From perfect liquid to viscous fluid: hydrodynamics at RHIC

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Heavy Ion Tea

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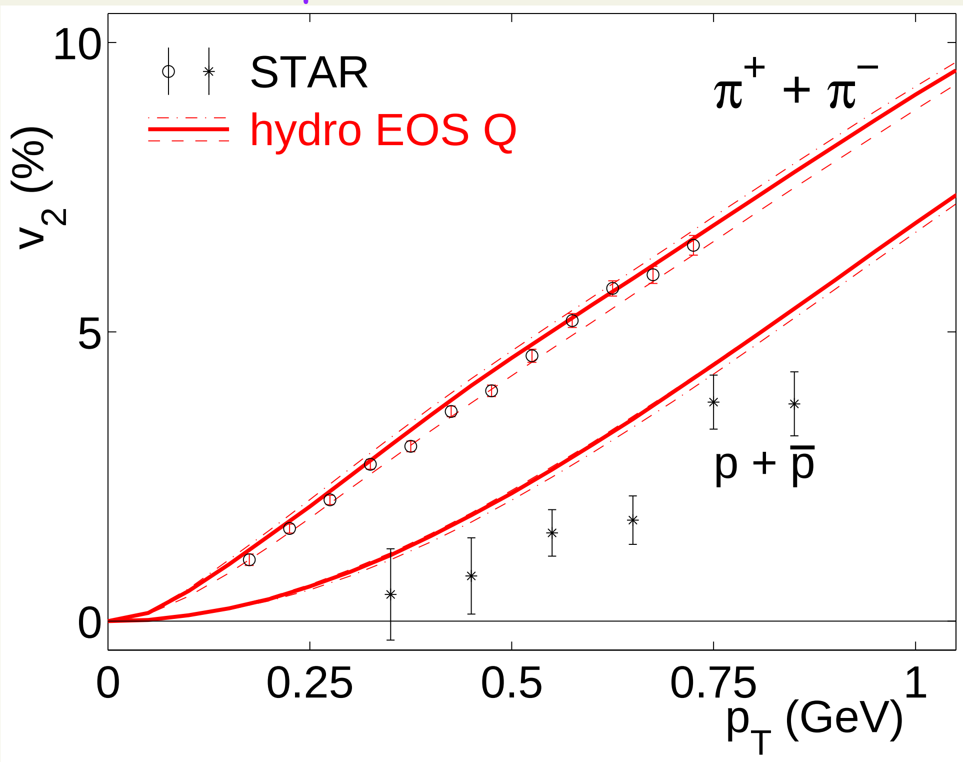
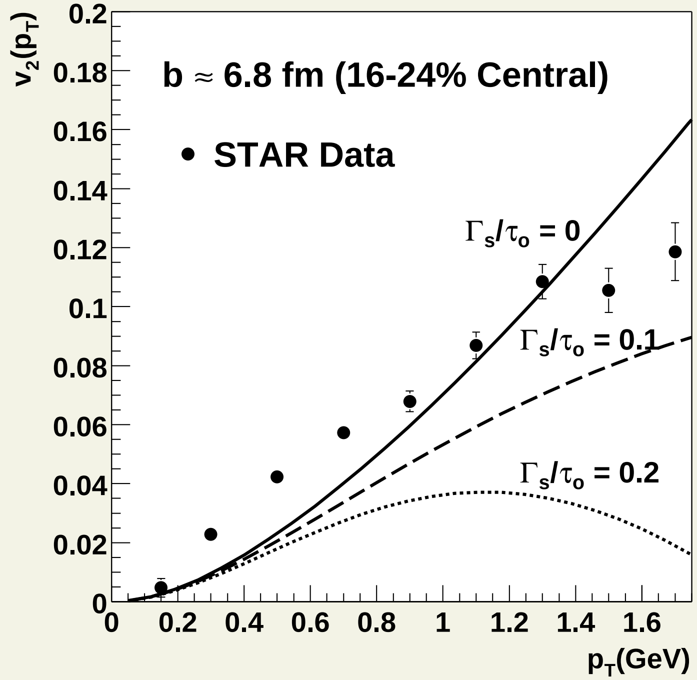
Success of ideal hydrodynamics

Kolb, Heinz, Huovinen et al ('01)

minbias Au+Au at RHIC



Teaney ('04): $\eta/s \propto \Gamma_s/\tau_0$



dissipation $\longrightarrow$ reduction of v_2

$\Rightarrow$ the idea of plasma as "perfect fluid"

- but how perfect? - what if $\eta/s > 0$, e.q. conjectured $\eta/s \geq 1/(4\pi)$
- tune the calculation to allow large viscosity?

Ideal hydrodynamics

space-time evolution given by: $\partial_\mu T^{\mu\nu}(x) = 0$
$\partial_\mu N^\mu(x) = 0$

- **local conservation of energy, momentum and baryon number**

local equilibrium, no dissipation: $T^{\mu\nu} = (e+p)u^\mu u^\nu - Pg^{\mu\nu}$
$N^\mu = nu^\mu$

local, macroscopic variables: **energy density** $e(x)$
pressure $P(x)$
flow velocity $u^\mu(x)$

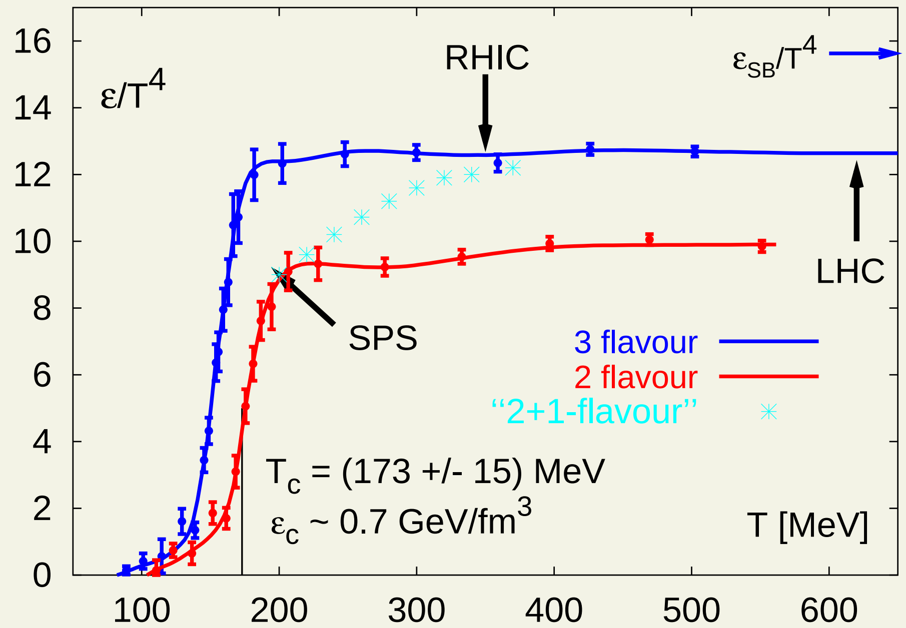
NO dissipation $\rightarrow$ conserves entropy $\partial_\mu(su^\mu) = 0$

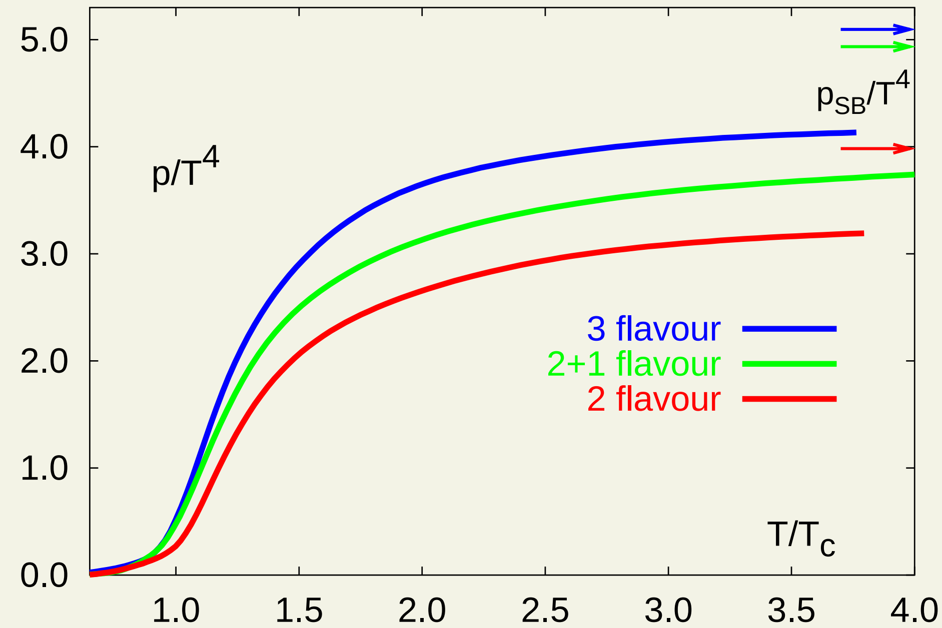
matter characterized by: equation of state $P = P(e,n)$

