 35mph. The results for the R^2 values between Xm and mass are shown in Figure 8. In general, the lowest correlation between mass and Xm is for the Van subgroup and the all vehicle group with the highest correlations for the Truck and SUV subgroups.

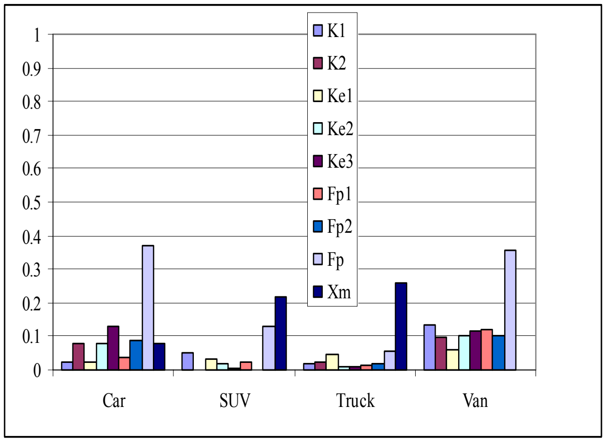

Figure 9. R^2 Values of Mass and All Stiffness Measures for the Subgroups

The results for the R^2 values between mass and all the stiffness measures, as well as mass and maximum crush for all the subgroups are shown in Figure 9. Either peak force Fp or maximum crush Xm has the highest correlation with mass. For Car and Van subgroups, peak force correlates the strongest with mass, while maximum crush Xm correlates with mass the strongest for the SUV and Truck subgroups.

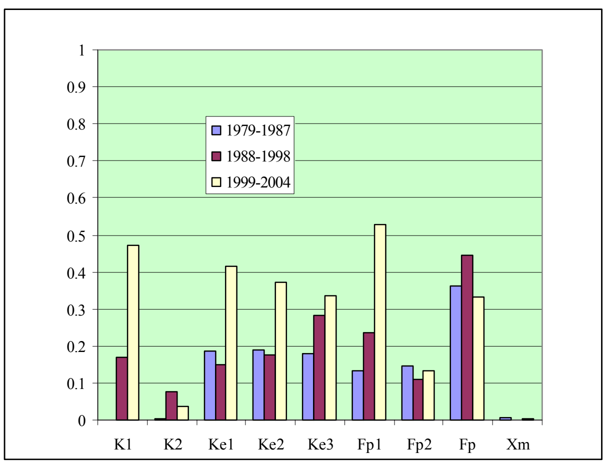

Figure 10. R^2 Values of Mass and all the Stiffness Measures for All Vehicle Group at Different Time Period

The results for the R^2 values between all the stiffness measures and mass at different time periods are shown in Figure 10. The whole time period from 1979 to 2004 is divided into three sub periods: from 1979 to 1987, from 1988 to 1989, and the last one from 1999 to 2004. It is observed from Figure 10 that peak forces Fp and Fp2 show the most consistent correlations with the mass across the different time periods. The correlation changes significantly between either K1 or Fp1 and mass, among these three different time periods. The correlations between average forces (Ke1 and Ke2) and mass change little from the period 1979-1987 to period1988-1998. But, they change significantly from period 1988-1998 to period 1999-2004.

Multiple Comparisons Procedure

The data used for the comparison are 1979 to 2004 model year NCAP tests. The results of the multiple comparisons procedure are shown in Figures 11-15. The absence of a solid line between two different vehicle types implies that a difference in stiffness has been found. The location of mean stiffness values, (the whisker-points in the upper half of the figure), indicates the direction of the difference. On the other hand, the presence of a solid line between two vehicle types indicates that the two vehicle types have statistically similar stiffness values.

For example, in Figure 11, the continuous solid lines between circles, squares, and diamonds, starting from Van type and passing Truck type and ending at SUV type, indicate that there are no substantial stiffness differences among these three vehicle types, by using Ke1, or Ke2, or Ke3. However, the absence of a solid line between Car type and any of the other types shows that Cars are different from all other type of vehicles.

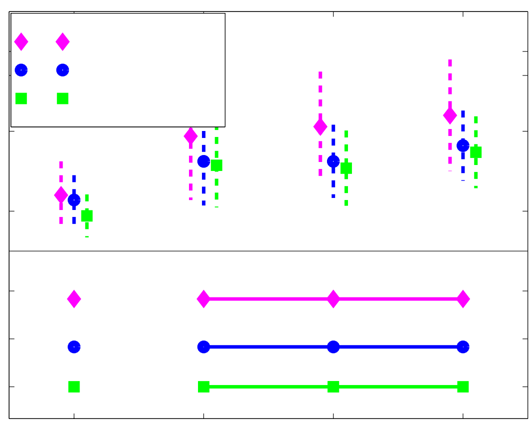

Figure 11. Ke1, Ke2 and Ke3 Stiffness Results. The Dashed Lines Indicate One Standard Deviation from The Mean Values. Each Non-Significant Difference between Two Vehicle Types Is Indicated by a Solid Line.

