Comparing viscous hydrodynamics to a parton cascade

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Viscous Hydrodynamics and Transport Models in Heavy Ion Collisions

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based on work in collaboration with Denes Molnar

Viscous hydrodynamics

relativistic Navier-Stokes hydro: small corrections linear in gradients

$$T^{\mu\nu}_{NS} = T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha$$

$$N^\mu_{NS} = N^\mu_{ideal} - \frac{n}{e+p}\kappa\nabla^\mu T$$

where $\Delta^{\mu\nu} \equiv u^\mu u^\nu - g^{\mu\nu}$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

η, ζ **shear and bulk viscosities,** κ **heat conductivity**

two problems:

parabolic equations $\rightarrow$ **acausal** Müller ('76), Israel & Stewart ('79) ...

instabilities Hiscock & Lindblom, PRD31, 725 (1985) ...

Causal viscous hydro

Müller, Israel & Stewart...

$$\Delta T^{\mu\nu} \equiv \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} \quad , \qquad \Delta N^{\mu} = -\tfrac{n}{e+p} q^{\mu}$$

bulk pressure Π, shear stress $\pi^{\mu\nu}$ heat flow q^{μ} treated as independent dynamical quantities that relax to their Navier-Stokes value on time scales $\tau_{\Pi}(e,n)$, $\tau_{\pi}(e,n)$, $\tau_{q}(e,n)$ - corresponds to keeping not only first but (certain) second derivatives.

Entropy four-flow including terms second order in dissipative fluxes:

$$S^{\mu} = \quad su^{\mu} + \tfrac{\mu}{T}\tfrac{q^{\mu}}{h} \quad -\left(\beta_0\Pi^2 - \beta_1 q_{\nu}q^{\nu} + \beta_2\pi_{\lambda\nu}\pi^{\lambda\nu}\right)\frac{u^{\mu}}{2T}$$
$$-\frac{\alpha_0 q^{\mu}\Pi}{T} + \frac{\alpha_1 q_{\nu}\pi^{\nu\mu}}{T}$$

Evolution equation for shear

Require non-decrease of entropy:

$$0 \geq \partial_\mu S^\mu = \Pi X + q_\mu X^\mu + \pi_{\mu\nu} X^{\mu\nu}$$

Identify $\pi^{\mu\nu} = 2\eta X^{\langle\mu\nu\rangle}$**:**

$$\begin{aligned}\pi^{\mu\nu} &= 2\eta \left[\nabla^{\langle\mu} u^{\nu\rangle} - \frac{1}{2}\pi^{\mu\nu} T \partial_\lambda \left(\frac{\tau_\pi u^\lambda}{2\eta T}\right)\right] - \tau_\pi \langle u^\lambda \partial_\lambda \pi^{\mu\nu}\rangle \\ &\quad + 2\eta \left[\alpha_1 \nabla^{\langle\mu} q^{\nu\rangle} + a_1' q^{\langle\mu} u^\lambda \partial_\lambda u^{\nu\rangle}\right]\end{aligned}$$

where

$$A^{\langle\mu\nu\rangle} = \left[\frac{1}{2}\left(\Delta^\mu_\sigma \Delta^\nu_\tau + \Delta^\nu_\tau \Delta^\mu_\sigma\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\sigma\tau}\right] A^{\sigma\tau}$$

and

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu.$$

Which terms to keep?

$$\text{Muronga, Romatschke :} \quad u^\alpha\partial_\alpha\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right)$$

$$-\left(u^\mu\pi^{\nu\alpha} + u^\nu\pi^{\mu\alpha}\right)u^\lambda\partial_\lambda u_\alpha$$

$$-\frac{1}{2}\pi^{\mu\nu}\left(u^\lambda\partial_\lambda \ln\frac{\beta_2}{T} + \partial_\lambda u^\lambda\right)$$

Which terms to keep?

$$\text{Muronga, Romatschke}: \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \left(u^\mu \pi^{\nu\alpha} + u^\nu \pi^{\mu\alpha}\right) u^\lambda \partial_\lambda u_\alpha - \frac{1}{2}\pi^{\mu\nu}\left(u^\lambda \partial_\lambda \ln\frac{\beta_2}{T} + \partial_\lambda u^\lambda\right)$$

$$\text{Song \& Heinz, Israel \& Stewart}: \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \left(u^\mu \pi^{\nu\alpha} + u^\nu \pi^{\mu\alpha}\right) u^\lambda \partial_\lambda u_\alpha$$

Which terms to keep?

Muronga, Romatschke :

$$u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \left(u^\mu\pi^{\nu\alpha} + u^\nu\pi^{\mu\alpha}\right)u^\lambda\partial_\lambda u_\alpha - \frac{1}{2}\pi^{\mu\nu}\left(u^\lambda\partial_\lambda \ln\frac{\beta_2}{T} + \partial_\lambda u^\lambda\right)$$

Song & Heinz, Israel & Stewart :

$$u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \left(u^\mu\pi^{\nu\alpha} + u^\nu\pi^{\mu\alpha}\right)u^\lambda\partial_\lambda u_\alpha$$

Chaudhuri :

$$u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right)$$

Potential caveat

Whereas Navier-Stokes is an expansion in λ/R (keeps only first derivatives), Israel-Stewart hydro is NOT a controlled approximation (retains certain second derivatives). For example in kinetic theory, it corresponds to Grad's 14-moment approximation

$$f(x,p) \approx [1 + C_{\alpha\beta}p^\alpha p^\beta] e^{(\mu - p^\mu u_\mu)/T}$$

while NS comes from the Chapman-Enskog expansion in small gradients

$$E\partial t f + \varepsilon \cdot \vec{p}\vec{\nabla} f = C[f] \quad , \qquad f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \ldots$$

If relaxation effects important, NS and IS are different

$\Rightarrow$ control against a nonequilibrium theory is crucial

Covariant transport

Boltzmann ..., Israel, Stewart, de Groot, ... Pang, Zhang, Gyulassy, Molnar, Vance, Csizmadia, Pratt, Cheng, Xu, Greiner ...

