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REAL TIME RELIABILITY ADEQUACY TOOLS (Carlos Martinez, Coordinator)

Time-space methods for determining locational reserves: A Framework for Location-based Pricing and Scheduling for Reserve Markets

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I INTRODUCTION

Currently there are no well functioning reserve markets in use in the US. There is some evidence that a well functioning reserve market can help mitigate price spikes and solve the capacity problem. Currently reserves are thought of as having time dependent properties, that is, they must be spinning or able to synchronize in ten minutes or able to synchronize in thirty minutes. However, it is well known that given a network that can become constrained on voltage or real power flows, reserves must also be spatially located in order to handle all the contingencies that could occur. To date, there is no credible science-based method for assigning reserves in this way. Virtually all methods are ad-hoc and are based on engineering judgment and experience. The purpose of this work is to develop a sound basis and a methodology for assigning both real energy as well as reactive reserves.

In the arena of deregulated electric power markets, the concept of location-based marginal pricing (LBMP) is well-established and is commonly used to set electricity prices at a nodal level, making it possible to assign the cost of congestion to the locations that create it in the first place. However, in the reserve power markets, no standard methods for location-based pricing of reserves exist. Part of the problem lies in the fact that reserves must be sized and located to help the system survive major changes in the configuration of nodes with injections and changes in network topology due to line outages. Traditionally, within each control area a certain amount of operating reserve is required, the assumption being that all generators in a given area cooperate to configure the flows to maintain the system in operation. In a deregulated market, however, no cooperation exists and it may be possible that redispatching becomes extremely expensive. Thus, it is hard to identify predefined areas, each with its own reserve requirements.

This project is exploring several options for the solution of the reserve scheduling problem that are also compatible with the idea of a deregulated reserve market structure. One of the major questions is whether it is possible to devise a cost-minimizing scheduling algorithm for the spatially distributed reserve problem that reveals the location-based shadow prices for the reserve requirements. The constraints imposed by grid security considerations should be taken into account in the procedure. If this major question is answered in the affirmative, a framework for a reserve power market based on the computed shadow prices should be derived automatically from it.

II PROBLEM FORMULATION

Reserves are necessary to provide for contingencies and changes in load over future time intervals. Contingencies involving both lines and generators must be provided for. In terms of the optimization problem we consider a base case plus a list of other cases including contingencies and different loads. It is common to formulate the unit commitment problem with an index i for generating units and t for time slice. Here we substitute the index k for contingencies (possibly including the time slice).

Rather than the unit commitment formulation

$$\underset{U,G}{\mathrm{Min}}\left\{K(U)+\sum_{t=1}^{n_t}\sum_{i=1}^{N}u^{i,t}f(G_{i,t})\right\}$$

we suppress the unit commitment portion of the problem and concerns with ramp rates until section III and focus on the concept of reserves by attempting to minimize the expected cost

$$\underset{G,r}{\mathrm{Min}}\sum_k p_k\left\{\sum_{i=1}^{N}f(G_{ik})+r_{ik}(R_{ik}(G))\right\}$$

where p_k is the probability of the k^{th} contingency, G is a matrix of generator powers and R_{ik} is the reserves needed from generator i for contingency k. The dependence of the reserve matrix R on the generation matrix G is important in constructing a suitable algorithm. Consider a simple example with three generators, a base case and three contingencies as shown in Table 1.

	Generation			*Reserves*		
	Gen1	Gen2	Gen3	Gen1	Gen2	Gen3
Base	50	250	100	100	0	50
Contingency1	50	200	150	100	50	0
Contingency2	100	150	150	50	100	0
Contingency3	150	150	100	0	100	50

Table 1 A Simple Example

The generation for the base and for each contingency is found from an Optimal Power Flow (OPF). This isn't a requirement but does provide a good initial guess for the optimization. The three generators represent different situations in terms of reserves. G3 produces 100 MW of energy in the base case and is required to increase its output to 150 MW for two of the contingencies. If one of those contingencies should occur G3 will be paid for generating 150 MW. In the other two cases, the base and contingency 3, G3 is paid (presumably at some other rate) for 100 MW of energy and 50 MW of reserve. The case for G1 has the minimum output from G1 as the base case with maximum reserves of 100 MW. Contingencies 2 and 3 result in more output from G1, more energy payments and less reserve payment. G2 has maximum output in the base case with decreased output for all three contingencies. Note that if contingency 2, for example, happens G2 is required to limit its output to 200 MW. Since we might return to the base case after a contingency, we regard the 50 MW required to return G2 to 250 MW as a reserve. Thus there is a reserve pattern for each contingency. It is assumed that given a generation pattern G that the reserve matrix R, is computed from G. For the example,

$$G = \begin{bmatrix} 50 & 250 & 100 \\ 50 & 200 & 150 \\ 100 & 150 & 150 \\ 150 & 150 & 100 \end{bmatrix}, \ R = \begin{bmatrix} 100 & 0 & 50 \\ 100 & 50 & 0 \\ 50 & 100 & 0 \\ 0 & 100 & 50 \end{bmatrix}$$

In general R can be computed from G as follows: Let R1=max(G) be a row vector containing the maximum of each column of G,

For the example.

$$R1 = \begin{bmatrix} 150 & 250 & 150 \end{bmatrix}$$

Then, R= ones(K,1)*R1-G, i.e.,

$$R = \begin{bmatrix} 150 & 250 & 150 \\ 150 & 250 & 150 \\ 150 & 250 & 150 \\ 150 & 250 & 150 \end{bmatrix} - \begin{bmatrix} 50 & 250 & 100 \\ 50 & 200 & 150 \\ 100 & 150 & 150 \\ 150 & 150 & 100 \end{bmatrix}$$

Given any G a matrix R can then be computed. The OPF minimizes energy costs but does not consider the reserve cost. In the lossless transmission example note that the reserve definition produces 150 MW of reserve for the base case and each contingency. If the cost of reserves were the same for all generators then there would be a constant cost for reserves and the generation pattern would not affect the reserve cost. The new optimization is necessary, and there are locational reserve prices if there are losses and line constraints.

