Open Questions – The Big Picture (I)

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Tentative Outline for this Lecture

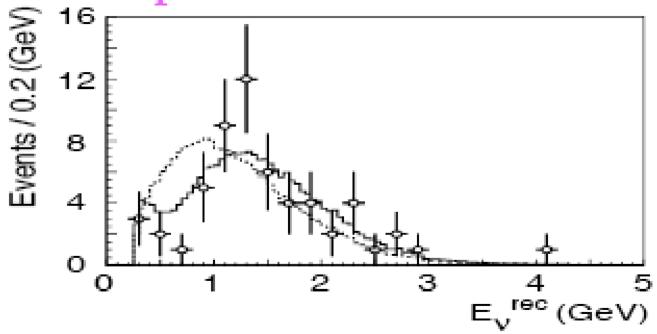
1. Brief Summary of What You Have Learned;
2. Prequel: Solar Neutrino "Oscillations";
3. Current Phenomenological Understanding of Neutral Leptons;
4. What We Know We Don't Know – Next-Generation ν Oscillations;
5. Hunting for θ_{13};
6. The Neutrino Mass Hierarchy;
7. CP-Violation in the Neutrino Sector.

[note: Questions are ALWAYS welcome]

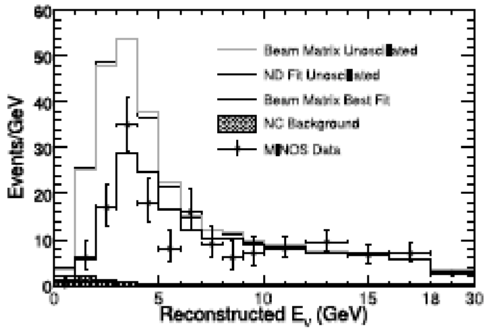
1 - Our Story So Far . . .

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

K2K 2004: spectral distortion

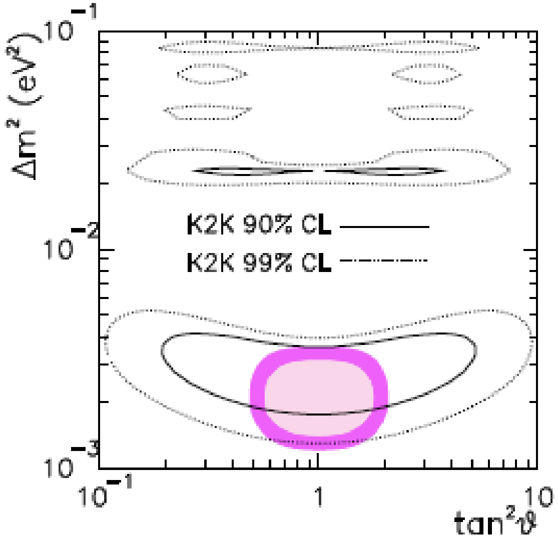


MINOS 2006: spectral distortion



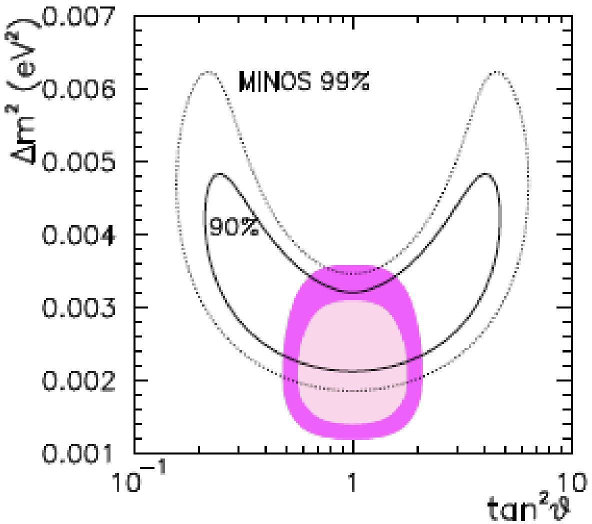
$$1 - P_{\mu\mu} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Confirmation of ATM oscillations



[Gonzalez-Garcia, PASI 2006]

Confirmation of ATM oscillations



Before We Can Talk About Solar Neutrinos, I'll Review

Two-Flavor Oscillation in the Presence of Matter [Boris]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix}=\left[\frac{\Delta m^2}{2E}\begin{pmatrix}\sin^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \cos^2\theta\end{pmatrix}+\begin{pmatrix}A & 0\\ 0 & 0\end{pmatrix}\right]\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix},$$

$A=\pm\sqrt{2}G_FN_e$ (+ for neutrinos, − for antineutrinos) → Matter Potential.

Note: Similar effect from neutral current interactions common to all (active) neutrino species → proportional to the identity.

In general, this is hard to solve, as A is a function of L: two-level non-relativistc quantum mechanical system in the presence of time dependent potential.

In some cases, however, the solution is rather simple.

Constant A: good approximation for neutrinos propagating through matter inside the Earth [exception: neutrinos that see Earth's internal structure (the crust, the mantle, the outer core, the inner core)]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} A & \Delta/2\sin 2\theta \\ \Delta/2\sin 2\theta & \Delta\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}, \quad \Delta \equiv \Delta m^2/2E.$$

$$P_{e\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta_M L}{2}\right),$$

where

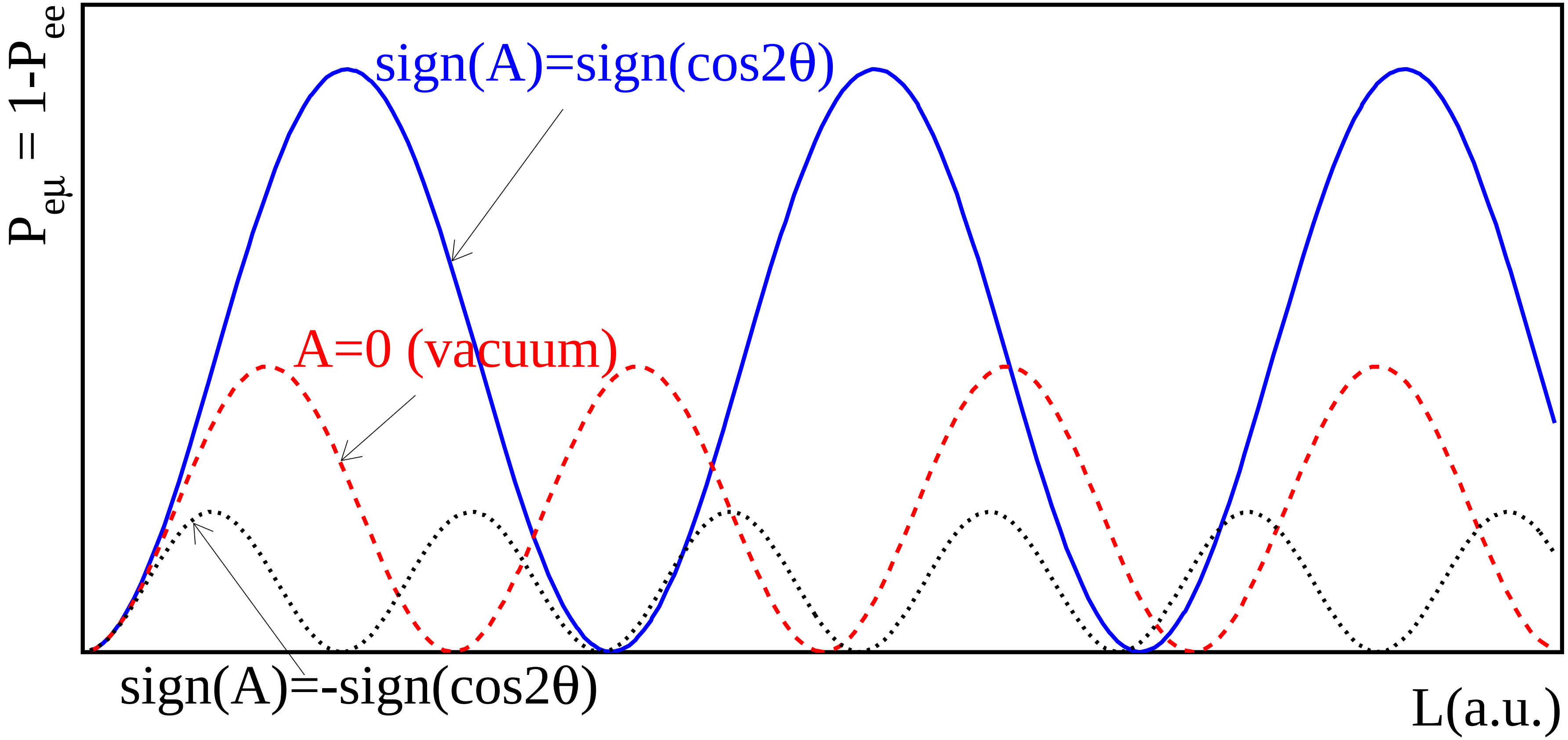
$$\begin{aligned} \Delta_M &= \sqrt{(A-\Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta}, \\ \Delta_M \sin 2\theta_M &= \Delta\sin 2\theta, \\ \Delta_M \cos 2\theta_M &= A - \Delta\cos 2\theta. \end{aligned}$$

The presence of matter affects neutrino and antineutrino oscillation differently.

Nothing wrong with this: CPT-theorem relates the propagation of neutrinos in an electron background to the propagation of antineutrinos in a positron background.

Enlarged parameter space in the presence of matter effects.

For example, can tell whether $\cos 2\theta$ is positive or negative.