Covariant, causal, nonequil. approach - formulated in terms of local rates.

$$\Gamma_{2\to 2}(x) \equiv \frac{dN_{scattering}}{d^4x} = \sigma v_{rel} \frac{n^2(x)}{2}$$

This theory has a hydrodynamic limit (i.e., it equilibrates) Boltzmann

Parameter σ controls transport coefficients and relaxation:

$$\eta \approx 0.8 \frac{T}{\sigma_{tr}} \qquad \tau_\pi = 1.2\lambda_{tr}$$

solvable numerically: HERE, utilize MPC algorithm DM, NPA 697 ('02)

η/s for transport

"minimal" viscosity - corresponds to $\lambda_{tr} \approx 1/(3T_{eff}) \approx 0.1$ fm at $\tau_0 = 0.1$ fm

estimate from average density: $\lambda_{tr} = \frac{1}{\langle n \rangle \sigma_{tr}}$

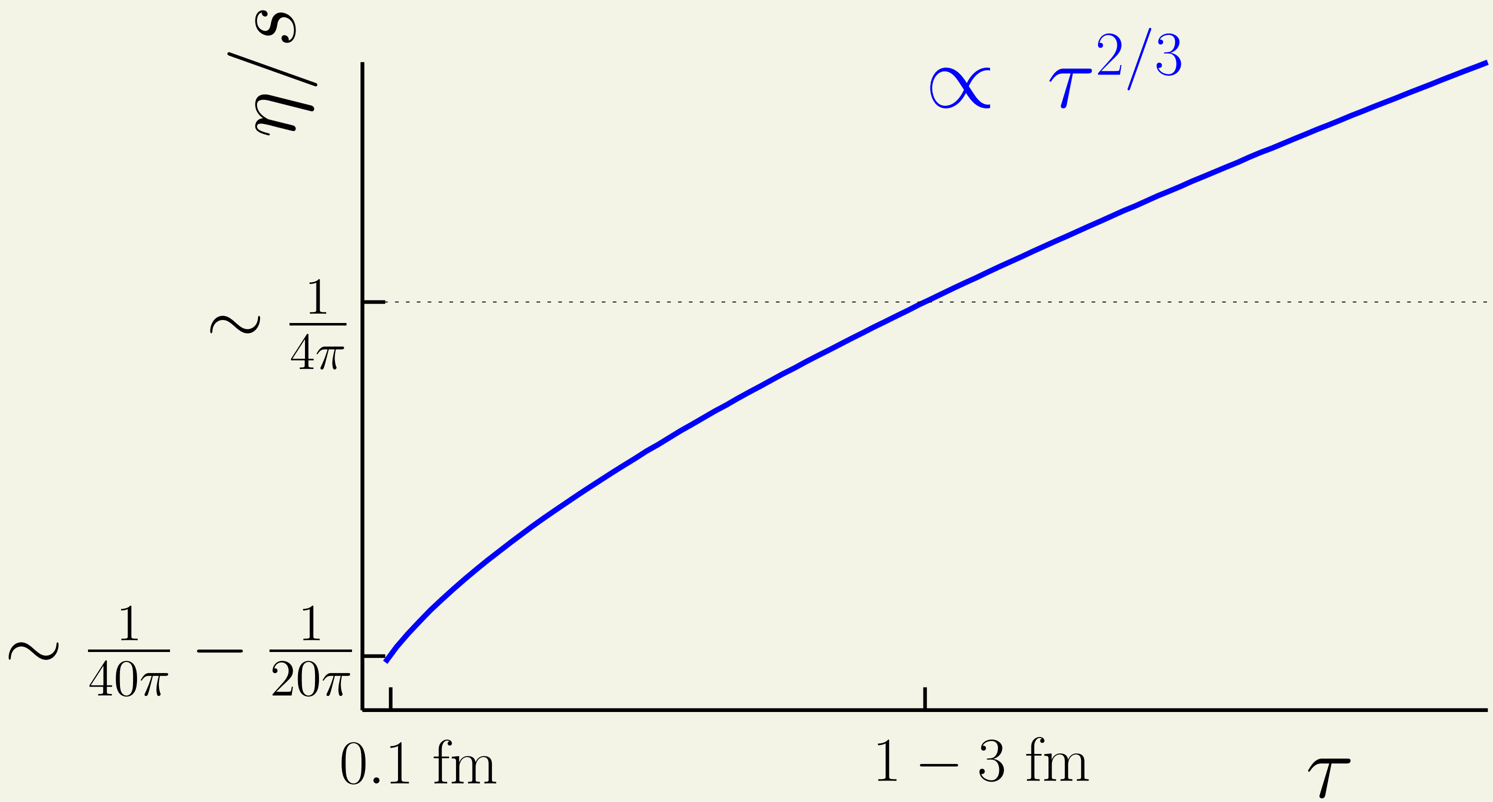
$$\lambda_{tr}(\tau_0) = \frac{\tau_0 A_T}{\sigma_{tr} dN/d\eta} \sim 1 - 2 \times 10^{-2} \text{ fm}$$

$\Rightarrow \sigma_{gg} \approx 50$ **mb is initially "better than perfect",**

after $\tau \sim 1 - 3$ fm "less than perfect"

