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Design Calculations for the LHC IR Quadrupole Magnet Beam Tubes

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Abstract—*This note briefly demonstrates our compliance with the ASME pressure vessel code for the design of beam tubes that would be inserted in the Large Hadron Collider Interaction Region (LHC IR) Quadrupole magnets and would form the inner wall for the magnet's cryogenic system which is designed for operation up to 20 bars. The note also describes other salient design features for the beam tubes.*

1. **Introduction**

Fermilab is responsible for the procurement and installation of beam tubes for all the LHC IR Quadrupole magnets, including the ones to be built at KEK, Japan. Due to the intense beam-induced energy deposition at the interaction region, absorbers (liners) need to be used to keep the local power density to an acceptable level and to prevent quenching of the superconducting magnets [1]. The LHC design requires that the physical aperture (including dispersion, alignment tolerances) in the IRs should be everywhere at least 12 σ, where σ is the rms beam size. Absorbers (liners) are placed at the outer radii of this 12 σ limit. Table 1 lists the inside diameter of the liners for both the high luminosity (IP1/IP5) and low luminosity (IP2/IP8) interaction points. These liners are copper/steel sleeves, with inside surface coated with a 50 μm thick copper layer, which would be inserted inside the beam tubes by CERN. The inside diameter of the beam tubes was fixed as 63 mm from the requirement of inside diameter for the low luminosity interaction point (IP2/IP8) magnets. Once the inside diameter is fixed, the outside diameter of the beam tube is determined by the wall thickness required to prevent the tube collapse (buckling) during quench. Table 1 presents the outside diameter, wall thickness, and other relevant physical dimensions for the LHC IR Quadrupole magnet beam tubes. A comparison is also provided with the CERN and BNL dipole magnets. This note presents the ASME design calculations for the wall thickness requirement. It should be noted that the presented dimensions provide sufficient thickness of the internal absorbers (liners) to reduce the energy deposited (P_{max}) to an acceptable level. Also, the current outer diameter of 66.7 mm provides the necessary cooling channel around the beam tube for cryogenic purposes.

Table 1: Physical dimensions for the LHC IR Quadrupole magnet beam tubes for both the high luminosity (IP1/IP5) and low luminosity (IP2/IP8) interaction points. Similar information is provided for the CERN main dipoles and Brookhaven separation dipoles for comparative purposes.

Magnets		Aperture (mm)	Beam Tube O.D. (mm)	Beam Tube I.D. (mm)	Wall Thickness (mm)	Liner Inside Diameter (mm)	Liner Thickness (mm)	Theoretical Radial Clearance (mm)
HGQ (IP1/ IP5)	Q2-Q3	70	66.7	63	1.85	60	1.5	1.65
	Q1	70	66.7	63	1.85	47	8	1.65
HGQ (IP2/ IP8)	Q1-Q3	70	66.7	63	1.85	N/A	N/A	1.65
LHC Main Dipoles		56	53	50	1.5			1.5
BNL LHC D1 Separation Dipoles		80	78	74	2			1
BNL LHC D2-D4 Separation Dipoles		80	73	69.08	1.96			3.5

2. Design Calculations

In this section, we present our design calculations for determining the necessary wall thickness. We first present the calculations that guided the initial design phase. We would later show our compliance with the ASME Pressure Vessel Code. The operational conditions for the beam tubes are summarized in Table 2. The beam tubes need to be designed for a pressure of 20 bar.

Table 2: Operational conditions for the LHC IR Quadrupole magnet beam tubes.

Condition	Temperature (K)	Internal Pressure (mbar)	External Pressure (bar)
Normal Operation	1.9	$< 10^{-11}$	1.1 abs
Cool down-Warm up, Quench	300-1.9	$< 10^{-11}$	20 abs

2.1 ***Stress Calculations***

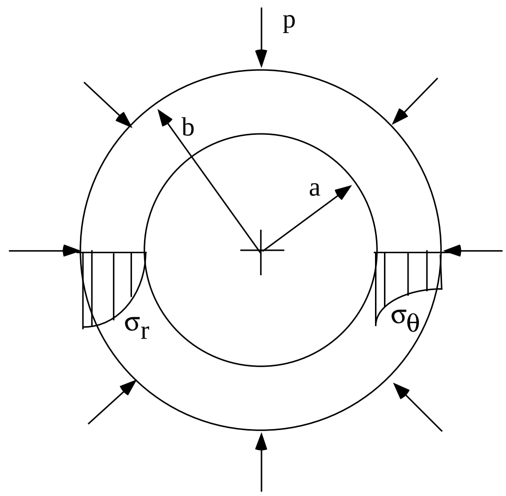


Figure 1: Stress distribution of a cylinder subjected to external pressure.

