On $t\bar{t}$ threshold and top quark mass definition

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Abstract. In this talk we discuss the process $e^+e^- \to t\bar{t}$ near threshold, calculated with NNLO accuracy as well as top quark mass definitions adequate for the threshold. As new definition we suggest the $\overline{\rm PS}$ mass which allows to calculate recoil corrections to the static PS mass. Using this result we calculate the cross section of $e^+e^- \to t\bar{t}$ near threshold at NNLO accuracy adopting three alternative approaches, namely (1) fixing the pole mass, (2) fixing the PS mass, and (3) fixing the new mass which we call the $\overline{\rm PS}$ mass. We demonstrate that perturbative predictions for the cross section become much more stable if we use the PS or the $\overline{\rm PS}$ mass for the calculations. A careful analysis suggests that the top quark mass can be extracted from a threshold scan at NLC with an accuracy of about 100 MeV.

I INTRODUCTION

In this talk we discuss two topics. We demonstrate how to calculate the cross section for process $e^+e^- \to t\bar{t}$ near threshold for NNLO accuracy and discuss a top quark mass definition adequate for the threshold.

Top quark physics will be one of the main subjects of studies at future e^+e^- and $\mu^+\mu^-$ colliders such as the Next Linear Collider (NLC) [1] and the Future Muon Collider (FMC). The goals are to measure and to determine the properties of the top quark which was first discovered at the Tevatron [2] with a mass of $m = 174.3 \pm 5\,\text{GeV}$ [3]. Although the top quark will be studied at the LHC and the Tevatron (RUN-II) with an expected accuracy for the mass of 3 GeV, the *most accurate measurement* of the mass with an accuracy of 0.1% (100 MeV) is expected to be obtained only at the NLC [4,5].

The top quark mass enters the relation between the electroweak precision observables indirectly through loops effects. The global electro-weak fit of the Standard Model requires to have very accurate input data in order to make a constraint for the masses of undiscovered particles, such as the Higgs boson or other particles. In addition we can study possible deviations from the Standard Model through anomalous couplings, CP violation, or extra dimensions.

II THE CROSS SECTION AT LO, NLO, AND NNLO

Due to the large top quark width ($\Gamma_t \approx 1.4\,\mathrm{GeV}$), the top-antitop pair cannot hadronize into toponium resonances. The cross section therefore appears to have a smooth line-shape showing only a moderate $1S$ peak. In addition the top quark width serves as an infrared cutoff and as a natural smearing over the energy. As a result, the nonperturbative QCD effects induced by the gluon condensate are small [7], allowing us to calculate the cross section with high accuracy by using perturbative QCD even in the threshold region. Many theoretical studies at LO and NLO have been done in the past, and recently for the NNLO as well. The results of the NNLO analysis are summarized in a review article [4]. To summarize the results for a standard approach using the pole mass, the NNLO corrections are uncomfortably large, spoiling the possibility for the top quark mass extraction at NLC with good accuracy because the $1S$ peak is shifted by about $0.5\,\mathrm{GeV}$ by the NNLO, the last known correction. One of the main reasons for this is the usage of the pole mass in the calculations. It was realized that such type of instability is caused by the fact that the pole mass is a badly defined object within full QCD. For more details we also refer the reader to Ref. [5].

Let us consider the cross section of the process $e^+e^- \to t\bar{t}$ in the near threshold region where the velocity v of the top quark is small. It is well-known that the conventional perturbative expansion does not work in the non-relativistic region because of the presence of the Coulomb singularities at small velocities $v \to 0$. The terms proportional to $(\alpha_s/v)^n$ appear due to the instantaneous Coulomb interaction between the top and the antitop quark. The standard technique for re-summing the terms $(\alpha_s/v)^n$ consists in using the Schrödinger equation for the Coulomb potential to find the Green function. The total cross section can then be related to the Green function by using the optical theorem,

$$R = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)} = e_Q^2 \frac{72\pi}{s} C(r_0) \mathrm{Im} \left[\left(1 - \frac{\vec{p}^{\,2}}{3m^2} \right) G(r_0, r_0 | E + i\Gamma) \right] \Bigg|_{r_0 \to 0} \tag{1}$$

where the Green function $G(\vec{r}, \vec{r}\,' | E + i\Gamma)$ satisfies the Schrödinger equation

$$(H - E - i\Gamma) G(\vec{r}, \vec{r}\,' | E + i\Gamma) = \delta(\vec{r} - \vec{r}\,'). \tag{2}$$

An obstacle for a straightforward calculation at NNLO are the UV divergences coming from relativistic corrections to the Coulomb potential (the so-called Breit-Fermi potential). This problem can be solved by a proper factorization of the amplitudes and by employing effective theories. The coefficient $C(r_0)$ can be fixed by using a direct QCD calculation of the vector vertex at NNLO in the so-called intermediate region [8,9] and by using the direct matching procedure suggested in Ref. [10]. In addition, the non-factorizable corrections cancel in the total cross section but modify the differential one [6]. For the numerical solution of the final equation we used the program derived in Ref. [11] by one of the authors.