In transport $\eta/s \sim \lambda_{tr}T \sim 1/(\sigma T^2)$

e.g., for $\sigma \approx 50$ **mb (**$\sigma_{tr} \approx 14$ **mb)**

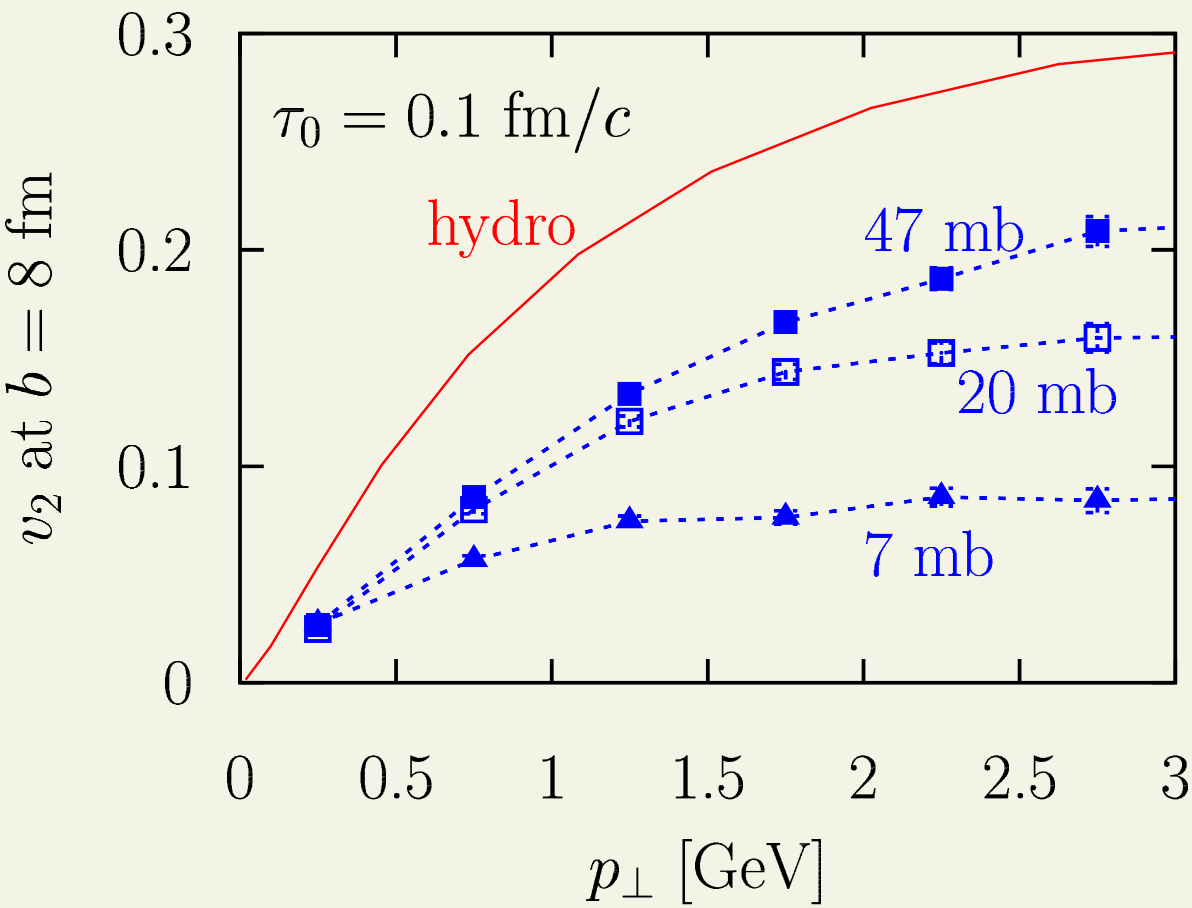


$\Rightarrow$ $\eta/s = const$ **needs** **growing** $\sigma(\tau) \propto 1/T^2 \propto \tau^{2/3}$

in perturbative QCD: $\sigma_{tr} \sim \frac{\alpha_s^2}{s} \ln \frac{s}{\mu_D^2} \sim \frac{g^4}{T^2} \ln \frac{1}{g^2}$

Not an ideal fluid

Molnar & Huovinen, PRL94 ('05): **final $v_2(p_T)$**



large gradients

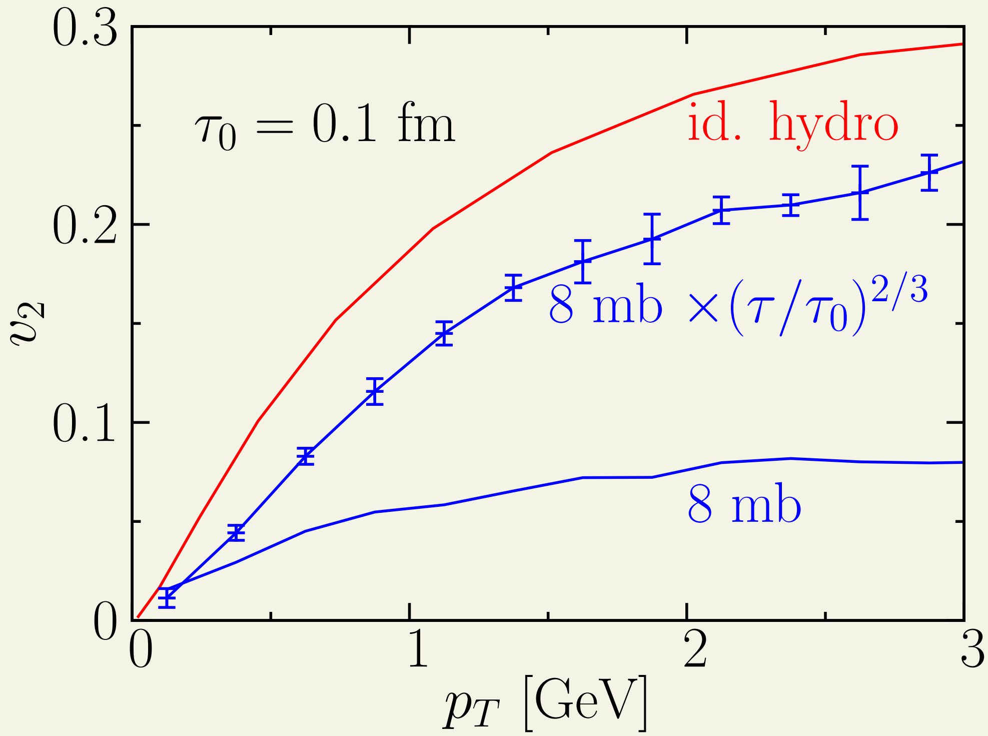
$\Rightarrow$ **even a tiny viscosity matters (need $\eta \nabla u \ll p$)**