Thus, the multiple comparison procedure for Ke1, Ke2 and Ke3 lead to infer, with a 90% family confidence coefficient, that essentially there are only two different stiffness groups: a less stiff Car group and a significantly stiffer group that includes Vans, Trucks and SUVs.

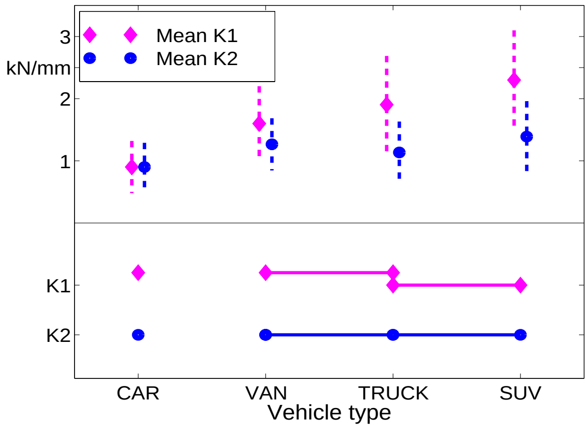

Figure 12. K1 and K2 Stiffness Results. The Dashed Lines Indicate One Standard Deviation from The Mean Values. Each Non-Significant Difference between Two Vehicle Types Is Indicated by a Solid Line.

The results from the stiffness measures Ke1, Ke2 and Ke3 are considered to be more informative than the ones from K1 and K2, since Ke1, Ke2 and Ke3 are based on energy relationship, while K1 and K2 have neither a momentum nor an energy relationship foundation. However, for completeness the results using K1 and K2 are also presented (see Figure 12). From the stiffness measures K1 and K2, it is possible to draw conclusions similar to the ones deduced using Ke1, Ke2, and Ke3. There is still evidence of only two different stiffness groups: a Car type less stiff group and a SUV type stiffer group. In particular, using K2, the same conclusion is reached by using Ke1, Ke2, and Ke3. While, using K1, the figure indicates that the Van and Truck groups show the similar stiffness value and the Truck and the SUV are similar too. But the SUV type is significantly stiffer than the Van type. One might see it as a contradiction; however, there is no contradiction, because the statistically similar property is not transitive.

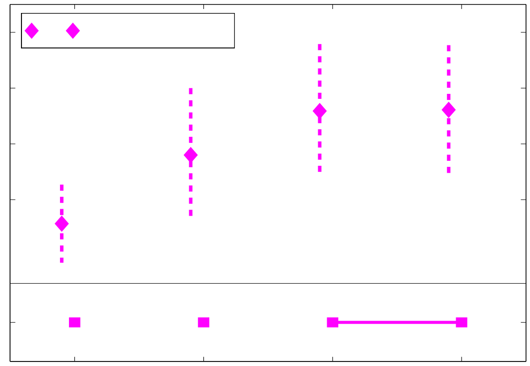

Figure 13. Peak Force Fp1 Results. The Dashed Lines Indicate One Standard Deviation from The Mean Value. Each Non-Significant Difference between Two Vehicle Types Is Indicated by a Solid Line.

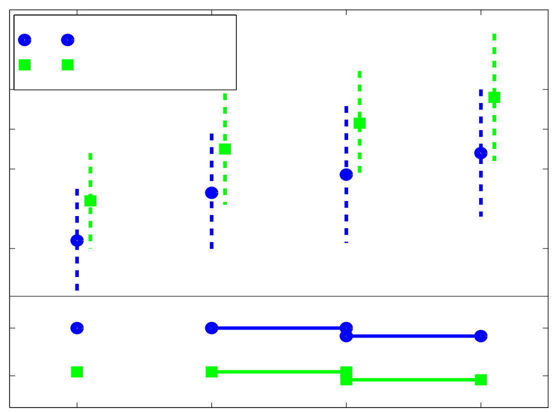

Figure 14. Peak Force Fp2 and Fp Results. The Dashed Lines Indicate One Standard Deviation from The Mean Value. Each Non-Significant Difference between Two Vehicle Types Is Indicated by a Solid Line.