Unknowns: initial state, final state, equation of state

QCD equation of state

lattice QCD **(Karsch & Laermann, hep-lat/0305025)**



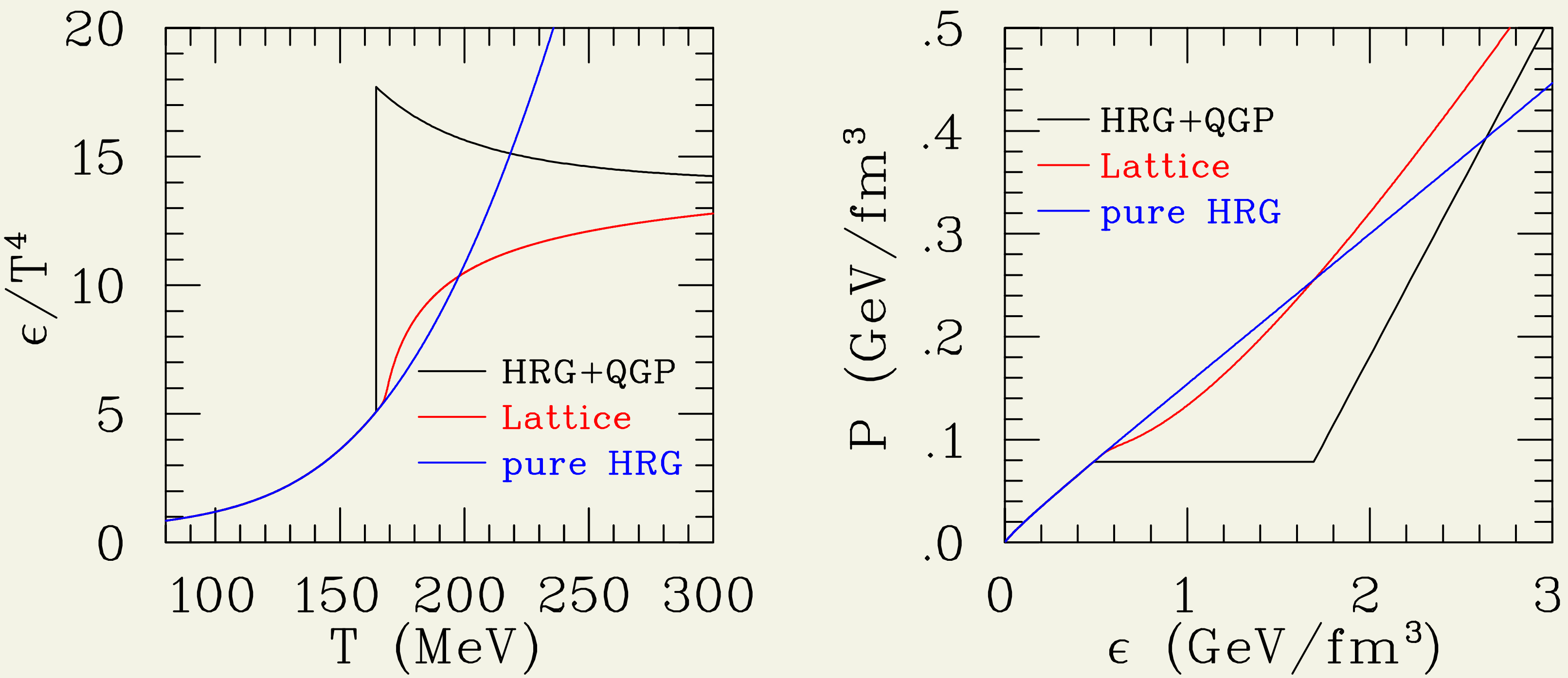


- jump in effective degrees of freedom at $T_c \approx 170 - 190$ **MeV** $\sim 2 \cdot 10^{12}$ **K**

- phase transition is a smooth crossover

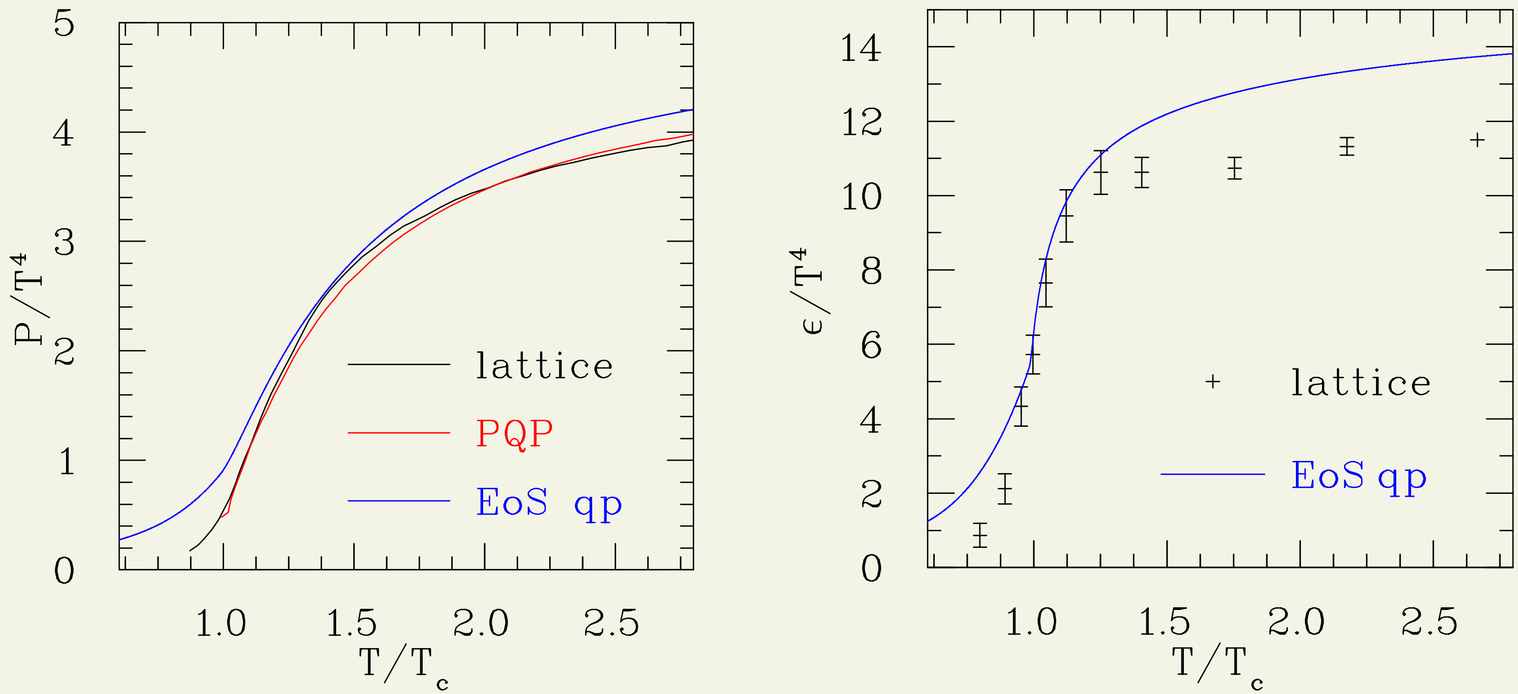
- hydro calculations: usually first order phase transition

Equation of state



- **HRG+QGP: ideal gas of partons and hadron resonance gas, first order PT**
- **Lattice: parametrized lattice EoS + hadron resonance gas, crossover**
- **pure HRG: hadron resonance gas only, no PT**

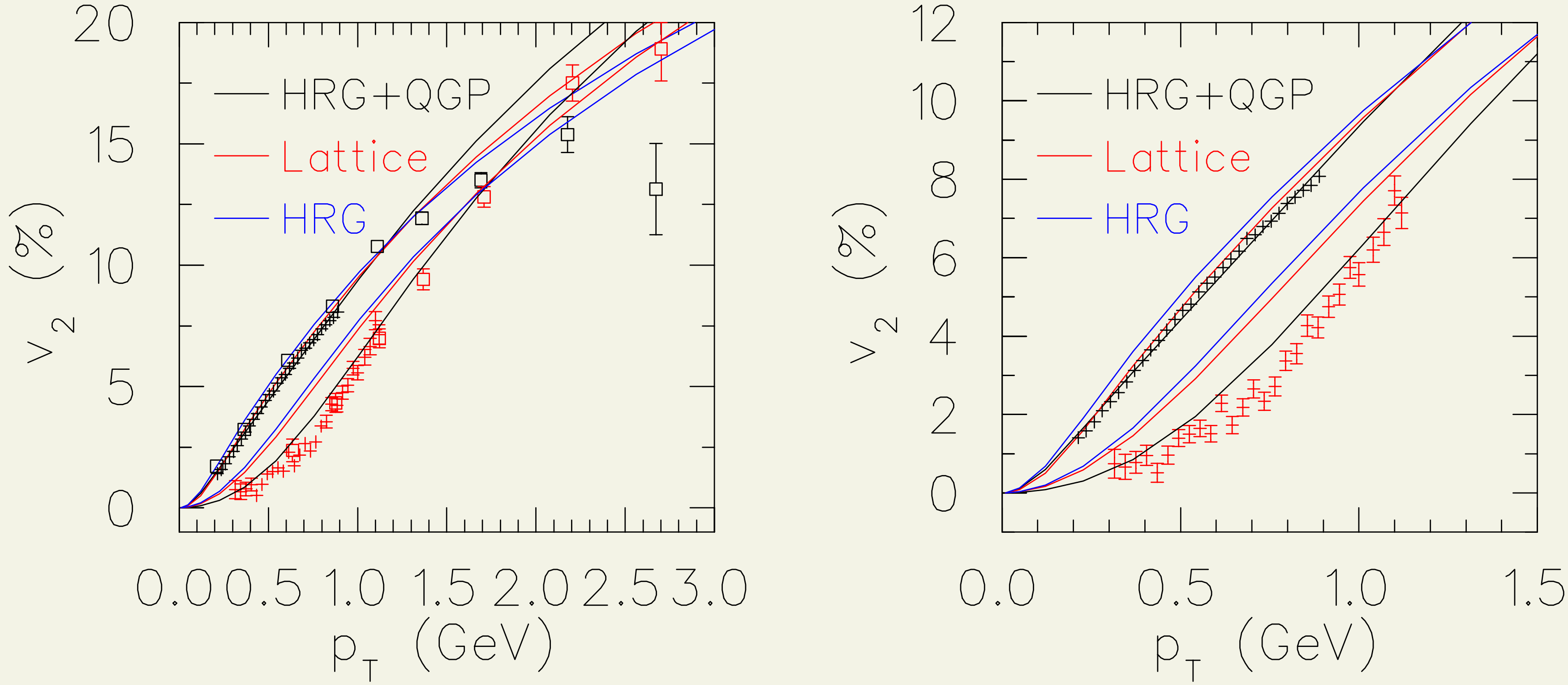
Parametrized lattice EoS



- **Extrapolation to finite quark masses?**
- **Schneider & Weise: quasiparticle model for plasma with known masses**
  - **Tune the model to fit lattice results with lattice masses**
  - **Change masses to more physical ones**