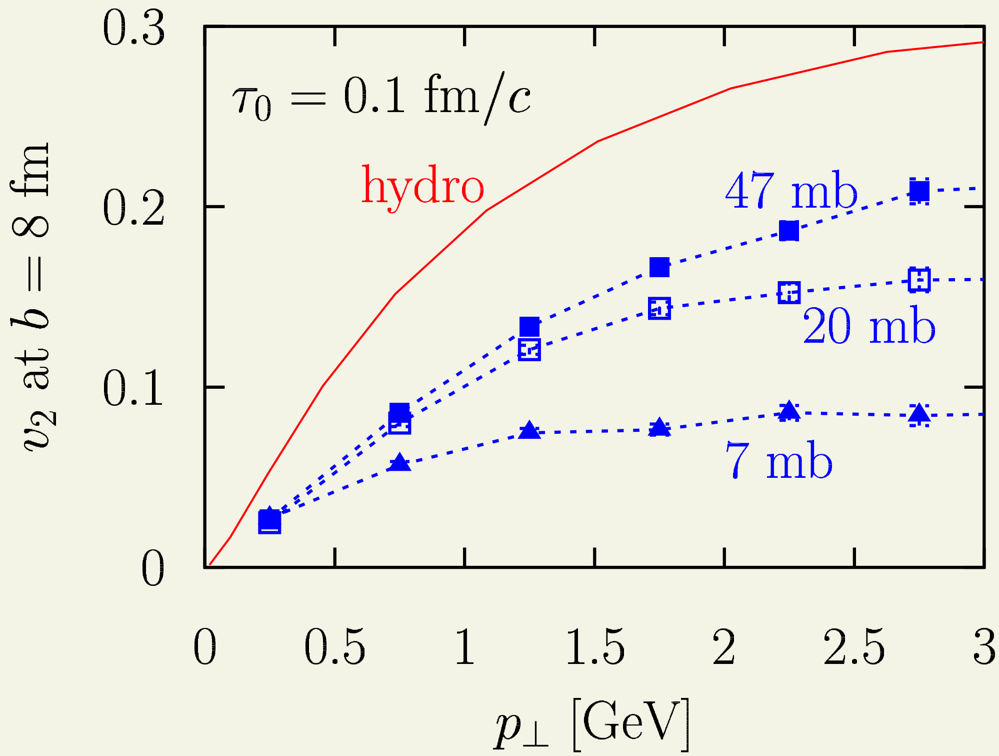
[identical RHIC Au+Au initconds, $b = 8$ fm, binary profile, $T_0 = 0.7$ GeV, e=3p EOS]

transport, now with "minimal viscosity" at RHIC

$\Rightarrow \sigma_{gg}(\tau = 0.1 \text{ fm}) \sim 8$ **mb**

Molnar '06: $b = 8$ **fm**





$\Rightarrow$ **almost identical to $\sigma_{gg} = 47$ mb**

IS hydro vs transport - 1D Bjorken

start with simplest of all cases - 1D Bjorken, massless $e = 3p$ EOS, $2 \to 2$

$\pi^{\mu\nu}_{LR} = diag(0, -\frac{\pi_L}{2}, -\frac{\pi_L}{2}, \pi_L)$, $\Pi \equiv 0$, $q^\mu \equiv 0$ **(reflection symm)**

$$\begin{aligned} \dot{p} + \frac{4p}{3\tau} &= -\frac{\pi_L}{3\tau} \\ \dot{\pi}_L + \frac{\pi_L}{\tau}\left(\frac{2K(\tau)}{3C} + \frac{4}{3} + \frac{\pi_L}{3p}\right) &= -\frac{8p}{9\tau} \ , \end{aligned}$$

