AGENT-BASED MODELS AS QUANTITATIVE SOCIOLOGICAL METHODOLOGY: CALIBRATING SIMULATION MODELS TO DATA AND FINDING CONFIDENCE INTERVALS FOR MODEL PARAMETERS

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ABSTRACT

A simulation model of neighborhood crime rates and its estimation using data illustrate how a simulation model can be employed as a supplement to prose-style sociological reasoning and how the simulation model can be used as an estimation methodology that replaces the traditional method of specifying regression models with mixtures of system-level and individual attributes as predictors. The core of the estimation framework is a generalization of the method of simulated moments (MSM) estimator of econometrics, which matches moments (i.e., expected values and variances) and a practical estimation methodology. A simulation meta-model giving the approximate relationship between model parameters and functions of the moments of model outputs, such as means and variances, is employed to calibrate the model to social data. The result is a simulation model-based replacement for the current paradigm for empirical sociological methodology that avoids reification and the ecological fallacy, which is applied to the estimation of a new model of neighborhood crime.

Keywords: Crime, quantitative sociological methodology, method of simulated moments, simulation model calibration, simulation meta-modeling

INTRODUCTION

To play off the title of a famous book on statistics (Christensen 2002), statistical methods in the social sciences often consist of "plane answers to complex questions." To answer questions about complex social phenomena, such as neighborhood crime rates, sociologists often commit the ecological fallacy by positing the reality of abstractions and then employing their measures in multiple regression models, which fit a hyperplane to the behavioral characteristics of a complex system. Usually these models pertain to a closely related group of dependent variables. These models are justified through discursive, philosophical-style social theory that makes ontological and behavioral claims about system dynamics and the relationship between the individual levels of analysis, but the degree of logical rigor achieved does not necessarily justify the epistemological claims made for the regression model specification. From the perspective of social simulation modeling, however, system models are available, and the question is one of choosing the right model, assessing where a candidate simulation deviates from validation data sets, and finding a good set of model coefficients in an efficient manner.

Many have noted the discursive nature of sociological argument, either as a good thing (Sica 2004) or a shortcoming (Mahoney 2004). Computational social science is emerging as an alternative paradigm, but it is time to go beyond demonstrations of promise and develop

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agent-based modeling as an alternative methodology on an equal basis to the traditional paradigms. Since quantitative sociological methodology operates by performing statistical tests on model parameters that are estimated from data, it is natural to wish to do the same thing with agent-based models and other social simulations. The process of theoretical development being considered here is to encode discursive theoretical arguments into agent-based models, and then evaluate their implications by executing the models. The next step is then to perform statistical tests on various model parameters, in order to test the theories under consideration.

We approach the issue of estimating parameters in social simulation models as a problem of model calibration or validation, in which the model employs parameters that cannot be estimated by direct observation in the real world. In order to make the process of theoretical testing work well, however, methods of parametric hypothesis testing are needed for a broader range of models. This advance is achieved by employing simulation meta-modeling and response surface methodology as a method to achieve the matching of sample moments from simulation runs to empirical data, and by employing a jackknife estimator of the standard errors of the estimated parameters. The resulting methodology brings together elements of econometrics, operations research, and computational statistics in a new way that is computationally feasible and suitable for use as a quantitative social science methodology.

In this paper, we showcase the methodology from a recent paper on crime in the sociological literature by Browning, Feinberg, and Dietz, to be referred to as BFD (Browning et al. 2004). Here we attempt to employ an agent-based model as a methodological improvement over the theoretical arguments and methodological approach of the subject paper, in which it is desired to substantiate a view of crime as occurring within a context of a network of social exchange relationships that tend to impede the suppression of crime. We consider a new agent-based model of neighborhood crime and apply the proposed methodology to reanalyze the data correlation matrix presented.

SIMULATION MODEL ESTIMATION METHODOLOGIES

The question of how to estimate the parameters of a simulation model, calibrate it, and/or validate it is not new. The case at hand is the presence of parameters (or processes) that are not directly observed in the data or that otherwise do not directly correspond to the observables, and the context is complex, stochastic simulations that are not susceptible to analytic (calculus-based) approaches. First we take a quick overview of the approaches that have been taken, and then we discuss the genealogy of the proposed methodology.

The basic principle of simulation or parameter estimation from outputs is to match such things as means and variances of model outputs to data from a real system. The conventional economic methodology of deriving analytical results has been applied to agent-based economic models, such as the application to exchange rates by Alfarano et al. (2005). Axtell et al. (2002) performed a systematic search of an eight-dimensional parameter space in order to find a match to the archeological record of the Anasazi of Long House Valley, but this approach has exponential computational complexity. From the nonconvex optimization perspective, Gilli and Winker (2003) investigated a heuristic for matching the moments of an agent-based simulation model of exchange rates by refining a set of points in a three-dimensional parameter space that bracket the optimal solution. Many others have employed a variety of approaches to the optimization problem of finding optimal parameters for a process that is modeled by using simulation. We now consider why the matching of simulation moments is also an econometric idea, and how to make it computationally feasible in the case of complex stochastic simulations through the application of simulation meta-modeling.