2 - PREQUEL: Solar Neutrinos and The MSW Effect

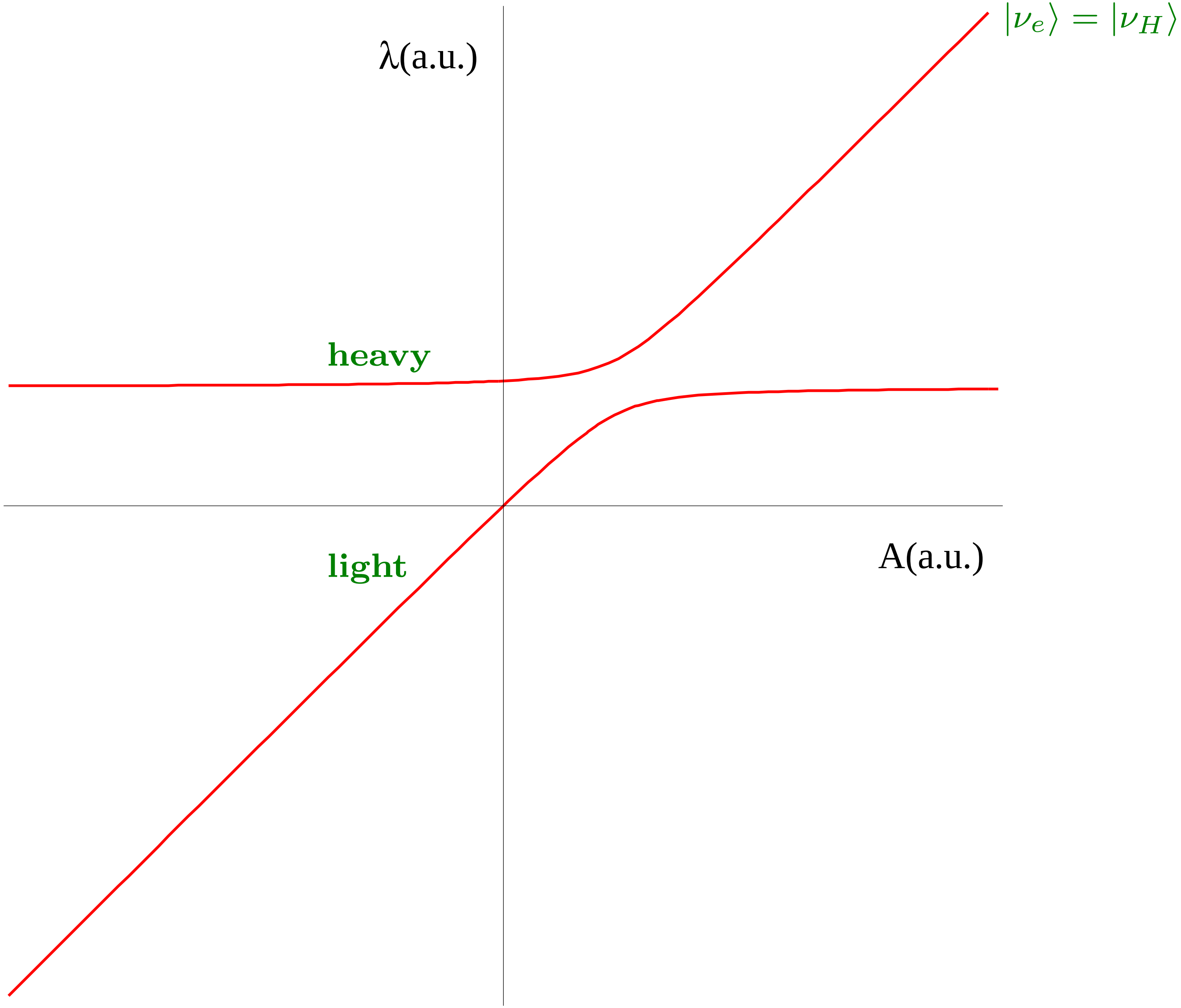
Curiously enough, the oldest neutrino puzzle is the one that is most subtle to explain. This is because solar neutrinos traverse a strongly varying matter density on their way from the center of the Sun to the surface of the Earth.

For the Hamiltonian

$$\left[\Delta\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)+A\left(\begin{array}{cc}1 & 0 \\ 0 & 0\end{array}\right)\right],$$

it is easy to compute the eigenvalues as a function of A:

(remember, $\Delta = \Delta m^2/2E$)



λ(a.u.)
$|\nu_e\rangle = |\nu_H\rangle$
heavy
light
A(a.u.)

A decreases "slowly" as a function of L $\Rightarrow$ system evolves adiabatically.

$$|\nu_e\rangle = |\nu_{2M}\rangle \text{ at the core } \rightarrow |\nu_2\rangle \text{ in vacuum,}$$

$$P_{ee}^{\rm Earth} = |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta.$$

Note that $P_{ee} \simeq \sin^2\theta$ applies in a wide range of energies and baselines, as long as the approximations mentioned above apply —ideal to explain the energy independent suppression of the ^{8}B solar neutrino flux!

Furthermore, large average suppressions of the neutrino flux are allowed if $\sin^2\theta \ll 1$. Compare with $\bar{P}_{ee}^{\rm vac} = 1 - 1/2\sin^2 2\theta > 1/2$.

One can expand on the result above by loosening some of the assumptions. $|\nu_e\rangle$ state is produced in the Sun's core as an *incoherent* mixture of $|\nu_{1M}\rangle$ and $|\nu_{2M}\rangle$. Introduce adiabaticity parameter P_c, which measures the probability that a $|\nu_{iM}\rangle$ matter Hamiltonian state will *not* exit the Sun as a $|\nu_i\rangle$ mass-eigenstate.

$$\begin{aligned} |\nu_e\rangle &\rightarrow |\nu_{1M}\rangle, \text{ with probability } \cos^2\theta_M, \\ &\rightarrow |\nu_{2M}\rangle, \text{ with probability } \sin^2\theta_M, \end{aligned}$$

where θ_M is the matter angle at the neutrino **production point**.

$$\begin{aligned} |\nu_{1M}\rangle &\rightarrow |\nu_1\rangle, \text{ with probability } (1-P_c), \\ &\rightarrow |\nu_2\rangle, \text{ with probability } P_c, \\ |\nu_{2M}\rangle &\rightarrow |\nu_1\rangle \text{ with probability } P_c, \\ &\rightarrow |\nu_2\rangle \text{ with probability } (1-P_c). \end{aligned}$$

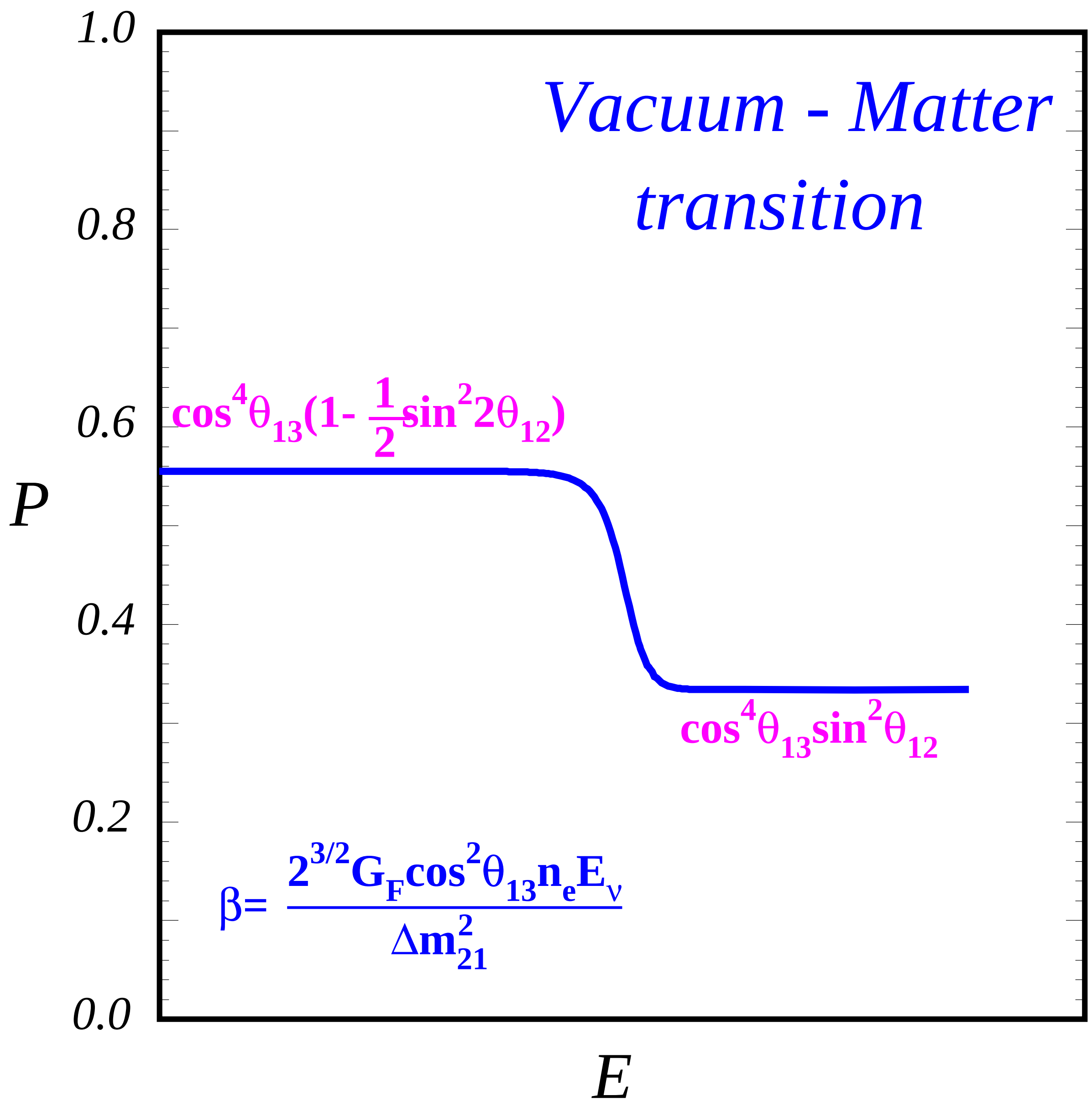
$P_{1e} = \cos^2\theta$ and $P_{2e} = \sin^2\theta$ so

$$\begin{aligned} P_{ee}^{\rm Sun} = &\cos^2\theta_M\left[(1-P_c)\cos^2\theta + P_c\sin^2\theta\right] \\ &+\sin^2\theta_M\left[P_c\cos^2\theta + (1-P_c)\sin^2\theta\right]. \end{aligned}$$

For $N_e = N_{e0}e^{-L/r_0}$, P_c, (crossing probability) is exactly calculable

$$P_c = \frac{e^{-\gamma\sin^2\theta} - e^{-\gamma}}{1-e^{-\gamma}}, \quad \gamma = 2\pi r_0\Delta. \tag{1}$$

Adiabatic condition: $\gamma \gg 1$, so that $P_c \rightarrow 0$.