The results for the three measures of peak barrier forces considered (Fp1, Fp2 and Fp) are presented in Figure 13 and 14. Figure 13 shows that, within the first 250 mm range (i.e., Fp1), the peak force Fp1 for the Truck type and SUV type is similar. But, there is substantial peak barrier force difference between the Cars and the Vans, and between the Vans and the Trucks. The peak barrier force for the Cars is substantially smaller than that for the Vans; and the peak force for the Vans is also substantially smaller than for the Trucks and SUVs. The results are different when the range is extended to 400 mm or further (i.e., Fp2 and Fp in Figure 14). In this case the peak barrier force value for the Car type is clearly the smallest. The SUVs and the Vans are similar, the Trucks and the Vans are similar, but the Trucks and the SUVs are not.

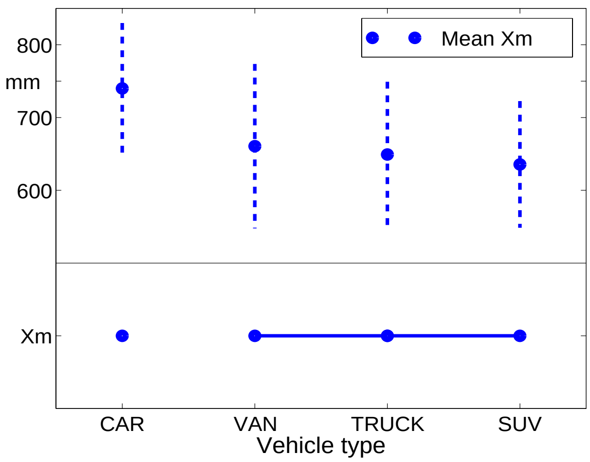

Figure 15. Maximum Crush Results. The Dashed Lines Indicate One Standard Deviation from The Mean Value. Each Non-Significant Difference between Two Vehicle Types Is Indicated by a Solid Line.

The results of maximum vehicle crush Xm are presented in Figure 15. Statistically there are only two groups. One is the Car group which has the largest maximum crush and the other group includes the Vans, Trucks and SUVs, which shows less maximum crush.

DISCUSSION

In this study NHTSA NCAP test data have been used to investigate the relationship between mass and stiffness. The primary reasons to use the NCAP tests are that there are enough samples to obtain reliable statistical analyses; the experimental procedures are robust and repeatable; and there is enough instrumentation for the analysis. In addition, there is significant crush of the vehicle front. These tests also approximate head-on crashes in the field. Nonetheless, by using multiple definitions of stiffness, the results should be more general than if only one definition was used. However, there are some limitations with using the NCAP data. Most notably, the stiffness measures derived with such data may not be directly useful for other modes of crashes.

Compared to the previous study [1], no significant changes have been observed in the relationships between mass and stiffness. But, there are some changes of the stiffness magnitudes and relative stiffness ranking among different vehicle types.

The sample size used in this paper is different from that in [1]. The sample size has been enlarged from model year 1999-2003 to model year 1979-2004. The number of samples were increased from 175 to 585. The distribution of different types of vehicles has been changed. The percentage of Cars increased from 55% to 66% and a higher percentage of large Cars (mass 1900kg and up) was included. The percentage of Vans and Trucks increased slightly. But the percentage of heavy (full sized) Vans has increased along with the percentage of light trucks. The percentage of SUV decreased significantly, from 25% to 14%, due to the fact that there were very few SUVs before the model year 1993.

It is observed that the correlation between mass and various stiffness measures has not changed significantly: Comparing Table 1 and Table 2 (from [1]), all the correlations are weak. The correlation for most of the stiffness measures, in the all-vehicle group is in general higher than for each individual subgroup. The R^2 values for K1, and K2, are lower with the larger sample size. Comparing the mass correlation with Ke1, Ke2, and Ke3, the R^2 value for the Ke1 is the highest and Ke3 is the lowest for the all vehicle group in previous study, but the trends are reversed in this study. Ke1 and Ke2 had a relatively high correlation with mass for the SUV and Truck subgroups in previous study, but it is not observed in this study. Due to a graphing error in [1] it is not possible to compare the peak forces from the previous study with those in the current study.