The generalized method of moments (GMM) econometric estimation methodology (Hansen 1982) is a powerful methodology for estimating the coefficients for a wide variety of estimation problems. GMM is instructive in that it performs a match between features of a model and features of the data and then defines asymptotic standard errors for the resulting parameter estimates. Following the textbook by Mátyás (1999, Chapters 1 and 10), the basic approach is to define a continuous, $q \times 1$ vector function

$$f(x_t, \theta) = s(y_t, z_t) - \sigma(z_t; \theta)$$

of a parameter vector, θ, and data vectors $x_t = (y_t, z_t) : t = 1, ..., T$, where x_t is divided into dependent variables y_t and independent variables z_t. We attempt to estimate θ_0, the true value of θ, by using the moment conditions

$$E\left[f(x_t, \theta)\right] = 0 .$$

A simple example of this is writing the mean and variance of X by using the notation of expected values and then subtracting their theoretical values, which are functions of θ. As we do not have these expected values, we employ instead the sample moments as a function of θ,

$$f_T(\theta) = T^{-1} \sum_{t=1}^{T} f(x_t, \theta) .$$

If there are q parameters to estimate, then the method of moments estimator is to solve the exactly identified system of equations, $f_T(\theta) = 0$, for $\hat{\theta}_{TM}$ in terms of the data. However, there are many possible moments from which to choose in this procedure. If there are fewer parameters to estimate than moment conditions, the problem is overidentified, and the GMM estimator defines a positive definite weighting matrix A_T, from which we obtain

$$Q_T(\theta) = f_T(\theta)' A_T f_T(\theta) ,$$

a measure of the "distance" away from satisfying the moment conditions. Assuming, among other regularity conditions, that $f_T(\theta)$ is continuously differentiable, the minimization of $Q_T(\theta)$ over θ yields the GMM estimator, $\hat{\theta}_T$, which is obtained via solving the first-order conditions.

For several varieties of complex econometric models, GMM estimation is computationally infeasible due to such problems as high-dimensional multiple integrals being required to compute the expected value of the dependent variable, as might be present as part of $\sigma(z_t; \theta)$. The method of simulated moments (MSM), which originated with Pakes and Pollard (1989) and McFadden (1989), addresses this issue. MSM has also been suggested by Richiardi (2004) as an estimation methodology for agent-based computational economics. In MSM, one performs a simulation that generates estimates of $\sigma(z_t; \theta)$, such as the natural Monte Carlo estimator based on a sample of size R,

$$\hat{\sigma}_R(z_t;\theta) = R^{-1}\sum_{r=1}^{R} s\left[y_t^{(r)}(\theta), z_t\right],$$

where $y_t^{(r)}(\theta)$ is the r'th simulated value of y_t. Then the MSM estimator $\hat{\theta}_{MSM}^{R}$ is obtained by minimizing the criterion,

$$\left[\sum_{t=1}^{T} f_R(y_t, z_t;\theta)\right]' A\left[\sum_{t=1}^{T} f_R(y_t, z_t;\theta)\right],$$

where A is a positive definite weighting function and

$$f_R(y_t, z_t;\theta) = B(z_t)'\left[s(y_t, z_t) - \hat{\sigma}_R(z_t;\theta)\right].$$

This optimization is facilitated by the formation of a Monte Carlo estimate of the derivatives of $\hat{\sigma}_R(z_t;\theta)$ with respect to θ, which may be readily available from the simulation as analytical derivatives conditional on the various pseudorandom number values.

The MSM procedure does not quite fit the envisioned application on two counts. First, a correlation matrix is not a generalized moment, but a function of moments. Second, it is not suitable for the general social or agent-based simulation model in that the derivatives of $\hat{\sigma}_R(z_t;\theta)$ are not necessarily available. While the former is accommodated in the discussion by Gelman (1995) by a modification to the normal equations, we address the latter through the use of least-squares models in order to estimate the relationships between the simulation parameters and the correlations or other features of interest. Using these relationships, or simulation meta-model, we then find an approximate minimum of the criterion function. Here we apply least-squares again to find the optimal set of parameters to minimize the "distance" between the simulation's correlation matrix and the real-world one. This procedure thus creates a point of contact with the literature on response surface methodology (Kleijnen 1998; Myers and Montgomery 2002) and simulation optimization (Andradóttir 1998; Fu 2002).

In outline, the approach proposed for agent-based modeling is as follows. For a model with Q parameters, define a set of N features, such as moments, functions of moments, and other definable functions of the model outputs that will be the basis of distinguishing good from poor models. The expected value of these should be a smooth function of the parameters, and their sampling variance should go down with sampling size. Then, running the model in M batches, collect the sets of feature vectors, s_i, $i = 1,...,M$. Using ordinary least-squares (OLS) or weighted least-squares (WLS), the regression equation for all feature vectors, consolidating the simulation meta-models, is given in matrix-vector form as

$$s_i = \Theta b_i + \varepsilon_i,$$

where Θ is the $M \times (Q + 1)$ matrix of parameters by batch number, including a column of ones; each b_i is a column vector consisting of Q coefficients plus an intercept term; and s_i is the M-vector of generalized moments of type i. At this point we check the meta-model with respect to the regression assumptions, such as linearity and homoscedasticity. It is also possible to perform multivariate regression tests of significance showing whether any of the features are affected by a given parameter (see Johnson and Wichern 1992, Chapter 7).

In aid of finding the desired estimates, define the concatenations of column vectors

$$\hat{B} = [\hat{b}_1, \hat{b}_2, \hat{b}_3 \ldots \hat{b}_M] \text{ and } S = [\hat{s}_1, \hat{s}_2, \hat{s}_3 \ldots s_N],$$

Then the predicted values from the regression model are given by the $M \times N$ matrix,

$$\hat{S} = \Theta \hat{B}.$$

We wish to find the values of θ that create the best fit to the actual feature data, s_0. $\hat{\theta}$ is obtained by minimizing

$$\left\| \theta' \hat{B} - s_0' \right\| = \left\| \hat{B}' \theta - s_0 \right\| .$$

This is a regression equation, which is estimated by using least-squares. Here, roles are reversed as the matrix of estimates from the first phase is transposed to become the set of predictor variables and as the target features form the dependent variable vector. We can think of this regression as finding the maximum likelihood estimator of θ conditional on the estimate $\hat{B}$ and assuming the accuracy of the meta-model for the relationship between the features and the parameters. From the perspective of inverse problem theory (Tarantola 2005), this regression is the solution to an inverse problem in which there is no prior information and the linear model is employed as an approximation to the parameter-data relationship. While inverse modeling is an established methodology in hydrology — as seen in Hill (1998) as well as Poeter and Hill (1998), for example — our setting has an additional complication in that the simulation model is stochastic.