Fig. 1 shows the stress distribution for a cylinder subjected to external pressure. The normal stresses in the radial and theta directions are given as:

$$\sigma_r = -\frac{pb^2}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_\theta = -\frac{pb^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

The variation of these stresses across the thickness are shown in Fig. 1. The most critical point lies on inner surface of the cylinder where the maximum shear stress is given by:

$$\tau_{\max} = p\frac{b^2}{b^2 - a^2}$$

For our case p=20 bars=20 x 10^5 N/m^2, b=66.7/2 mm and a=63/2 mm and therefore τ_{max}=18.6 MPa. Assuming a safety factor of 4, the beam tubes should be designed for a maximum shear stress (τ_{max}) of 74.3 MPa. This implies that the yield strength of the material should be at least 148.6 MPa.

Since the beam tube material is stainless steel and most of the stainless steel have a yield strength greater than 200 MPa, we are safe from the strength point of view. Note that the strength increases further at low temperatures. It should also be noted that we have not accounted for stresses due to the Lorentz forces produced by the eddy currents in the thin copper layer since the tubes would be fitted with sleeves by CERN, which would have a copper coating to take the eddy currents.

2.2 Buckling Calculations

The main requirement for wall thickness comes from buckling considerations. For a cylinder shown in Fig. 1, the critical buckling pressure is given by:

$$p_{cr} = \frac{E\,t^3}{4\,R_m{}^3\left(1-\nu^2\right)}$$

where E is the Young's modulus of the beam tube material, ν is the Poisson's ratio, t is the wall thickness and R_m is the mean radius. Assuming E=200 GPa and ν=0.28 for stainless steel, for our beam tube dimensions we obtain a critical buckling pressure of 10.08 MPa, which gives a safety factor of 5.04 for a design pressure of 20 bars. Table 3 provides a comparison of the safety factors used for the different magnets for a design pressure of 20 bars. The table also lists the minimum and maximum safety factors accounting for the manufacturing tolerances. The higher safety factor used for the CERN dipoles when compared to IR Quadrupoles is partly due to the fact that in quenching quadrupoles, eddy current pressure is smaller than quenching dipoles by a factor of $\sim\left(\frac{Kb}{B}\right)^2$, where K in T/m is the quad strength. Also note that the CERN design provides a minimum safety factor of 4.3 after accounting for the manufacturing tolerances.

Table 3: Safety factors against buckling for the different magnets for a design pressure of 20 bars.

Magnet	D_0 (mm)	Wall thickness (mm)	D_i (mm)	Nominal Safety Factor	Minimum/ Maximum Safety Factor
LHC IR Quad	66.7 ± 0.15	1.85 ± 0.1	63	5.04	4.2/6.0
BNL D1	78 ± 0.4	2 ± 0.2	74	3.95	2.8/5.4
BNL D2-D4	73 ± 0.38	1.96 ± 0.18	69.08	4.6	3.3/6.1
CERN Dipoles	53 ± 0.15	1.5 ± 0.1	50	5.4	4.3/6.6

The effect of variation in wall thickness (say due to manufacturing tolerances) or outer diameter on safety factor can be investigated using the Excel worksheet located at http://tdpc02.fnal.gov/yadav/LHC/BeamTube/buckling_pressure_calculations.xls.

2.3 ASME Calculations

We now present evidence of our compliance with the ASME Pressure Vessel Code. Article AF-105 on *Permissible Mill Underthickness Tolerances* from *1998 ASME Boiler and Pressure Vessel Code Section VIII—Division 2* provides guideline to account for the manufacturing tolerances. In particular, according to Article AF 105.2—For Pipes and Tubes—"*If pipe or tube is ordered from its nominal wall thickness, the manufacturing undertolerance on wall thickness shall be taken into account. The manufacturing undertolerances are given in Part AM. After the minimum wall thickness is determined, it shall be increased by an amount sufficient to provide the manufacturing undertolerance allowed in the pipe or tube specification.*"

For our case the nominal wall thickness is 1.85 mm and the manufacturing undertolerance on the wall thickness is 0.1 mm. Therefore, we would demonstrate that a wall thickness of 1.75 mm is the minimum wall thickness that satisfies the requirements of the ASME Pressure Vessel Code. We follow Article UG-28 on *Thickness of Shells and Tubes under External Pressure* of *ASME Boiler and Pressure Vessel Code Section VIII—Division 1*. The relevant sections of the code are attached as Appendix along with this note. The ASME calculations can be performed by using the Excel worksheet located at http://tdpc02.fnal.gov/yadav/LHC/BeamTube/asme_calculations.xls.