III ON THE MASS DEFINITIONS

The top quark mass is an input parameter of the Standard Model. Although it is widely accepted that the quark masses are generated due to the Higgs mechanism, the value of the mass cannot be calculated from the Standard Model. Instead, quark masses have to be determined from the comparison of theoretical predictions and experimental data. There is no unique definition of the quark mass. Because the quark cannot be observed as a free particle like the electron, the quark mass is a purely theoretical notion and depends on the concept adopted for its definition. The best known definitions are the pole mass and the $\overline{\rm MS}$ mass. However, both definitions are not adequate for the analysis of top quark production near threshold. The pole mass should not be used because it has the renormalon ambiguity and cannot be determined more accurately than $300-400\,{\rm MeV}$. The $\overline{\rm MS}$ mass is an Euclidean mass, defined at high virtuality, and therefore destroys the non-relativistic expansion. Instead, it was recently suggested to use threshold masses. There are many such definitions: the low scale (LS) mass [13], the potential subtracted (PS) mass [14], and one half of the perturbative mass of a fictitious 1^3S_1 ground state (called $1S$ mass) [15]. All of them could be used in application but have to be related to the fixed notation, e.g. the $\overline{\rm MS}$ mass. We suggest a new definition which we think is more physically motivated. The objectives in defining such a mass are that the mass should be "short distance", being free from soft QCD effects and adequate for the threshold calculations. Furthermore, the definition should be gauge independent and well-defined within quantum field theory so that radiative and relativistic corrections can be calculated in a systematic way. Our definition given in [5] is $m_{\overline{\rm PS}}=m_{\rm pole}-\delta m_{\overline{\rm PS}}$ with $\delta m_{\overline{\rm PS}}=\Sigma_{\rm soft}(\not p)|_{\not p=m_{\rm pole}}$ where $\Sigma_{\rm soft}$ is the soft part of the heavy quark self energy which is defined as the part where at least one of the heavy quark propagators is on-shell. Summarizing all contribution up to NNLO accuracy, we obtain for the difference $m_{\overline{\rm PS}}(\mu_f)-m_{\rm pole}$ [5]

$$-\frac{\alpha_s(\mu)C_F}{\pi}\mu_f\left\{1-\frac{\mu_f^2}{2m^2}+\frac{\alpha_s(\mu)}{4\pi}\left(C_1+C_A\frac{\pi^2}{2}\frac{\mu_f}{m}\right)+C_2\left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\right\} \tag{3}$$

where the coefficients C_1 and C_2 are given in Ref. [14], m is the pole mass, μ is the renormalization scale, and μ_f is the factorization scale. We have calculated the coefficients to higher orders in $1/m$ which are explicitly given in Eq. (3) and which are recoil corrections to the previous one. Remark that there is no contribution of the leading order to $1/m$. Our result can be represented in a condensed form as $m_{\overline{\rm PS}}(\mu_f)-m_{\rm pole}=-\frac{1}{2}\int^{\mu_f}\left(V_C(|\vec k|)+V_R(|\vec k|)+V_{NA}(|\vec k|)\right)d^3k/(2\pi)^3$ where the first term V_C is the static Coulomb potential, V_R is the relativistic correction (which is related to the Breit-Fermi potential but does not coincide with it), and V_{NA} is the non-abelian correction. Using three-loop relations between the pole mass and the $\overline{\rm MS}$ mass given in Ref. [16,17], we fix the $\overline{\rm MS}$ mass to take the values $\overline{m}(\overline{m})=160\,{\rm GeV}$, $165\,{\rm GeV}$, and $170\,{\rm GeV}$ and determine the pole mass at LO, NLO, and NNLO. This pole mass is then used as input parameter m in Eq. (3) to

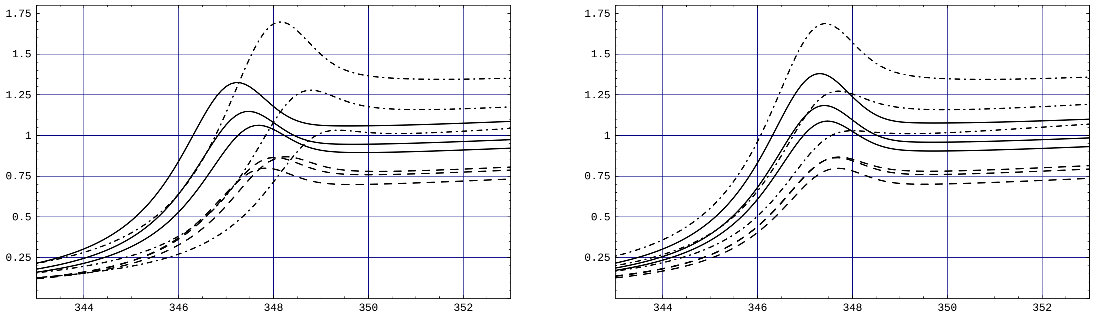

FIGURE 1. NNLO threshold analysis for pole mass (left) and $\overline{\text{PS}}$ mass (right)

determine the PS and $\overline{\text{PS}}$ masses at LO, NLO, and NNLO. The obtained values for the PS and $\overline{\text{PS}}$ mass differ only in NNLO. Fig. 1 shows the results of the analysis of the cross section. The triple of curves indicates different values for the renormalization scale, shown for LO (dashed-dotted), NLO (dashed), and NNLO (solid). It is obvious that in using the $\overline{\text{PS}}$ mass (as well as the other threshold masses) we gain a remarkable improvement of the stability in going from LO to NLO to NNLO (shifts are below 0.1 GeV). This understanding removes one of the obstacles for an accurate top mass measurement and one can expect that the top quark mass will be extracted from a threshold scan at NLC with an accuracy of about 100 MeV.

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