We need:

- $P_{ee} \sim 0.3$ (^{8}B neutrinos)
- $P_{ee} \sim 0.6$ (^{7}Be, pp neutrinos)

$\Rightarrow \quad \sin^2\theta \sim 0.3$

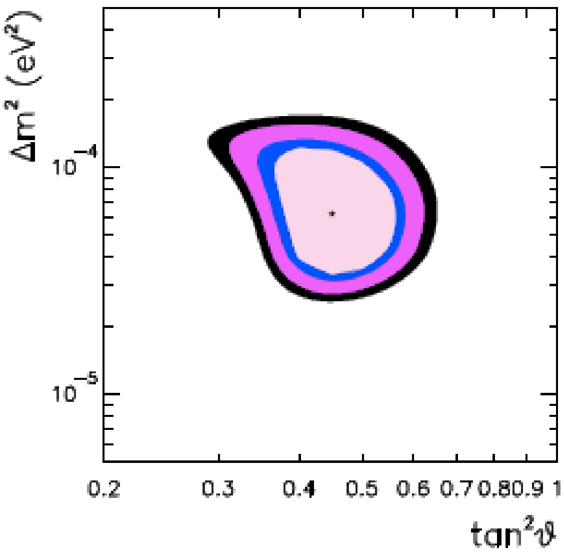
$\Rightarrow \quad \Delta m^2 \sim 10^{-(5 \text{ to } 4)}$ eV^2

for a long time, there were many other options!

(LMA, LOW, SMA, VAC)

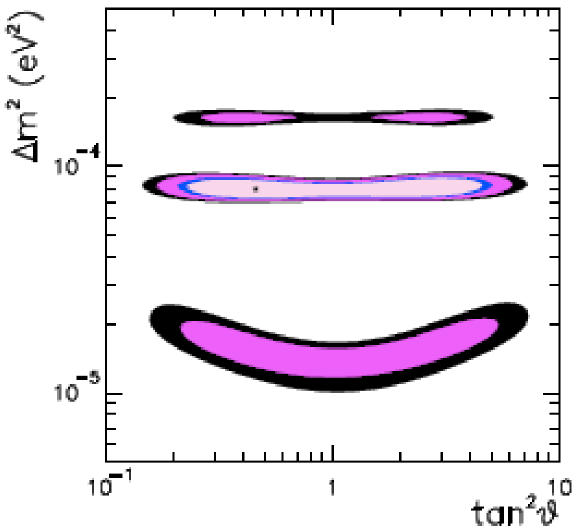
Solar

$\nu_e \rightarrow \nu_{\rm active}$

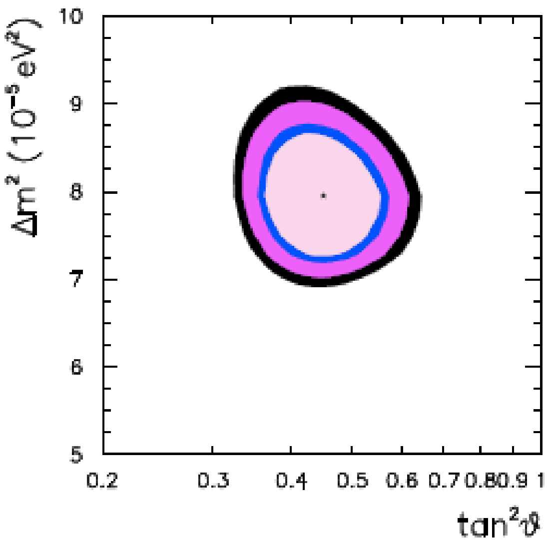


+ KamLAND

$\bar{\nu}_e \not\rightarrow \bar{\nu}_e$



ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$



$$\Delta m^2_{\odot} = (8^{+0.4}_{-0.5}) \times 10^{-5}\ \mathrm{eV}^2 \quad (1\sigma)$$
$$\tan^2\theta_{\odot} = 0.45^{+0.05}_{-0.05}$$

[Gonzalez-Garcia, PASI 2006]

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of two-flavor neutrino oscillations:

- **solar**: $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4}$ eV^2, $\sin^2\theta \sim 0.3$.
- **atmospheric**: $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.5$ ("maximal mixing").

3 - Putting it all together – 3 flavor mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ — $\Delta m^2_{13} < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ — $\Delta m^2_{13} > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

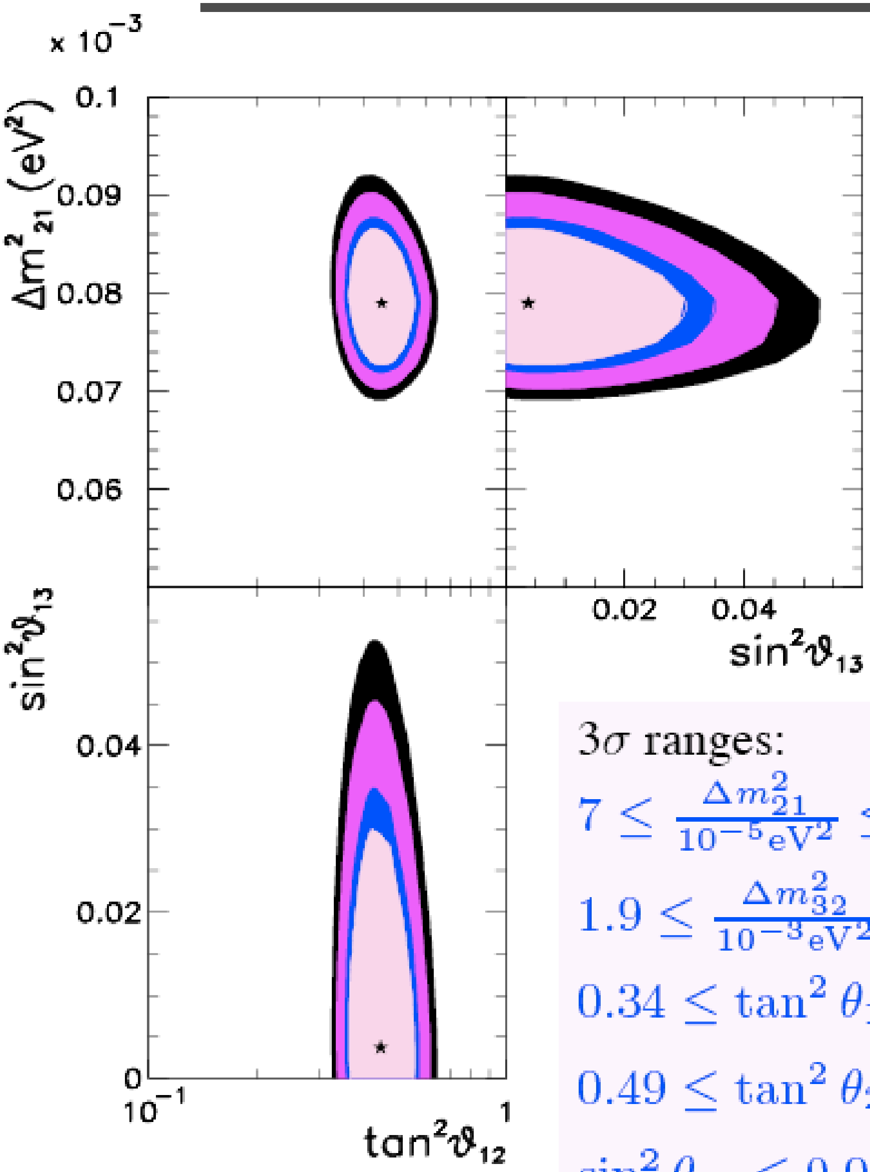
It Turns Out That ...

- Two Mass-Squared Differences Are Hierarchical, $\Delta m^2_{12} \ll |\Delta m^2_{13}|$;
- One of The Mixing Angles Turns Out to Be Small, $\sin^2\theta_{13} < 0.04$;

$\Rightarrow$ Two Puzzles Decouple, and Two-Flavor Interpretation Captures Almost All the Physics:

- Atmospheric Neutrinos Determine $|\Delta m^2_{13}|$ and θ_{23};
- Solar Neutrinos Determine Δm^2_{12} and θ_{12}.

(small θ_{13} guarantees that $|\Delta m^2_{13}|$ effects governing electron neutrinos are small, while $\Delta m^2_{12} \ll |\Delta m^2_{13}|$ guarantees that Δm^2_{12} effects are small at atmospheric and accelerator experiments).



3σ ranges:

$7 \leq \frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2} \leq 9.1$

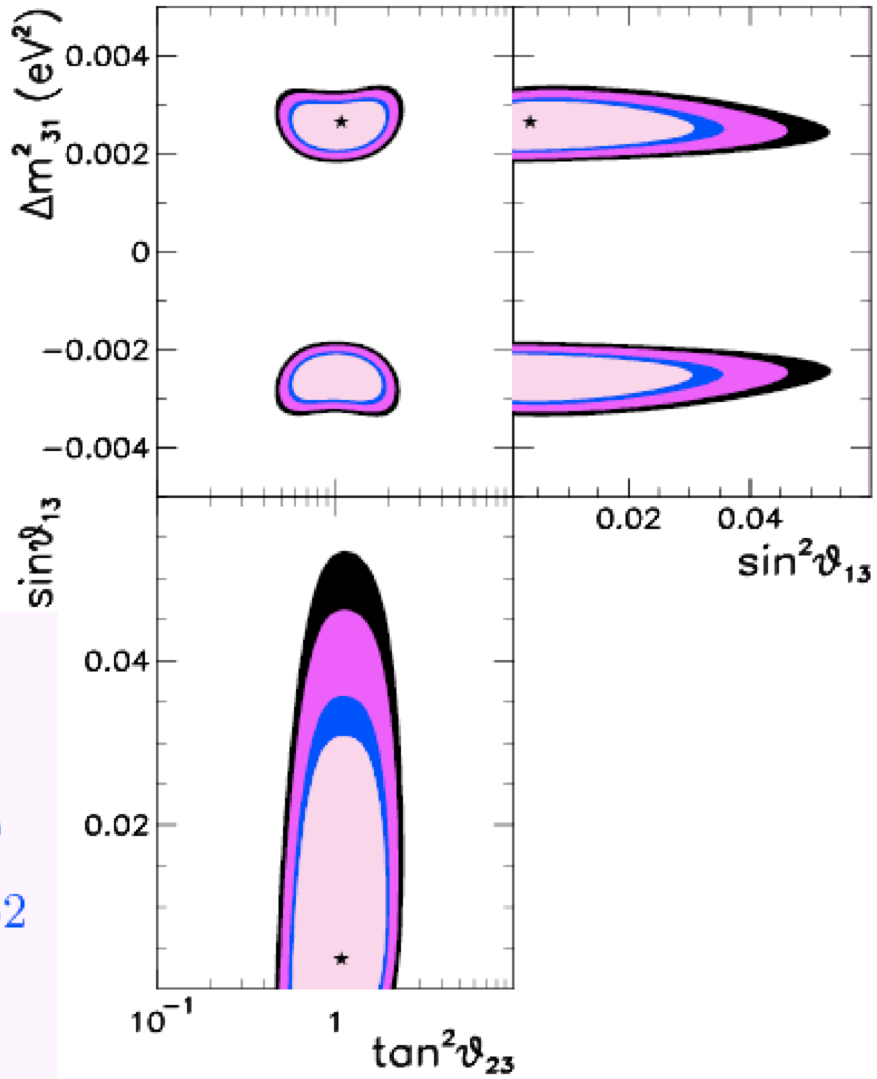
$1.9 \leq \frac{\Delta m^2_{32}}{10^{-3}\mathrm{eV}^2} \leq 3.25$