We now follow the code step by step:

(c) *Cylindrical Shells and Tubes*. The required minimum thickness of a cylindrical shell or tube under external pressure, either seamless or with longitudinal butt joints, shall be determined by the following procedure.

(1) Cylinders having D_0/t *values* ≥ 10:

Step 1. Assume a value for t and determine the ratio L/D_0 and D_0/t.

For our case t=1.75 mm, D_0=66.7 mm and L=11 m.
Then, $L/D_0 > 50$ and D_0/t=38.1

Step 2. Enter Fig. 5-UGO-28.0 in Appendix 5 at the value of L/D_0 determined in Step 1. For values of L/D_0 greater than 50, enter the chart at a value of L/D_0 =50.

Step 3. Move horizontally to the line for the value of D_0/t determined in Step 1. Interpolation may be made for intermediate values of D_0/t. From this point of intersection move vertically downward to determine the value of factor A.

From the chart A=0.000788

Step 4. Using the value of A calculated in Step 3, enter the applicable material chart in Appendix 5 for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature.

Step 5. From the intersection obtained in Step 4, move horizontally to the right and read the value of factor *B*.

From Fig. 5-UHA-28.4 for 316 L stainless steel, for *A*=0.0007888 and for operation up to 100 F, we obtain *B*=8610.

Step 6. Using the value of B, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{4\,B}{3\,(D_0\,/\,t)}$$

This gives P_a=301.3 psi or 2.07 MPa (20.7 bar). This demonstrates that the chosen wall thickness is sufficient for the design operating pressure.

3. Summary

In this note we have presented the design calculations for determining the relevant physical dimensions for the LHC IR Quadrupole magnet beam tubes. It is observed that a wall thickness of 1.75 mm satisfies the requirements of the ASME Pressure Vessel code. However, a wall thickness of 1.85 mm is necessary to account for the manufacturing undertolerances.

References

[1] N.V. Mokhov and J.B Strait, "Towards the Optimal LHC Interaction Region: Beam-Induced Energy Deposition."

Appendix

R = inside radius of the shell course under consideration,[15] in.

S = maximum allowable stress value, psi (see applicable table of stress values in Subsection C, and the stress limitations specified in UG-24)

E = joint efficiency for, or the efficiency of, appropriate joint in cylindrical or spherical shells, or the efficiency of ligaments between openings, whichever is less.

For welded vessels, use the efficiency specified in UW-12.

For ligaments between openings, use the efficiency calculated by the rules given in UG-53.

(c) Cylindrical Shells. The minimum thickness or maximum allowable working pressure of cylindrical shells shall be the greater thickness or lesser pressure as given by (1) or (2) below.

(1) Circumferential Stress (Longitudinal Joints). When the thickness does not exceed one-half of the inside radius, or P does not exceed $0.385SE$, the following formulas shall apply:

$$t = \frac{PR}{SE - 0.6P} \quad \text{or} \quad P = \frac{SEt}{R + 0.6t} \qquad (1)$$

(2) Longitudinal Stress (Circumferential Joints).[16] When the thickness does not exceed one-half of the inside radius, or P does not exceed $1.25SE$, the following formulas shall apply:

$$t = \frac{PR}{2SE + 0.4P} \quad \text{or} \quad P = \frac{2SEt}{R - 0.4t} \qquad (2)$$

(d) Spherical Shells. When the thickness of the shell of a wholly spherical vessel does not exceed $0.356\ R$, or P does not exceed $0.665SE$, the following formulas shall apply:

$$t = \frac{PR}{2SE - 0.2P} \quad \text{or} \quad P = \frac{2SEt}{R + 0.2t} \qquad (3)$$

(e) When necessary, vessels shall be provided with stiffeners or other additional means of support to prevent overstress or large distortions under the external loadings listed in UG-22 other than pressure and temperature.

(f) A stayed jacket shell that extends completely around a cylindrical or spherical vessel shall also meet the requirements of UG-47(c).