Note that the usual MSM approach calculates standard errors for the coefficients on the basis of the availability of an accurate derivative of the moment function conditional on the various random number instantiations that occurred in the simulation. We replace this derivative with a least-squares estimate, whose sampling error affects the standard error of the estimates in addition to the consequences of any lack of fit in the meta-models.

As an alternative to an involved matrix formula for an asymptotic approximation to the standard error of the estimates, we consider utilizing the Quenouille-Tukey jackknife, whose use as a variance estimator is discussed in Efron and Stein (1981). The jackknife is a feasible and general-purpose method of conservatively estimating standard errors while also reducing estimation bias. To utilize the jackknife, form the pseudo-values

$$P_j = \hat{\theta} + (N-1)(\hat{\theta} - \hat{\theta}_{-j}) \qquad \text{for } j = 1, \ldots, N,$$

where $\hat{\theta}_{-j}$ is the estimator of choice computed with observation j removed. In this case, the "observations" that are removed are one of the several generalized moments or features being matched and its associated regression coefficients. The observation thus deleted embodies the sampling error for the original data as well as the estimation error of the simulation meta-model coefficients for that parameter. The jackknife estimators of the mean and variance of the estimate are the mean of the P_j and the customary formula for the sampling variance of a mean:

$$\hat{\hat{\theta}} = \frac{1}{N}\sum_{j=1}^{N} P_j$$

and

$$Var(\hat{\hat{\theta}}) = \frac{1}{N(N-1)}\sum_{j=1}^{N}\left(P_j - \hat{\hat{\theta}}\right)^2 .$$

For simplicity, these are the equations for a single component of the estimate vector, but a covariance matrix of the estimates can also be computed in the usual fashion from the P_j as well.

NEIGHBORHOOD CRIME MODELING

Theoretical Discussion

In BFD's review of the crime literature, it is clear that sociological theories of crime are based on hypotheses about social interactions in a neighborhood network. Not well-covered are economic theories of crime, such as rational choice theory (Becker 1968, and many others to follow) and the conception of peer effects as positive and negative externalities (Glaeser et al. 1996; Calvó-Armengol and Zenou 2004), which are also relevant.

Social disorganization theory (Shaw and McKay 1969; Kasarda and Janowitz 1974; Kornhauser 1978) views interpersonal social attachment as a good thing. According to Shaw and McKay, poverty, residential instability, and ethnic heterogeneity promote crime by inhibiting the formation of neighborly networks and attenuating community-level action against crime. According to Kasarda and Janowitz, extensive friendship and kinship bonds strengthen neighborhood attachment, and Kornhauser finds that weak bonds mediate the effect of disadvantage on the capacity for social control.

The cultural transmission model (Whyte 1937; Wilson 1996; Crane 1991) focuses on the legitimate social networks as bulwarks against a counterculture of crime. The criminal subculture emerges in opposition to mainstream culture, and strong networks in socially disadvantaged communities may facilitate its spread. Thus, there is a contagion of problem behaviors, for which gang culture provides social support.

BFD propose and empirically support a negotiated co-existence model, in which social networks are a source of general social capital for offenders, which tends to protect them. Thus the attitude of neighborly efficacy to fight crime tends to suppress criminal behavior but is offset to some degree by social capital. Thus, social disorganization theory is not quite right, but BFD wish to avoid attributing too much organizational capacity to the criminal networks as well.

The economic literature on peer effects in crime is intriguing as a contrast because it is inherently agent-oriented. The analyses by Glaeser et al. (1996) and Calvó-Armengol and Zenou (2004) highlight the importance of heterogeneity in agents' toleration for crime as a moderator of peer influence effects, which act as source of training and facilitation. The positive externalities due to the interactions between criminals contrast with their aggregate competition for resources. These competing phenomena help explain the variability of crime rates across time and space.

Critique of BFD's Model

BFD support the negotiated coexistence model by estimating a regression model by using data defined at the neighborhood level. In it, an interaction term between the level of the attitude of efficacy to fight crime and an attitudinal measure of network exchange shows up as a significant predictor of the crime rate. Individual-level attitude and household-level victimization data are aggregated into an area measure, by using hierarchical linear models to obtain empirical Bayes residuals as the dependent variable and main independent variables. This effectively partials out gender, age, race/ethnicity, education, employment status, marital status, years of residency, home ownership, and number of recent moves. The dependent variables are violent crime victimization (log odds) and the logarithm of the homicide rate. Control variables include measures of disadvantage, residential stability, population density, immigrant concentration, and the lagged homicide rate. Support is found for the negotiated coexistence model.

BFD's results may be critiqued in that using neighborhood-level data to support theories of agents in social networks succumbs to the ecological fallacy. Also, using attitudes as "independent variables" is questionable, since attitudes may be an accommodation to facts rather than their cause. Attitudes are clearly endogenous as a class, and there is always a question concerning the direction of causality. There is also the problem of completeness when reasoning in discursive ways. It is not always clear that the prose theory supports a certain sign of regression coefficient, since something may be left out of the reasoning.

Building an agent-based form of the theory of crime has promise for addressing the issues that arise in the consideration of BFD's analysis. By building the model at the agent level, the ecological fallacy can be avoided. By using an agent-based model to reason about the way in which different phenomena interact to produce an expected result, we avoid the problem of incomplete reasoning, although the problem may occur at a higher level in the form of the choice of models or perspective. Within the limits of causal reasoning, the endogeneity of attitudes can be addressed in an agent-based model by specifying path models with appropriate loops.

Agent-based Model Development

The model developed here and presented in Figure 1 incorporates a two-dimensional analogue of the circular social influence network considered by Glaeser et al. (1996), in which the features of nonhomogeneous occupational preferences and competition among criminals for scarce economic resources create a situation in which disparate equilibria are possible and the crime rate can vary significantly over time. In addition, it includes the interplay between the attitude of neighborly anti-crime efficacy and the behavior of being a criminal.