The optimization problem is to minimize the expected cost

$$\underset{G}{\mathrm{Min}} \sum_k p_k \left\{ \sum_{i=1}^{N} f(G_{ik}) + r_{ik}(R_{ik}(G)) \right\}$$

subject to line constraints and energy balance constraints. As an example consider the system in Figure 1 with the two tie lines (4-12 and 23-24) constrained to flows of 10 MVA and with offers from the six generators in the form of Figure 2. (note that f.(.) and r (.) are then piecewise linear) The optimization is performed with the additional constraint that for each generating unit the sum of generation and reserve is less than or equal to the unit's capacity. If DC load flow assumptions are used for the line constraints then the piecewise linear cost function can be manipulated so that a linear programming problem is produced. The alternative is to use AC power flows for the line constraints in order to incorporate reactive power and voltage issues into the reserve environment. The modified work statement (section IV) involves the development of software modeling the system in Figure 1 with offers as shown in Figure 2. At this size it seems reasonable that the AC power flows can be used even if this turns out not to be the case for very large systems

A numerical simulation was run to test the linear program version using the system shown in Figure 3 . The simulation involves a base-case and 30 line-out contingencies The contingencies were chosen such that if they occurred they would not cause the system to island. Figure 3 shows the optimal dispatch for each generator as a function of the contingencies. The "biggest" contingency is contingency #21, in which line No.28 (the line connecting bus 15 and bus 23) is out. In that case, Generator #6 must supply most of the demand in area 2 because of the tie-line limit. So the answer to the locational reserves problem for this case is that generator #6 must supply all the reserves. Clearly then, generator #6 has reserve market power and could offer any price (presumably a high one) into the reserve market and get it This is consistent with the situation observed in New York last year.

