DEPARTMENT
OF
PHOTOGRAMMETRY & SURVEYING

UNIVERSITY COLLEGE LONDON

LATTICE PROGRAM

DOCUMENTATION

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DESCRIPTION AND DOCUMENTATION FOR PROGRAM

LATTICE

1.0 Program Description

The Lattice Program is a C program designed to perform a transformation of spatial and mapping plane coordinates between any two of the coordinate systems which will necessarily be in use at the Fermi National Accelerator Labortory (Fermilab). It is loosely defined in concept upon a similar program written for use at the SSCL, but the structure of the software is considerably different.

The software is currently implemented under the Windows 3.11 operating system, and is compiled using the Microsoft Visual C++ compiler, using the QuickWin compiler options for a 386 PC.

The program is designed to be run any number of times, with the user defining the controlling parameters of the transformation at each stage. It is possible to transform between any two of the coordinate systems defined for use at Fermilab, and any number of coordinates may be transformed. The coordinates are processed in batches of 500. Testing of the software is by way of a baseline set of coordinates provided by Fermilab, together with tests of the internal precision of the program, and tests of transformations using independent algorithms or known test data.

The data controlling any specific run of the software is input interactively, in response to a number of prompts to the user. The user is initially asked for the input device, which may be the console or a data file, the coordinate system and the units of the points in this system. The user is then prompted for the same information regarding the output device, output coordinate system, and output units. Finally a request is made for the number of decimal places to which the data is to be output.

A full listing of the program is given in Appendix A, supplemented by flow charts in Appendix B. Annotated examples of the Input and Output files are given in Appendix C, and results of the software tests in Appendix D.

2.0 Using the Program

The program may be executed in the same way as any Windows 3.11 program. The user is presented with a window, in which the user responses will appear, and in which the first of several prompts to define the controlling parameters of the current transformation is given.

The valid responses at each stage are indexed, and the user is asked to input the appropriate index value. If the user inputs an invalid value the user is once again prompted to input a different response.

If data is to be input or output to a file, the program will prompt for a filename. Any problems encountered opening the file will result in the user being asked to enter a new filename, or choose to terminate the program run. Naming restrictions of the Windows 3.11 operating system apply. Any data already in a specified output file will be overwritten.

The formats for input data are given in Appendix C. The point ID, and x, y, z (or their equivalent) should be input, with one point given on a data line. Blank lines, and lines whose first string is the US pound sign (#) will be ignored. Additional data appended to the end of each data line will also be ignored.

If the input is to be through the console window, the data formats will be supplied just before the user is asked to input the point coordinate data. Rudimentary cut and paste is available. Each paste is followed by a carriage return/linefeed, so if incomplete lines are left after copying in data, the program will record an error in the input data. Results output to this console may be copied to the clipboard and pasted elsewhere.

The user is not presented with the input coordinate system in the list of possible output coordinate systems.

If the input or output point data is indicated to be in a user specified local geodetic coordinate system (LGS), the user is prompted for the name and geodetic coordinates of the origin of the system. The ellipsoidal height should be given in metres.

At the end of each requested transformation, the user is prompted to rerun the program for a new transformation, or to terminate the program. The user may stop the program executing by choosing Exit from the File Menu, which will close the Lattice window. The effects of doing this in the middle of a transformation are not defined.

3.0 Coordinate Systems

There are nine coordinate systems which will be used by Fermilab, some of which are merely intermediate steps in the transformation between two other essential coordinate systems. A brief description of each of these coordinate systems may be found in the FMI Document [MI-0209].