Since neighborhoods are geographical, this model represents the attitudes and behavior of residents on a two-dimensional 50×50 lattice but uses the toroidal topology to avoid edge effects. In a real neighborhood, one has more than the eight neighbors present in many lattice models. Here we use the Moore neighborhood of radius 4, giving a set of 80 neighbors with

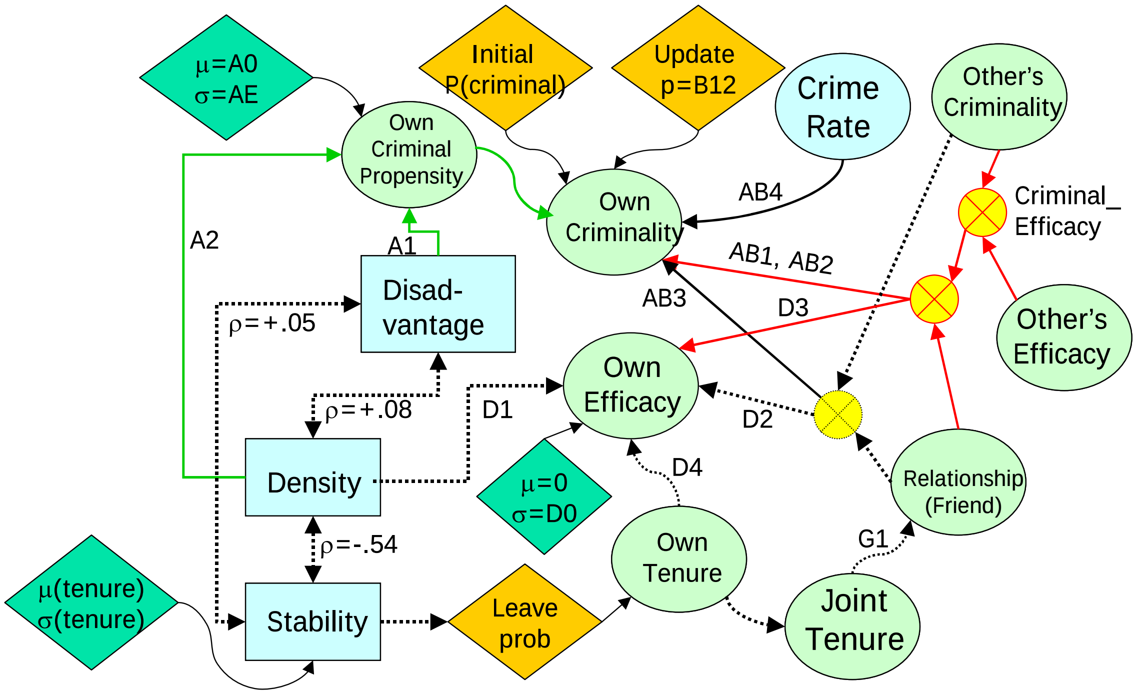

FIGURE 1 Path diagram of the agent-based model

whom an agent may form friendships and by whom an agent may be influenced. However, the crime rate's economic effect on occupational choice is taken over the entire set of 2,500 agents.

In this model, agents may be either criminals or law abiding, and they possess a level of "efficacy." Their underlying criminal propensities are heterogeneous within the neighborhood. Social influence is conveyed at three levels: friends, nonfriends in the Moore neighborhood, and the community as a whole. Agents may move out of the neighborhood and thus be immediately replaced. They also may change status between criminal and noncriminal and form friends. All friendships are two-way.

In the blue boxes, *Density*, *Stability*, and *Disadvantage* are unit-free exogenous variables that drive residents' *Criminal Propensities*, *Efficacy*, and *Probability of Leaving*, seen in parameters *A1*, *A2*, and *D1*. In the green diamonds are scale and location parameters related to these exogenous variables. *Tenure* is influenced by a yearly leaving probability that is derived from the stability parameter. *Criminality* is a choice that is initialized and then reviewed with probability *B12*. If it is to updated, the agent is a criminal according to a criminal propensity, which is heterogeneous between agents, as well as to the influences of friends and neighbors. In accordance with the peer effects literature, the global crime rate has a suppressing effect *(AB4)* on an individual's propensity to choose criminality, while friends who are criminals have a separate effect *(AB3)*. The value of efficacy used in the influence calculations depends on whether the person doing the influencing is a criminal, in which case the *Criminal_Efficacy* parameter is used. The effective efficacy of the other thus computed has two different effects *(AB1* and *AB2)* on the other's occupational choice, depending on whether the other is a friend or just a neighbor. The update of *Efficacy* is affected by the crime rate among the agent's friends according to parameter *D2*, and their *Efficacy* according to parameter *D3*. *Efficacy* is also influenced by tenure *(D4)* and density *(D1)*. Finally, friendships grow with joint tenure, according to probability *G1* each year.

Implementation Details

Each iteration of the simulation is nominally one year, during which each agent's status is stochastically updated in fixed order:

1. Moving out of the neighborhood,

2. Building friendships (which accumulate),

3. Deciding whether to reevaluate one's occupation, and

4. Updating attitude of efficacy.

First it is determined whether the agent will leave on the basis of the tenure and stability-related calculations of the probability of leaving detailed above. The probability of leaving does not depend on the crime rate. If the agent leaves, it is replaced with a new agent; otherwise, the agent is updated.

A new agent is given a criminal status and propensity according to the probability utilized in the initialization of criminal status, and it is given values of baseline efficacy, friendship probability, and friendship status according to the initialization procedure. The starting value of tenure is 1 year.

An agent who does not move out of the neighborhood is influenced by the *efficacy* of both friends and nonfriends in his Moore neighborhood, but according to separate coefficients. If the other is a criminal, the effective *efficacy* is set at a parameter; otherwise, the effective *efficacy* is the actual *efficacy*. For both classes of Moore neighbors, the influence of the other on one's own efficacy is on a per-agent basis, making influence of the two categories proportional to their numbers divided by the total number of Moore neighbors.