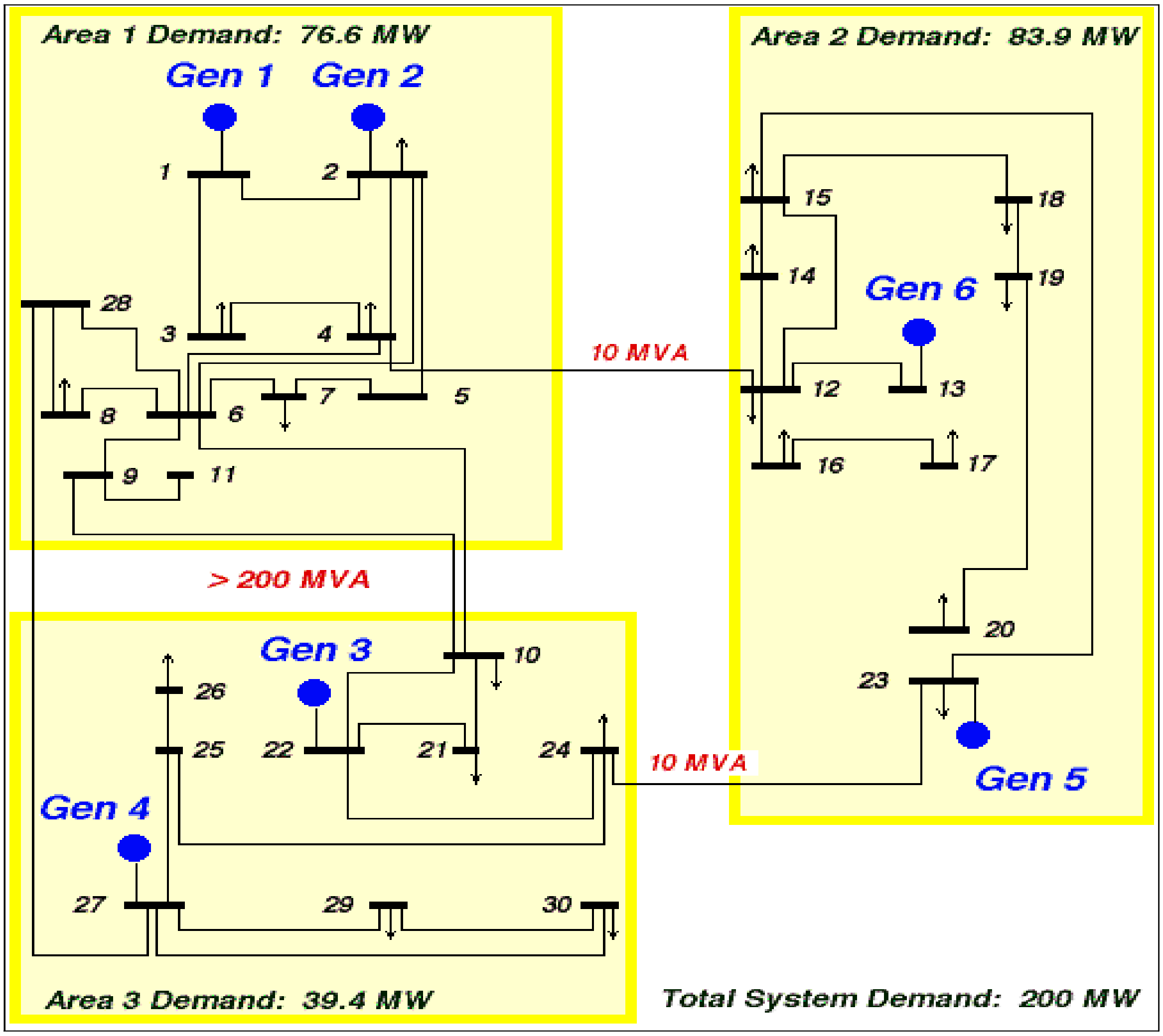

Figure 1. Sample System

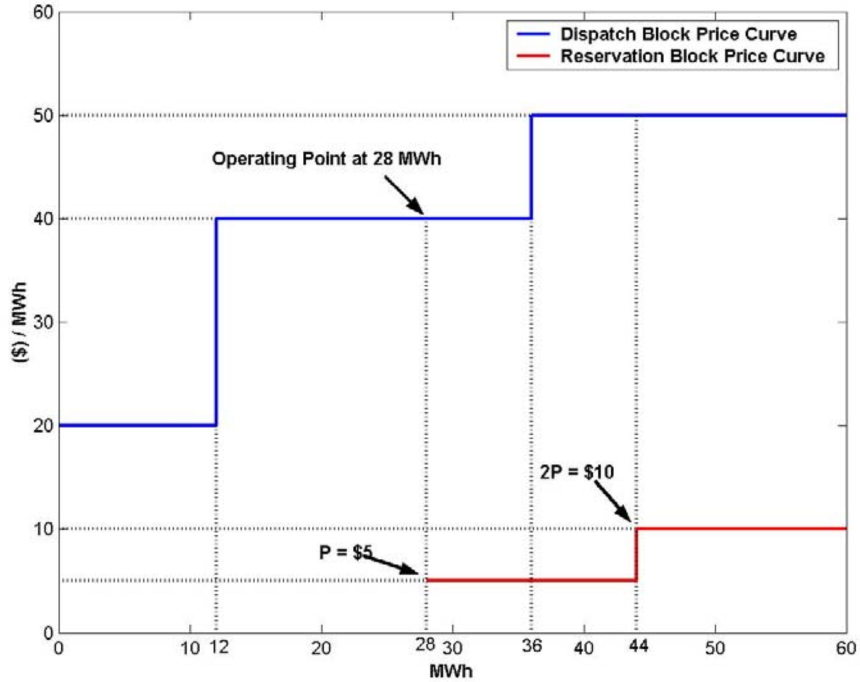

Figure 2. Block Prices for Energy and Reserves

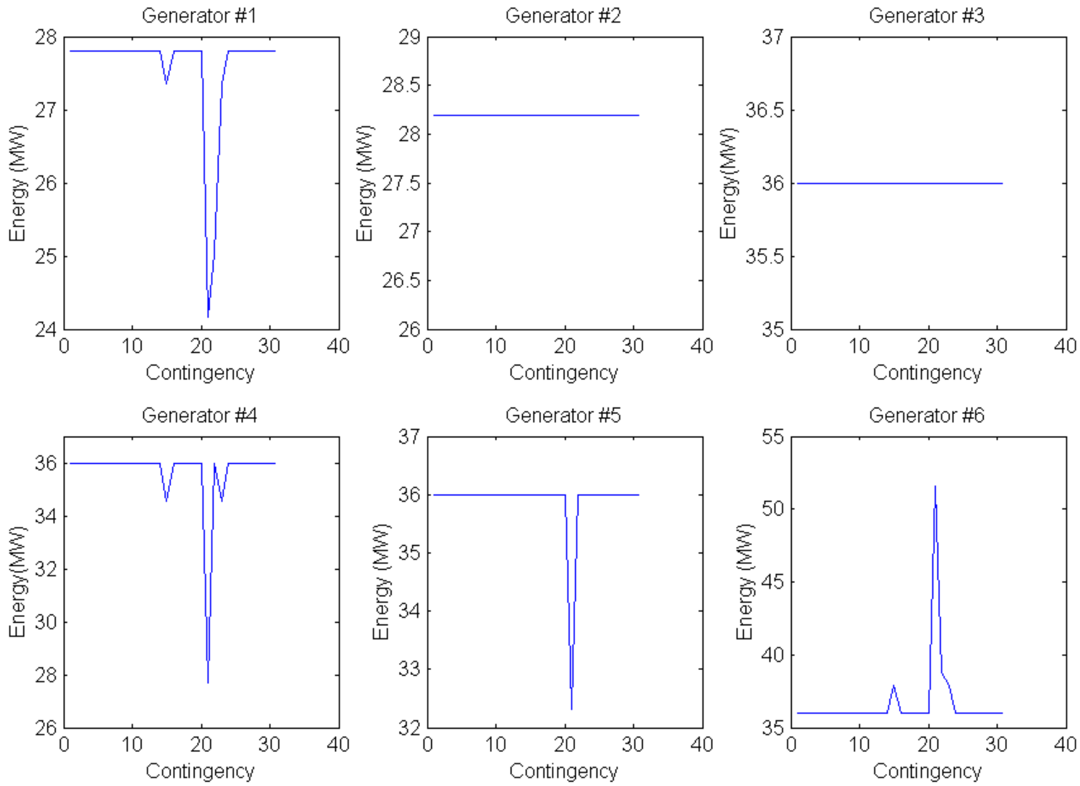

Figure 3: Optimal generation dispatch for the six generators in Figure 1 and 30 line contingencies

III. A Security-Constrained Unit Commitment Treatment

1 Introduction

As the computational power available to industry operators increases, it becomes possible to solve ever more complex problems at the production planning (or market clearing) stage. The basic economic dispatch problem has been refined through the decades to include loss approximations, hydrothermal coordination considerations, commitment decision integer variables, linear network flow models, nonlinear AC network flow models and so on, all the way up to integrated unit commitment-nonlinear optimal power flows as in [50]. This work is an attempt to incorporate security considerations in the form of post-contingency redispatch feasibility to the production planning problem. In doing so, it addresses the problem of finding the optimal geographic distribution of reserve, because a flawed choice of reserve distribution would result in an infeasible post-contingency redispatch.

Security-abiding dispatch has been an important topic in the industry, and even early papers such as [13] are still continuously referred to. Security considerations are now an integral part of commercial software, and continues to be an active are of research. Both probabilistic and deterministic methods abound. Deterministic methods usually consider specific contingencies and provide a test by running power flows. If the resulting power flow solution is inadequate the nominal dispatch is modified. In the planning stage, it is possible to include some restrictions so that the resulting dispatch is more secure [10, 13, 19]. Some rules of thumb have been adopted in the operational procedures advocated by several technical bodies, such as having a certain percentage of available reserve. In solving the planning problem, sometimes it is forgotten that these rules of thumb are there to ensure safe operation and they are just included in the model without taking a look at actual security of the system [38, 39, 43], or, if security is inspected it is usually in a diagnostic mode [44, 44, 46]. In this sense, there hasn't been a lot of improvement since the early works [10, 13, 19]. Now, the need arises to dispatch reserve in a secure manner in a market environment, which implies a price-clearing procedure as well. This work tries to solve the security problem first and then prices can be determined using extra procedures.

2 The Base Problem

Consider a network description given by

$$I = B\theta + I_o \tag{1}$$

where $I \in \mathbb{R}^{n_b}$ is the vector of bus power injections, $\theta \in \mathbb{R}^{n_b}$ are the bus voltage angles, B describes the "nominal" network topology and I_o is a "quiescent" power injection due to non-zero branch flows when any phase shifter angles are different from zero. Consider

$$I = G - D = M_G(u * P_g) - P_D \tag{2}$$

where G, $D \in \mathbb{R}^{n_b}$ are net generation and demand at each bus, $P_g \in \mathbb{R}^{n_g}$ are the generators' power outputs, P_D is the vector of bus loads, and $M_G \in \mathbb{R}^{n_b \times n_g}$ is a "placement" matrix that assigns a given generator's output to the bus where the generator is placed. Also, u is a vector of 0-1 variables representing each generator's commitment status and $*$ is the element-wise multiplication of two vectors. We consider n_t time periods and when appropriate the indexes i and t are used to refer to generator number or a particular time period respectively. Then, an unit commitment plus optimal flow problem (UCOPF) with this network model is stated as