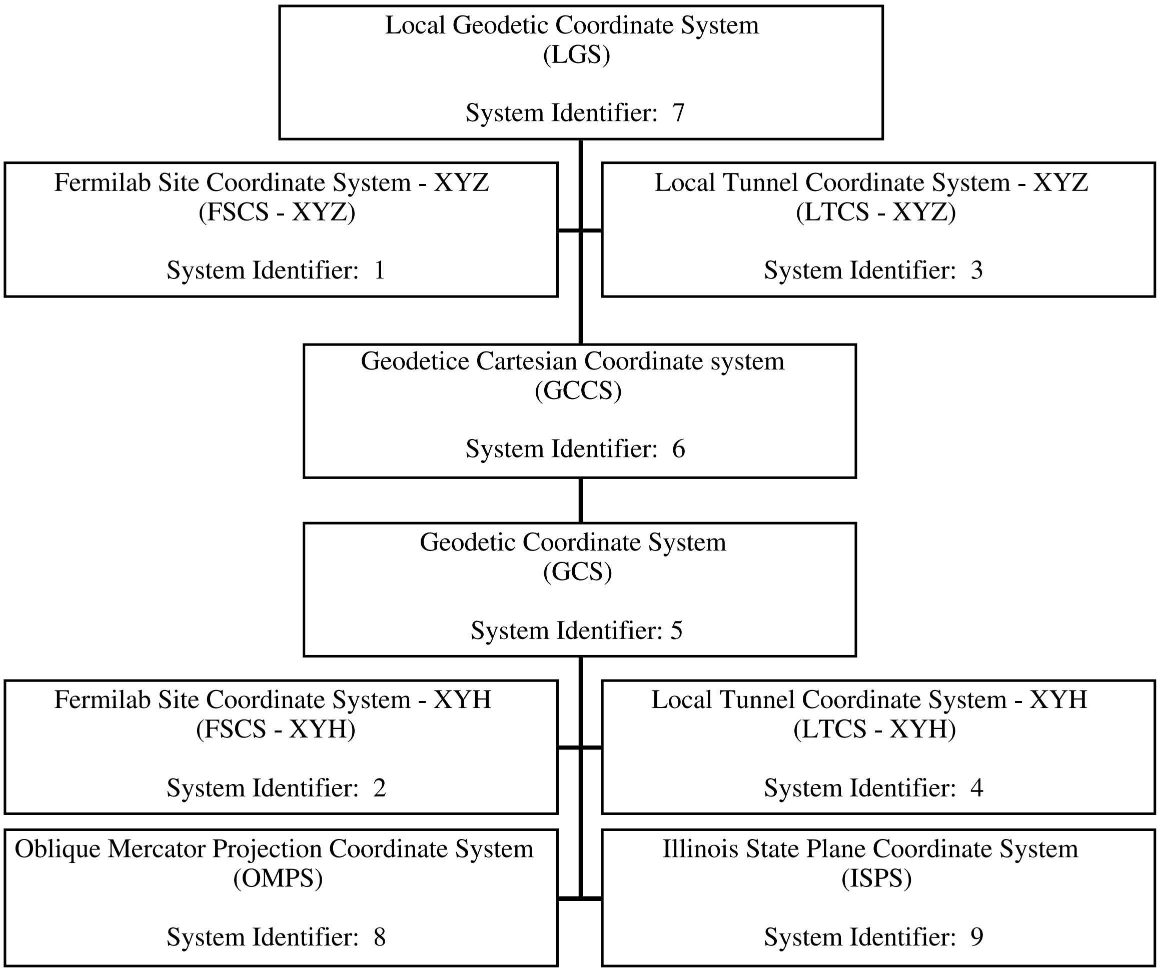

Figure 1.0 Coordinate Systems and Relationships between them

4.0 Coordinate Transformations

The transformation of coordinates from one of the coordinate systems to another one, is a step wise process with a dictated route for a given transformation. Each of the described coordinate systems is directly related to at most two other systems, and this program links the transformations between neighbouring coordinate systems to provide the desired result.

The links between the various coordinate systems are shown in Figure 1.0, with the coordinate systems identified by number, corresponding to the listing provided by the program when it is run.

All combinations of different start and finish coordinate systems are permitted by the program, but they must be different.

The relationship between FSCS -XYZ and FSCS -XYH is shown in diagramatic form in Figure 2.0. It is noted that for the LTCS systems there is an additional rotation applied about CMFI to bring the xy-plane parallel to that defined in MI-0209.

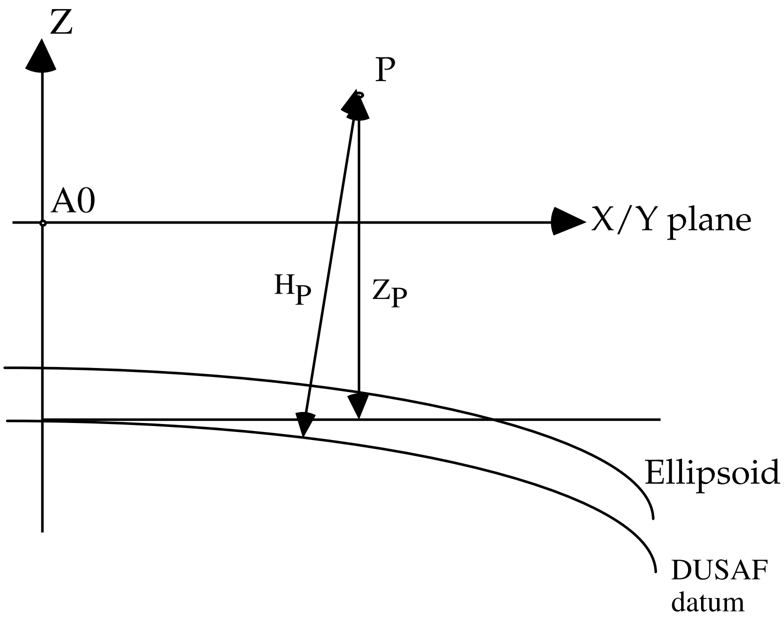

Figure 1

Figure 2.0 Relationship between FSCS -XYZ and FSCS -XYH

4.1 Transformation Formulae

The governing equations for each of the trasformation steps depicted in Figure 1.0, are given below for both the forward and reverse directions.

4.1.1 Transformation GCCS : GCS

Reference: P., Vanicek, & E.J., Krakiwsky: Geodesy the Concepts, 1982. North-Holland Publishing Company.

This transformation between the two Geodetic coordinate systems may be explicitly expressed for the reverse transformation, but the geodetic latitude and geodetic height are determined iteratively for the forward transformation.