(g) Any reduction in thickness within a shell course or spherical shell shall be in accordance with UW-9.

UG-28 THICKNESS OF SHELLS AND TUBES UNDER EXTERNAL PRESSURE

(a) Rules for the design of shells and tubes under external pressure given in this Division are limited to cylindrical shells, with or without stiffening rings, tubes, and spherical shells. Three typical forms of cylindrical shells are shown in Fig. UG-28. Charts used in determining minimum required thicknesses of these components are given in Appendix 5.

(b) The symbols defined below are used in the procedures of this paragraph:

A = factor determined from Fig. 5-UGO-28.0 in Appendix 5 and used to enter the applicable material chart in Appendix 5. For the case of cylinders having D_o/t values less than 10, see UG-28(c)(2).

B = factor determined from the applicable material chart in Appendix 5 for maximum design metal temperature, psi [see UG-20(c)]

D_o = outside diameter of cylindrical shell course or tube, in.

E = modulus of elasticity of material at design temperature, psi. For external pressure design in accordance with this Section, the modulus of elasticity to be used shall be taken from the applicable materials chart in Appendix 5.[17] (Interpolation may be made between lines for intermediate temperatures.)

L = total length, in., of a tube between tubesheets, or design length of a vessel section between lines of support (see Fig. UG-28.1). A line of support is: A89

(1) a circumferential line on a head (excluding conical heads) at one-third the depth of the head from the head tangent line as shown on Fig. UG-28;

[15]For pipe, the inside radius R is determined by the nominal outside radius minus the nominal wall thickness.

[16]These formulas will govern only when the circumferential joint efficiency is less than one-half the longitudinal joint efficiency, or when the effect of supplementary loadings (UG-22) causing longitudinal bending or tension in conjunction with internal pressure is being investigated. An example illustrating this investigation is given in L-2(a) and (b).

[17]Note that the modulus of elasticity values listed in UF-27 of this Division shall not be used for external pressure design.

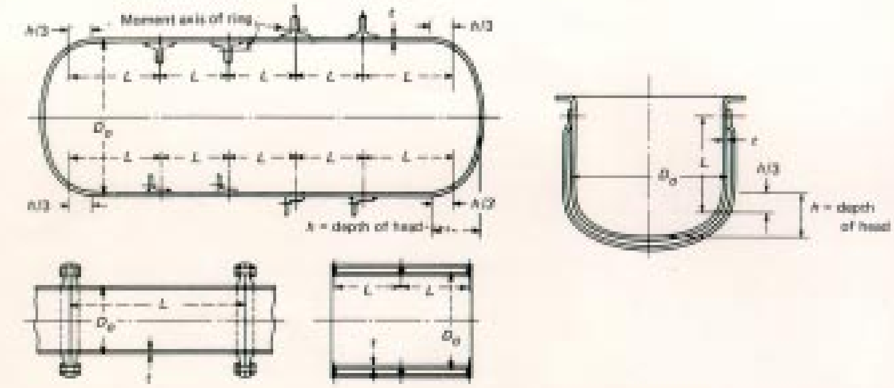


FIG. UG-28 DIAGRAMMATIC REPRESENTATION OF VARIABLES FOR DESIGN OF CYLINDRICAL VESSELS SUBJECTED TO EXTERNAL PRESSURE

(2) a stiffening ring that meets the requirements of UG-29;

(3) a jacket closure of a jacketed vessel that meets the requirements of 9-5;

(4) a cone-to-cylinder junction or a knuckle-to-cylinder junction of a toriconical head or section which satisfies the moment of inertia requirement of 1-8.

P = external design pressure, psi [see Note in UG-28(f)]

A89 P_a = calculated value of maximum allowable external working pressure for the assumed value of t, psi [see Note in (f) below]

R_o = outside radius of spherical shell, in.

t = minimum required thickness of cylindrical shell or tube, or spherical shell, in.

t_s = nominal thickness of cylindrical shell or tube, in.

(c) Cylindrical Shells and Tubes. The required minimum thickness of a cylindrical shell or tube under external pressure, either seamless or with longitudinal butt joints, shall be determined by the following procedure:

(1) Cylinders having D_o/t values $\geq$ 10:

Step 1. Assume a value for t and determine the ratios L/D_o and D_o/t.

Step 2. Enter Fig. 5-UGO-28.0 in Appendix 5 at the value of L/D_o determined in Step 1. For values of L/D_o greater than 50, enter the chart at a value of L/D_o = 50. For values of L/D_o less than 0.05, enter the chart at a value of L/D_o = 0.05.