$$\min_{u,P_g} \quad u^T f(P_g) + K(u) \tag{3}$$

such that

$$\begin{aligned}
u &\in \mathcal{U} && (4)\\
u^{i,t} P^i_{min} \leq \; P_g^{i,t} &\leq u^{i,t} P^i_{max}, \; i = 1 \ldots n_g, t = 1 \ldots n_t && (5)\\
M_G(u^t * P_g^t) - P_D^t &= B\theta^t + I_o^t, \; t = 1 \ldots n_t && (6)\\
-L \leq \; C\theta^t &\leq L, \; t = 1 \ldots n_t && (7)
\end{aligned}$$

where f and K are separable cost functions of the decision variables, $\mathcal{U}$ is the set of feasible commitment schedules that respects individual generators' minimal up and down times, P_{min} and P_{max} represent generation operating limits, C models individual line flows and L are the thermal limits for all branches. Now consider a "post-contingency" system in which a branch or a generator has gone offline. We are not going to consider the dynamic behavior of the system immediately after the contingency and before it settles in a new equilibrium. Rather, we assume that the system survives the transient and the time period until the (usually automatic) redispatch takes place, and that the problem to be solved is that a feasible redispatch of the system must exist for all contingencies being considered.

Let $\hat{B}$, $\hat{I}_o$, $\hat{M}_G$, $\hat{P}_{min}$, $\hat{P}_{max}$, $\hat{C}$ and $\hat{L}$ model the new topology and limits, and define the post-contingency state variables and their increments from the nominal case

$$\hat{P}_g = P_g + \Delta P_g, \quad \hat{\theta} = \theta + \Delta\theta \tag{8}$$

Then, the post-contingency state variables must satisfy, in every time period

$$\begin{aligned}
\hat{M}_G(u * \hat{P}_g) - \hat{P}_D &= \hat{B}(\theta + \Delta\theta) + \hat{I}_o && (9)\\
u * P_{min} \leq \; \hat{P}_g &\leq u * P_{max} && (10)\\
-\hat{L} \leq \; \hat{C}\hat{\theta} &\leq \hat{L} && (11)\\
u * \hat{P}^- \leq \; \Delta P_g &\leq u * \hat{P}^+ && (12)
\end{aligned}$$

with P^- and P^+ being limits placed on the incremental injections after a contingency. These limits can be zero if a given generator is not available for redispatch, or can be limited by ramp rate restrictions or by stability considerations.

If the post-contingency state variables and their constraints are now incorporated into the original UC problem, yielding a problem described by (3-12), the resulting mathematical program is a Security Constrained Unit Commitment with Optimal Flow problem (SCUCOPF) which takes into account a single contingency. More eventualities can be accounted for by including the post-contingency state variables and constraints to the original formulation. For example, if a second contingency is being considered, another set of post-contingency variables ΔP_g" and $\Delta\theta$" and another constraint characterization set, similar to (9-12) but written in terms of the second post-contingency variables will be added to the formulation, which now would include two post-contingency sets of variables and sets of constraints in addition to the base dispatch problem. For the time being, we restrict our treatment to the one-contingency case to avoid the very cumbersome notation that would result, in the understanding that the mechanism for considering more contingencies is conceptually straightforward, even if it does increase the size of the problem by adding more variables and constraints.

A note regarding the nature of the post-contingency flow is in order. Following a generator outage, for example, other generators used for load following will increase their output until frequency recovers and all zonal transfers achieve their prescribed setpoints. If we want to model this behavior in the post-contingency flow, we run into a more complex problem. Let us ellaborate: Within the zone of the failed generator, the subset of units that implement load-following usually pick up a share of the lost generation according to pre-specified participation factors. If those units are also being considered for the unit commitment decision, then we have a problem in which the participation factors depend on the commitment schedule. Participation factors are linear restrictions on the incremental share assigned to each load-following generator. If they are tied to the unit commitment schedule, they become in effect different (in cardinality and in coefficients) linear constraints depending on the choice of u. Let us ellaborate: Suppose that three generators G1–G3 in a given area are being considered for commitment. If , say, G1 and G2 are chosen, then the load-pickup participation factors impose the restriction $\Delta P_g^1 = \Delta P_g^2 = 50\%$, which can be modeled using one linear post-contingency restriction. If all three units are committed, then the postcontigency restriction is $\Delta P_g^1 = \Delta P_g^2 = \Delta P_g^3 = 33\%$, which can be modeled using two linear post-contingency restrictions. Thus, the number of post-contingency constraints, their coefficients and structure are dependent on the commitment decision, which is a difficult situation to model. If, on the other hand, we knew ahead of time which load-following units are selected (i.e., prescribed to be turned on as part of the problem data) then the portion of generation that each would pick up could easily be incorporated into the post-contingency flow by choice of $\hat{P}^+$. It is only when load-following units are also available for commitment choice that we have a more complex problem. More on this topic after the formulation for the problem and solution method are addressed.

3 Lagrangian Relaxation Solution

Lagrangian Relaxation algorithms have been very popular for the solution of the mixed-integer unit commitment problem since they were introduced in the power production planning literature in [17]. Before, the dominant technique was dynamic programming [1]; see [14] for a thorough review of the state of the art prior to the advent of Lagrangian Relaxation, and [4, 7, 9, 10, 16, 22] to sample the historical progression of the dynamic programming treatment.

The Lagrangian relaxation technique, first used in the operations research community to attack mixed-integer problems [6, 11, 12, 15], yields optimization algorithms that attack the dual functional derived from the Lagrangian when the non-convex constraints are relaxed with multipliers. Historically, they are derived from earlier works on smooth dual optimization [2, 3]. The salient features of these algorithms are twofold: (1) the dual maximization is straightforward because the subgradients with respect to the dual variables are readily obtained, and (2) the evaluation of the dual functional, itself a minimization problem on the primal variables, has a separable structure that greatly simplifies its computation and that cuts into the combinatoric complexity exhibited by other solution techniques such as straight dynamic programming. In the past 25 years, the technique first been proved to be appropriate (in fact, performing better) for large-scale problems [23, 24]; it has been improved by inclusion of linear network flow models [27]; refined to include improved convergence properties [28]; augmented with ever more kinds of constraints [30, 35, 39], and even used to integrate the thermal unit commitment problem and the AC, nonlinear optimal power flow problem [47, 50].

To solve the SCUCOPF problem, we can resort to a variable duplication, Augmented Lagrangian Relaxation-like algorithm of the sort first reported in [28]. This kind of algorithm has been widely used in the literature and it rests on two basic pillars: a formulation using two copies of the optimization variables—a trick that reduces the number of constraints to be relaxed, while still permitting the separability of the resulting dual objective, and reformulation of the dual optimization process using the Auxiliary Problem Principle developed in [18, 20, 25]. The basic methodology in [28] has been further developed in [29, 31] and in [47] and [50].