4.1.1.1 Forward Transformation

The equations describing the transformation from GCCS to GCS are:

Taking,

$$\mathbf{X}_{GC} = \begin{pmatrix} X_{GC} \\ Y_{GC} \\ Z_{GC} \end{pmatrix} \rightarrow \text{Geodetic Cartesian Coordinates}$$

$$\boldsymbol{\phi}_{GPL} = \begin{pmatrix} \phi_G \\ \lambda_G \\ h_G \end{pmatrix} \rightarrow \text{Geodetic Coordinates}$$

then

$$\lambda_G = 2 \arctan\left(\frac{Y_{GC}}{X_{GC} + p_{GC}}\right)$$

an iterative solution, for ϕ_G , h_G

$$h_{i_G} = \frac{p_{GC}}{\cos \phi_{i-1_G}} - \nu_{i-1} \qquad ; i = 0, 1, 2, \cdots$$

$$\phi_{i_G} = \arctan\left[\frac{Z_{GC}}{p_{GC}\left(1 - \frac{e^2 \nu_{i-1}}{\nu_{i-1} + h_{i_G}}\right)}\right] \qquad ; i = 0, 1, 2, \cdots$$

stopping criterion; suitable values for the precision of the transformation

$$|h_i - h_{i-1}| < \varepsilon_h$$
$$|\phi_i - \phi_{i-1}| < \varepsilon_\phi$$

initial values

$$h_0 = 0$$

$$\Rightarrow \qquad \phi_0 = \arctan\left[\frac{Z_{GC}}{p_{GC}(1 - e^2)}\right]$$

where

$$p_{GC} = \left(X_{GC}^2 + Y_{GC}^2\right)^{\frac{1}{2}}$$

$$\nu_i = \frac{a}{\left(1 - e^2 \sin^2 \phi_{i_G}\right)^{\frac{1}{2}}} \qquad ; \text{for any given } i$$

$$e^2 = \frac{a^2 - b^2}{a^2}$$

$a, b =$ parameters of the reference ellipsoid

In the program,

$$\varepsilon_h = 1.0e - 9$$
$$\varepsilon_\phi = 1.0e - 14$$

4.1.1.2 Reverse Transformation

The equations describing the transformation from GCS to GCCS are:

$$\begin{pmatrix} X_{GC} \\ Y_{GC} \\ Z_{GC} \end{pmatrix} = \begin{bmatrix} (\nu + h_G)\cos\phi_G \cos\lambda_G \\ (\nu + h_G)\cos\phi_G \sin\lambda_G \\ \left(\nu \frac{b^2}{a^2} + h_G\right)\sin\phi_G \end{bmatrix}$$

with notation as before.

4.1.2 Transformation LGS : GCCS

Reference: P., Vanicek, & E.J., Krakiwsky: Geodesy the Concepts, 1982. North-Holland Publishing Company.

This transformation involves a change from a right handed system (GCCS) to a left handed coordinate system (LGS), and therefore also involves a reflection in the plane Y = 0. In addition there is a rotation from the Geodetic coordinates of the User Specified Origin, and a translation from this point.

4.1.2.1 Forward Transformation

The equations describing the transformation from GCCS to LGS are:

$$\mathbf{X}_{LG} = \mathbf{P}_Y \mathbf{R}_Y^T\left(\frac{\pi}{2} - \phi_C\right)\mathbf{R}_Z^T(\pi - \lambda_C)\left[\mathbf{X}_{GC} - \mathbf{X}_{0_{LG}}\right]$$

where

$$\mathbf{X}_{GC} = \begin{pmatrix} X_{GC} \\ Y_{GC} \\ Y_{GC} \end{pmatrix} \rightarrow \text{Geodetic Cartesian Coordinates}$$

$$\mathbf{X}_{LG} = \begin{pmatrix} X_{LG} \\ Y_{LG} \\ Y_{LG} \end{pmatrix} \rightarrow \text{Local Geodetic Coordinates}$$

$$\mathbf{X}_{0_{LG}} = \begin{pmatrix} X_{0_{LG}} \\ Y_{0_{LG}} \\ Z_{0_{LG}} \end{pmatrix} \rightarrow \text{Geodetic Cartesian Coordinates of User Specified Origin}$$

$$\mathbf{P}_Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_Z(\pi - \lambda_C) = \begin{bmatrix} \cos(\pi - \lambda_C) & \sin(\pi - \lambda_C) & 0 \\ -\sin(\pi - \lambda_C) & \cos(\pi - \lambda_C) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_Y\left(\frac{\pi}{2} - \phi_C\right) = \begin{bmatrix} \cos\left(\frac{\pi}{2} - \phi_C\right) & 0 & \sin\left(\frac{\pi}{2} - \phi_C\right) \\ 0 & 1 & 0 \\ -\sin\left(\frac{\pi}{2} - \phi_C\right) & 0 & \cos\left(\frac{\pi}{2} - \phi_C\right) \end{bmatrix}$$

$$\phi_C = \text{Geodetic Latitude of User Specified Origin}$$
$$\lambda_C = \text{Geodetic Longitude of User Specified Origin}$$

4.1.2.2 Reverse Transformation

The equations describing the transformation from LGS to GCCS is:

$$\mathbf{X}_{GC} = \mathbf{R}_Z(\pi - \lambda_C)\mathbf{R}_Y\left(\frac{\pi}{2} - \phi_C\right)\mathbf{P}_Y\mathbf{X}_{LG} - \mathbf{X}_{0_{LG}}$$

with notation as before.