The latent probability of becoming a criminal is the $\exp(x)/[1 + \exp(x)]$ function of the sum of the agent's *criminal propensity* and products of parameters with the efficacies of friends and nonfriends in the agent's Moore neighborhood, as well as the crime rates among friends and globally. If a Bernoulli trial against the criminal status update probability is successful, criminal status is updated according to the latent probability.

Friendship cumulatively increases, and new friends are added from the Moore neighborhood according to the friendship creation probability. Tenure is incremented by one each iteration. Efficacy is based on the baseline efficacy for the agent calculated at initialization, plus the products of parameterized coefficients multiplied by the friend *crime rate*, the friend *efficacy*, and *tenure*.

One may comment on the model's complexity. The path model implied by theory is fairly complex yet incomplete, and things had to be added. The effect of time on relationship building is common sense, but not explicitly stated as theory. Cognitive consistency theories could be further exploited to suggest additional relationships between one's own attitudes and own behavior. The model is also too simple in that there is no distinction between the rates of victimization and the populations of criminals.

Model Parameterization

Table 1 presents the 19 parameters corresponding to the path model in Figure 1 to describe the simulation process. In the path diagram, the prefix containing the affected node is omitted, but it is included here for clarity.

DATA ANALYSIS

Empirical Data Reanalyzed

BFD report a correlation matrix of the data analyzed in their regression analyses, which form the feedstock for the demonstration of the proposed analytical method. The correlations reported by BFD and utilized in this paper are presented in Table 2. The correlations highlighted in orange are the exogenous variables. The presence of the 4-year lag of the crime rate gives us a reading on the level of consistency over time possessed by the phenomenon of crime, which is

TABLE 1 Agent-based model parameters

Number	Name	Type of Parameter	What It Is Multiplied by	What It Affects
1	pcriminal	Initial probability	-----	Initial criminal status
2	CriminalPropensity_A0	Constant	-----	*Criminal Propensity*
3	CriminalPropensity_AE	Variability	-----	*Criminal Propensity*
4	CriminalPropensity_A1	Coefficient	*Disadvantage*	*Criminal Propensity*
5	CriminalPropensity_A2	Coefficient	*Density*	*Criminal Propensity*
6	CriminalImpulse_AB1	Coefficient	*UnrelatedEfficacy*	*CriminalImpulse*
7	CriminalImpulse_AB2	Coefficient	*FriendEfficacy*	*CriminalImpulse*
8	CriminalImpulse_AB3	Coefficient	*FriendCrimeRate*	*CriminalImpulse*
9	CriminalImpulse_AB4	Coefficient	*CrimeRate*	*CriminalImpulse*
10	Efficacy_D0	Variability	-----	*Efficacy (initial)*
11	Efficacy_D1	Coefficient	*Density*	*Efficacy (initial)*
12	Efficacy_D2	Coefficient	*FriendCrimeRate*	*Efficacy*
13	Efficacy_D3	Coefficient	*FriendEfficacy*	*Efficacy*
14	Efficacy_D4	Coefficient	*Tenure*	*Efficacy*
15	Criminal_Efficacy	Constant	-----	*Effective efficacy*
16	Criminal_B12	Update probability	-----	*Criminality*
17	Friend_G1	Probability of Formation	-----	*Friendship*
18	Tenure_mean	Constant	-----	*Probability of Leaving*
19	Tenure_std	Variability	-----	*Probability of Leaving*

TABLE 2 Correlations Reported by Browning et al. (2004)

	Crime_Rate (Homicide)	*Disadvantage*	*Residential_stability*	*Density*	*Previous Crime_Rate (Homicide)*	*Collective_Efficacy*	*Network_Interaction*	*High_interaction*	*High_interaction*Collective_Efficacy*
Crime_Rate (Homicide)	1.00								
Disadvantage	0.76	1.00							
Residential_stability	0.04	0.05	1.00						
Density	-0.03	0.08	-0.54	1.00					
Previous Crime_Rate (Homicide)	0.81	0.77	-0.06	0.09	1.00				
Collective_Efficacy	-0.54	-0.56	0.38	-0.44	-0.60	1.00			
Network_Interaction	-0.14	-0.13	0.05	-0.18	-0.13	0.47	1.00		
High_interaction	-0.07	-0.02	0.08	-0.17	-0.07	0.36	0.75	1.00	
*High_interaction*Collective_Efficacy*	-0.30	-0.33	0.27	-0.26	-0.37	0.68	0.38	0.39	1.00

important with respect to the economic literature, as the level of instability has been the difficult part to explain. The means of the variables are not given. Many of the means and standard deviations are meaningless, since they are empirical Bayes residuals or attitude measures.

Since they are survey estimates and the outcome of empirical Bayes purification, many of the subject variables are subject to estimation error themselves. This creates the problem of unmodeled measurement error in the predictor variables, which causes bias in the regression estimates. For the present purposes, it would also be helpful to have estimates of the reliability of the predictor measures.

Simulation Model Data Collection

Ecological data are collected from the 2,500 persons in the simulated neighborhood. After initialization, the model is executed for 20 iterations prior to the collection of the lagged log odds crime rate, and then executed for 4 more iterations prior to the collection of the rest of the data. Residential stability is collected as the mean of tenure (in simulation iterations). Network interaction is the mean of the friendship status. Efficacy is also the mean of this variable before its adjustment for criminal status. The log odds of the crime rate are estimated by using an accommodation for the possibility of zero crime rates. The agent-based model was implemented in Matlab 5.3 (The Mathworks, Inc., 1999).

Correlations of interest were calculated from batches of 30 independent model runs and used as data for the later method of simulated moments analysis. To achieve this, variables were calculated to support the interaction effects in the regression analysis BFD report. From the network interaction variable, the 70th percentile was calculated in order to create the indicator for high network interaction and thus its product with collective efficacy.