Consider two sets of variables both representing the power outputs of the generators, s and d, together with the original commitment decision variables u. Proceed to rewrite the problem statement using the two sets of variables

$$\min_{u,d,s} u^T f(d) + K(u) \tag{13}$$

such that, for every time period $t = 1 \ldots n_t$ and every generator $i = 1 \ldots. n_g$

$$\begin{aligned} M_G s - P_D &= B\theta + I_o && (14)\\ -L \leq\ C\theta &\leq L && (15)\\ u &\in \mathcal{U} && (16)\\ u * P_{min} \leq\ d &\leq u * P_{max} && (17)\\ s - u * d &= 0 && (18)\\ \hat{d} &= d + \Delta d && (19)\\ \hat{s} &= s + \Delta s && (20)\\ \hat{\theta} &= \theta + \Delta\theta && (21) \end{aligned}$$

$$\hat{M}_g\hat{s} - \hat{P}_D \quad = \quad \hat{B}(\theta + \Delta\theta) + \hat{I}_o \tag{22}$$

$$-\hat{L} \leq \quad \hat{C}\hat{\theta} \quad \leq \hat{L} \tag{23}$$

$$u * P_{min} \leq \quad \hat{d} \quad \leq u * P_{max} \tag{24}$$

$$u * \hat{P}^- \leq \quad \Delta d \quad \leq u * \hat{P}^+ \tag{25}$$

$$u * \Delta d = \Delta s \quad \text{or} \quad u * \hat{d} = \hat{s} \tag{26}$$

Note that (14-18) are the restrictions for the base problem, with (14-15) being the network restrictions, (16-17) being the generator-wise dynamic restrictions and (18) being the equality of the d and s sets of variables. Then, (19-21) are definitions for postcontingency quantities, (22-23) are static (i.e., network) post-contingency restrictions, (24-25) are dynamic post-contingency restrictions, and (26) is the equality of the two sets of post-contingency variables. We call constraints (16,17,19,24) and (25) the $\mathcal{D}$ constraints, and constraints (14,15,20,21,22) and (23) the $\mathcal{S}$ constraints. The remaining constraints, (18) and (26) are the *coupling* constraints. It is rather important to note that the $\mathcal{D}$ constraints are expressed only in terms of $(u, d, \hat{d})$ while the $\mathcal{S}$ constraints are expressed only in terms of $(s, \hat{s}, \theta, \hat{\theta})$. Thus, we use $D \in \mathcal{D}$, with $D = (u, d, \hat{d})$ to represent feasible $(u, d, \hat{d})$ in light of the $\mathcal{D}$ constraints and $S \in \mathcal{S}$ to represent feasible $S = (s, \hat{s}, \theta, \hat{\theta})$ in light of the $\mathcal{S}$ constraints. There is even more structure in $\mathcal{D}$ and $\mathcal{S}$: The constraints in $\mathcal{D}$ group several time periods, but only one generator because the only constraints that span more than one time period are in (16). Similarly, the constraints in $\mathcal{S}$ span all generators and angles, but only one time period. This is important because it will result in problems in the D variables that are separable by generator and problems in the S variables that are separable by time period.

The next step is to write a Lagrangian for this problem, relaxing only the equalities (18) and (26) with multipliers λ and β:

$$\begin{aligned}
\mathcal{L}(u, d, s, \hat{d}, \hat{s}, \lambda, \beta) \; = & \\
& \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}[u^{i,t}f^i(d^{i,t}) + K^{i,t}(u^{i,\cdot})] \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\lambda^{i,t}(s^{i,t} - u^{i,t}d^{i,t}) \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\beta^{i,t}(\hat{s}i,t - u^{i,t}\hat{d}^{i,t}) & (27) \\
= \; & \sum_{i=1}^{n_g}\sum_{t=1}^{n_t}\left\{u^{i,t}f^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot}) - \lambda^{i,t}u^{i,t}d^{i,t} - \beta^{i,t}u^{i,t}\hat{d}^{i,t}\right\} \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}(\lambda^{i,t}s^{i,t} + \beta^{i,t}\hat{s}^{i,t}) & (28) \\
= \; & \mathcal{L}_1(D, \lambda, \beta) + \mathcal{L}_2(S, \lambda, \beta) & (29)
\end{aligned}$$

A dual objective can now be defined as follows:

$$\begin{aligned}
q(\lambda, \beta) \; &= \; \min_{D\in\mathcal{D}, S\in\mathcal{S}} [\mathcal{L}_1(D, \lambda, \beta) + \mathcal{L}_2(S, \lambda, \beta)] \\
&= \; \min_{D\in\mathcal{D}} \mathcal{L}_1(D, \lambda, \beta) + \min_{S\in\mathcal{S}} \mathcal{L}_2(S, \lambda, \beta) \qquad (30)
\end{aligned}$$

Thus, this dual objective can be computed by solving two separate minimization problems. Furthermore, the minimization of $\mathcal{L}_1$ can be further separated by generator, resulting in n_g dynamic programs, one for each generator, and the minimization of $\mathcal{L}_2$ can be further separated by time period, resulting in n_t security-constrained optimal power flows. Then, this objective can be maximized by adjusting (λ, β) according to a subgradient method; the subgradient can be readily computed from (27).

An algorithm could actually be proposed at this point, except for one fault: the Lagrangian is unstable because the cost reflected in the dynamic program subproblems for the $\hat{d}$ variables is linear, not strictly convex. The standard regularization technique involves using strictly convex penalty functions of the relaxed constraints, resulting in the following Lagrangian:

$$
\begin{aligned}
\mathcal{L}(u, d, s, \hat{d}, \hat{s}, \lambda, \beta) \quad = & \\
& \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} [u^{i,t} f^i(d^{i,t}) + K^{i,t}(u^{i,\cdot})] \\
& + \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} \lambda^{i,t}(s^{i,t} - u^{i,t} d^{i,t}) \\
& + \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} \beta^{i,t}(\hat{s}^{i,t} - u^{i,t} \hat{d}^{i,t}) \qquad (31) \\
& + \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} \frac{c}{2}(s^{i,t} - u^{i,t} d^{i,t})^2 \\
& + \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} \frac{c}{2}(\hat{s}i, t - u^{i,t} \hat{d}^{i,t}) \qquad (32)
\end{aligned}
$$