4.1.3 Transformation GCCS : FSCS (XYZ) & GCCS : LTCS (XYZ)

The transformation between both the FSCS (XYZ) and LTCS (XYZ) and the GCCS are essentially the same, with only the parameter values of the transformation changing. The FSCS (XYZ) and the LTCS (XYZ) are in reality modifications of two specific Local Geodetic Systems, designed for Fermilab.

The LTCS system now also incorporates an additional rotation to make the xy-plane of the final system parallel to the plane defined in MI-0209.

The forward transformation involves a reflection in the X = Y plane, to give a right handed system, followed by a rotation and translation of the coordinates. This process is reversed to transform the points the other way.

4.1.3.1 *Forward Transformation*

The equations describing the transformation from the GCCS to a Fermilab (XYZ) Coordinate System, are initially those described for the transformation from the GCCS to a LGS (see § 4.1.2) with the origin at A0 (FSCS), or CFMI (LTCS). Following those transformations:

$$\mathbf{X}_{\mathrm{FXYZ}} = \mathbf{R}_{\mathrm{Z}\alpha}\mathbf{P}_{\mathrm{XY}}\mathbf{X}_{\mathrm{LG}} + \mathbf{X}_0$$

where

$$\mathbf{X}_{\mathrm{FXYZ}} = \begin{pmatrix} \mathrm{X}_{\mathrm{FXYZ}} \\ \mathrm{Y}_{\mathrm{FXYZ}} \\ \mathrm{Y}_{\mathrm{FXYZ}} \end{pmatrix} \rightarrow \text{Fermilab (XYZ) Coordinates}$$

$$\mathbf{X}_{\mathrm{LG}} = \begin{pmatrix} \mathrm{X}_{\mathrm{LG}} \\ \mathrm{Y}_{\mathrm{LG}} \\ \mathrm{Y}_{\mathrm{LG}} \end{pmatrix} \rightarrow \text{Local Geodetic Coordinates at Fermilab (XYZ) Origin}$$

$$\mathbf{X}_0 = \begin{pmatrix} \mathrm{X}_0 \\ \mathrm{Y}_0 \\ \mathrm{Z}_0 \end{pmatrix} \rightarrow \text{False Coordinates at Fermilab (XYZ) Origin}$$

$$\mathbf{P}_{\mathrm{XY}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_{\mathrm{Z}\alpha} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

X_0 = False Easting at Fermilab (XYZ) Origin

Y_0 = False Northing at Fermilab (XYZ) Origin

$\mathrm{Z}_0 = \mathrm{H}_0 - \mathrm{dH}$

H_0 = Orthometric Height at Fermilab (XYZ) Origin

dH = Height Correction (DUSAF to NAVD88)

a = Geodetic Azimuth of the Fermilab (XYZ) Origin

4.1.3.2 *Reverse Transformation*

The equations describing the transformation from a Fermilab (XYZ) Coordinate System to the GCCS are initially as follows:

$$\mathbf{X}_{\mathrm{LG}} = \mathbf{P}_{\mathrm{XY}}\mathbf{R}^{\mathrm{T}}_{\mathrm{Z}\alpha}\left(\mathbf{X}_{\mathrm{FXYZ}} - \mathbf{X}_0\right)$$

This transformation is then followed by the transformation from the LGS at the Fermilab (XYZ) origin, to the GCCS, (see § 4.1.2).

Notation is as before.

4.1.4 Transformation GCS : FSCS (XYH) & GCS : LTCS (XYH)

Reference: D.B., Thomson, M.P., Mepham, R.R., Steeves: The Stereographic Double Projection, July 1977. Department of Surveying and Engineering, University of New Brunswick, Fredericton, N.B., Canada. Technical Report No. 46.

The transformation between both the FSCS (XYH) and LTCS (XYH) and the GCS are essentially the same, with only the parameter values of the transformation changing. The FSCS (XYH) and the LTCS (XYH) are both Double Stereographic Mapping Planes, designed for Fermilab.

The LTCS system now also incorporates an additional rotation to the xy-coordinates to match the additional rotation incorporated into the transformation from the GCCS to the LTCS -XYZ system (see § 4.1.3).

The transformation between Geodetic coordinates and Double Stereographic coordinates is a combination of two conformal transformations, the first between ellipsoidal coordinates and spherical coordinates, and the second between these spherical coordinates and the mapping plane. The transformation from the sphere to the ellipsoid again requires an iterative solution, performed here by the Newton-Raphson Method.

The resulting coordinates are re-scaled by the scale factor, F_0 (defined in MI-0209), to give true scale at the origin of the Fermilab 720ft datum (Note that the point scale factors at the origin of the two projections are both 1.0 for the two Fermilab coordinate systems).

The determination of the orthometric height for each point uses geoid height values estimated using the GEOID93 Model (see MI-0209). These heights are then shifted to give the height above the DUSAF datum.