The experimental design employed in data collection is to uniformly generate the simulation parameters within the upper and lower bounds determined by the experimenter. An initial set of test runs yielded a set of confidence bands, which were employed in the subsequent batch of data collection runs. A total of 117 batches were collected after the deletion of those with missing values due to taking the logarithm of zero.

Data Analysis

Statistical Procedures

Standard tools of multivariate analysis and regression model checking are employed to estimate and assess the meta-models of correlations in the simulated data. SAS version 9 (SAS Institute, Inc. 2002) was employed for the bulk of the post-simulation analysis, although the estimator was also implemented in Matlab (by using standard least-squares formulas taken from Judge et al. 1988). This being the first application of the application of meta-modeling to MSM, the difficulties encountered are instructive. Since correlations are the dependent variable in the meta-models, a transformation to correct heteroscedasticity was needed. Fisher (1915) suggests the

$$\tanh^{-1}(r) = \log_e\left(\frac{1+r}{1-r}\right)$$

transformation, but this was modified to

$$\log_e\left(\frac{1.02+r}{1.02-r}\right)$$

in order to avoid difficulties with the logarithm of zero, which was encountered in some of the regressions. Other difficulties arise when not all of the correlations are predicted equally as well from the model parameters during the first least-squares estimation. This results in heteroscedasticity in the second phase regression, which is addressed by using a vector of weights calculated as the inverse of the residual variance estimates in phase 1. For this exploratory analysis, the standard errors are computed by using weighted least-squares regression rather than using the jackknife variance estimate.

Model Critique

An advantage of the new methodology is its ability to assess whether model parameters affect the measurements being made. In this study, the first-stage regression analyses exhibited strong effects of some parameters on the sampled correlations, but not others, as determined by using multivariate tests of the parameters. High points of the significance parade include *Efficacy_D1*, which is the effect of *density* on *efficacy*, and *Efficacy_D3*, which is the effect of having criminals as friends on one's feeling of efficacy. The mean and standard deviation of neighborhood stability (average tenure) also get high marks. Table 3 presents multivariate tests of the effects of the simulation parameters on the correlation statistics collected. Parameters 6–8 of the simulation concern the effects of one's efficacy and criminal behavior on another's criminal behavior. A multivariate test that the correlations generated by the model were not related to these parameters rejected this null hypothesis with a statistically significant Wilks' Lambda p-value. However, it seems surprising that so many of the parameters appear to be immaterial when examined in this fashion, although one can always look to increasing sample size. The one hoped-for lack of statistical significance is that of *pcriminal*, which is an initialization constant for agents, but this was marginally significant.

TABLE 3 Multivariate tests of significance for model parameters

Hypothesized Zero Effects	Name	Wilks' Lambda	FValue	NumDF	DenDF	ProbF
1	*pcriminal*	0.536767	1.7	33	65	0.0343
2	*CriminalPropensity_A0*	0.651701	1.05	33	65	0.4198
3	*CriminalPropensity_AE*	0.625851	1.18	33	65	0.2826
4	*CriminalPropensity_A1*	0.763691	0.61	33	65	0.9391
5	*CriminalPropensity_A2*	0.736327	0.71	33	65	0.8626
6	*CriminalImpulse_AB1*	0.465699	2.26	33	65	0.0025
7	*CriminalImpulse_AB2*	0.370912	3.34	33	65	<.0001
8	*CriminalImpulse_AB3*	0.627128	1.17	33	65	0.2888
9	*CriminalImpulse_AB4*	0.495998	2	33	65	0.0086
10	*Efficacy_D0*	0.595334	1.34	33	65	0.1569
11	*Efficacy_D1*	0.234221	6.44	33	65	<.0001
12	*Efficacy_D2*	0.572457	1.47	33	65	0.0923
13	*Efficacy_D3*	0.387803	3.11	33	65	<.0001
14	*Efficacy_D4*	0.527811	1.76	33	65	0.0259
15	*Criminal_Efficacy*	0.599631	1.32	33	65	0.1719
16	*Criminal_B12*	0.46647	2.25	33	65	0.0026
17	*Friend_G1*	0.514365	1.86	33	65	0.0166
18	*Tenure_mean*	0.347084	3.71	33	65	<.0001
19	*Tenure_std*	0.409598	2.84	33	65	0.0002
6 to 9	*Influence_on_crime*	0.065114	1.95	132	261.32	<.0001
6 to 8	*Interpersonal_Influence*	0.119368	2.04	99	195.5	<.0001

The power of the meta-modeling approach is balanced by the need for model checking. Consider, for example, in Figure 2 (the residual plot versus the predicted values for w183), the transform of the correlation between *residential stability* and *network interaction*. There appears to be a curvilinear effect, which, however, is not the case universally, as seen in the residual plot for w117 in Figure 3 (the transformation of the correlation between *density* and *collective efficacy*). Here the plot is basically acceptable, except for some question about the narrowing of the residuals toward the right side boundary. The plots for the other variables show some combination of these issues as well. This indicates the need for an ad hoc approach to assuring that the statistical meta-model fits, rather than relying on an automated procedure.

Estimated Model Parameters

Table 4 shows the estimated model parameters from the method of simulated moments by using weighted least-squares. The parameter estimates and p-values show some disappointments and some surprises. The tolerance values are included as an indication of an identification problem.

While most parameters are not statistically significant based on the estimated t statistic, three stand out as having statistics of greater than 2.5 in magnitude. Parameter 7, *CriminalImpulse_AB2*, which is the effect of the efficacy of friends on the agent's criminal impulse, is positive, which is contrary to expectation. This might be explained by omitting the effect of behavior on attitudes in the path diagram, but it is not obvious how this explanation might apply in the current case.