Unfortunately, this augmented Lagrangian is not separable because there are cross products of variables that need to be separated. Here is where the framework of the Auxiliary Problem Principle is invoked, which, in the context of an iterative algorithm, allows us to substitute the augmentation terms by the following at the kth iteration:

$$
\begin{aligned}
& \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} c(s_{k-1}^{i,t} - u_{k-1}^{i,t} d_{k-1}^{i,t})(s^{i,t} - u^{i,t} d^{i,t}) \\
& + \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} \frac{b}{2}\{(s^{i,t} - s_{k-1}^{i,t})^2 + (u^{i,t} d^{i,t} - u_{k-1}^{i,t} d_{k-1}^{i,t})^2\} \\
& + \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} c(\hat{s}_{k-1}^{i,t} - u_{k-1}^{i,t} \hat{d}_{k-1}^{i,t})(\hat{s}^{i,t} - u^{i,t} \hat{d}^{i,t}) \\
& + \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} \frac{b}{2}\{(\hat{s}^{i,t} - \hat{s}_{k-1}^{i,t})^2 + (u^{i,t} \hat{d}^{i,t} - u_{k-1}^{i,t} \hat{d}_{k-1}^{i,t})^2\} \qquad (33)
\end{aligned}
$$

where the $(k-1)$ subindexes denote the values obtained in the previous dual iteration. Convergence in the convex case has been proved for $b \geq 2c$ with a limit on the step size depending on c [18, 25]. The resulting Lagrangian can be rearranged in the following manner:

$$
\mathcal{L}(u, d, s, \hat{d}, \hat{s}, \lambda_{k-1}, \beta_{k-1}) \; =
$$

$$\begin{aligned}
& \sum_{i=1}^{n_g}\sum_{t=1}^{n_t}\left\{\begin{array}{c} u^{i,t}f^i(d^{i,t}) + K^{i,t}(u^i) \\ +\frac{b}{2}(u^{i,t}d^{i,t})^2 - (\lambda_{k-1}^{i,t} + c\Delta_{k-1}^{i,t} + bu_{k-1}^{i,t}d_{k-1}^{i,t})u^{i,t}d^{i,t} \\ +\frac{b}{2}(u^{i,t}\hat{d}^{i,t})^2 - (\beta_{k-1}^{i,t} + c\hat{\Delta}_{k-1}^{i,t} + bu_{k-1}^{i,t}\hat{d}_{k-1}^{i,t})u^{i,t}\hat{d}^{i,t} \end{array}\right\} \\
+ & \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\left\{\begin{array}{c} \frac{b}{2}(s^{i,t})^2 + (\lambda_{k-1}^{i,t} + c\Delta_{k-1}^{i,t} - bs_{k-1}^{i,t})s^{i,t} \\ +\frac{b}{2}(\hat{s}^{i,t})^2 + (\beta_{k-1}^{i,t} + c\hat{\Delta}_{k-1}^{i,t} - b\hat{s}_{k-1}^{i,t})\hat{s}^{i,t} \end{array}\right\} \\
+ & \frac{b}{2}\left\{(s_{k-1}^{i,t})^2 + (u_{k-1}^{i,t}d_{k-1}^{i,t})^2 + (\hat{s}_{k-1}^{i,t})^2 + (u_{k-1}^{i,t}\hat{d}_{k-1}^{i,t})^2\right\} \qquad (34)
\end{aligned}$$

with $\Delta_{k-1}^{i,t} = s_{k-1}^{i,t} - u_{k-1}^{i,t}d_{k-1}^{i,t}$ and $\hat{\Delta}_{k-1}^{i,t} = \hat{s}_{k-1}^{i,t} - u_{k-1}^{i,t}\hat{d}_{k-1}^{i,t}$. Then, the Lagrangian becomes separable with the following components in the resulting dual program:

1. A security-augmented optimal power flow for each time period on the $(s, \theta, \hat{s}, \Delta\theta)$ variables with quadratic costs associated to the penalty function terms, the relaxation terms and with restrictions (14,15,22), and (23).

2. Several dynamic programs, one for each generator, on the variables $(u, d, \hat{d})$, with costs factored both from the original cost function and the augmentation and relaxation terms, together with constraints (16,17,24), and (25).

3. Multipliers that accompany the relaxed constraints (18) and (26), and which can be updated using a simple subgradient rule in order to maximize the dual, i.e.

$$\begin{aligned}
\lambda_k &= \lambda_{k-1} + \alpha_k\Delta_{k-1} \\
\beta_k &= \beta_{k-1} + \alpha_k\hat{\Delta}_{k-1}
\end{aligned}$$

At this point, the resulting dual optimization algorithm exhibits the separation properties that we expect from a Lagrangian relaxation treatment.

3.1 Observations

It is appropriate to make several observations now.

1. The algorithm involves the solutions, at each dual iteration, of potentially very large security-constrained optimal linear power flows, one for each time-period. These problems are quadratic programs with linear constraints, which can be solved efficiently by modern interior point methods such as those implemented in *BPMPD* [32]. The dynamic programming subproblems are relatively simple and include the logic to enforce the minimum up and down-time constraints [23, 47]; no special software is required to solve them. Also note that in principle, it is possible to include a nonlinear AC network flow model using the same formulation, as in [47, 50]. However, the very large nonlinear optimal flow problems that would result when we include several post-contingency sets of variables would be difficult to solve with current OPF technology for more than just a few contingencies. As large-scale nonlinear optimization techniques continue to evolve, it may be possible to include the full AC power flow in the formulation.

2. The β multipliers at the solutions will convey information about the price tag attached to making the dispatch secure against a given contingency. This is useful information that can be used to price reserve.

3. At this point, it is appropriate to continue the discussion on the load-following units. The conclusion so far is that the linear restrictions that are needed to enforce fixed participation factors in the post-contingency load flow change in coefficients and cardinality when such units are considered for the commitment decision-a rather difficult problem. However, it is actually possible to enforce such restrictions on the "dynamic" variables by resorting to joint dynamic programs involving all such units within a zone. Indeed, when the dynamic programs are joined, when computing the cost for, say $(u_1 = 1, u_2 = 0, u_3 = 1)$ it is easy to include the restriction that $\Delta d_1 = \Delta d_3$. However, since the state space of the resulting joint dynamic program is the cartesian product of the state spaces for the single-generator programs, this method can become impractical if there are many load-following generators being considered for commitment in a given zone. Thus, further consideration of the specific commitment scenario is needed. Let us examine several possibilities. First consider the case of a central dispatcher in charge of a large geographic area which is broken in zones for historical reasons or perhaps because the instrumentation required for the implementation of load-following is already zonified, so that each zone has at most a few load-following generators which are also being considered for unit commitment (as opposed to pre-committed). Then the joint dynamic program approach might be feasible. Consider now a second scenario: If there is no technical reason to discount a system-wide load following mechanism, perhaps it would be wiser to avoid the fixed participation factor scheme and use instead all available load-following capacity; this would take the problematic proportional restrictions out of the picture and make use of individual limits on Δs_i, which are already considered in the formulation. If, on the other hand, we insist on using any proportional allocation rules for load following on a large set of units that are also being considered for the UC decision, we are again facing the large joint dynamic program problem as a fundamental issue.