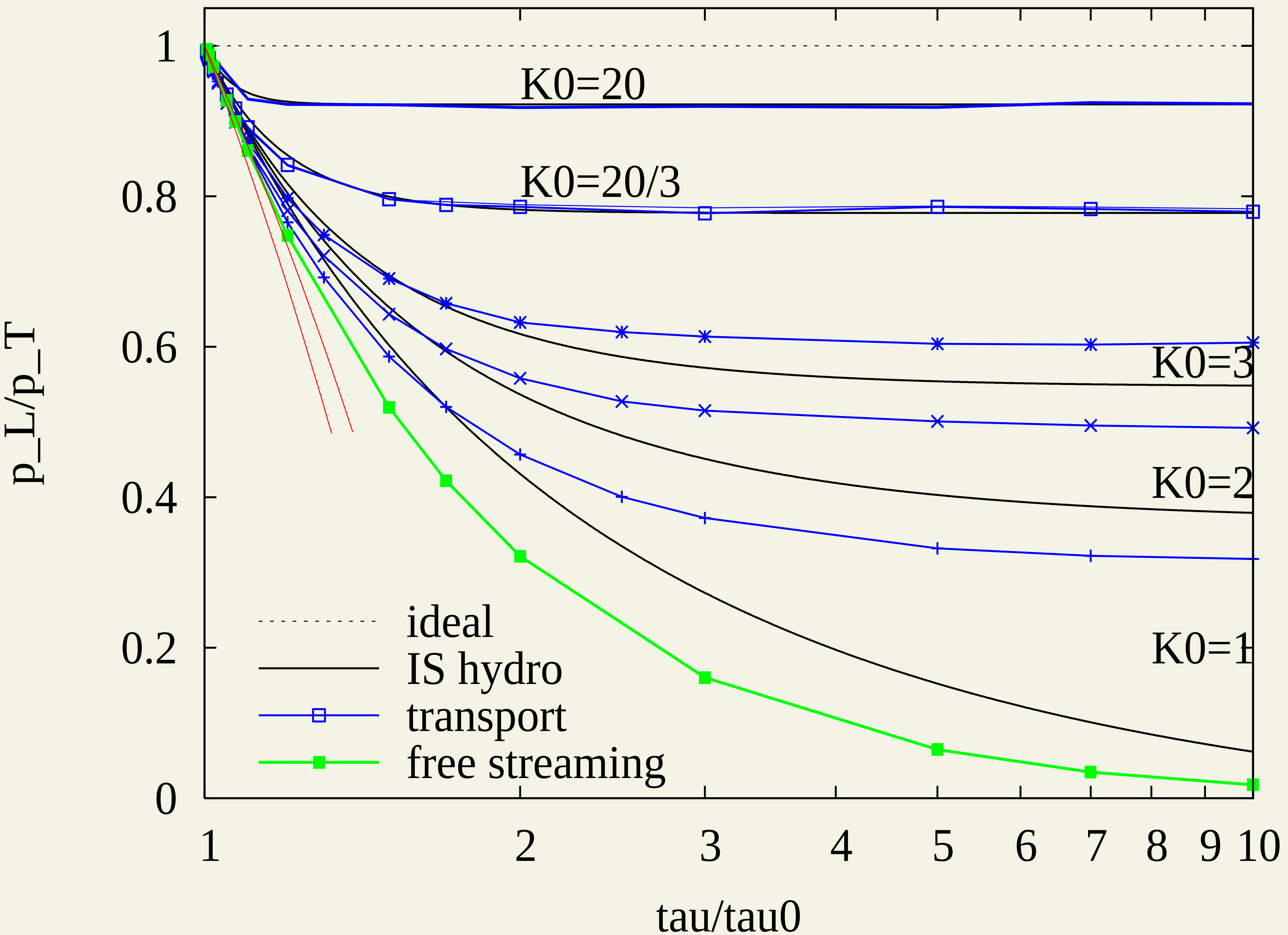
where

$$K(\tau) \equiv \frac{\tau}{\lambda_{tr}(\tau)} \ , \quad C \approx \frac{4}{5} \ .$$

For $\sigma = const$: $\lambda_{tr} = 1/n\sigma_{tr} \propto \tau \quad \Rightarrow \quad K = const$

for $\sigma \propto 1/T^2$: $K \propto \tau^{\sim 2/3}$

pressure anisotropy $T_{zz}/T_{xx} = (1 + \pi_L)/(1 - \pi_L/2)$



early evolutions differ but for $K \gtrsim 3$ Navier-Stokes sets in by $\Delta\tau/\tau_0 \gtrsim 2/K$

K **value at RHIC: driven by nuclear geometry and cross section, (remember,** $K = \tau/\lambda_{tr} = \tau n \sigma_{tr}$**)**

Au+Au, $\sigma_{gg} = 47$ **mb, collision center:** $b = 0$ fm - $K \approx 20$
$b = 8$ fm - $K \approx 12$
$b = 11$ fm - $K \approx 6$

On the other hand: $b = 8$ fm**:** $K = 3$ **at** $r \sim 3 - 4$ fm
system size $R \sim 4 - 6$ fm

$\Rightarrow$ **IS should work well for central, expect** $10 - 20$**% corrections at** $b = 8$ **fm**

Viscous hydro vs transport

We solve the full Israel-Stewart-Muronga equations, including vorticity terms from kinetic theory, in a 2+1D boost-invariant scenario. **Shear stress only.**

Mimic a known reliable transport model:

- massless Boltzmann particles $\Rightarrow \epsilon = 3P$
- only $2 \leftrightarrow 2$ **processes**, i.e. conserved particle number
- $\eta = 4T/(5\sigma_{\rm tr})$
- either $\sigma_{\rm tr} =$ **const.** $= 47$ **mb** $(\sigma_{tr} = 14$ **mb)** $\leftarrow$ the simplest in transport
  or $\sigma_{\rm tr} \propto \tau^{2/3}$ close to $\eta/s = 1/(4\pi)$