4.1.4.1 Forward Transformation

The equations describing the transformation from GCS to DS are:

Transformation from the Ellipsoid to the Sphere

$$\Lambda = c_1 \lambda_G$$

$$\chi = 2\left\{\arctan\left(c_2\left[\tan\left(\frac{\pi}{4}+\frac{\phi_G}{2}\right)\left(\frac{1-e\sin\phi_G}{1+e\sin\phi_G}\right)^{\frac{e}{2}}\right]^{c_1}\right)-\frac{\pi}{4}\right\}$$

$$k_s = c_1 k_0 \frac{R\cos\chi}{\nu_G \cos\phi_G}$$

where

$$\phi_G = \begin{pmatrix}\phi_G\\ \lambda_G\\ h_G\end{pmatrix} \rightarrow \text{Geodetic Coordinates}$$

$$\begin{pmatrix}\chi\\ \Lambda\end{pmatrix} = \text{Spherical Coordinates}$$

k_s = Point Scale Factor
k_0 = Point Scale Factor at Origin of Projection (Ellipsoid to Sphere)

$$\begin{pmatrix}\phi_0\\ \lambda_0\end{pmatrix} = \text{Geodetic Coordinate of the Origin for the Transformation}$$

$$\chi_0 = \arcsin\left(\frac{\sin\phi_0}{c_1}\right)$$

$$\Lambda_0 = c_1 \lambda_0$$

$$c_1 = \left[1+\left(\frac{e^2}{1-e^2}\right)\cos^4\phi_0\right]^{\frac{1}{2}}$$

$$c_2 = \tan\left(\frac{\pi}{4}+\frac{\chi_0}{2}\right)\left[\tan\left(\frac{\pi}{4}+\frac{\phi_0}{2}\right)\left(\frac{1-e\sin\phi_0}{1+e\sin\phi_0}\right)^{\frac{e}{2}}\right]^{-c_1}$$

$$e^2 = \frac{a^2-b^2}{a^2}$$

$$R = (\nu_0 \rho_0)^{1/2}$$

$$\nu_i = \frac{a}{\left(1-e^2\sin^2\phi_i\right)^{\frac{1}{2}}} \quad ;\text{ for any given } i$$

$$\rho_0 = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\phi_0\right)^{\frac{3}{2}}}$$

a, b = parameters of the reference ellipsoid

Transformation from the Sphere to the Mapping Plane

$$x_{DS} = 2k_0^{'}k_0R\left\{\frac{\cos(\chi)\sin(\Delta\Lambda)}{1+\sin(\chi_0)\sin(\chi)+\cos(\chi_0)\cos(\chi)\cos(\Delta\Lambda)}\right\}$$

$$y_{DS} = 2k_0^{'}k_0R\left\{\frac{\cos(\chi_0)\sin(\chi)-\sin(\chi_0)\cos(\chi)\cos(\Delta\Lambda)}{1+\sin(\chi_0)\sin(\chi)+\cos(\chi_0)\cos(\chi)\cos(\Delta\Lambda)}\right\}$$

where

$$\mathbf{x}_{DS} = \begin{pmatrix} x_{DS} \\ y_{DS} \end{pmatrix} \rightarrow \text{Double Stereographic Coordinates}$$

$$k_0^{'} = \text{Point Scale Factor at the Origin of the Projection (Sphere to Plane)}$$

$$\Delta\Lambda = \Lambda - \Lambda_0$$

with unspecified notation as before.

Transformation into the Fermilab Double Stereographic Projection

$$\mathbf{x}_{FXY} = F_0\mathbf{R}_\alpha\mathbf{x}_{DS} + \mathbf{x}_0$$

$$\mathbf{x}_{FXY} = \begin{pmatrix} x_{FXY} \\ y_{FXY} \end{pmatrix} \rightarrow \text{Fermilab Double Stereographic Coordinates}$$

$$\mathbf{R}_\alpha = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

$$F_0 = \text{Height Scale Factor}$$

$$\mathbf{x}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \text{False Coordinates at the Origin}$$

$$x_0 = \text{False Easting at Origin}$$

$$y_0 = \text{False Northing at Origin}$$

Orthometric Height in Fermilab Double Stereographic Mapping Plane

$$
\begin{aligned}
H_{FXY} &= H - dH \\
H &= h + N \\
H_{FXY} &= \text{Fermilab Orthometric Height above DUSAF datum} \\
H &= \text{Orthometric Height above NAVD88 datum} \\
dH &= \text{Height Correction (DUSAF to NAVD88)} \\
h &= \text{Ellipsoidal Height} \\
N &= \text{Geoidal Height (derived from GEOID93)}
\end{aligned}
$$

4.1.4.2 Reverse Transformation

The equations governing the transformation from DS to GCS are given below.

Transformation from the Fermilab Double Stereographic Projection

$$
\mathbf{x}_{DS} = \frac{1}{F_0}\mathbf{R}_{\alpha}\left(\mathbf{x}_{FXY} - \mathbf{x}_0\right)
$$