   It is suggested that at this point we assume that the situation is best described by the second scenario above and that we avoid the joint dynamic programs altogether in the treatment, unless consultation with industry deems them necessary.

3.2 Progress report

Coding of the algorithm continues and is reaching completion. It is presently in a debugging stage. The data organization and the engine to build the constraints describing the many possible contingencies have proved a challenge, but it should soon be possible to make some test runs using nontrivial systems. There is no reason why the algorithm would not work, being that this treatment has worked for theoretically more complicated problems such as the nonlinear flow + unit commitment case. No convergence problems are expected.

References

[1] R.E. Bellman, *Dynamic Programming*, Princeton University Press, Princeton, New Jersey, 1957.

[2] K.J. Arrow & L. Hurwicz, "Gradient Method for Concave Programming, I: Local Results", in *Studies in Linear and Nonlinear Programming, K.J. Arrow, L. Hurwicz & H. Uzawa, Stanford University Press, 1958.*

[3] H. Uzawa, "Iterative Methods for Concave Programming", in *Studies in Linear and Nonlinear Programming, K.J. Arrow, L. Hurwicz & H. Uzawa, Stanford University Press, 1958.*

[4] W.H. Osterle, J. Geiser, K.M. Dale & C.A. DeSalvo, "Computer Selection of Generating Units to be Operated, Part II: by Dynamic Programming on a Large Computer", presented at the PICA Conference, St. Louis, Mo., November 9–11, 1960.

[5] J. Carpentier, "Contribution à 'l Étude du Dispatching Économique", *Bulletin de la Societé Française des Electriciens*, Vol. 3, August 1962, pp. 431–447.

[6] H. Everett, "Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources", *Operations Research*, Vol. 11, 1963, pp. 399–417.

[7] P.G. Lowery, "Generating Unit Commitment by Dynamic Programming", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-85, May 1966, pp. 422–426.

[8] J.A. Muckstadt & R.C. Wilson, "An Application of Mixed-Integer Programming Duality to Scheduling Thermal Generating Systems", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-87, No. 12, December 1968, pp. 1968–1978.

[9] A.K. Ayoub & A.D. Patton, "Optimal Thermal Generating Unit Commitment", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-90, No. 4, July/August 1971, pp. 1752–1756.

[10] J.D. Guy, "Security Constrained Unit Commitment", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-90, No. 3, May–June 1971, pp. 1385–1390.

[11] M. Fisher, "Optimal Solution of Scheduling Problems Using Lagrange Multipliers: Part I", *Operations Research*, Vol. 21, No. 5, September-October 1973, pp. 1114–1127.

[12] A.M. Geoffrion, "Lagrangean Relaxation for Integer Programming", in *Mathematical Programming Study 2*, North-Holland, 1974, pp. 82–114.

[13] O. Alsac & B. Stott, "Optimal Load Flow with Steady-State Security", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS 93, No. 3, 1974, pp. 745–751.

[14] J. Gruhl, F. Schweppe & M. Ruane, "Unit Commitment Scheduling of Electric Power Systems", in *Systems Engineering for Power: Status and Prospects*, Henniker, New Hampshire, pp. 116–129, August 17–22, 1975.

[15] M.L. Fisher, W.D. Northup & J.F. Shapiro, "Using Duality to Solve Discrete Optimization Problems: Theory and Computational Experience", *Mathematical Programming Study 3*, North-Holland, 1975, pp. 56–94.

[16] C.K. Pang & H.C. Chen, "Optimal Short-Term Thermal Unit Commitment", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-95, No. 4, July/August 1976, pp. 1336–1346.

[17] J. A. Muckstadt & S.A. Koenig, "An Application of Lagrange Relaxation to Scheduling in Power–Generation Systems", *Operations Research*, Vol. 25, No. 3, May–June 1977, pp. 387–403.

[18] G. Cohen, "Optimization by Decomposition and Coordination: A Unified Approach", *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 2, April 1978, pp. 222-232.

[19] B. Stott, J.L. Marinho & O. Alsac, "Review of Linear Programming Applied to Power System Rescheduling", *Proceedings of the 1979 Power Industry Computer Applications Conference*, pp. 142–154.

[20] G. Cohen, "Auxiliary Problem Principle and Decomposition of Optimization Problems", *Journal of Optimization Theory and Applications*, Vol. 32, No. 3, November 1980, pp. 277–305.

[21] M.L. Fisher, "The Lagrangian Relaxation method for Solving Integer Programming Problems", *Management Science*, Col. 27 No. 1, January 1981, pp. 1–18.

[22] C.K. Pang, G.B. Sheblé & F. Albuyeh, "Evaluation of Dynamic Programming Based Methods and Multiple Area Representation for Thermal Unit Commitments", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-100, No. 3, March 1981, pp. 1212–1218.

[23] G.S. Lauer, D.P. Bertsekas, N.R. Sandell & T.A. Posbergh, "Solution of Large–Scale Optimal Unit Commitment Problems", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-101, No. 1, January 1982, pp. 79–85.

[24] D.P. Bertsekas, G.S. Lauer, N.R. Sandell & T.A. Posbergh, "Optimal Short-Term Scheduling of Large-Scale Power Systems", *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 1, January 1983, pp. 1–11.

[25] G. Cohen & D.L. Zhu, "Decomposition Coordination Methods in Large Scale Optimization Problems: The Nondifferentiable Case and the Use of Augmented Lagrangians", in *Advances in Large Scale Systems, Vol. 1; J.B. Cruz, Ed.*, JAI Press Inc, 1984, pp. 203–266.

[26] A.I. Cohen & V.R. Sherkat, "Optimization-Based Methods for Operations Scheduling", *Proceedings of the IEEE*, Vol. 75, No. 12, December 1987, pp. 1574–1591.