Step 3. Move horizontally to the line for the value of D_o/t determined in Step 1. Interpolation may be made for intermediate values of D_o/t. From this point of intersection move vertically downward to determine the value of factor A.

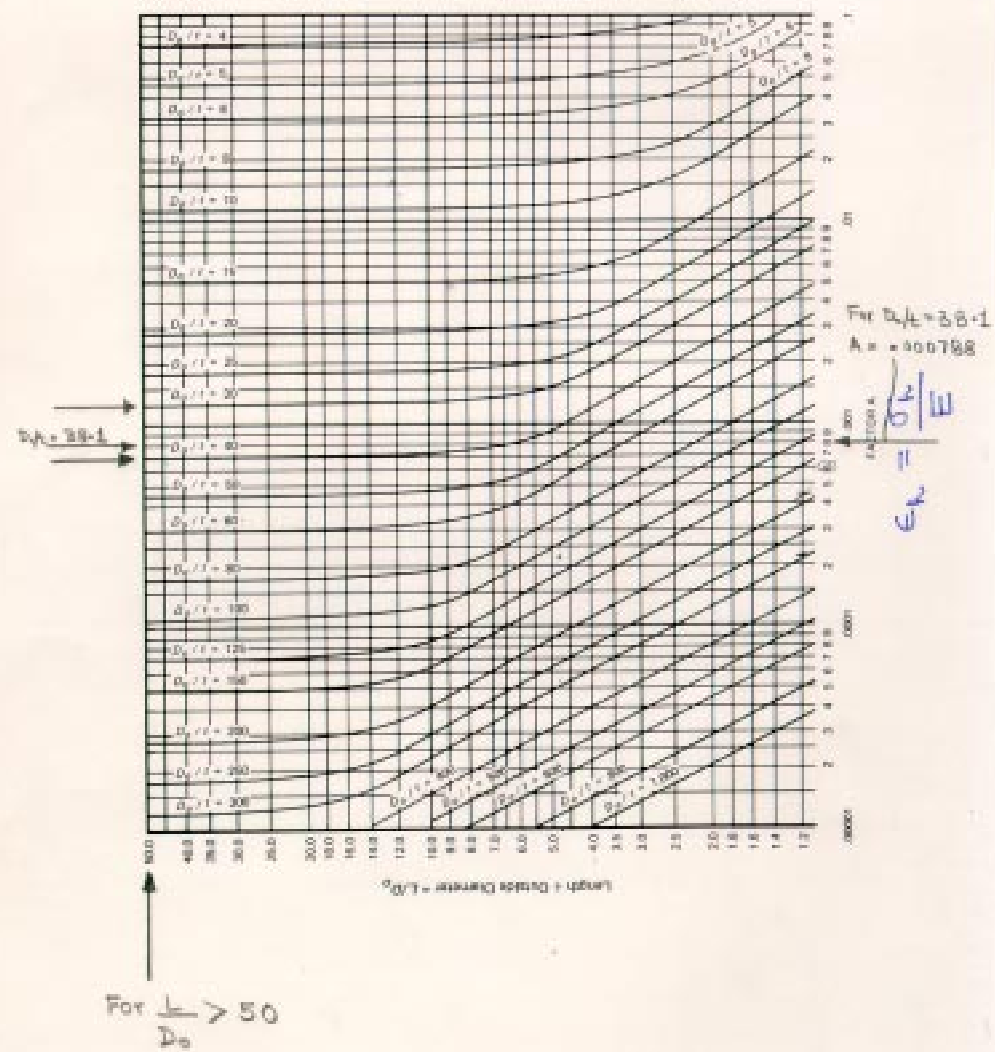
Step 4. Using the value of A calculated in Step 3, enter the applicable material chart in Appendix 5 for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature (see UG-20). Interpolation may be made between lines for intermediate temperatures.

In cases where the value of A falls to the right of the end of the material/temperature line, assume an intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see Step 7.

Step 5. From the intersection obtained in Step 4, move horizontally to the right and read the value of factor B.