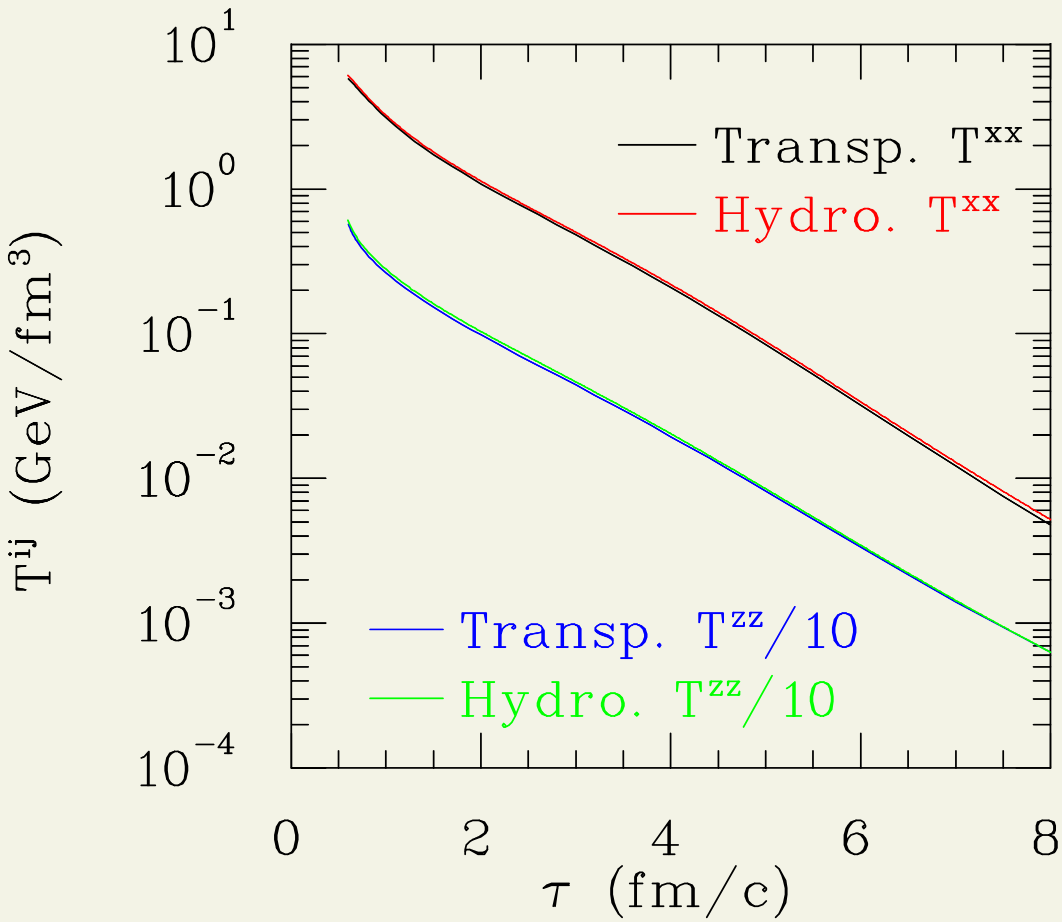
Our "RHIC-like" initialization:

- $\tau_0 = 0.6$ **fm/**c
- $b = 8$ **fm**
- $T_0 = 385$ **MeV** and $dN/d\eta|_{b=0} = 1000$
- freeze-out at **constant** $n = 0.365$ **fm**$^{-3}$

Pressure evolution in the core

T^{xx} **and** T^{zz} **averaged over the core of the system,** $r < 1$ **fm:**

$\eta/s \approx 1/(4\pi)$ $(\sigma_{\rm tr} \propto \tau^{2/3})$



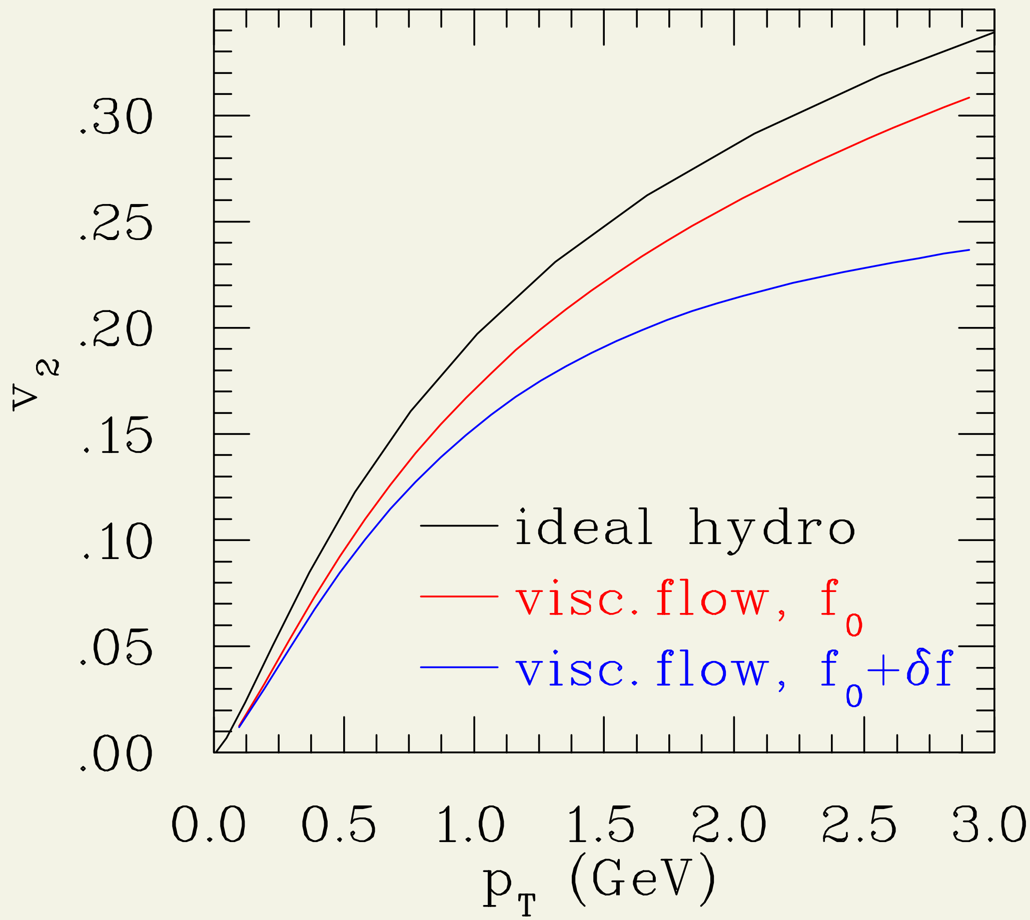
remarkable similarity!