Transformation from the Mapping Plane to the Sphere

$$
\chi = \arcsin\left(\sin(\chi_0)\cos(\delta) + \cos(\chi_0)\sin(\delta)\sin(\beta)\right)
$$

$$
\Lambda = \Lambda_0 + \arcsin\left(\frac{\cos(\beta)\sin(\delta)}{\cos(\chi)}\right)
$$

where

$$
\beta = \arcsin\left(\frac{y_{DS}}{s}\right)
$$

$$
s = \sqrt{x_{DS}^2 + y_{DS}^2}
$$

$$
\delta = 2\arctan\left(\frac{s}{2k_0 k_0' R}\right)
$$

Transformation from the Sphere to the Ellipsoid

$$
\lambda_G = \frac{\Lambda}{c_1}
$$

Newton-Raphson, iterative solution for ϕ_G

$$\phi_i = \phi_{i-1} - \frac{f(\phi_{i-1})}{f'(\phi_{i-1})} \qquad ; \ i = 0,\ 1,\ 2, \cdots$$

stopping criterion

$$|\phi_i - \phi_{i-1}| < \varepsilon$$

initial value

$$\phi_0 = \chi$$

where

$$f(\phi_i) = c_2 \left[\tan\left(\frac{\pi}{4} + \frac{\phi_i}{2}\right) \left(\frac{1 - e \sin \phi_i}{1 + e \sin \phi_i}\right)^{\frac{e}{2}} \right]^{c_1} - \tan\left(\frac{\pi}{4} + \frac{\chi}{2}\right)$$

$$f'(\phi_i) = c_1\, c_2 \left[\tan\left(\frac{\pi}{4} + \frac{\phi_i}{2}\right) \left(\frac{1 - e \sin \phi_i}{1 + e \sin \phi_i}\right)^{\frac{e}{2}} \right]^{c_1 - 1}$$

$$\times \left(\frac{1 - e \sin \phi_i}{1 + e \sin \phi_i}\right)^{\frac{e}{2}} \left[\frac{1}{2} \sec^2\left(\frac{\pi}{4} + \frac{\phi_i}{2}\right) - \left(\frac{e^2 \cos\phi_i}{1 - e^2 \sin^2\phi_i}\right) \tan\left(\frac{\pi}{4} + \frac{\phi_i}{2}\right) \right]$$

in the program,

$$\varepsilon = 1.0e - 14$$

with notation as before.

Ellipsoidal Heights

$$h = H_{FXY} + dH - N$$

4.1.5 Transformation GCS : OMPS

Reference: J.P., Snyder: Map Projections -A Working Manual, 1987. U.S. Geological Survey Professional Paper 1395, United States Goverment Printing Office, Washington.

The transformation between the three dimensional Geodetic Coordinates and the two dimensional Oblique Mercator Projection System in use at Fermilab, as defined by a central point, and the azimuth of the central line through the central point. A series is used in the reverse transformation to avoid an iterative procedure to determine the latitude.

The determination of the orthometric height for each point uses geoid height values estimated using the GEOID93 Model (see MI-0209). These heights are then shifted to give the height above the DUSAF Datum (see § 4.1.3).

4.1.5.1 Forward Transformation

The equations describing the transformation from GCS to OMPS are:

$$\mathbf{x}_{FOM} = F_0 \mathbf{x}_{OM} + \mathbf{x}_0^{'}$$

where

$$\mathbf{x}_{FOM} = \begin{pmatrix} x_{FOM} \\ y_{FOM} \end{pmatrix} \rightarrow \text{Fermilab Oblique Mercator Projection Coordinates}$$

$$\mathbf{x}_{OM} = \begin{pmatrix} x_{OM} \\ y_{OM} \end{pmatrix} \rightarrow \text{Standard Oblique Mercator Projection Coordinates}$$

$$\mathbf{x}_0^{'} = \begin{pmatrix} x_0^{'} \\ y_0^{'} \end{pmatrix} \rightarrow \text{Adapted False Coordinates at the Origin}$$

$$x_{OM} = \frac{A}{2B} \ln\left[\frac{1-U}{1+U}\right]$$

$$y_{OM} = \frac{A}{B} \arctan\left[\frac{S\cos(\gamma_0) + V\sin(\gamma_0)}{\cos\left[B(\lambda - \lambda_0)\right]}\right]$$

$$U = \frac{S\sin(\gamma_0) - V\cos(\gamma_0)}{T}$$

$$V = \sin\left[B(\lambda - \lambda_0)\right]$$

$$T = \frac{1}{2}\left(Q + \frac{1}{Q}\right)$$

$$S = \frac{1}{2}\left(Q - \frac{1}{Q}\right)$$

$$Q = \frac{E}{t^B}$$

$$t = \frac{\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\left[(1 - e\sin(\phi))/(1 + e\sin(\phi))\right]^{\frac{e}{2}}}$$

$$\lambda_0 = \lambda_c - \frac{\arcsin\left[G\tan(\gamma_0)\right]}{B}$$

$$\gamma_0 = \arcsin\left[\frac{\sin(\alpha_c)}{D}\right]$$

$$G = \frac{1}{2}\left(F - \frac{1}{F}\right)$$

$$E = Ft_0^B$$

$$F = D \pm (D^2 - 1)^{1/2} \text{ , taking the sign of } \phi_0$$

$$D = \frac{B(1-e^2)^{1/2}}{\cos(\phi_0)\left[1 - e^2\sin^2(\phi_0)\right]^{1/2}}$$

$$t_0 = \frac{\tan\left(\frac{\pi}{4} - \frac{\phi_0}{2}\right)}{\left[(1 - e\sin(\phi_0))/(1 + e\sin(\phi_0))\right]^{\frac{e}{2}}}$$