Step 6. Using this value of B, calculate the value of A89
the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{4B}{3(D_o/t)}$$


$D_o/t = 4$
$D_o/t = 5$
$D_o/t = 6$
$D_o/t = 10$
$D_o/t = 20$
$D_o/t = 25$
$D_o/t = 30$
$D_o/t = 40$
$D_o/t = 50$
$D_o/t = 60$
$D_o/t = 80$
$D_o/t = 100$
$D_o/t = 125$
$D_o/t = 150$
$D_o/t = 200$
$D_o/t = 250$
$D_o/t = 300$
$D_o/t = 1{,}000$
Length ÷ Outside Diameter = L/D_o
FACTOR A
$D_o/t = 38.1$
For $D_o/t = 38.1$
A = .000788
For $\frac{L}{D_o} > 50$

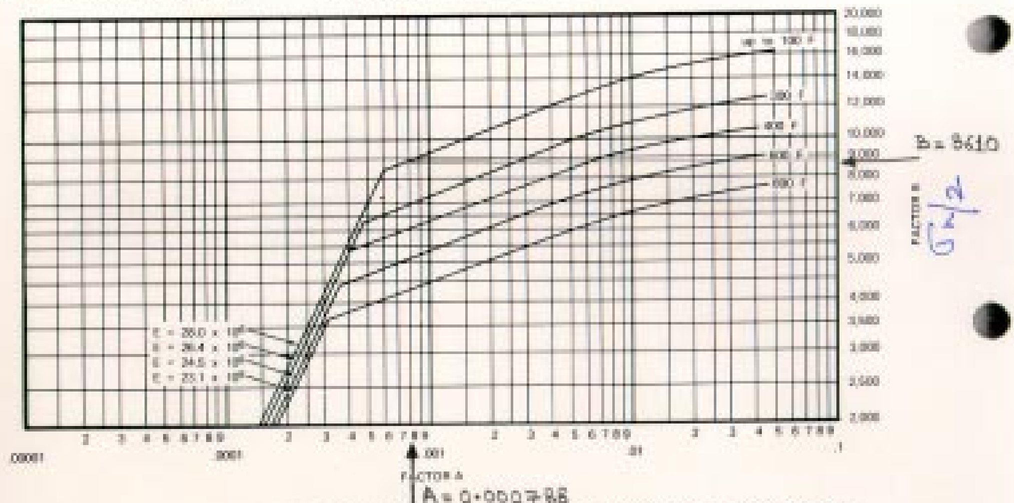


FIG. 5-UHA-28.4 CHART FOR DETERMINING SHELL THICKNESS OF CYLINDRICAL AND SPHERICAL VESSELS UNDER EXTERNAL PRESSURE WHEN CONSTRUCTED OF AUSTENITIC STEEL (18Cr-8Ni-Mo-0.03 MAXIMUM CARBON, TYPES 316L AND 317L) [NOTE (8)]

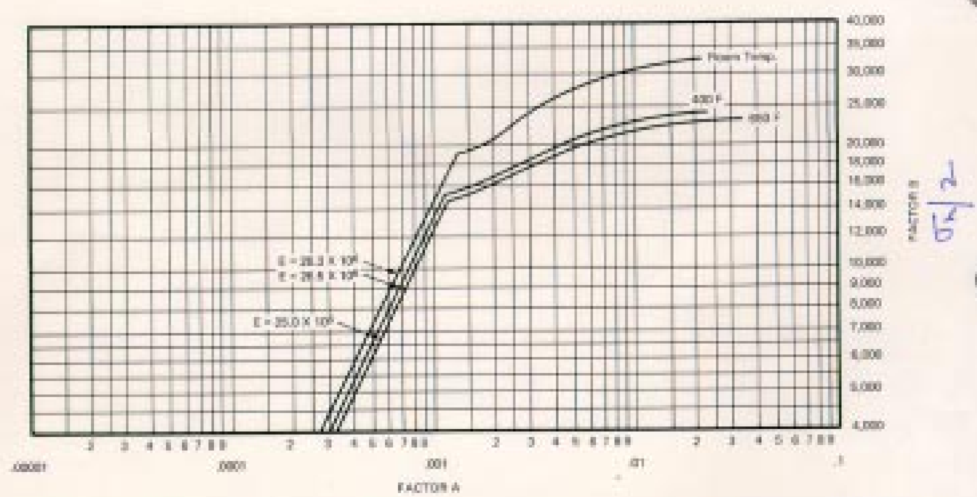


FIG. 5-UHA-28.5 CHART FOR DETERMINING SHELL THICKNESS OF CYLINDRICAL AND SPHERICAL VESSELS UNDER EXTERNAL PRESSURE WHEN CONSTRUCTED OF Cr-Ni-Mo ALLOY (S31500) SA-669