$0.34 \leq \tan^2\theta_{12} \leq 0.62$

$0.49 \leq \tan^2\theta_{23} \leq 2.2$

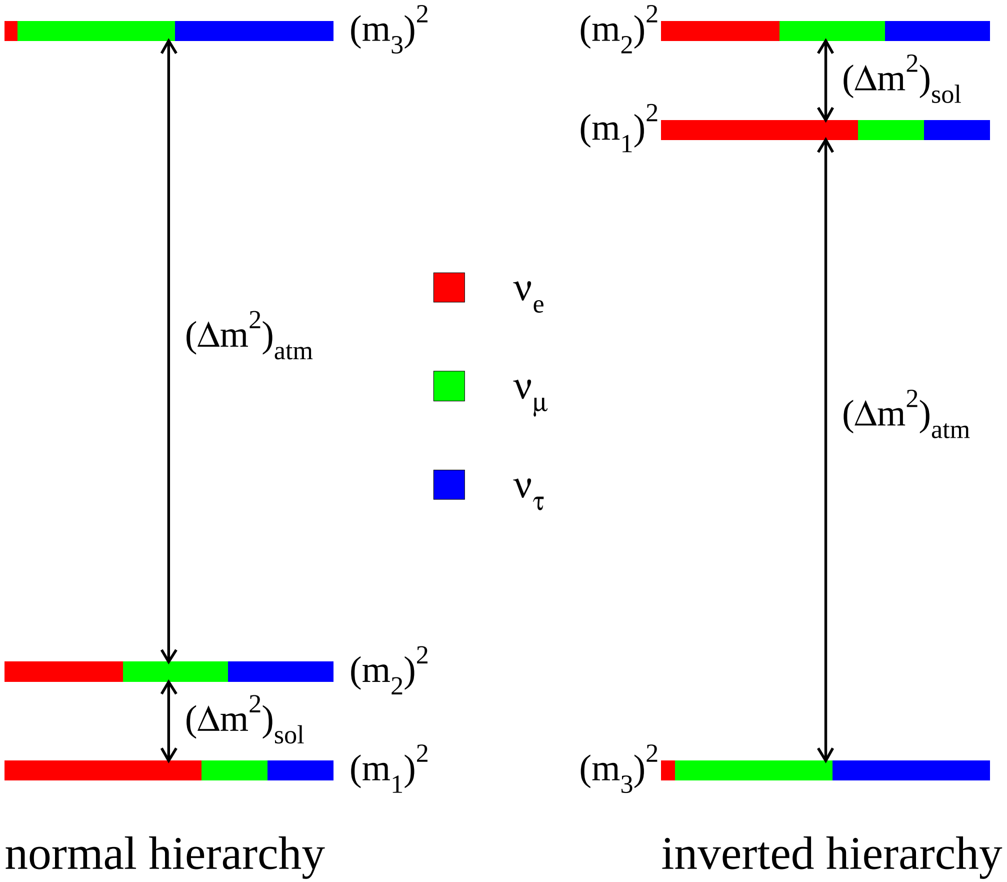
$\sin^2\theta_{13} \leq 0.045$

$-\pi \leq \delta \leq \pi$



[Gonzalez-Garcia, PASI 2006]

4 - What We Know We Don't Know, Oscillation Edition



- What is the ν_e component of ν_3? ($\theta_{13} \neq 0$?)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m^2_{13} > 0$?)

$\Rightarrow$ All of these can be addressed in neutrino oscillation experiments **if** we get lucky, that is if θ_{13} is large enough.

Why Do We Care About Answering These Questions?

1. Because they are there!

2. Fundamental pieces of the flavor puzzle – what are the values of fermion masses and mixing angles trying to tell us about fundamental physics?

3. "Guaranteed" access to new form of CP-invariance violation. Window to matter–antimatter asymmetry of the Universe?

4. This is what Nature seems to be telling us to do. We have recently uncovered a new phenomenon – mass-driven neutrino oscillations – that led to a new picture of the lepton sector of the standard model. We need to fill in the blanks in order to make sure that the picture is complete!

5. – [your favorite reason here] –

5 - Hunting For θ_{13} (or U_{e3})

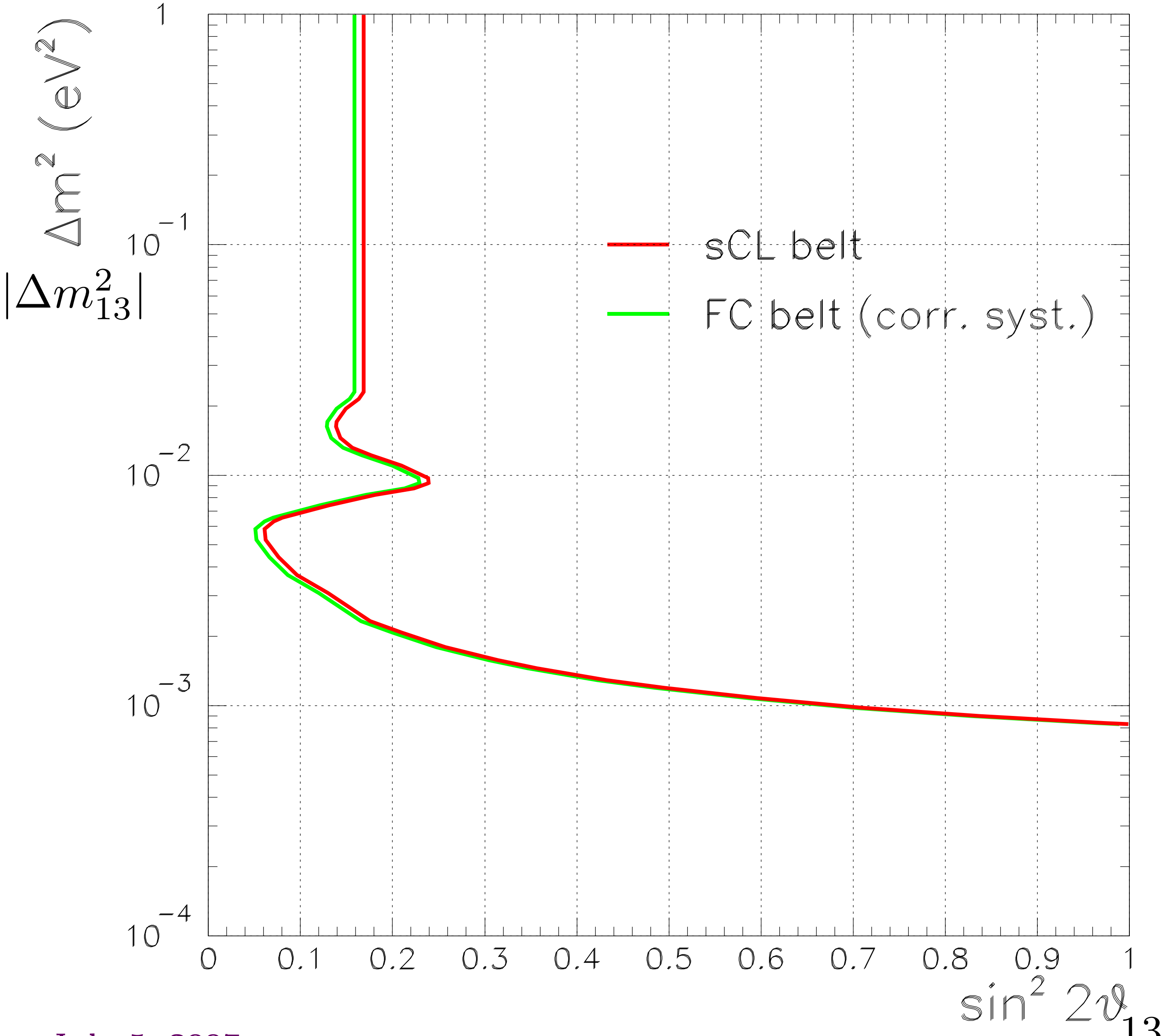
The best way to hunt for θ_{13} is to look for oscillation effects involving electron (anti)neutrinos, governed by the atmospheric oscillation frequency, Δm^2_{13} (other possibility, precision measurement of ν_μ disappearance. . .).

One way to understand this is to notice that if $\theta_{13} \equiv 0$, the ν_e state only participates in processes involving Δm^2_{12}.