Table 2. Mass-Stiffness R^2 Value (1999 to 2003)

	K1	Ke1	K2	Ke2	Ke3	Fp1	Fp2	Fp
All	0.43	0.46	0.02	0.41	0.28	0.49	0.16	0.32
Car	0.04	0.08	0.07	0.14	0.09	0.07	0.12	0.27
SUV	0.31	0.28	0.09	0.07	0.01	0.18	0.00	0.03
Truck	0.23	0.39	0.03	0.25	0.02	0.33	0.04	0.05
Van	0.32	0.27	0.31	0.28	0.09	0.25	0.38	0.34

The stiffness relationships are slightly different from the previous study .For example, the Cars have no relationships to the other three vehicle groups in this study; however, in the previous study [1] the Vans sometimes are similar to the Cars (Ke1, and Ke3), and sometimes similar to the Trucks and SUVs. This could be due to the addition of the heavy Vans (full size). For K1, the Trucks and the SUVs have similar stiffness in both studies, but the Vans are dissimilar to SUVs, which is different from the previous study [1]. For K2, the Vans and the SUVs have the similar stiffness in both studies, but the Trucks are similar to the Cars, which is different from this study.

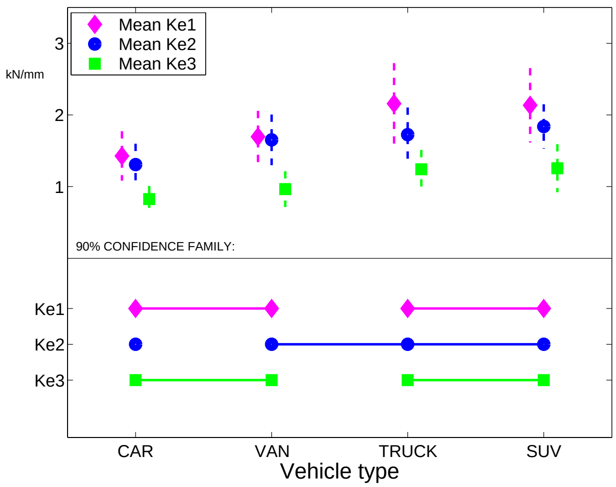

Figure 16. Ke1, Ke2 and Ke3 Stiffness Results for 1999 to 2003 MY. The Dashed Lines Indicate One Standard Deviation from The Mean Values. Each Non-Significant Difference between Two Vehicle Types Is Indicated by a Solid Line.

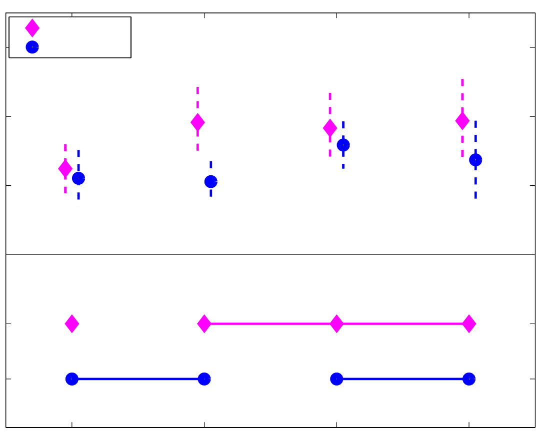

Figure 17. K1 and K2 Stiffness Results For 1999 to 2003 MY. The Dashed Lines Indicate One Standard Deviation from The Mean Values. Each Non-Significant Difference between Two Vehicle Types Is Indicated by a Solid Line.

The correlation analysis shows that there is no significant linear correlation between the different stiffness measures considered and the vehicle mass. To each vehicle type, a significant relationship between mass and stiffness does not exist. Moreover, it is also noticed that for the overall vehicle population when such a relationship exists, it is weak. The reason could be with the enlargement of the range of mass values. Though a large number of samples have been added to this study, the mass ratio has not changed significantly. Therefore, no significant change in mass-stiffness relationship has been seen in this study. However, it is clear that as the mass range increases the relationship between mass and stiffness also increases: compare the all vehicle group to any of the subgroup.

There are two parameters that correlate with the mass Best: For the Truck and SUV groups, the maximum crush Xm correlates with mass the best, while the initial peak force Fp1 correlates with mass the best for the Car and Van groups. This may be because of the structure of the vehicles.

The correlation of mass and some of the stiffness measures shows significant differences for the vehicles manufactured in different time periods: vehicle stiffness relationship with mass are time period dependent.. These may result from the regulations on passive restraint requirements and/or the Corporate Average Fuel Economy. Future regulations may also force a change in the trend of the relationship between mass and stiffness.