$$A = \frac{aBk_0(1-e^2)^{1/2}}{\left[1 - e^2\sin^2(\phi_0)\right]}$$

$$B = \left[1 + \frac{e^2\cos^4(\phi_0)}{(1-e^2)}\right]^{1/2}$$

ϕ = geodetic latitude of the point

λ = geodetic longitude of the point

ϕ_0 = latitude of the selected centre of the projection

λ_c = longitude of the selected centre of the projection

α_c = azimuth of the central line

k_0 = Point Scale Factor at the Origin

x_0 = False Easting at the Origin

$y_0' = y_0 - y_c$

$$y_c = \pm\frac{A}{B}\arctan\left[\frac{(D^2-1)^{1/2}}{\cos(\alpha_c)}\right] \text{ , taking the sign of } \phi_0$$

y_0 = False Northing at the origin

a, e = parameters of the reference ellipsoid

4.1.5.2 Reverse Transformation

The equations describing the transformation from OMPS to GCS are:

$$\phi = \chi + \text{term1} + \text{term2} + \text{term3} + \text{term4}$$

$$\lambda = \lambda_0 - \frac{1}{B}\arctan\left[\frac{S'\cos(\gamma_0) - V'\sin(\gamma_0)}{\cos\left(\frac{By_{OM}}{A}\right)}\right]$$

where

$$\chi = \frac{\pi}{2} - 2\arctan(t)$$

$$\text{term1} = \left(\frac{e^2}{2} + \frac{5e^4}{24} + \frac{e^6}{12} + \frac{13e^8}{360}\right)\sin(2\chi)$$

$$\text{term2} = \left(\frac{7e^4}{48} + \frac{29e^6}{240} + \frac{811e^8}{11520}\right)\sin(4\chi)$$

$$\text{term3} = \left(\frac{7e^6}{120} + \frac{81e^8}{1120}\right)\sin(6\chi)$$

$$\text{term4} = \left(\frac{4279e^8}{161280}\right)\sin(8\chi)$$

$$t = \left\{\frac{E}{\left[(1+U')/(1-U')\right]^{1/2}}\right\}^{1/B}$$

$$U' = \left[V'\cos(\gamma_0) + S'\sin(\gamma_0)\right]/T'$$

$$V' = \sin(By_{OM}/A)$$

$$T' = \frac{1}{2}\left(Q' + \frac{1}{Q'}\right)$$

$$S' = \frac{1}{2}\left(Q' - \frac{1}{Q'}\right)$$

$$Q' = e^{-(Bx_{OM}/A)} \quad , \quad e = 2.71828.... \text{ , the base of natural logarithms}$$

$$\mathbf{x}_{OM} = \frac{1}{F_0}\left(\mathbf{x}_{FOM} - \mathbf{x}'_0\right)$$

with unspecified notation as before.

4.1.6 Transformation GCS : ISPS

Reference: J.P., Snyder: Map Projections -A Working Manual, 1987. U.S. Geological Survey Professional Paper 1395, United States Goverment Printing Office, Washington.

The transformation between the three dimensional Geodetic Coordinates and the two dimensional ISPS mapping plane coordinates. The reverse transformation uses an iterative procedure for the latitude.

The determination of the orthometric height for each point uses geoid height values estimated using the GEOID93 Model (see MI-0209). For this mapping plane these heights are not shifted to give the height above the DUSAF, but are left as heights above the NAVD88 Datum.

4.1.6.1 Forward Transformation

The equations describing the transformation from GCS to ISPS are:

$$\mathbf{x}_{\mathrm{ISPS}} = k_0 \mathbf{x}_{\mathrm{TM}} + \mathbf{x}_0$$

where

$$\mathbf{x}_{\mathrm{ISPS}} = \begin{pmatrix} x_{\mathrm{ISPS}} \\ y_{\mathrm{ISPS}} \end{pmatrix} \rightarrow \text{Illinois State Plane Coordinates}$$

$$\mathbf{x}_{\mathrm{TM}} = \begin{pmatrix} x_{\mathrm{TM}} \\ y_{\mathrm{TM}} \end{pmatrix} \rightarrow \text{Transverse Mercator Projection Coordinates}$$

$$\mathbf{x}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \rightarrow \text{False Coordinates at the Origin}$$

$$k_0 = \text{ point scale factor at origin of projection}$$

$$y_{TM} = M + (\Delta\lambda)^2 \text{term2} + (\Delta\lambda)^4 \text{term3} + (\Delta\lambda)^6 \text{term3a}$$

$$x_{TM} = \Delta\lambda \text{term4} + (\Delta\lambda)^3 \text{term5} + (\Delta\lambda)^5 \text{term6}$$

$$M = b\left\{\begin{array}{l} \left[\left(1 + n + \frac{5}{4}n^2 + \frac{5}{4}n^3\right)(\phi - \phi_0)\right] \\ -\left[\left(3n + 3n^2 + \frac{21}{8}n^3\right)\sin(\phi - \phi_0)\cos(\phi + \phi_0)\right] \\ +\left[\left(\frac{15}{8}n^2 + \frac{15}{8}n^3\right)\sin 2(\phi - \phi_0)\cos 2(\phi + \phi_0)\right] \\ -\frac{35}{24}n^3 \sin 3(\phi - \phi_0)\cos 3(\phi + \phi_0) \end{array}\right\}$$