Example – "short" baseline reactor neutrino experiments:

$$P_{ee} \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + O\left(\frac{\Delta m^2_{12}}{\Delta m^2_{13}}\right)$$

Reactor Neutrino Searches for θ_{13}



- $L \sim 1$ km
- $E_\nu \sim 5$ MeV

next-generation: aim at improving CHOOZ bound by an order of magnitude.

e.g. Double CHOOZ,
Daya Bay, etc

[lectures by Mike Shaevitz]

$\nu_\mu \leftrightarrow \nu_e$ at Long-Baseline Experiments

REQUIREMENTS: ν_μ beam, detector capable of seeing electron appearance. This is the case of "Superbeam Experiments" like T2K and NOνA. [D. Harris]

or

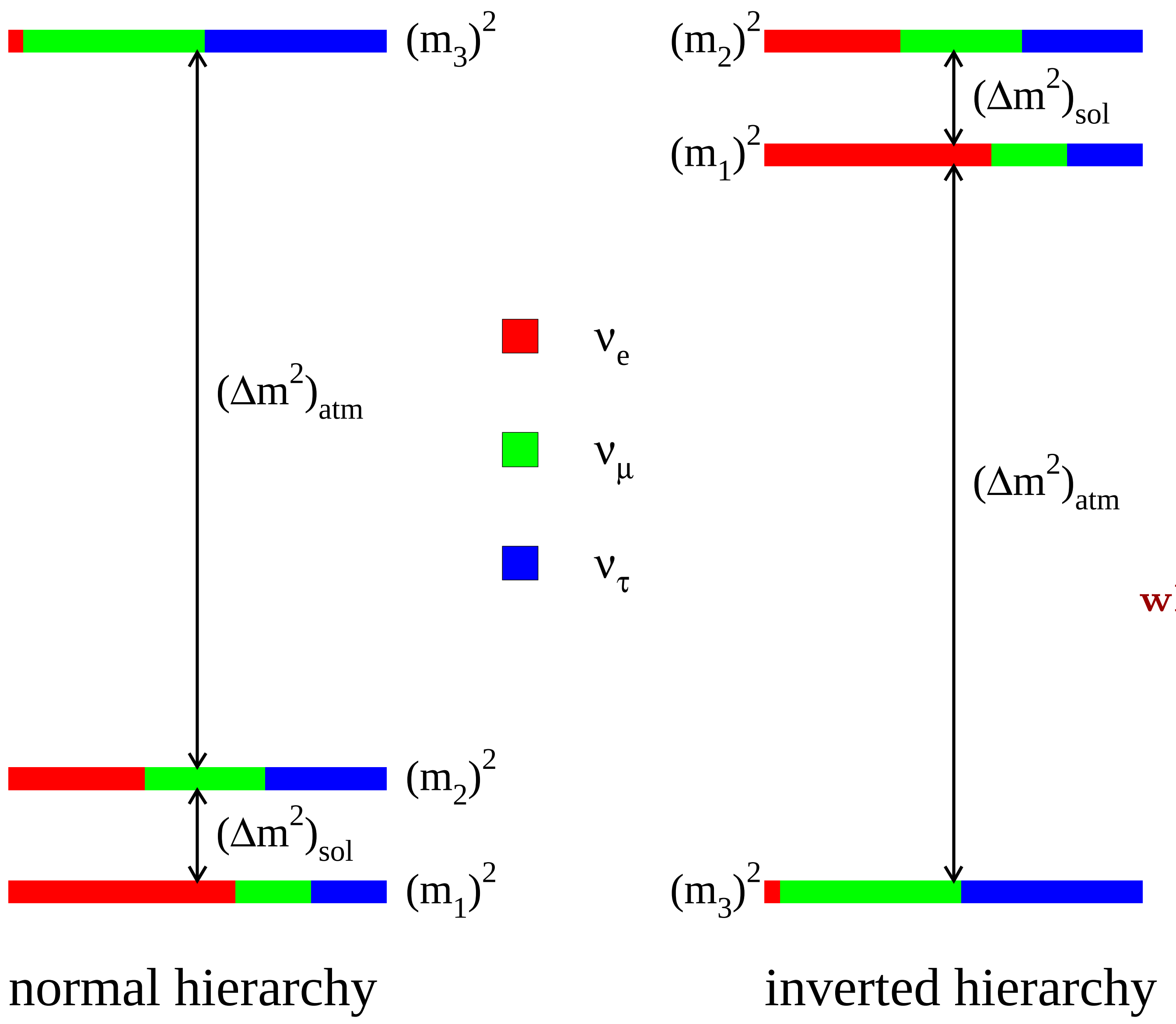
ν_e beam and detector capable of detecting muons (usually including sign). This would be the case of "Neutrino Factories" ($\mu^+ \to e^+\bar{\nu}_\mu\nu_e$) and "Beta Beams" ($Z \to (Z\pm 1)e^\mp\nu_e$).

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ "subleading"}.$$

- Sensitivity to $\sin^2\theta_{13}$. More precisely, $\sin^2\theta_{23}\sin^2 2\theta_{13}$. This leads to one potential degeneracy.

6 - The Neutrino Mass Hierarchy



which is the right picture?

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm^2_{13} comes from atmospheric neutrino experiments (SuperK). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + \text{ subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm^2_{13}.

On the other hand, because $|U_{e3}|^2 < 0.05$ and $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} < 0.06$ are both small, we are yet to observe the subleading effects.

Determining the Mass Hierarchy via Oscillations – the large U_{e3} route

Again, necessary to probe $\nu_\mu \to \nu_e$ oscillations (or vice-versa) governed by Δm^2_{13}. This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including the next generation experiments T2K and NOνA.

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”},$$

so that, again, this is insensitive to the sign of Δm^2_{13} at leading order. However, in this case, matter effects may come to the rescue.

As discussed already, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m^2_{12}}{2E}$ terms are ignored, the $\nu_\mu \to \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2\theta_{23}\sin^2 2\theta^{\rm eff}_{13}\sin^2\left(\frac{\Delta^{\rm eff}_{13}L}{2}\right),$$

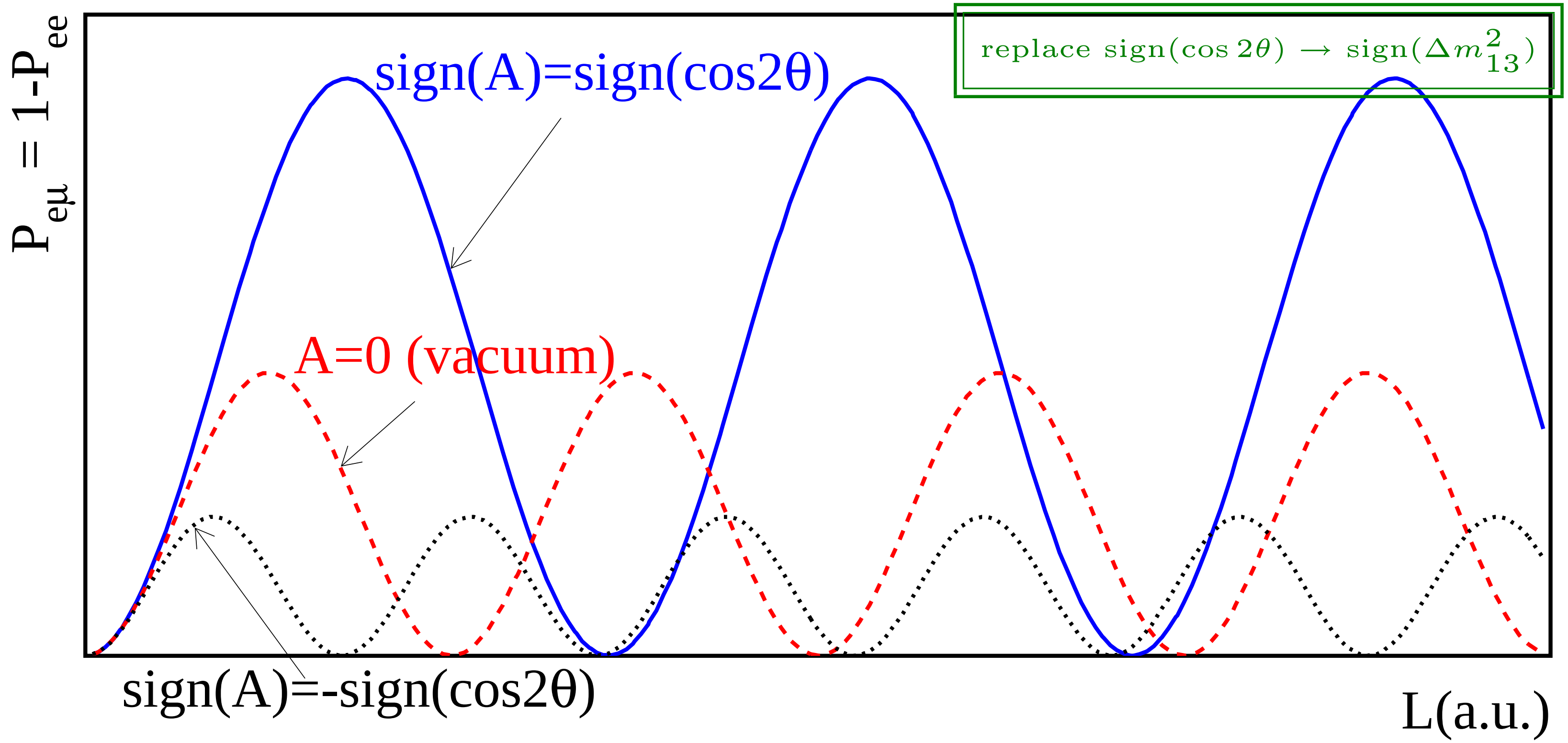
$$\sin^2 2\theta^{\rm eff}_{13} = \frac{\Delta^2_{13}\sin^2 2\theta_{13}}{(\Delta^{\rm eff}_{13})^2},$$

$$\Delta^{\rm eff}_{13} = \sqrt{(\Delta_{13}\cos 2\theta_{13} - A)^2 + \Delta^2_{13}\sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m^2_{13}}{2E},$$

$A \equiv \pm\sqrt{2}G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A. It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.