Viscous hydro elliptic flow

TWO effects: **- dissipative corrections to hydro fields** u^μ, T, n
- dissipative corrections to thermal distributions $f \to f_0 + \delta f$

$\eta/s \approx 1/(4\pi)$ $(\sigma_{\rm tr} \propto \tau^{2/3})$

$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^6}\right]$$



Calculation for $\sigma_{\rm tr} =$ **const.** ~ 15 **mb shows similar behaviour**

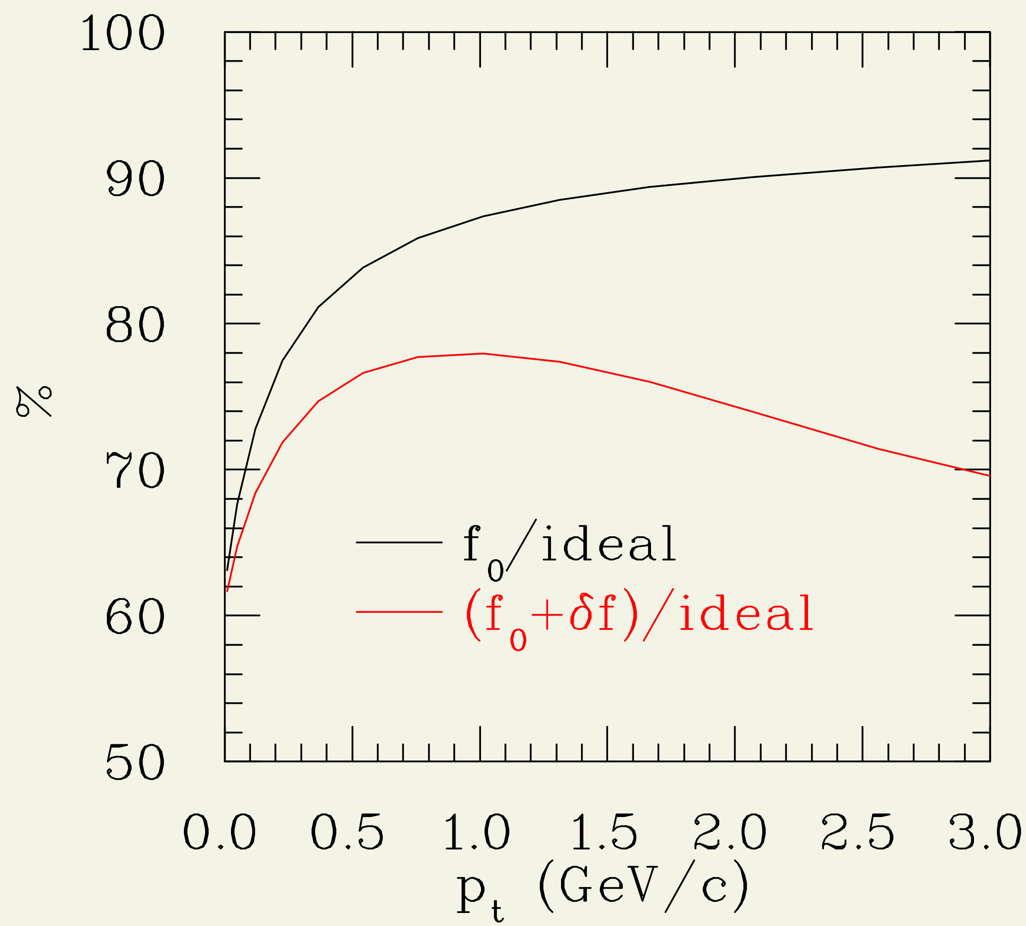
Viscous hydro elliptic flow

TWO effects: - **dissipative corrections to hydro fields** u^μ, T, n

- **dissipative corrections to thermal distributions** $f \rightarrow f_0 + \delta f$