Anisotropy as function of p_{T}

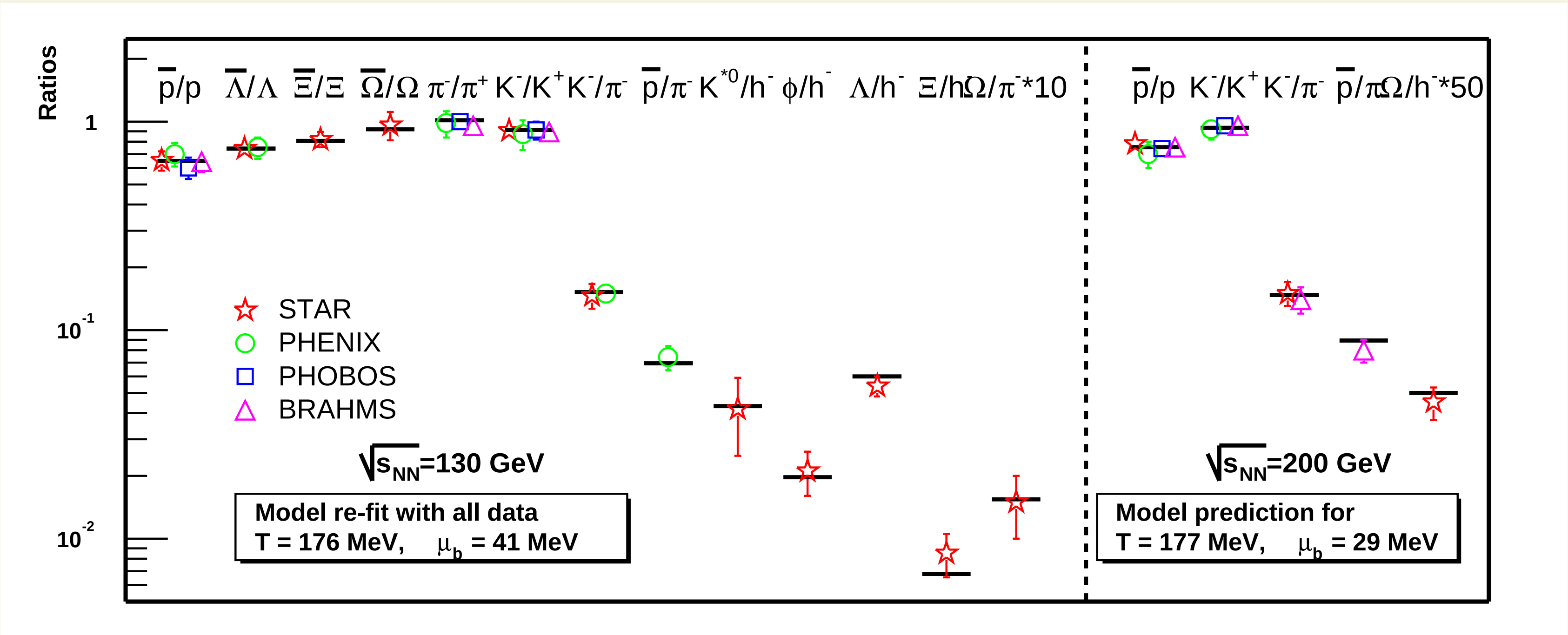
minbias Au+Au at RHIC ($\sqrt{s_{\mathrm{NN}}} = 200$ GeV)



- **All EoSs lead to similar pion $v_2(p_T)$**
- **Proton $v_2(p_T)$ is sensitive to EoS:**
  - **Lattice EoS gives as bad fit than EoS without any phase transition!**
  - **An EoS with a first order phase transition is closest to the data**

Thermal models

- Hadronic phase: ideal gas of massive hadrons and resonances
- in chemical equilibrium



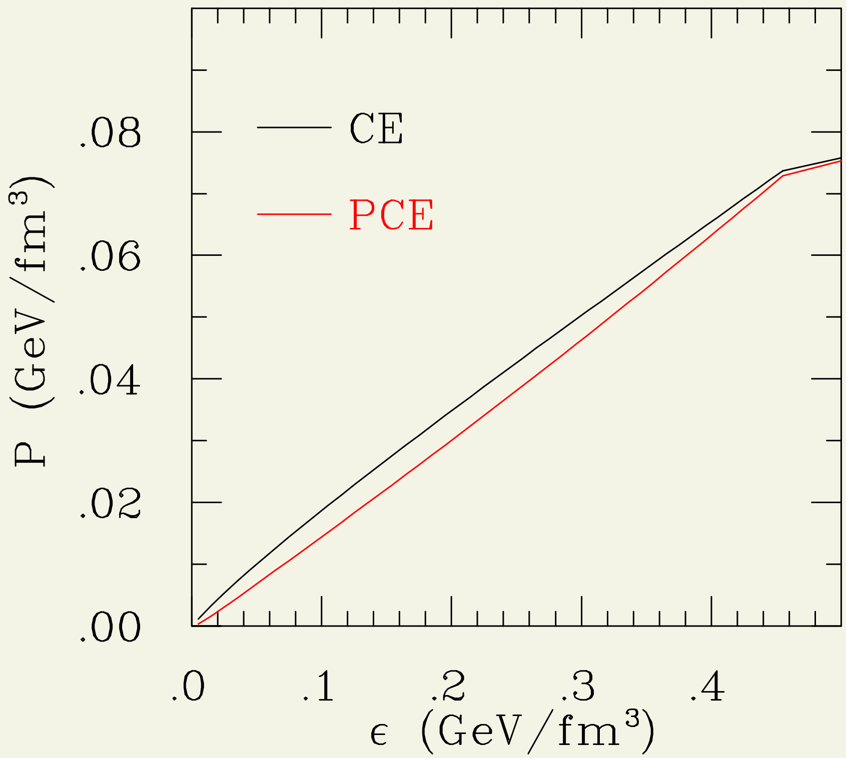
- Particle ratios (approximately) correspond to a system in $T \approx 160$–170 MeV temperature

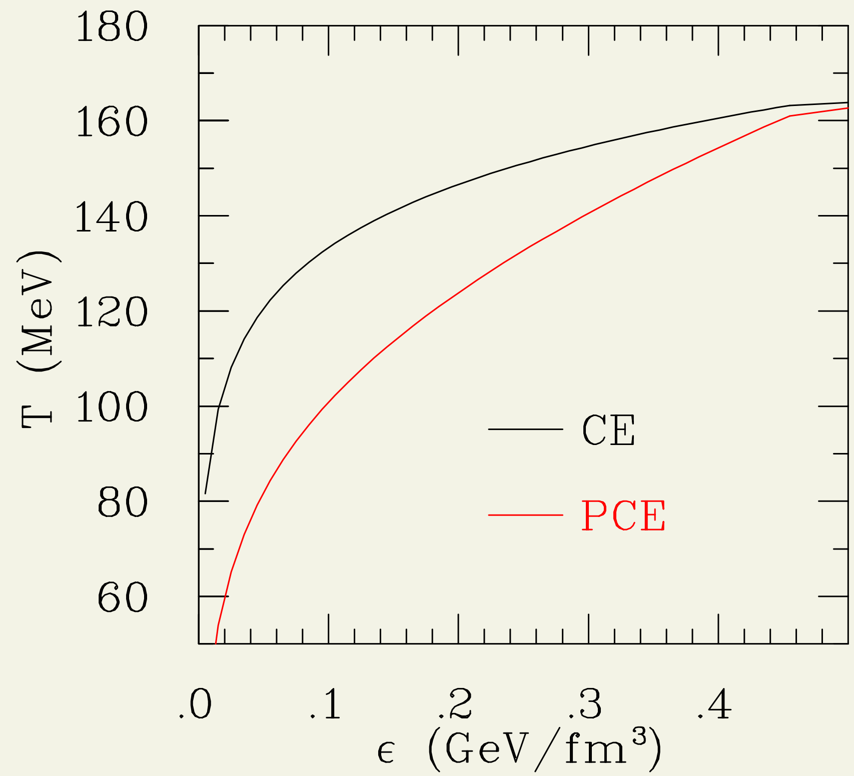
- Evolution to $T \approx 100$–120 MeV temperature

$\Rightarrow$ In hydro particle ratios become wrong

Chemical non-equilibrium

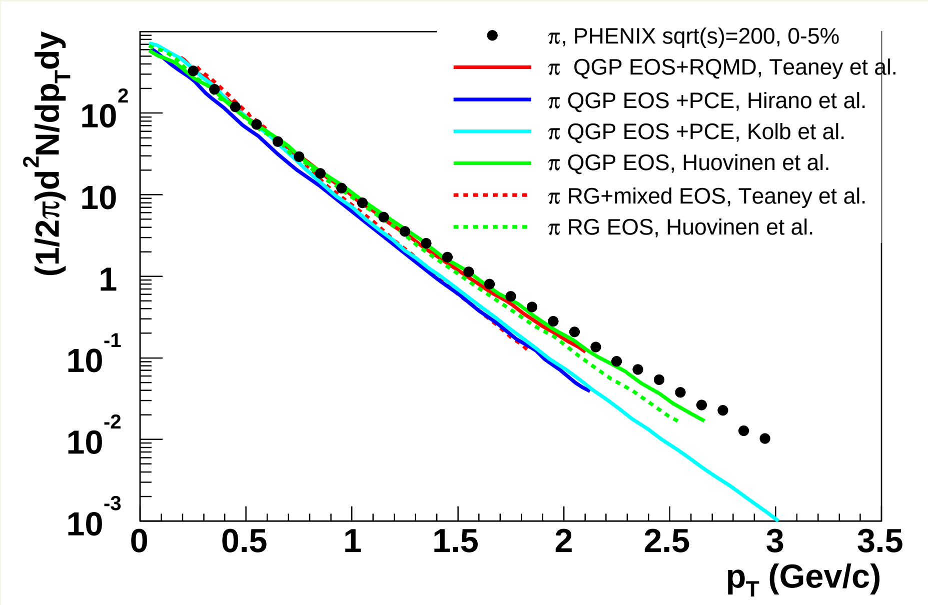
- **Treat number of pions, kaons etc. as conserved quantum numbers below T_{ch} (Bebie et al, Nucl.Phys.B378:95-130,1992)**
- $P = P(\epsilon, n_b)$ **changes very little, but** $T = T(\epsilon, n_b)$ **changes. . .**