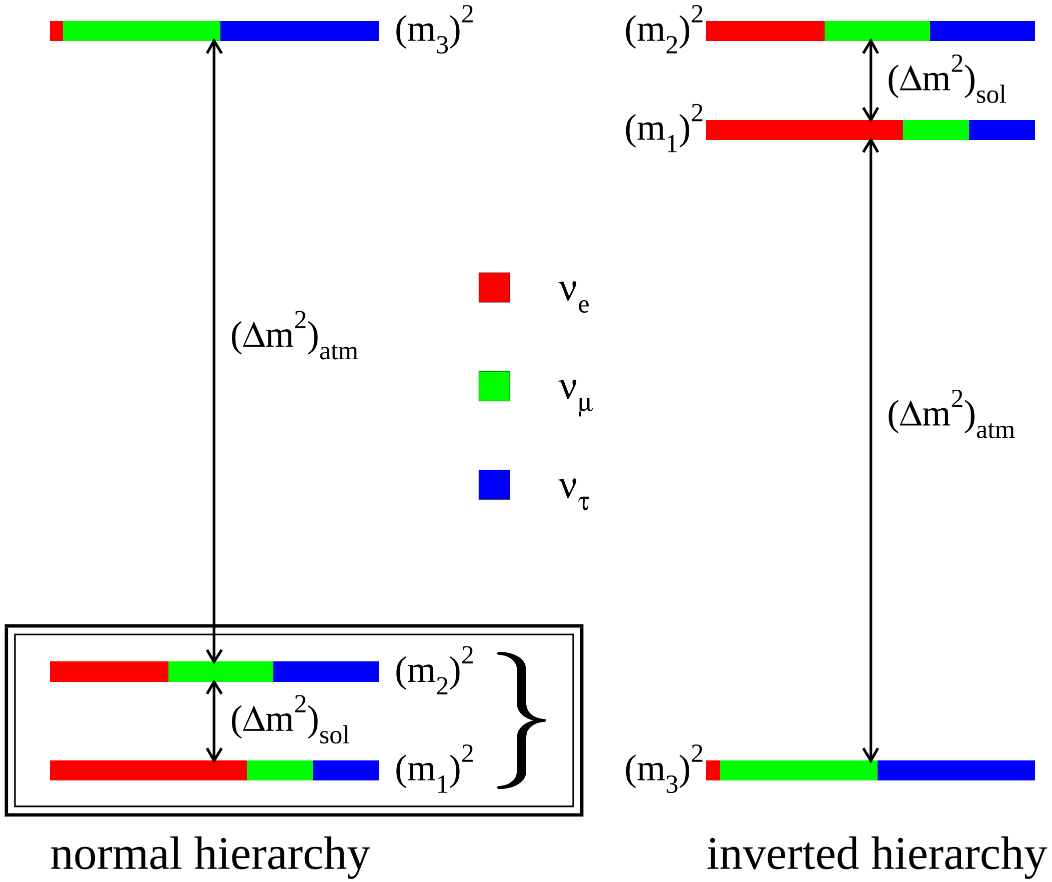


replace $\text{sign}(\cos 2\theta) \to \text{sign}(\Delta m^2_{13})$

Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta^{\rm eff}_{13} L$ large enough – matter effects are absent near the origin.

ASIDE – this is how the solar mass ordering was determined



Of course, Δm^2_{12} is positive-definite. What I mean by the solar mass ordering is whether ν_e is "mostly heavy" (ν_2) or "mostly light" (ν_1).

Matter effects in the Sun have uniquely determined that the electron-type neutrino is "mostly light."

NOTE: this is a "two-flavor" effect!

7 - The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

$\Rightarrow$ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Recent experimental developments, however, provide strong reason to believe that this is not the case: neutrinos have mass, and leptons mix!

[a] modulo the QCD θ-parameter, which will be "willed away" henceforth.

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

$$A_{\mu e} = U_{e1}^* U_{\mu 1} + U_{e2}^* U_{\mu 2} e^{i\Delta_{12}}, + U_{e3}^* U_{\mu 3} e^{i\Delta_{13}}$$

where $\Delta_{1i} = \frac{\Delta m_{1i}^2 L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process is

$$\bar{A}_{\mu e} = U_{e1} U_{\mu 1}^* + U_{e2} U_{\mu 2}^* e^{i\Delta_{12}}, + U_{e3} U_{\mu 3}^* e^{i\Delta_{13}}.$$

In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^* U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: Δ_{12}, $\Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, **we need $|U_{e3}| \neq 0$.**

The goal of next-generation neutrino experiments is to determine the magnitude of $|U_{e3}|$. We need to know this in order to understand how to study CP-invariance violation in neutrino oscillations!

In the real world, life is much more complicated. The lack of knowledge concerning the mass hierarchy, θ_{13}, θ_{23} leads to several degeneracies.

Note that, in order to see CP-invariance violation, we **need** the "subleading" terms!

In order to ultimately measure a new source of CP-invariance violation, we will need to combine different measurements:
– oscillation of muon neutrinos and antineutrinos,
– oscillations at accelerator and reactor experiments,
– experiments with different baselines,
– etc.

[Lectures By S. Parke]

To Conclude: More on the "Origin" Of Lepton Mixing.

How many new CP-violating parameters in the neutrino sector?

If the neutrinos are Majorana fermions, there are more physical observables in the leptonic mixing matrix.

Remember the parameter counting in the quark sector:

9 (3 $\times$ 3 unitary matrix)

-5 (relative phase rotation among six quark fields)

4 (3 mixing angles and 1 CP-odd phase).

If the neutrinos are Majorana fermions, the parameter counting is quite different: there are no right-handed neutrino fields to "absorb" CP-odd phases:

9 (3 × 3 unitary matrix)

−3 (three right-handed charged lepton fields)

6 (3 mixing angles and 3 CP-odd phases).

There is CP-invariance violating parameters even in the 2 family case: $4 - 2 = 2$, one mixing angle, one CP-odd phase.

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \overline{\nu_L^c}(M_\nu)\nu_L + H.c.$$

Write $U = E^{-i\xi/2} U' E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \mathrm{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \overline{\nu_L^c}(M_\nu) E^{-i\alpha}\nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

on the other hand

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \bar{\nu}_R(M_\nu)\nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \bar{\nu}_R(M_\nu) E^{-i\alpha/2}\nu_L + H.c.$$

ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$

END LECTURE # 1

(To Be Continued . . .)

BONUS MATERIAL:

- Measuring the deviation of the atmospheric mixing from maximal (is $\theta_{23} \neq \pi/4$?)
- How do you determine the neutrino mass hierarchy if θ_{13} turns out to be too small?

On measuring $\sin^2\theta_{23}$ (the atmospheric mixing angle)

More specifically, we would like to ask whether it is possible to determine:

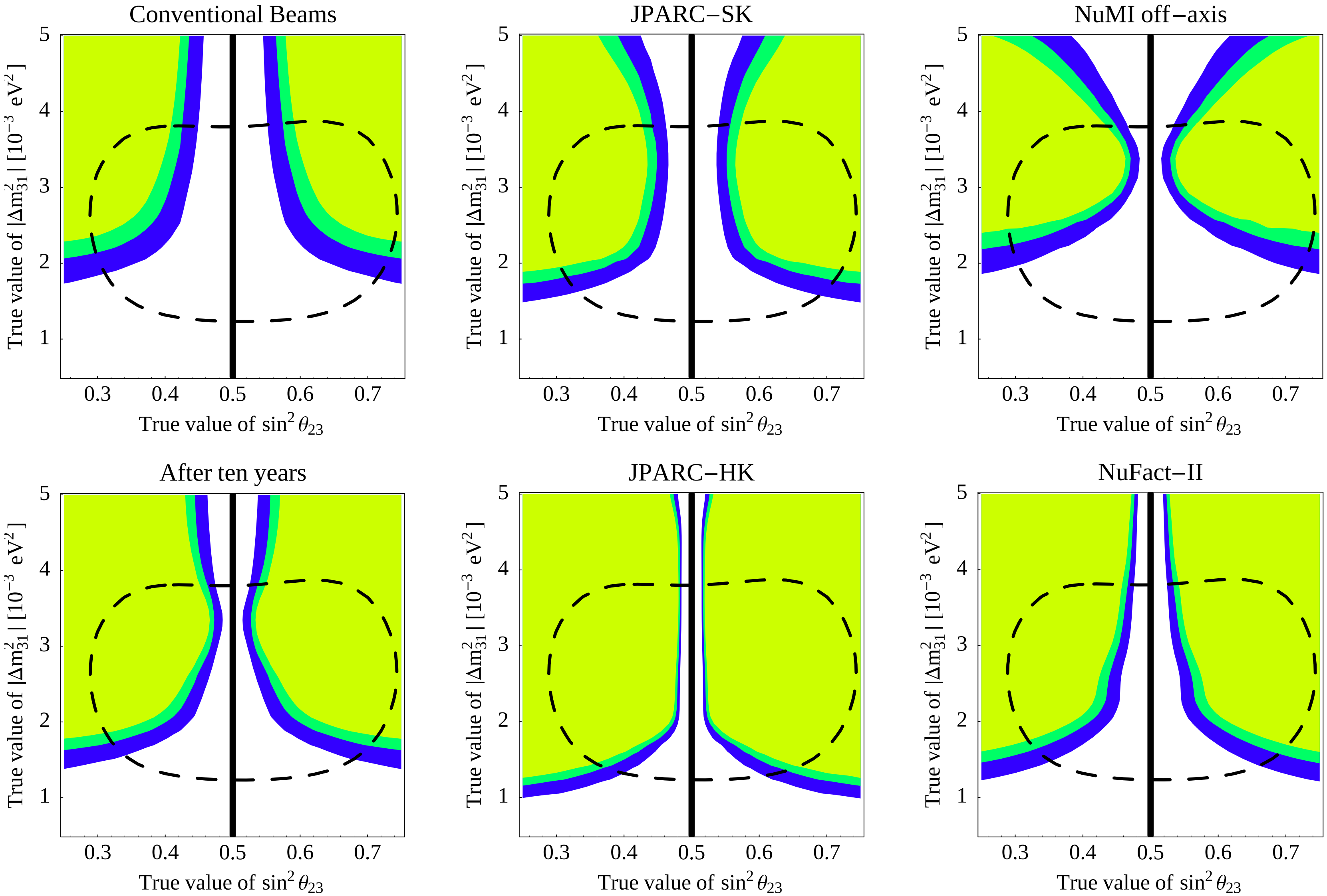
1. Is it maximal ($\sin^2\theta_{23} = 1/2$)?
2. Is $\sin^2\theta_{23} > 1/2$ or $\sin^2\theta_{23} < 1/2$?

Limited information regarding (2) from disappearance channel. — $P_{\mu\mu} \propto \sin^2 2\theta_{23}$. Simply adding $P_{\mu e} \propto \sin^2\theta_{23}\sin^2 2\theta_{13}$ does not help!