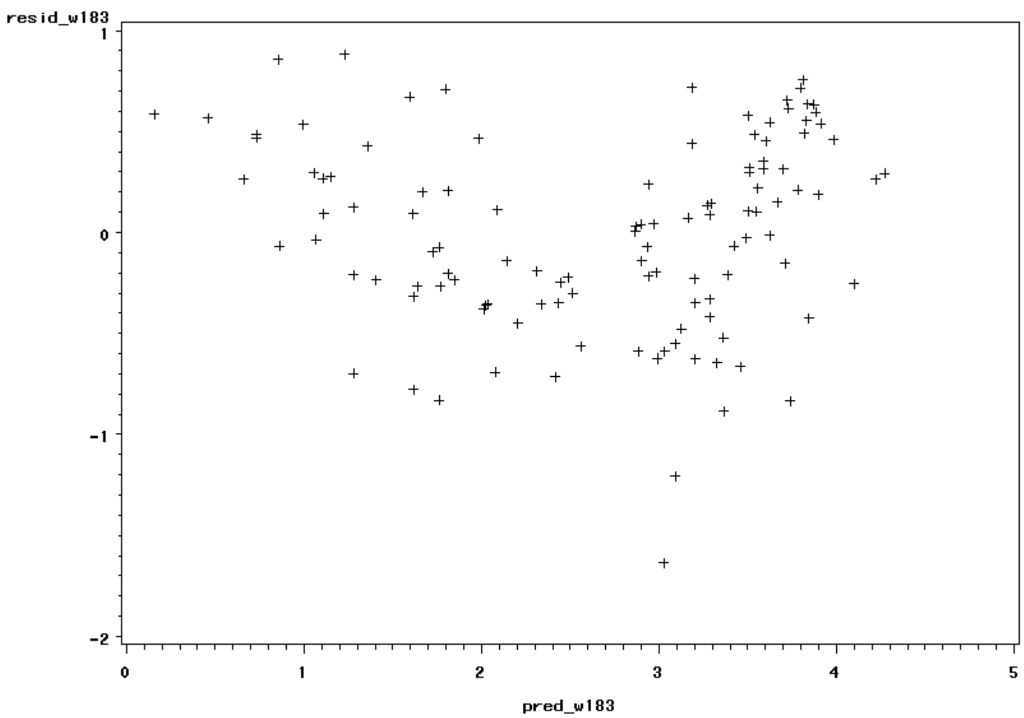

FIGURE 2 Residual analysis of the meta-model for w183

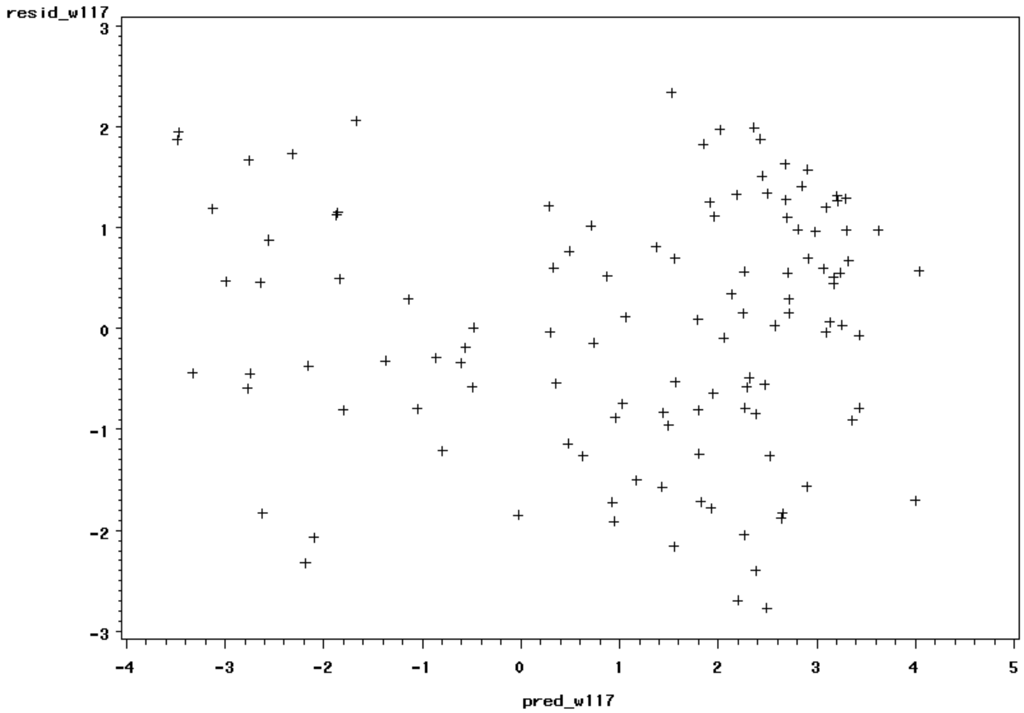

FIGURE 3 Residual analysis of the meta-model for w117

TABLE 4 Regression results

	Name	Sampled Minimum	Sampled Maximum	Parameter Estimate	Standard Error	t Value	Pr > |t|	Tolerance
1	*pcriminal*	-0.163	0.05	0.05817	0.2423	0.24	0.8137	0.04317
2	*CriminalPropensity_A0*	-2.85	1	-8.23298	5.40891	-1.52	0.1502	0.13064
3	*CriminalPropensity_AE*	-4.63	1	3.96582	3.855	1.03	0.321	0.18601
4	*CriminalPropensity_A1*	0.0018	3.34	5.76952	2.73867	2.11	0.0537	0.13844
5	*CriminalPropensity_A2*	-3.05	1.87	12.10151	7.95447	1.52	0.1504	0.07677
6	*CriminalImpulse_AB1*	-1.09	2.46	0.67283	5.25193	0.13	0.8999	0.07619
7	*CriminalImpulse_AB2*	-9.83	34.1	70.94446	23.66379	3	0.0096	0.06765
8	*CriminalImpulse_AB3*	-0.0804	2.28	6.61994	2.28734	2.89	0.0118	0.41749
9	*CriminalImpulse_AB4*	-22	14.8	-48.5363	36.60198	-1.33	0.206	0.12967
10	*Efficacy_D0*	-6.44	0.959	-4.32025	4.33556	-1	0.3359	0.04991
11	*Efficacy_D1*	-1.51	1.4	-0.76677	0.99723	-0.77	0.4547	0.1348
12	*Efficacy_D2*	-4.11	-0.0057	-2.6532	4.13226	-0.64	0.5312	0.12534
13	*Efficacy_D3*	-2.88	1.98	3.80379	3.3113	1.15	0.2699	0.03767
14	*Efficacy_D4*	-0.028	0.251	-0.4344	0.31996	-1.36	0.196	0.02597
15	*Criminal_Efficacy*	-20.7	-0.0151	29.16331	15.13305	1.93	0.0745	0.08524
16	*Criminal_B12*	-0.123	0.919	1.0151	0.77115	1.32	0.2092	0.14942
17	*Friend_G1*	-0.001	0.335	0.61942	0.22094	2.8	0.0141	0.08399
18	*Tenure_mean*	1.01	23.8	4.35497	12.99332	0.34	0.7425	0.05683
19	*Tenure_std*	-1.99	0.991	0.04896	1.7355	0.03	0.9779	0.03722

Parameter 8, the effect of the crime rate among the agent's friends on his own criminal impulse, also was estimated as being positive. However, the estimate is outside the range of the data, which was in part based on preliminary estimates based on the first 60 observations. Parameter 17, which is the rate of friend formation per year, was also outside the range of the data. Parameter 4, which is the effect of disadvantage on criminal propensity, was marginally statistically significant and positive. This is no surprise, but the estimate was also outside the range of the data.

The tolerance values indicate an approximate lack of full rank in the parameter estimates from the meta-models, which may indicate an identification problem with regard to estimating the original path diagram by using the available correlations. As an alternative, we may consider the stepwise weighted regression results (using the default settings) from SAS Proc Reg presented in Table 5.

As a subset of the original variables, the average criminal propensity, the effects of *urban density* and criminal friends on *criminal propensity*, the rate at which the criminal choice is updated, the probability rate of friendship formation, and neighborhood stability suffice to model the observed data as well as can be expected from the agent-based model developed here. Interestingly, it does not seem necessary to include the parameters associated with the particular interaction effect that was the centerpiece of the article by BFD. However, there is an issue with regard to the calibration of the time clock, as a mean neighborhood tenure of 35 years is too long, and a rate of friendship formation of 61% per year for the closest 80 neighbors seems high.

DISCUSSION

The application of simulation meta-modeling to estimate simulation parameters by using the MSM is feasible and scalable. With the methodology, it should be possible to extend agent-based models into the practice of quantitative sociological methodology by performing statistical tests of agent-based model parameters instead of regression parameters. The results of this approach are more accurate reasoning about the agents and activities reasoned about in substantive research. As befits a methodological pilot study, however, a number of critiques and research opportunities need to be addressed in further work. These issues are detailed below.