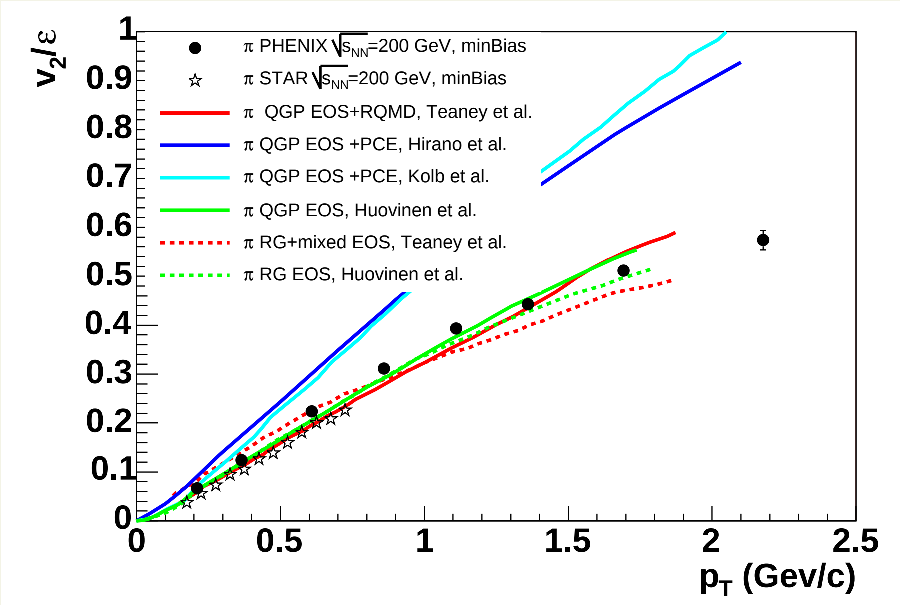


PHENIX whitepaper, NPA 757, 184 (2005)

π p_T-spectrum:



π $v_2(p_T)$:



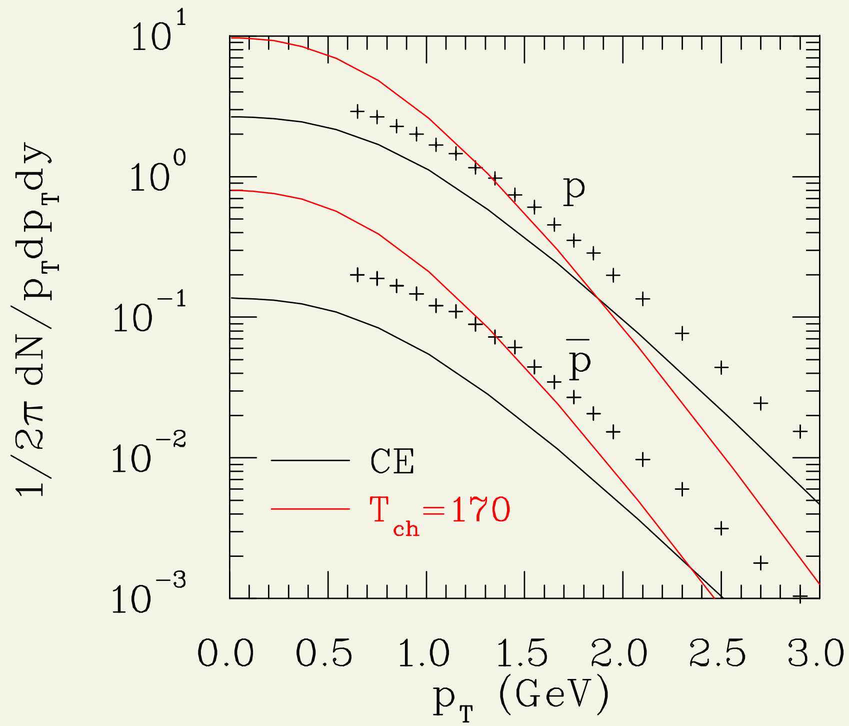
- **Blue curves**: Hirano & Tsuda and Kolb & Rapp, ideal hydro, no chemical equilibrium

$\Rightarrow$ **IT DOES NOT WORK!**

- If ideal hydro reproduces particle yields, it does not reproduce their distributions nor elliptic flow

Closer look at (anti)protons

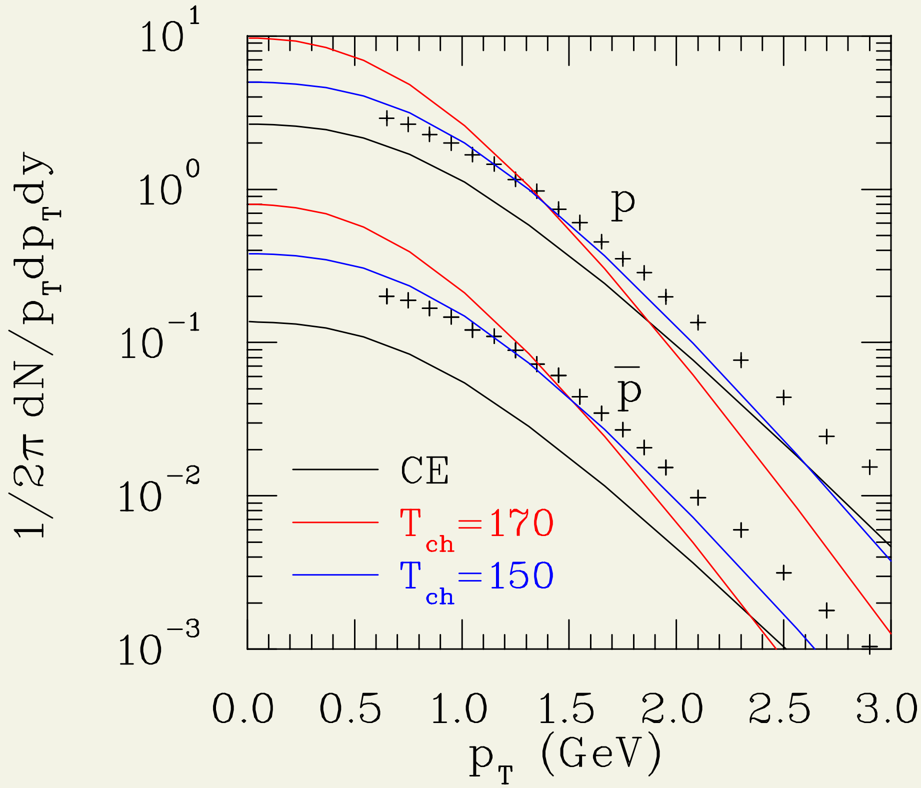
- **Too many protons and antiprotons:**



$\mathrm{d}N/\mathrm{d}y$	π^+	p	$\bar{p}$
PHENIX	286.4 ± 24.2	18.4 ± 2.6	13.5 ± 1.8
$T_{\mathrm{ch}} = 170$	**268**	**25**	**20**

- **Thermal models slightly overestimate p/π ratio**
- **Fit of π, K, p and $\bar{p}$ ratios $\Rightarrow$ $T \approx 150$ MeV (Cleymans et al., Phys.Rev.C71:054901,2005)**

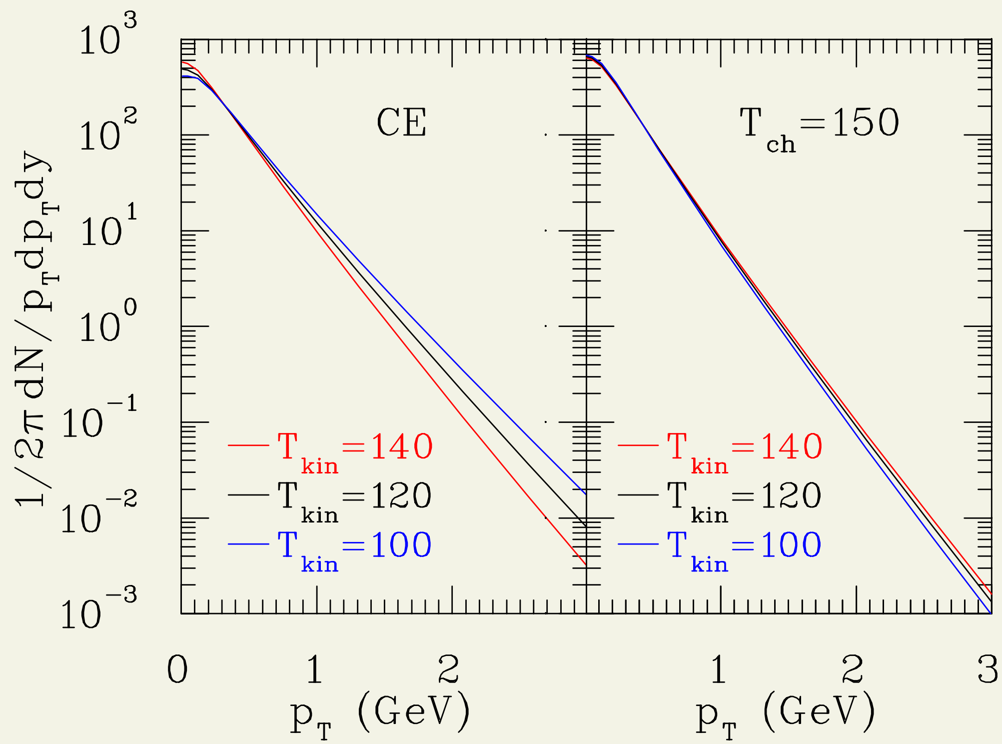
Second look at (anti)protons



- **Cooler chemical freeze-out temperature $T_{ch} = 150$ MeV**
  $\Rightarrow$ yields OK.