Since there was no clear indication of any trend between the stiffness measures and vehicle type, it was decided to estimate all pairwise comparisons. This statistical analysis gives evidence (at level alpha 0.9) that essentially there are only two different stiffness groups: a Car-type group and a stiffer Truck/SUV type group. However, for all but one definition of stiffness the Van and Truck types could be considered to have the same stiffness as the SUV type. The Cars appear to be in the lowest stiffness group with the SUV/Trucks as the highest stiffness group and the Vans close to the SUV and Truck group. However, this may be the result of including full size vans in the Van group. Minivans by themselves may have a different stiffness than SUV and Trucks.

The finding that the strong stiffness and mass in general do not have a good correlation has implications to a number of aspects for the crash safety field. One example is given here. Vehicle Mass stiffness, and geometry are often included as independent parameters in regression models [2, 3].

The correlation between these parameters, especially that between stiffness and mass has always been an issue of concern [5]. This is because the quality of the regression model (and therefore that of the risk prediction), which depends on the statistical sampling-related uncertainty of the regression coefficients, depends on the extent of the correlation between the independent parameters. This is demonstrated in Figure 18 through both theoretical prediction and a statistical simulation (Monte Carlo) for a linear regression with two independent parameters.

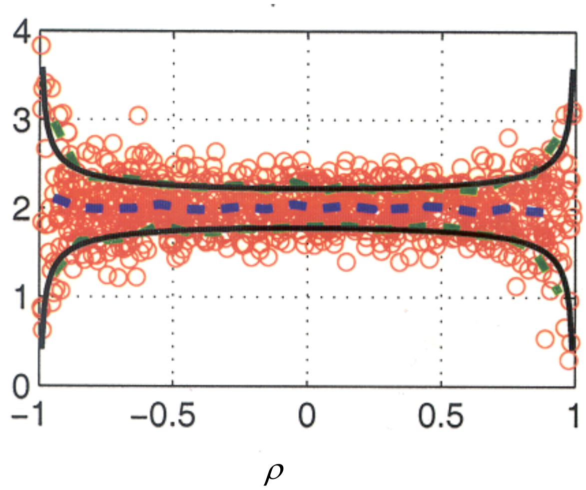

Figure 18. Theoretical prediction and Monte Carlo simulation of the dependence of variability of one of the two regression coefficients of a 2-independent variable linear regression on the extent of correlation between the independent variables. The abscissa is the coefficient of correlation, and the ordinate is the value of the coefficient estimate (The actual value of the coefficient is 2 units. The two continuous lines are theoretical prediction of the standard deviation of the estimate and each red dot is result from one of 1000 Monte Carlo regression simulations. The dashed line in the middle is the mean of the statistical simulation. The theoretical model and simulations use 2 units as the standard deviation of the independent variable; and 2 for the random sampling noise.)

Figure 18 shows that only when the Pearson's coefficient of correlation (equaling the square-root of the R^2 value in the case of two variables) is less than 0.8 (R^2 of 0.64), does the variance of the estimated coefficients stay small enough for the regression model to be useful (in theory, the variance is proportional to $1/\sqrt{1-\rho^2}$, where ρ is the coefficient of correlation). Because of the close resemblance in the underlying structure of the models this applies to both linear regression and the logistical regression. With this basic statistical observation, the finding of the current study that the stiffness and mass in general have R^2 values below 0.5 (ρ values less than 0.7) provides a basis for establishing and applying risk models based on regression involving stiffness and mass as independent variables.

CONCLUSIONS

This study employs all the available/reliable NCAP data from model years 1979 to 2004 and represents a reasonable estimate of the current fleet. The trends may change if the fleet changes significantly. But from this study, using 4 different definitions of stiffness for a total of 9 different measures, it is concluded that:

- There is no significant correlation between mass and stiffness for the vehicles in the current fleet.
- In general, for most of the stiffness definitions considered, the correlation for all vehicles together is higher than those for any of the vehicle subgroups. The qualitative relationship between stiffness and mass is theoretically sound for a large vehicle mass range (mass ratio). As the mass ratio increases the correlation increases. However, the current fleet does not have a significant mass ratio for the mass to be significantly correlated with stiffness.
- The relationship between mass and stiffness is not significant enough to contaminate any reasonable risk analysis using crash data.
- The mass correlates with the maximum crush better than stiffness for the SUV and Truck groups,
- From the definitions of stiffness used in this study, it is reasonable to assume that Cars and SUV/Trucks have different stiffness, with SUV/Trucks significantly stiffer than Cars. Vans (including full size vans are

closer to SUV/Trucks, but may not be if only minivans are included

- The correlations between mass and some stiffness measures may vary for different time periods.

ACKNOWLEDGEMENTS

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