[27] S. Ružić & N. Rajaković, "A New Approach for Solving Extended Unit Commitment Problem", *IEEE Transactions on Power Systems*, Vol. 6, No. 1, February 1991, pp. 269–277.

[28] J. Batut & A. Renaud, "Daily Generation Scheduling Optimization with Transmission Constraints: A New Class of Algorithms", IEEE Transactions on Power Systems, Vol. 7, No. 3, August 1992, pp. 982–989.

[29] R. Baldick, "The Generalized Unit Commitment Problem", *IEEE Transactions on Power Systems*, Vol. 10, No. 1, February 1995, pp. 465–475.

[30] W.L. Peterson & S.R. Brammer, "A Capacity Based Lagrangian Relaxation Unit Commitment with Ramp Rate Constraints", *IEEE Transactions on Power Systems*, Vol. 10, No. 2, May 1995, pp. 1077–1084.

[31] P. Carpentier, G. Cohen, J.C. Culioli & A. Renaud, "Stochastic Optimization of Unit Commitment: A New Decomposition Framework", *IEEE Transactions on Power Systems*, Vol. 11, No. 2, May 1996, pp. 1067–1073.

[32] Cs. Mészáros, "The efficient implementation of interior point methods for linear programming and their applications", Ph.D. Thesis, Eötvös Loránd University of Sciences, 1996.

[33] H. Ma & S.M. Shahidehpour, "Decomposition Approach to Unit Commitment with Reactive Constraints", *IEE Proceedings—Generation, Transmission and Distribution*, Vol. 144, No. 2, March 1997, pp. 113–117.

[34] B.A. Murtagh & M.A. Saunders, "MINOS 5.5 User's Guide", *Stanford University Systems Optimization Laboratory Technical Report SOL 83-20R.*

[35] A.J. Svoboda, C.L. Tseng, C.A. Li & R.B. Johnson, "Short-Term Resource Scheduling with Ramp Constraints", *IEEE Transactions on Power Systems*, Vol. 12, No. 1, February 1997, pp. 77–83.

[36] C.L. Tseng, S.S. Oren, A.J. Svoboda & R.B. Johnson, "A Unit Decommitment Method in Power Systems Scheduling", *Electrical Power & Energy Systems*, Vol. 19, No. 6, June 1997, pp. 357–365.

[37] R. Zimmerman & D. Gan, "*MATPOWER:* A Matlab Power System Simulation Package", *http://www.pserc.cornell.edu/matpower/* .

[38] M. Aganagic, K.H. Abdul-Rahman & J.G. Wright, "Spot Pricing of Capacities for Generation and Transmission of Reserve in an Extended Poolco Model" *IEEE Transactions on Power Systems*, Vol. 13, No. 3, August 1998.

[39] A.G. Bakirtzis, "Joint Energy and Reserve Dispatch in a Competitive Pool Using Lagrangian Relaxation", *IEEE Power Engineering Review*, November, 1998.

[40] J. Zollweg, "MultiMATLAB for NT Clusters", Cornell Theory Center Software Documentation, *http://www.tc.cornell.edu/UserDoc/Intel/Software/ multimatlab/*.

[41] C.L. Tseng, S.S. Oren, A.J. Svoboda & R.B. Johnson, "Price-Based Adaptive Spinning Reserve Requirements in Power System Scheduling", *Electrical Power & Energy Systems*, Vol. 21, pp. 137–145, 1999.

[42] L. Pereira & D. DeBerry, "The Role of Less-Economic Generating Units in the Open Access Transmission Era", *IEEE Transactions on Power Systems*, Vol. 14, No. 3, August 1999.

[43] X. Ma, D. Sun & K. Cheung, "Energy and Reserve Dispatch in a Multi-Zone Electricity Market", *IEEE Transactions on Power Systems*, Vol. 14, No. 3, August 1999.

[44] S. Halilčević & F. Gubina, "Optimal Space Distribution of Generation Ready-Reserve Capacity", *IEEE Transactions on Power Systems*, Vol. 14, No. 4, November 1999.

[45] S. Halilčević & F. Gubina, "An On-Line Determination of the Ready Reserve Power", *IEEE Transactions on Power Systems*, Vol. 14, No. 4, November 1999.

[46] H.B. Gooi, D.P. Mendes, K.R.W. Bell & D.S. Kirschen, "Optimal Scheduling of Spinning Reserve", *IEEE Transactions on Power Systems*, Vol. 14, No. 4, November 1999.

[47] C.E. Murillo-Sánchez, "On the Integration of Unit Commitment and Optimal Power Flow", Ph. D. Thesis, School of Electrical Engineering, Cornell University, 1999.

[48] N. Alguacil & A.J. Conejo, "Multiperiod Optimal Power Flow Using Benders Decomposition", *IEEE Transacions on Power Systems*, Vol. 15, No. 1, February 2000.

[49] K. Xie, Y.H. Song, J. Stonham, E. Yu & G. Liu, "Decomposition Model and Interior Point Methods for Optimal Spot Pricing of Electricity in Deregulation Environments", *IEEE Transacions on Power Systems*, Vol. 15, No. 1, February 2000.

[50] C.E. Murillo-Sánchez & R.J. Thomas, "Thermal Unit Commitment with a Nonlinear AC Power Flow Network Model", in *The Next Generation of Electric Power Unit Commitment Models*, B.F. Hobbs, M.H. Rothkopf, R.P. O'Neill and H.P. Chao, Eds., Kluwer Academic Publishers, 2001.

IV WORKPLAN.

The modified workplan is:

Task 1. Develop a combined economic and technical model for New York State Thomas. Jan 1, 2001 - July 1, 2001

Task 2. Formulate the reserve commitment problem as an optimization problem. Thomas and Thorp April 1, 2001 - Sept 1, 2001

a) Conceptually as a nonlinear (stochastic) optimization problem
b) Approximate linear problem formulation

The problem is related to the unit commitment problem but must include a set of credible contingencies. The value of an accurate linearization is that some form of Linear Programming could be used and the dual variables obtained from the solution could yield marginal prices.

Task 3. Develop algorithms to solve the reserve commitment problem. Thorp. Sept 1, 2001 - March 31, 2002

Task 4. Experimental testing of multidimensional energy/reserve markets using the software and human decision makers. Thomas and Thorp. Jan 2002- Dec 2002

Task 5. Develop techniques for reducing the number of contingencies used in the reserve calculation. Thomas July 2001- Sept 2002

Task 6. Test and modify the algorithms on the New York model. Sept 2002 - Dec 2002

Task 7, Prepare final report Thomas and Thorp Sept 1, 2002 - Dec 31, 2002