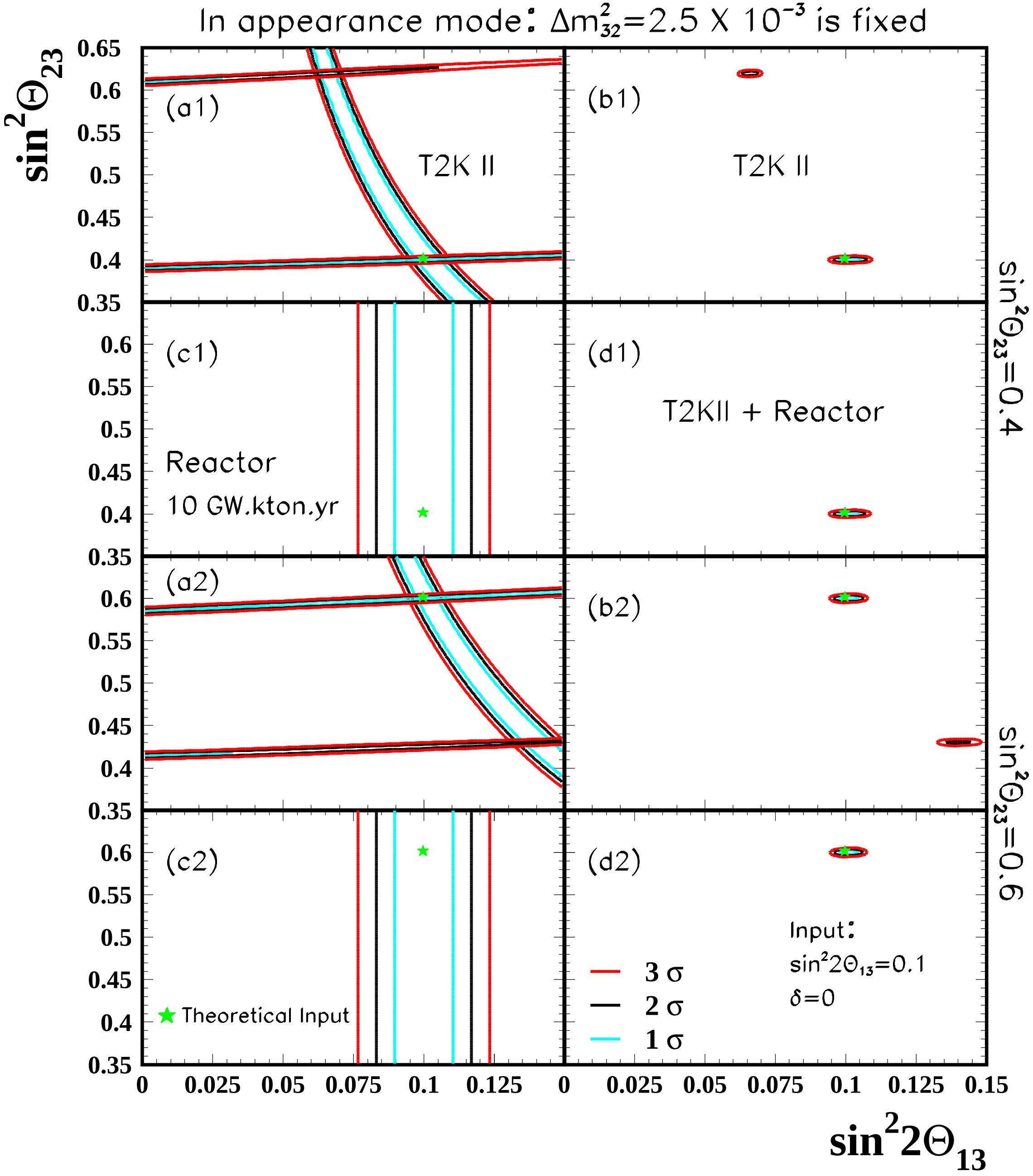
$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{subleading}.$$

In order to resolve this issue, need more information from reactors, atmospheric neutrinos, $P_{e\tau} \propto \cos^2\theta_{23}$ (which required τ appearance and is beyond the reach of "standard" next-generation LBL experiments – usually requires Neutrino Factory).

Deciding that θ_{23} is not maximal with LBL experiments



Antusch *et al.*, PRD70, 097302 (2004).



⇐ Appearance + Disappearance not Enough

⇐ Reactors Can Resolve Degeneracy

Hiraide *et al.*, hep-ph/0601258

Determining the Mass Hierarchy via Oscillations – vanishing U_{e3} route

hep-ph/0503079, hep-ph/0507021, hep-ph/0509359

In the case of two-flavors, the "mass-hierarchy" can only be determined in the presence of matter effects: vacuum neutrino oscillations are not sensitive to the mass hierarchy.

In the case of three-flavors, this is not the case: vacuum neutrino oscillation probabilities are sensitive to the neutrino mass hierarchy. This does not depend on whether U_{e3} vanishes or not.

How does one compare the two mass hierarchies and determines which one is correct?

The question I address is the following:

For a positive choice of $\Delta m^2_{13} = \Delta m^{2+}_{13}$, is there a negative choice for $\Delta m^2_{13} = \Delta m^{2-}_{13}$ that yields identical oscillation probabilities?

If the answer is 'yes,' then one cannot tell one mass hierarchy from the other. If the answer is 'no,' then one can, in principle, distinguish the two possibilities.

More concretely: fix Δm^{2+}_{13} (which I'll often refer to as Δm^2_{13}) and define x so that

$$\Delta m^{2-}_{13} = -\Delta m^{2+}_{13} + x.$$

Question: Is there a value of x that renders $P(\Delta m^{2+}_{13}) = P(\Delta m^{2-}_{13})$?

Note: x is such that Δm^2_{13} is negative. It turns out that x's that almost do the job are of order Δm^2_{12}.

I will concentrate on survival probabilities (which will be the only relevant ones in the $U_{e3} \to 0$ limit):

$$P_{\alpha\alpha} = 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta_{12}L}{2}\right) - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta_{13}L}{2}\right) - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta_{23}L}{2}\right),$$

$\Delta_{ij} \equiv \Delta m^2_{ij}/2E$. Note that $\Delta_{23} = \Delta_{13} - \Delta_{12}$.

It is easy to see how the different hierarchies lead to different results. In the normal case, $|\Delta_{13}| > |\Delta_{23}|$, while in the inverted case $|\Delta_{13}| < |\Delta_{23}|$. Hence, "all" one needs to do is establish which frequency is associated to which amplitude (governed by the $U_{\alpha i}$'s).

More detail:

$$P^+_{\alpha\alpha} - P^-_{\alpha\alpha} = -4|U_{\alpha 3}|^2 \left\{ |U_{\alpha 1}|^2 \left[\sin^2\left(\frac{\Delta_{13}L}{2}\right) - \sin^2\left(\frac{(\Delta_{13}-X)L}{2}\right)\right] + |U_{\alpha 2}|^2 \left[\sin^2\left(\frac{(\Delta_{13}-\Delta_{12})L}{2}\right) - \sin^2\left(\frac{(\Delta_{13}+\Delta_{12}-X)L}{2}\right)\right]\right\},$$

$X = x/2E$.

There is no choice of x that renders this zero for all L and E,

unless (i) $|U_{\alpha 2}|^2 = |U_{\alpha 1}|^2$ (known not to happen) or (ii) $\Delta_{12} = 0$ (also does not happen) or (iii) one of the $U_{\alpha i}$'s vanishes (could happen in the case of P_{ee}).

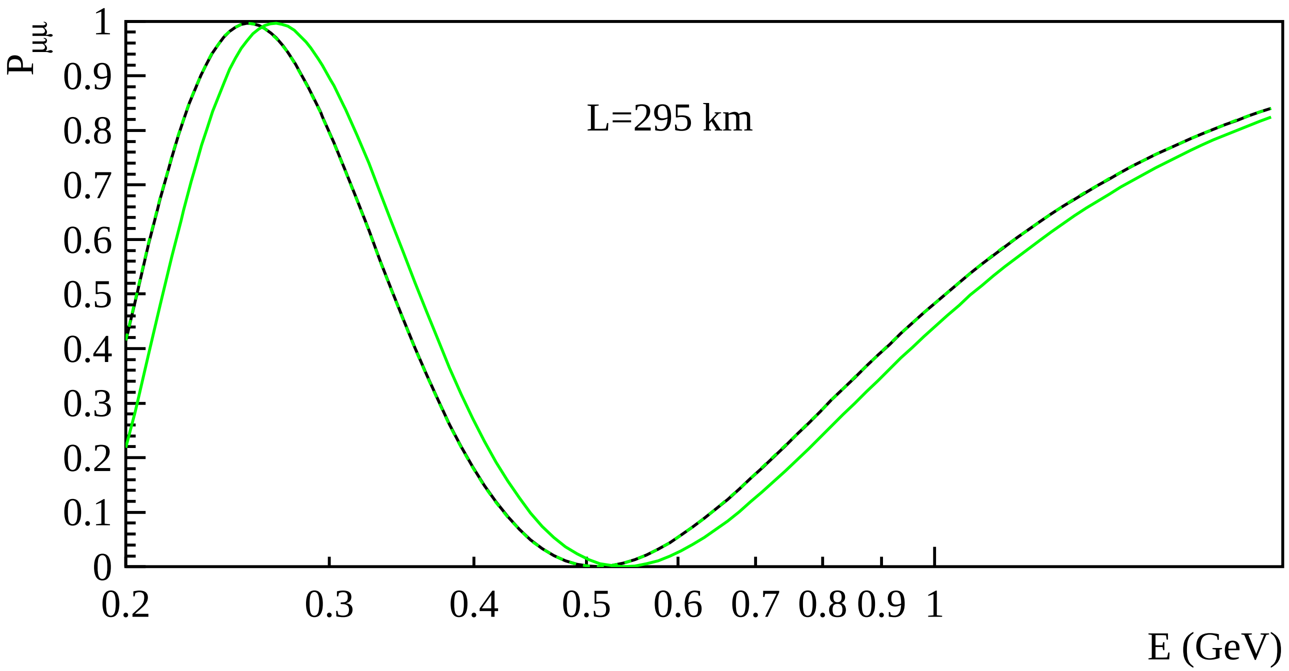
Life is not this simple. Most experimental set-ups looking for U_{e3} effects concentrate on L and E so that $\Delta_{13}L \sim 1$. This means that $\Delta_{12}L \ll 1$.

It turns out that

$$x = \frac{2|U_{\alpha 2}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2}\Delta m^2_{12},$$

renders $P^+_{\alpha\alpha} - P^-_{\alpha\alpha} = \mathcal{O}(\Delta_{12}L)^2$.

There are two ways around this problem. One is to make sure you consider large $\Delta_{12}L$ values. The other is to note that different α's yeild different values of x. Both are very, very challenging. . .