- **Slope too steep**
  - **Lower freeze-out temperature does not work**

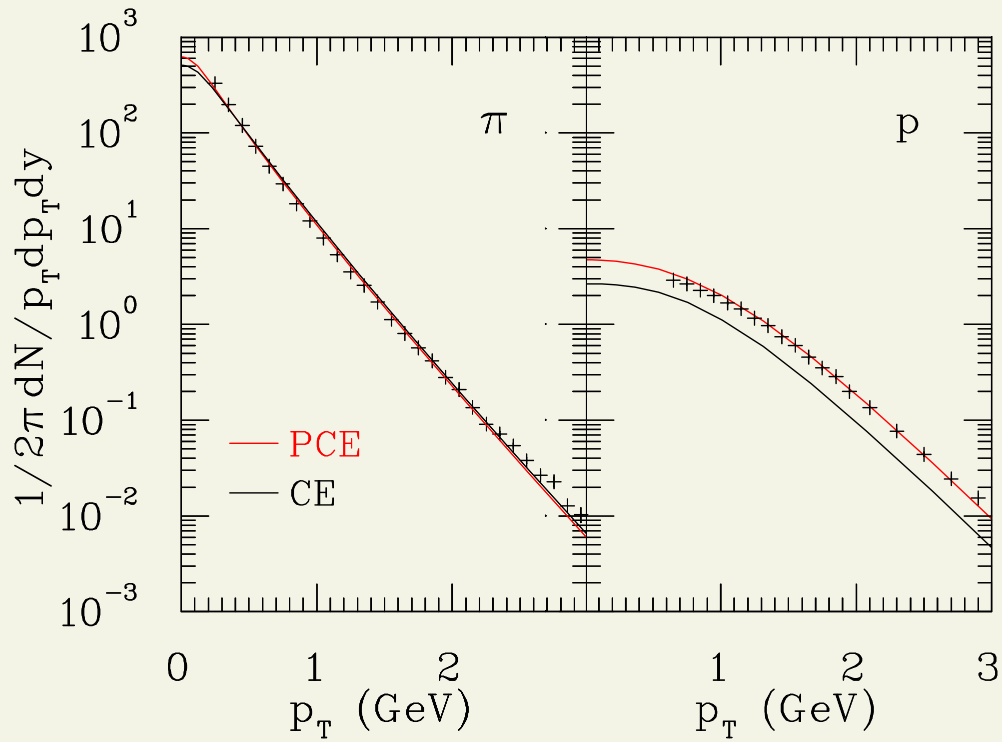
Effect of T_{kin} on pions



- **Longitudinal expansion does work ($p\,\mathrm{d}V$) $\Rightarrow$ $\frac{\mathrm{d}E_T}{\mathrm{d}y}$ decreases**
- **If particle # is conserved, $\langle p_T \rangle$ decreases**
- **In chemical equilibrium mass energy of baryon-antibaryon pairs is converted to kinetic energy $\Rightarrow$ $\langle p_T \rangle$ increases!**

Initial state

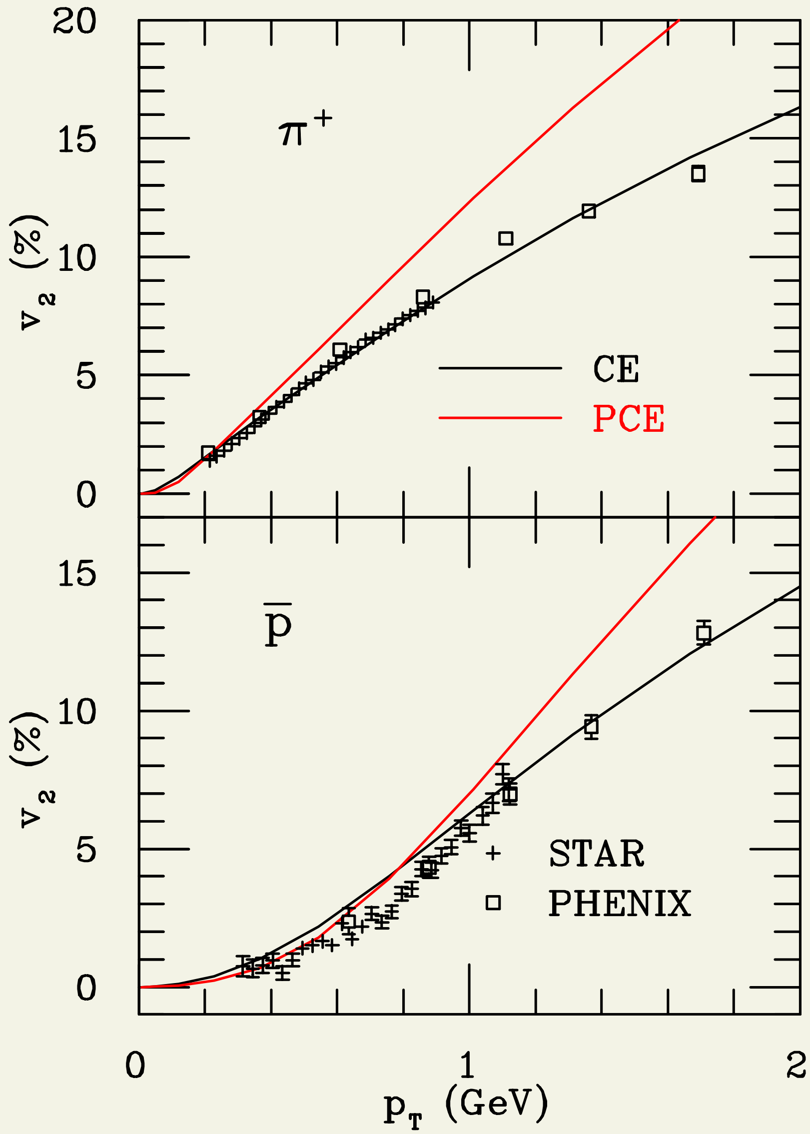
- **Need more transverse flow**
  - **Steeper initial density profile**
  - **Short initial time $\tau_0 = 0.2$ fm/c instead of $\tau_0 = 0.6$–1.0 fm/c**



- **If momentum distribution is isotropic, $\epsilon = 3P$ holds**

- **No need for exact thermalization**

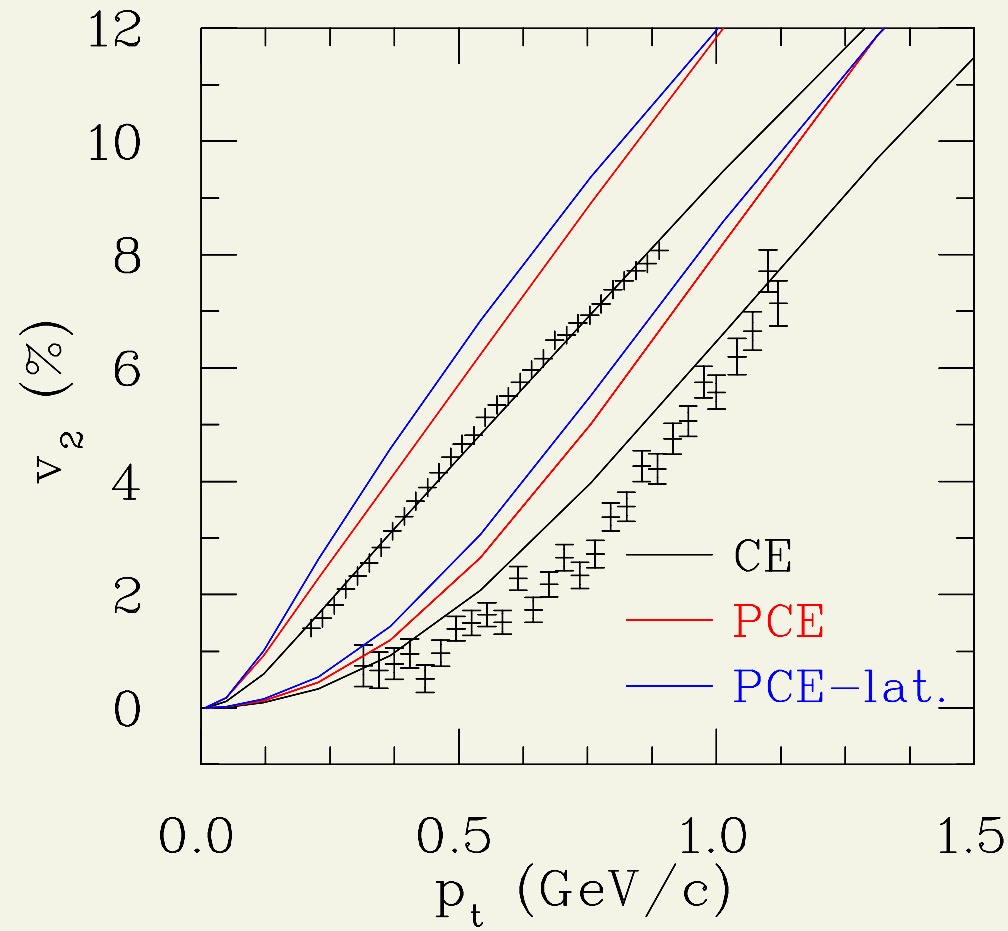
Elliptic flow $v_2(p_T)$



Failed!

- **Dissipation required**
- **But where?**
  - **in hadronic phase?**
  - **in plasma?**
  - **or both?**

And phase transition?



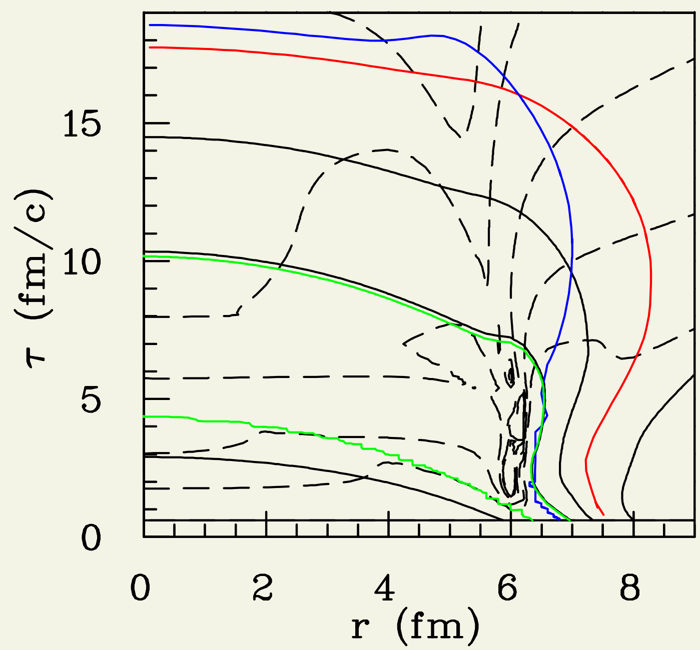
- **smaller but still visible effect**
- **even worse fit than with a first order phase transition**