TABLE 5 Trimmed regression results

Number	Name	Parameter Estimate	Standard Error	T Value	Pr > |t|	Tolerance
2	*CriminalPropensity_A0*	-5.11923	2.55393	-2	0.0551	0.6836
5	*CriminalPropensity_A2*	10.84781	2.43872	4.45	0.0001	0.95291
8	*CriminalImpulse_AB3*	3.78883	1.70443	2.22	0.0348	0.87717
16	*Criminal_B12*	0.88839	0.36351	2.44	0.0213	0.78448
17	*Friend_G1*	0.61142	0.0881	6.94	<.0001	0.61627
18	*Tenure_mean*	35.42161	3.50078	10.12	<.0001	0.91326

There is much computational statistical work to be performed on the application of meta-models to MSM, including equations for the asymptotic variance matrix of the parameter estimates and simulation studies of the performance of the jackknife estimator and some alternatives. In this case study, a weighted least-squares estimate of the final parameters was employed as an approximation on the grounds that the meta-model is misspecified to some degree anyway. Another issue is the nonlinearity of some of the relationships between parameters and the generalized moments. While Fisher's transformation helped, the one-size-fits-all approach had its limits in terms of addressing heteroscedasticity and nonlinearity. A difficulty with estimating nonlinear relationships is inverting them to determine the final parameter estimates, which is possible, but not as easy as solving a regression equation. A related difficulty is that the meta-model did not predict all the correlations equally well.

The agent-based model developed here had at its heart a path model of social influence. Any issue with such path models, such as identification, can be expected to present difficulties in this context as well. Further work with attitudes and behavior in relationship to crime would need to take care concerning model identification with respect to the underlying causal path model. The technique being explored here is not a substitute for collecting the right data and matching the right features of the data.

Employing an agent-based simulation as a replacement for theory increases the precision of one's arguments, but at a price. With the simulation, the domain of modeling concern increases as one examines the arguments and aligns the theory. Because of the increased rigor of this process, the need for elaboration is made clear beyond what was apparent from the prose expression of the theory.

There are also additional substantive issues as well as issues with the simulation model that may be addressed. Calvó-Armengol and Zenou (2004) find that the equilibrium crime rates are sensitive to the geometry of the social network among criminals. Thus it is fair to ask how the social network assumptions made here affect the results. There is also a tendency for friends to be selected to match one's choices, as can be seen in the case of adolescent sexual behavior (Billy and Udry 1985a,b). This may affect the friendship network insofar as influence effects are concerned. The current model has issues pertaining to the details of the simulation of friendship formation and the rate of leaving, which are a priority for model refinement.

In this paper, we have considered MSM by using weighted least-squares as a methodologically superior alternative to ecological regression models and prose sociological theory, and we employed a recent sociological journal article on neighborhood crime rates as a case study. Although in using this case study, a number of methodological issues and areas where hand statistical labor is required arose, the theoretical advantages of the methodology and its basic practicality are an important forward step in sociological methodology.

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