$\Delta m^2_{13} = +2.20 \times 10^{-3}$ eV^2

$\Delta m^2_{13} = -2.20 \times 10^{-3}$ eV^2 (s)

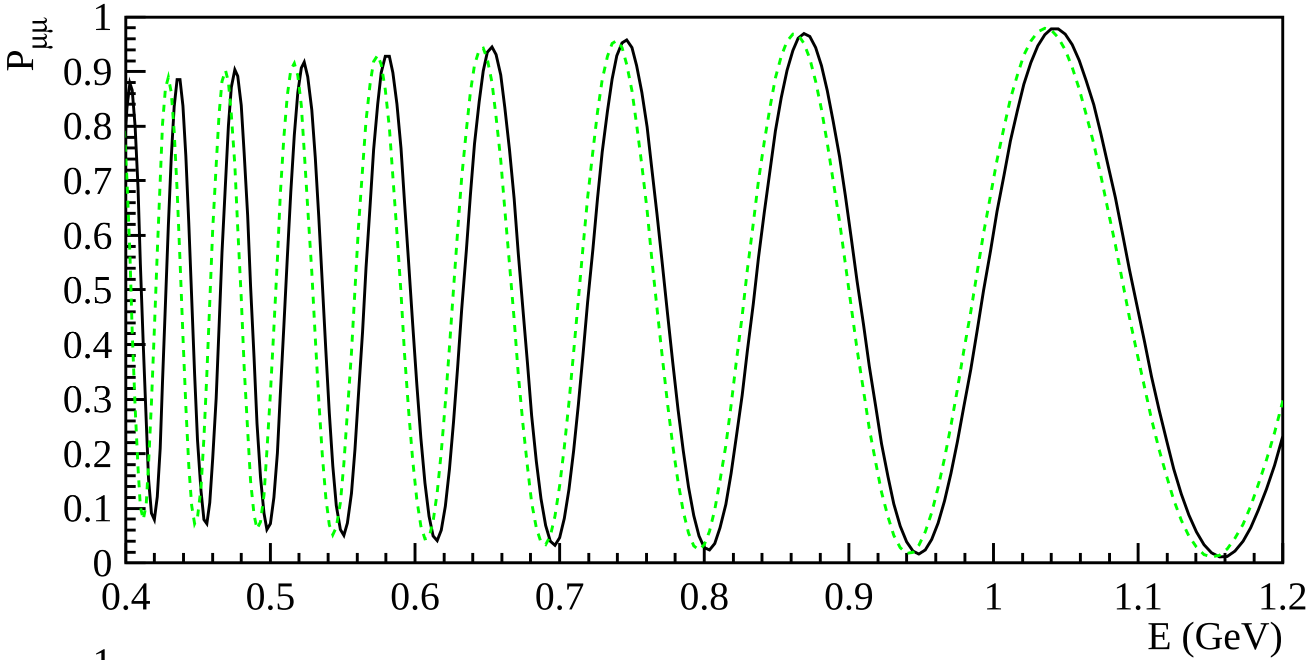
$\Delta m^2_{13} = -2.08 \times 10^{-3}$ eV^2 (d)

$\Delta m^2_{12} = 8.2 \times 10^{-5}$ eV^2,

$\sin^2 2\theta_{12} = 0.83$, $\sin^2 2\theta_{23} = 1$

The small $\Delta_{12}L$ problem: in this case $x = 2\Delta m^2_{12}\cos^2\theta_{12}$ ($= 1.16 \times 10^{-4}$ eV^2).

This would be the situation at a "short" baseline experiment: even with quasi-infinite statistics one would still end up with two different values of Δm^2_{13}, one for each hierarchy hypothesis.



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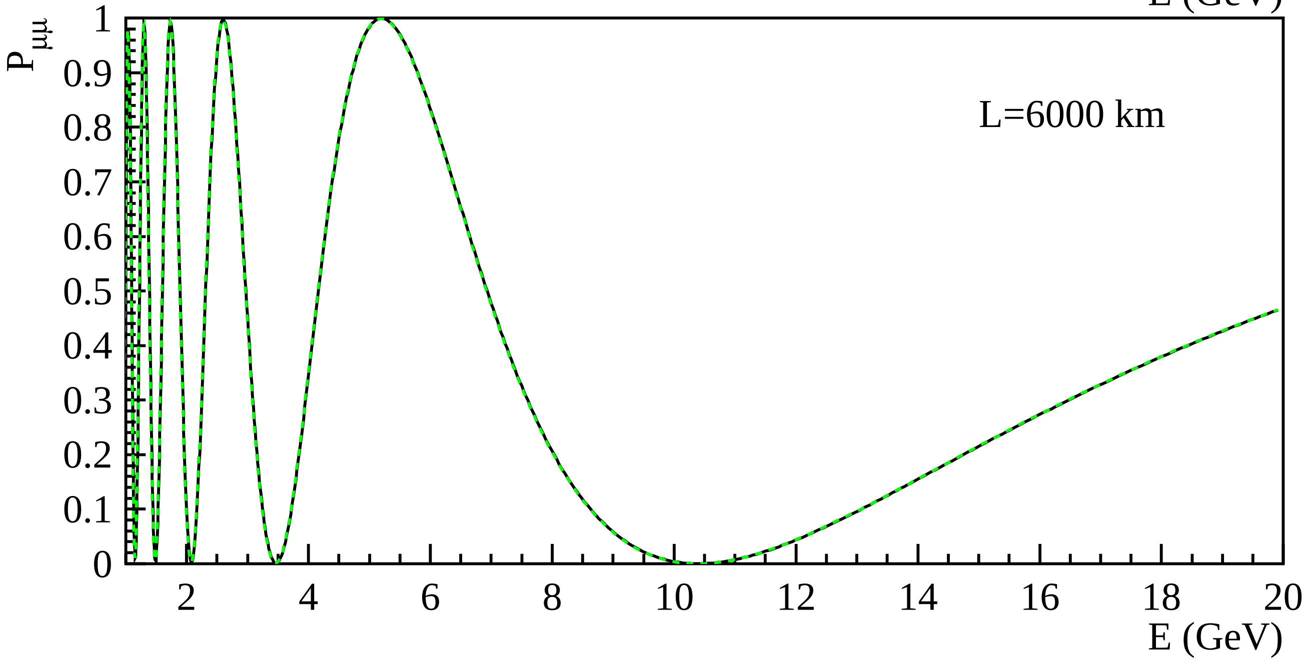
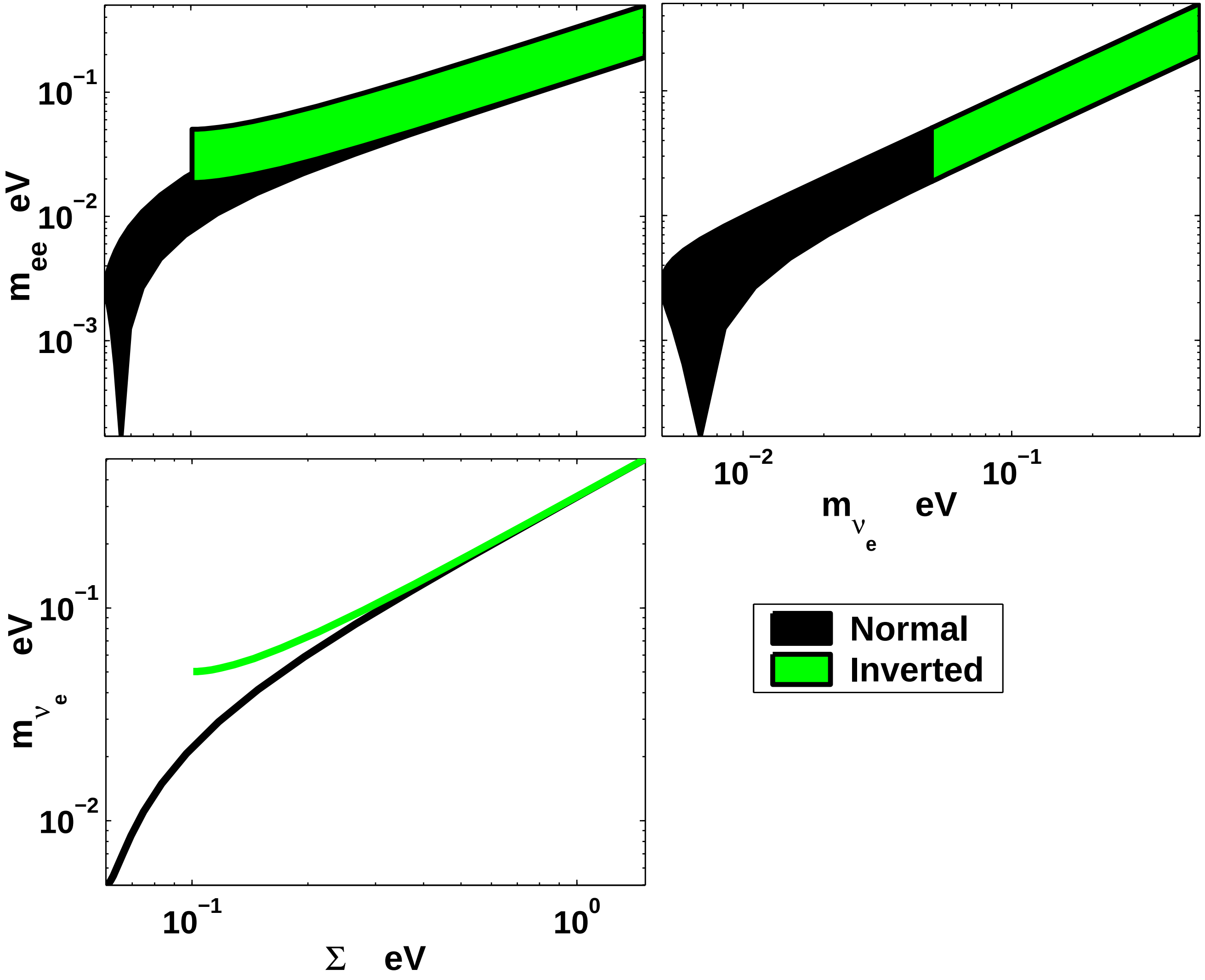
$\sin^2 2\theta_{12} = 0.83$, $\sin^2 2\theta_{23} = 1$



There is hope! But can we "see" the fast oscillations at low energies?

Other Tool? – Non-Oscillation Experiments

$(U_{e3}=0,\ \Delta m^{2+}_{13}=+2.50\times10^{-3}\ \text{eV}^2,\ \Delta m^{2-}_{13}=-2.44\times10^{-3}\ \text{eV}^2)$



$$\Sigma = m_1 + m_2 + m_3$$

$$m^2_{\nu_e} \equiv \sum_i |U_{ei}|^2 m_i^2$$

$$m_{ee} \equiv \sum_i U_{ei}^2 m_i \equiv m_1|U_{e1}|^2 e^{i\alpha_1} + m_2|U_{e2}|^2 e^{i\alpha_2} + m_3|U_{e3}|^2 e^{-2i\delta}$$