Freeze-out

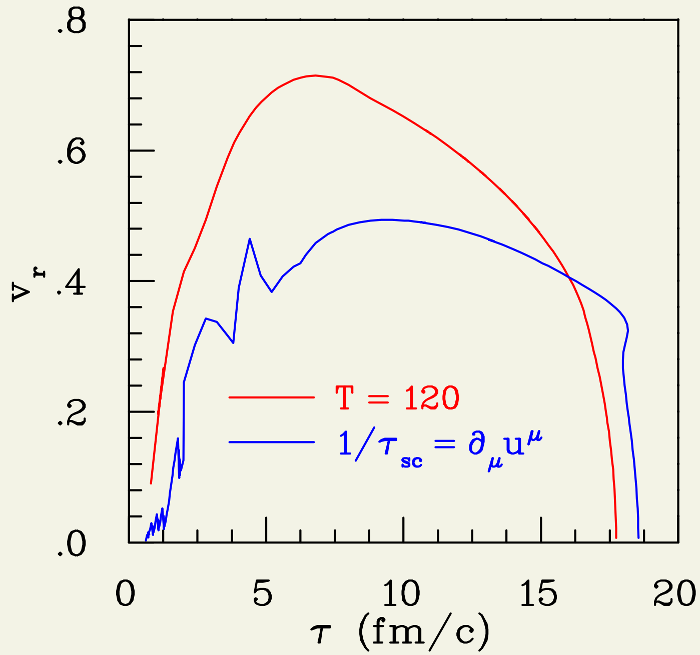
- when system no longer behaves as fluid but as free streaming particles
- momentum distributions cease to evolve → they "freeze-out"
- criterion: expansion rate larger than scattering rate:

$$\tau_{\rm scat}^{-1} \approx \partial_\mu u^\mu$$

- $\tau_{\rm scat}^{-1} \propto T^4 \rightarrow$ freeze-out at $T =$ const.
- not so good approximation:

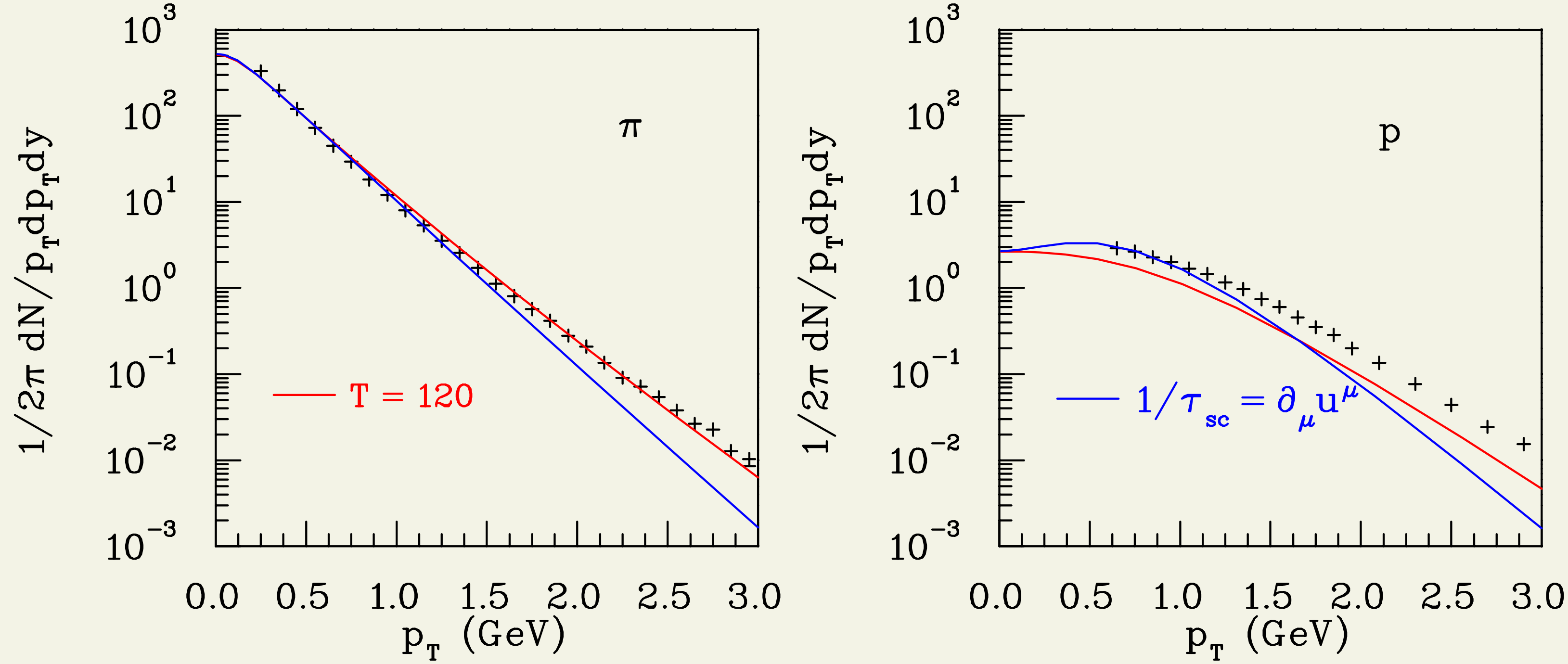


temperature and expansion rate contours



flow velocity on freeze-out surface

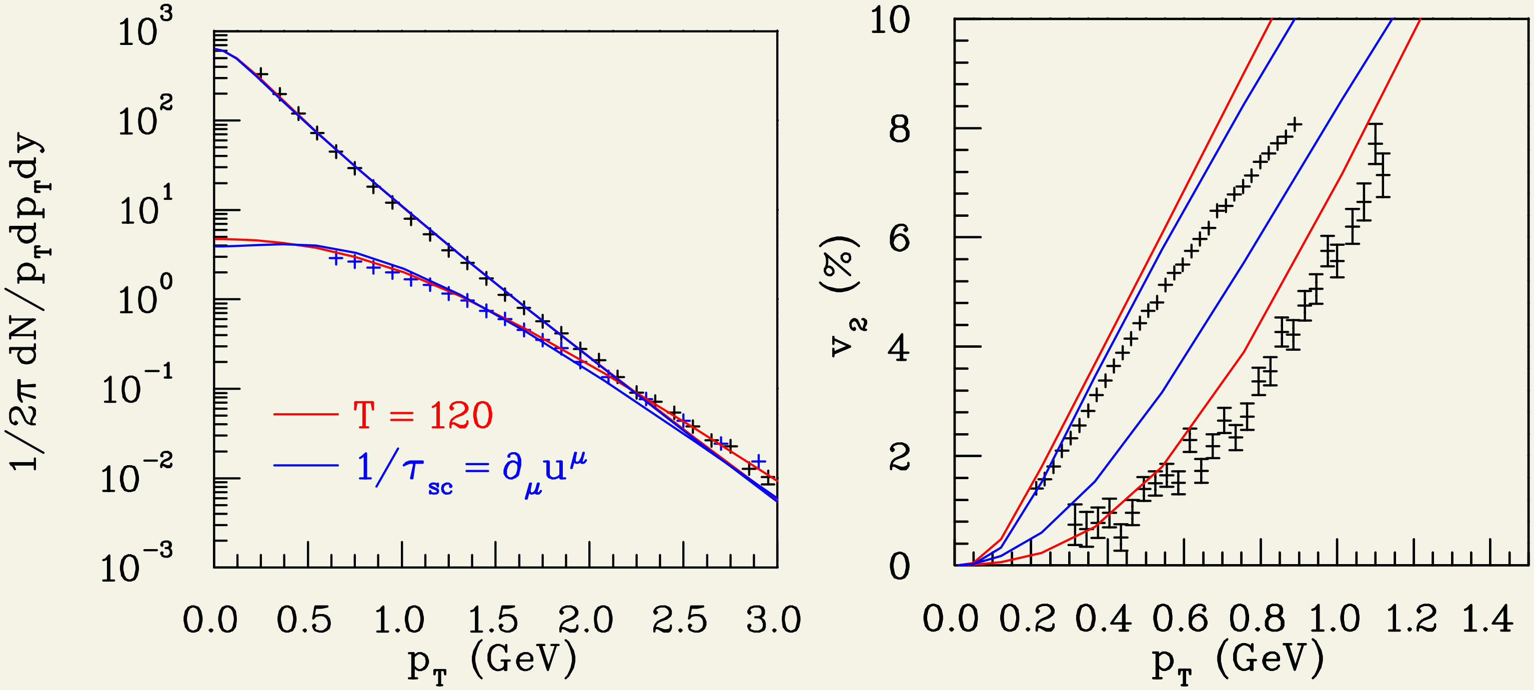
Effect on p_T spectra:



- **Need more transverse flow**

Early start?

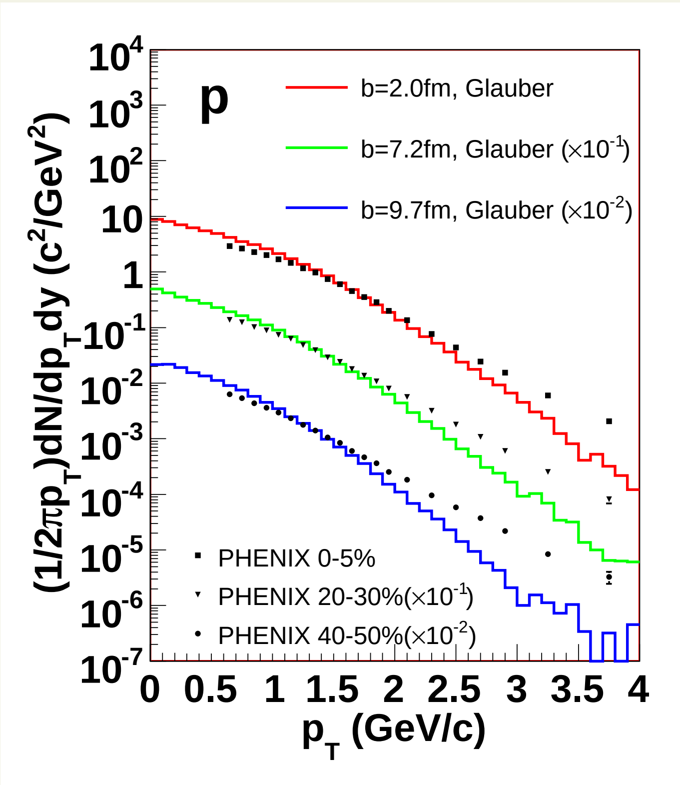
- freeze-out has only a small effect
- system freezes out soon after phase transition anyway
- results for chemical non-equilibrium and $\tau_0 = 0.2$ fm/c:



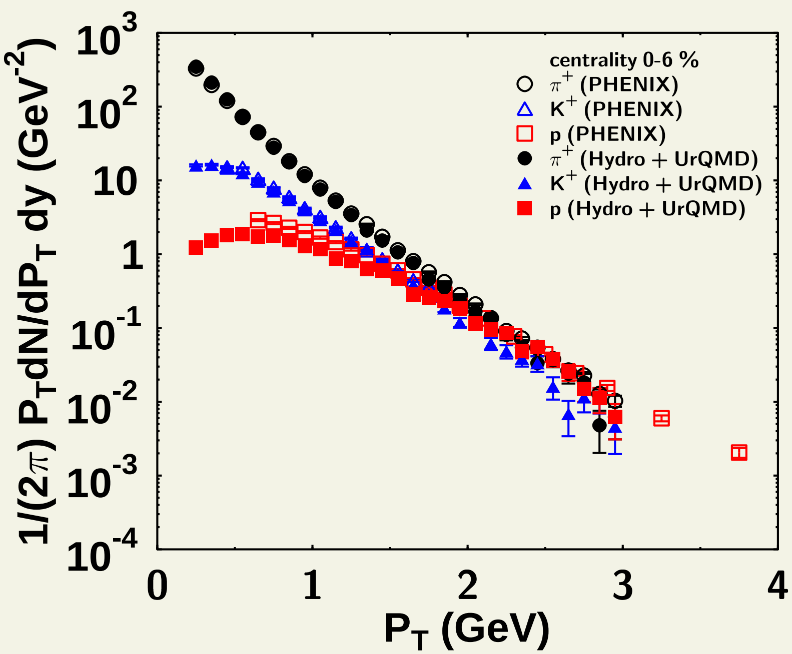
Hydro+cascade

- **Cascade description describes chemistry and freeze-out**
- **Reproduces both spectra and v_2**
- **$n \to 2$ processes not included**

Hirano et al., arXiv:0710.5795:



Nonaka & Bass, Phys.Rev.C75:014902,2007:



Viscous hydrodynamics

relativistic Navier-Stokes hydro: small corrections linear in gradients

$$T^{\mu\nu}_{NS} = T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha$$

$$N^{\mu}_{NS} = N^{\mu}_{ideal} - \frac{n}{e+p}\kappa\nabla^\mu T$$

where $\Delta^{\mu\nu} \equiv u^\mu u^\nu - g^{\mu\nu}$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

η, ζ shear and bulk viscosities, κ heat conductivity

two problems:

parabolic equations $\rightarrow$ acausal Müller ('76), Israel & Stewart ('79) ...

instabilities Hiscock & Lindblom, PRD31, 725 (1985) ...

Causal viscous hydro

Müller, Israel & Stewart...

$$\Delta T^{\mu\nu} \equiv \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} \quad , \qquad \Delta N^{\mu} = -\tfrac{n}{e+p} q^{\mu}$$

bulk pressure Π, shear stress $\pi^{\mu\nu}$ heat flow q^{μ} treated as independent dynamical quantities that relax to their Navier-Stokes value on time scales $\tau_{\Pi}(e,n)$, $\tau_{\pi}(e,n)$, $\tau_{q}(e,n)$ - corresponds to keeping not only first but (certain) second derivatives.

Entropy four-flow including terms second order in dissipative fluxes:

$$S^{\mu} = \quad su^{\mu} + \tfrac{\mu}{T}\tfrac{q^{\mu}}{h} \quad - \left(\beta_0\Pi^2 - \beta_1 q_{\nu}q^{\nu} + \beta_2 \pi_{\lambda\nu}\pi^{\lambda\nu}\right)\frac{u^{\mu}}{2T}$$
$$-\frac{\alpha_0 q^{\mu}\Pi}{T} + \frac{\alpha_1 q_{\nu}\pi^{\nu\mu}}{T}$$

Require non-decrease of entropy:

$$0 \geq \partial_{\mu}S^{\mu} = \Pi X + q_{\mu}X^{\mu} + \pi_{\mu\nu}X^{\mu\nu}$$

Which terms to keep?

$$\text{Muronga, Romatschke}: \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta \nabla^{\langle\mu} u^{\nu\rangle}\right)$$
$$- \left(u^\mu \pi^{\nu\alpha} + u^\nu \pi^{\mu\alpha}\right) u^\lambda \partial_\lambda u_\alpha$$
$$-\frac{1}{2}\pi^{\mu\nu}\left(u^\lambda \partial_\lambda \ln \frac{\beta_2}{T} + \partial_\lambda u^\lambda\right)$$

Which terms to keep?

$$\text{Muronga, Romatschke}: \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \left(u^\mu \pi^{\nu\alpha} + u^\nu \pi^{\mu\alpha}\right) u^\lambda \partial_\lambda u_\alpha - \frac{1}{2}\pi^{\mu\nu}\left(u^\lambda \partial_\lambda \ln\frac{\beta_2}{T} + \partial_\lambda u^\lambda\right)$$

$$\text{Song \& Heinz}: \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \left(u^\mu \pi^{\nu\alpha} + u^\nu \pi^{\mu\alpha}\right) u^\lambda \partial_\lambda u_\alpha$$

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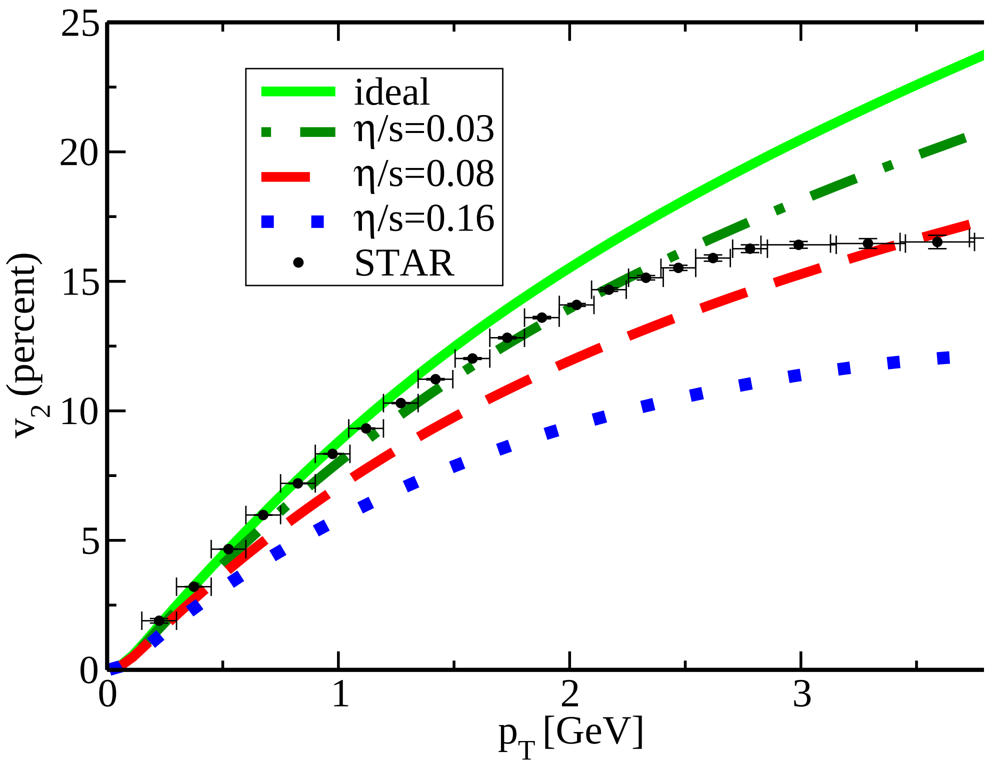
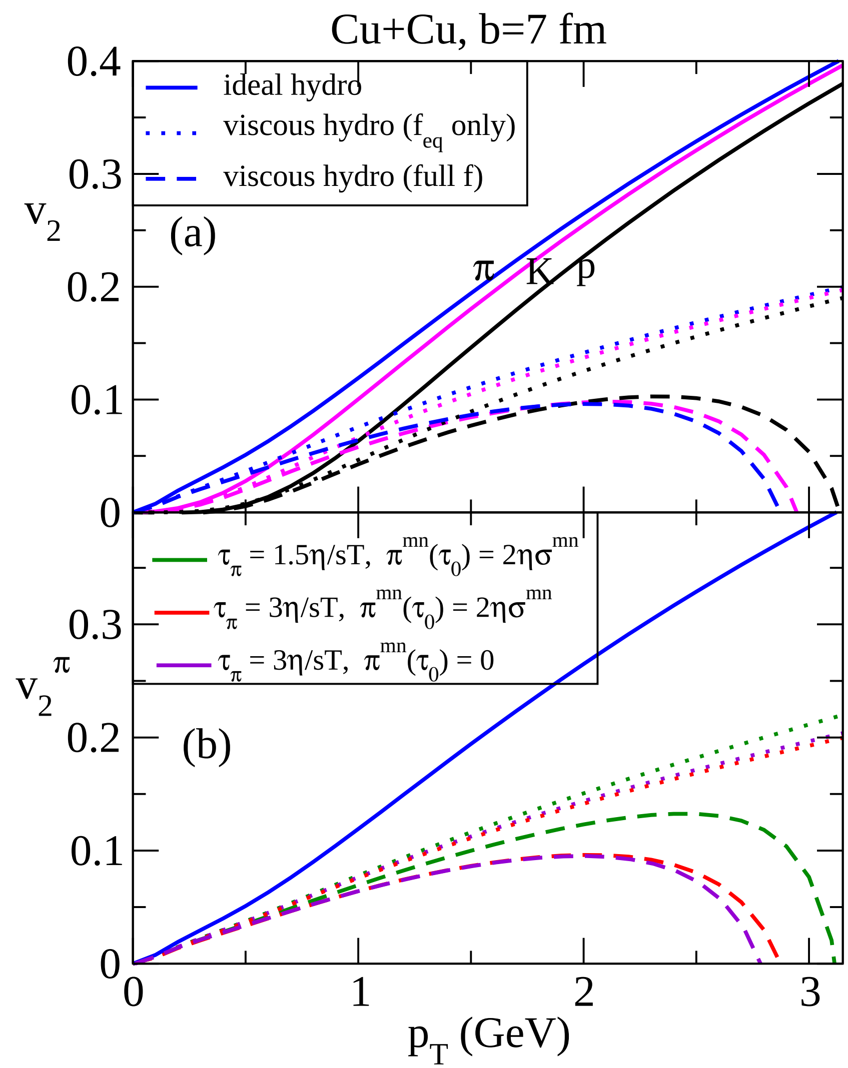
$$\text{Chaudhuri}: \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right)$$

$\eta/s = 1/(4\pi)$ corrections from IS hydro





Romatschke & Romatschke, arxiv:0706.1522 **Song & Heinz, arxiv:0709.0742**

10-20% **or** **50+% reduction??**

Viscous hydro vs transport

We solve the full Israel-Stewart-Muronga equations, including vorticity terms from kinetic theory, in a 2+1D boost-invariant scenario. Shear stress only.