$$\text{term2} = \frac{\nu}{2}\sin(\phi)\cos(\phi)$$

$$\text{term3} = \frac{\nu}{24}\sin(\phi)\cos^3(\phi)\left[5 - \tan^2(\phi) + 9\eta^2\right]$$

$$\text{term3a} = \frac{\nu}{720}\sin(\phi)\cos^5(\phi)\left[61 - 58\tan^2(\phi) + \tan^4(\phi)\right]$$

$$\text{term4} = \nu\cos(\phi)$$

$$\text{term5} = \frac{\nu}{6}\cos^3(\phi)\left[\frac{\nu}{\rho} - \tan^2(\phi)\right]$$

$$\text{term6} = \frac{\nu}{120}\cos^5(\phi)\left[5 - 18\tan^2(\phi) + \tan^4(\phi) + 14\eta^2 - 58\tan^2(\phi)\eta^2\right]$$

$$\Delta\lambda = \lambda - \lambda_0$$

$$n = \frac{a - b}{a + b}$$

$$\nu = \frac{a}{\left(1 - e^2\sin^2(\phi)\right)^{1/2}}$$

$$\rho = \frac{a\left(1 - e^2\right)}{\left(1 - e^2\sin^2(\phi)\right)^{3/2}}$$

$$\eta^2 = \frac{\nu}{\rho} - 1$$

$$\begin{aligned}
\phi &= \text{geodetic latitude of the point}\\
\lambda &= \text{geodetic longitude of the point}\\
\phi_0 &= \text{latitude of the origin of the projection}\\
\lambda_0 &= \text{longitude of the origin of the projection}\\
x_0 &= \text{False Easting at the Origin}\\
y_0 &= \text{False Northing at the origin}\\
a, b, e &= \text{parameters of the reference ellipsoid}
\end{aligned}$$

Orthometric Height in Illinois State Plane Coordinate System

$$\begin{aligned}
H &= h + N\\
H &= \text{Orthometric Height above NAVD88 datum}\\
h &= \text{Ellipsoidal Height}\\
N &= \text{Geoidal Height (derived from GEOID93)}
\end{aligned}$$

4.1.6.2 Reverse Transformation

The equations describing the transformation from ISPS to GCS are:

$$\phi = \phi' - x_{TM}^2\,\text{term}1 + x_{TM}^4\,\text{term}2 - x_{TM}^6\,\text{term}3$$

$$\lambda = \lambda_0 + x_{TM}\text{term}4 - x_{TM}^3\,\text{term}5 + x_{TM}^5\,\text{term}6 - x_{TM}^7\,\text{term}6\text{a}$$

where

we determine ϕ' through an iterative procedure

$$\phi'_{i+1} = \phi'_i + \frac{y_{TM} - M_i}{a} \quad ; \; i = 0,\ 1,\ 2,\ \ldots$$

stopping criterion

$$\left|\phi'_{i+1} - \phi'_i\right| < \varepsilon$$

initial value

$$\phi'_0 = \phi_0 + \frac{y_{TM}}{a}$$

in the program,

$$\varepsilon = 1.0e - 14$$

$$M_i = b\left\{\begin{array}{l} \left[\left(1+n+\frac{5}{4}n^2+\frac{5}{4}n^3\right)\left(\phi_i^{'}-\phi_0\right)\right] \\ -\left[\left(3n+3n^2+\frac{21}{8}n^3\right)\sin\left(\phi_i^{'}-\phi_0\right)\cos\left(\phi_i^{'}+\phi_0\right)\right] \\ +\left[\left(\frac{15}{8}n^2+\frac{15}{8}n^3\right)\sin 2\left(\phi_i^{'}-\phi_0\right)\cos 2\left(\phi_i^{'}+\phi_0\right)\right] \\ -\frac{35}{24}n^3\sin 3\left(\phi_i^{'}-\phi_0\right)\cos 3\left(\phi_i^{'}+\phi_0\right) \end{array}\right\}$$

$$\mathrm{term1} = \frac{\tan(\phi')}{2\rho\nu}$$

$$\mathrm{term2} = \frac{\tan(\phi')}{24\rho\nu^3}\left(5+3\tan^2(\phi')+\eta^2-9\tan^2(\phi')\eta^2\right)$$

$$\mathrm{term3} = \frac{\tan(\phi')}{720\rho\nu^5}\left(61+90\tan^2(\phi')+45\tan^4(\phi')\right)$$

$$\mathrm{term4} = \frac{\sec(\phi')}{\nu}$$

$$\mathrm{term5} = \frac{\sec(\phi')}{6\nu^3}\left(\frac{\nu}{\rho}+2\tan^2(\phi')\right)$$

$$\mathrm{term6} = \frac{\sec(\phi')}{120\nu^5}\left(5+28\tan^2(\phi')+24\tan^4(\phi')\right)$$

$$\mathrm{term7} = \frac{\sec(\phi')}{5040\nu^7}\left(61+662\tan^2(\phi')+1320\tan^4(\phi')+720\tan^6(\phi')\right)$$

$$\mathbf{x}_{TM} = \frac{1}{k_0}\left(\mathbf{x}_{ISPS}-\mathbf{x}_0\right)$$

with unspecified notation as before.

Ellipsoidal Heights

$$h = H - N$$

Appendix A Program Listing

Appendix B Program Flow Charts

Appendix C Program Test Results