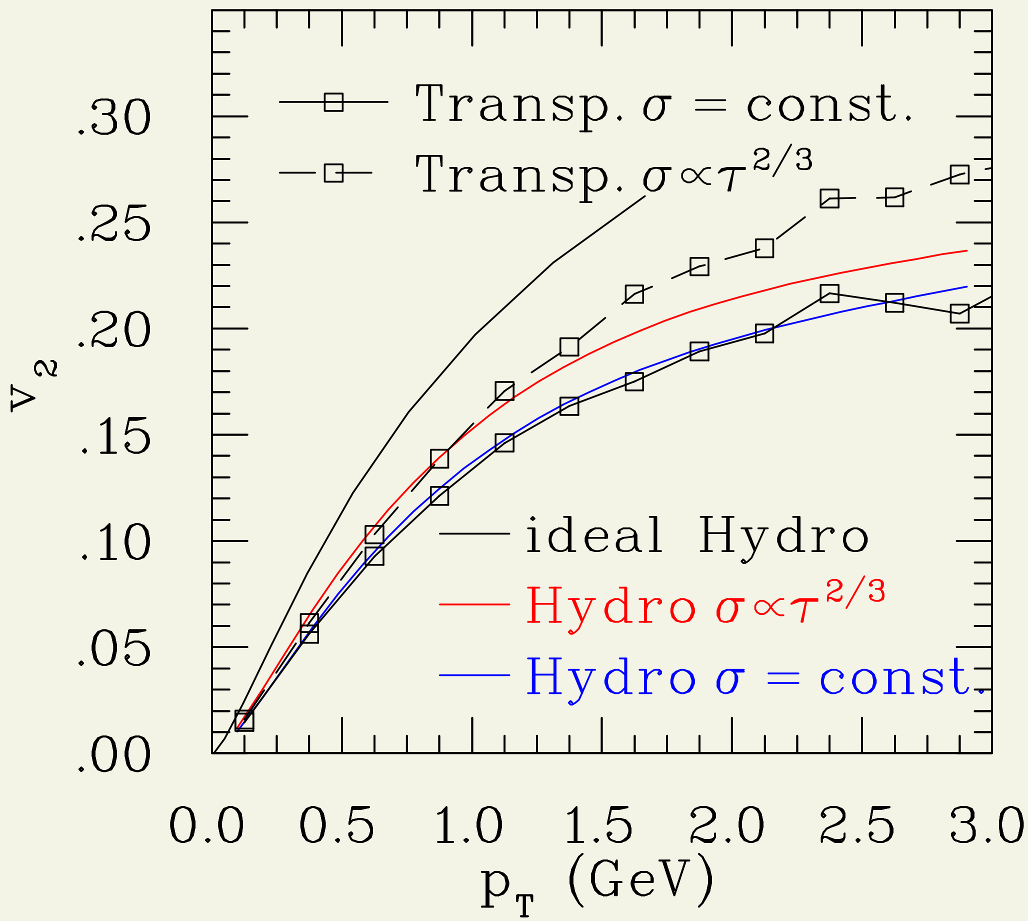
Mimic a known reliable transport model:

- **massless Boltzmann particles** $\Rightarrow \epsilon = 3P$
- **only** $2 \leftrightarrow 2$ **processes, i.e. conserved particle number**
- $\eta = 4T/(5\sigma_{\rm tr})$
- **either** $\sigma_{\rm tr} =$ **const.** the simplest in transport
  or $\sigma_{\rm tr} \propto \tau^{2/3}$ close to $\eta/s = 1/(4\pi)$

Our "RHIC-like" initialization:

- $\tau_0 = 0.6$ **fm**$/c$
- $b = 8$ **fm**
- $T_0 = 385$ **MeV** and $dN/d\eta|_{b=0} = 1000$
- **freeze-out at constant** $n = 0.365$ **fm**$^{-3}$

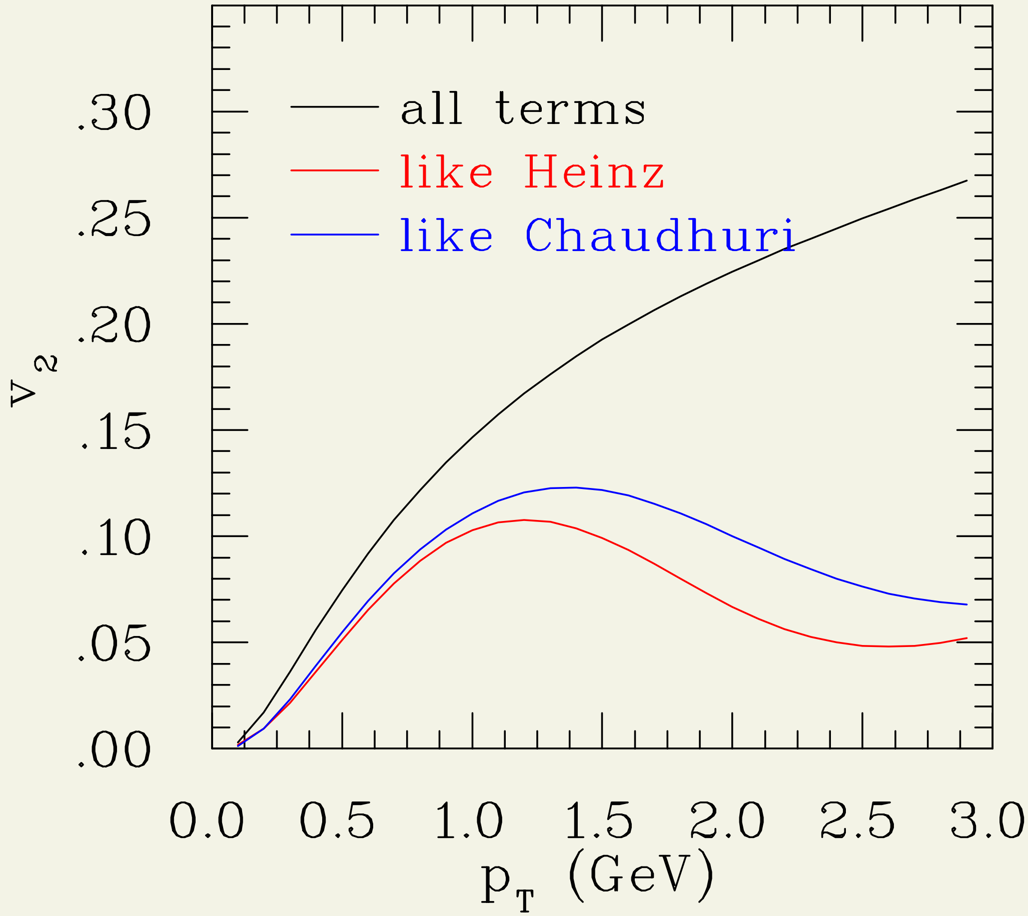
Viscous hydro vs transport v_2



- **excellent agreement when $\sigma =$ const ~ 47 mb**
- **good agreement fo $\eta/s \approx 1/(4\pi)$, i.e. $\sigma \propto \tau^{2/3}$**

- **BUT results sensitive to freeze-out criterion, especially at high p_T**

Viscous hydro v_2



- **Important to keep all terms!**

Conclusions

improvements to ideal hydrodynamical models

- crossover phase transition
- correct particle ratios
- more physical freeze-out

increase elliptic flow

dissipative corrections required to fit the data

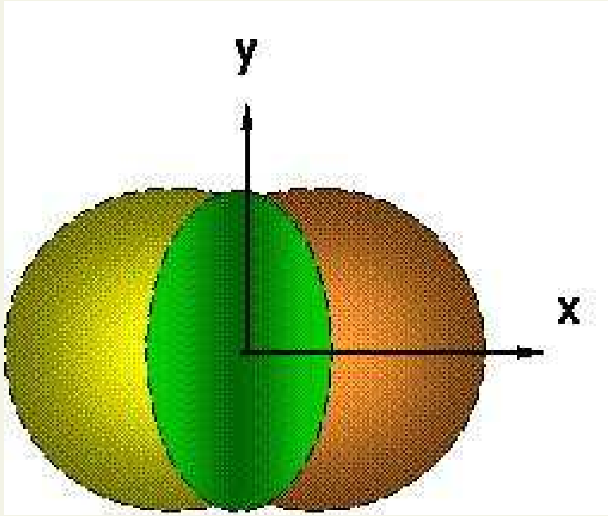
how large viscosity allowed in plasma unknown

prospects for applicability of Israel-Stewart causal hydrodynamics at RHIC look promising, based on comparisons with covariant transport in 2+1D Bjorken scenario

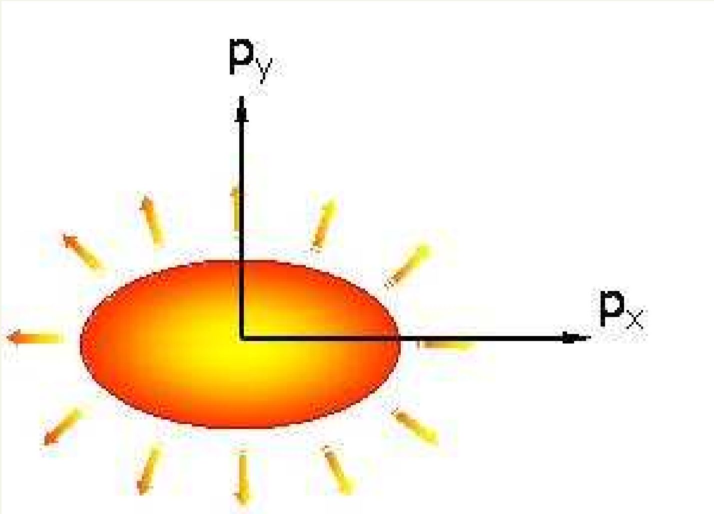
we expect $20 - 30$% reduction to $v_2(p_T)$ due to dissipation

Elliptic flow v_2

spatial anisotropy $\quad\rightarrow\quad$ final azimuthal momentum anisotropy



$\longrightarrow$



$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} \qquad\qquad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

sensitive to speed of sound $c_s^2 = \partial p / \partial e$ **and shear viscosity** η