$\eta/s \approx 1/(4\pi)$ $(\sigma_{\rm tr} \propto \tau^{2/3})$

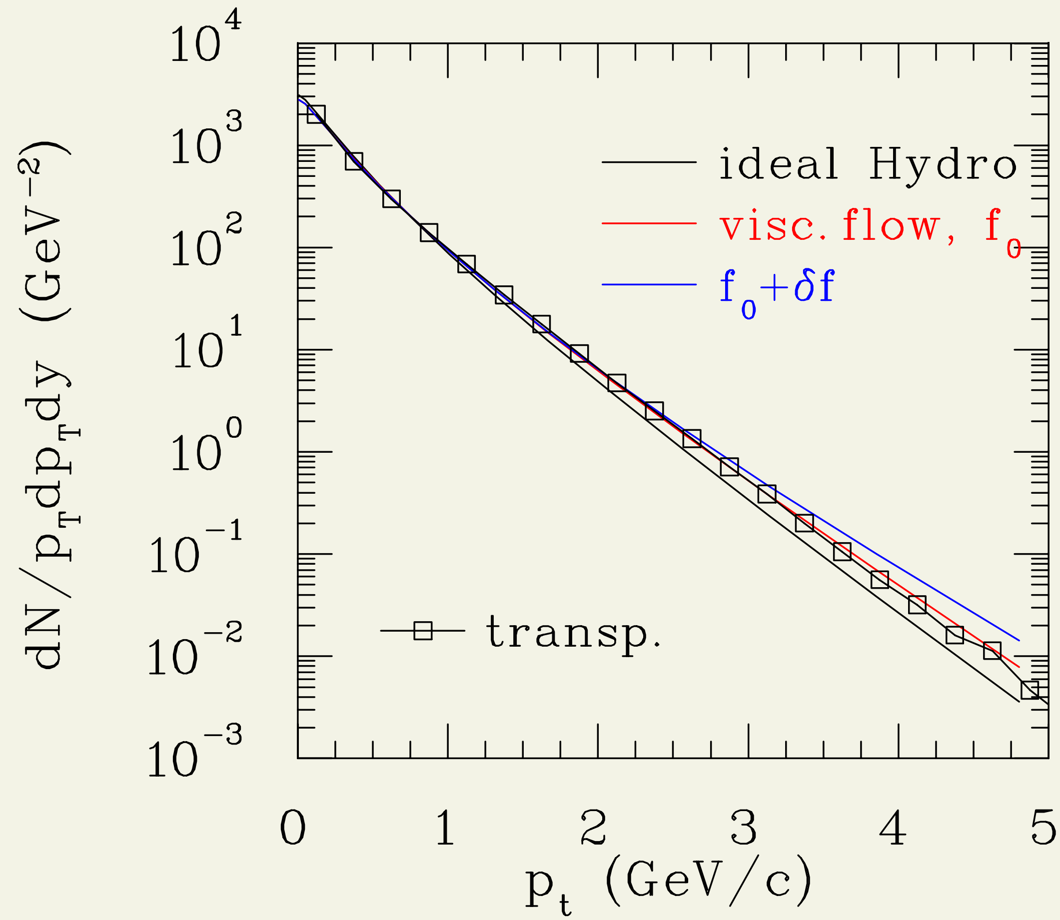
$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^6}\right]$$



Viscous hydro p_T distribution

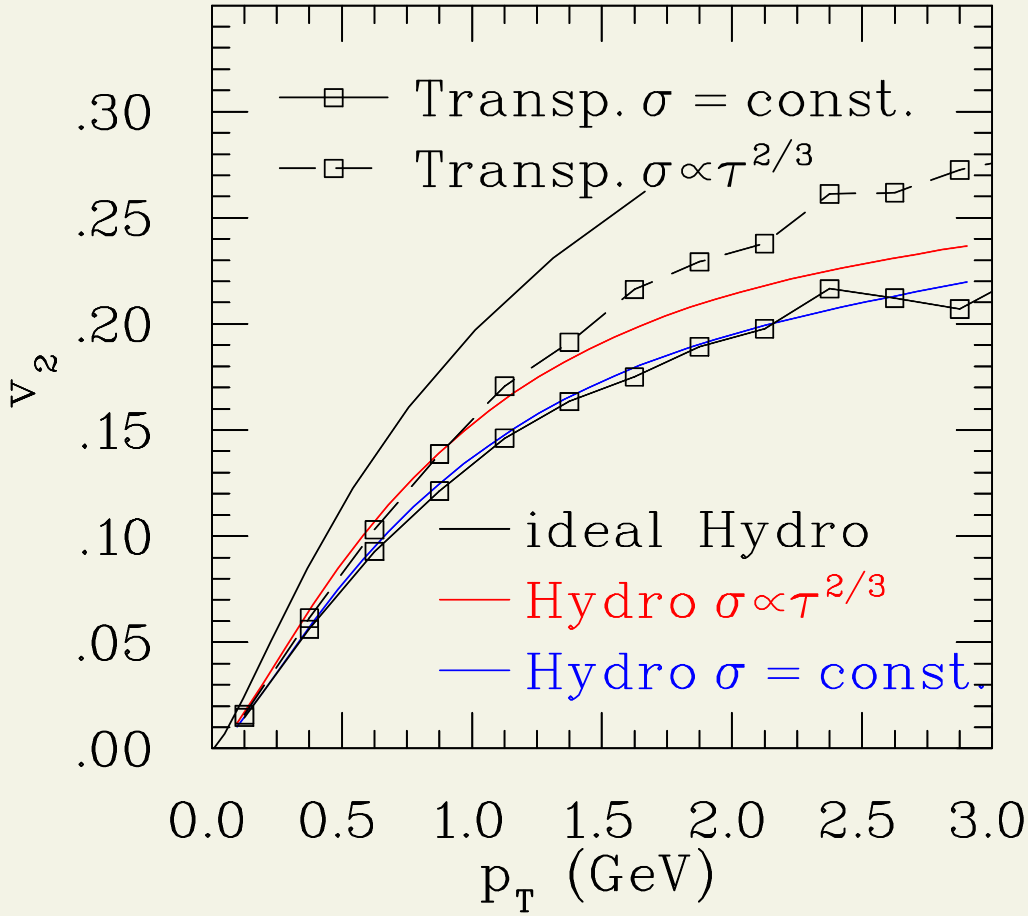
$\eta/s \approx 1/(4\pi)$ $(\sigma_{\rm tr} \propto \tau^{2/3})$

$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^6}\right]$$



- **Ideal hydro slightly below viscous and transport**
- **Viscous hydro and transport very close**

Viscous hydro vs transport v_2

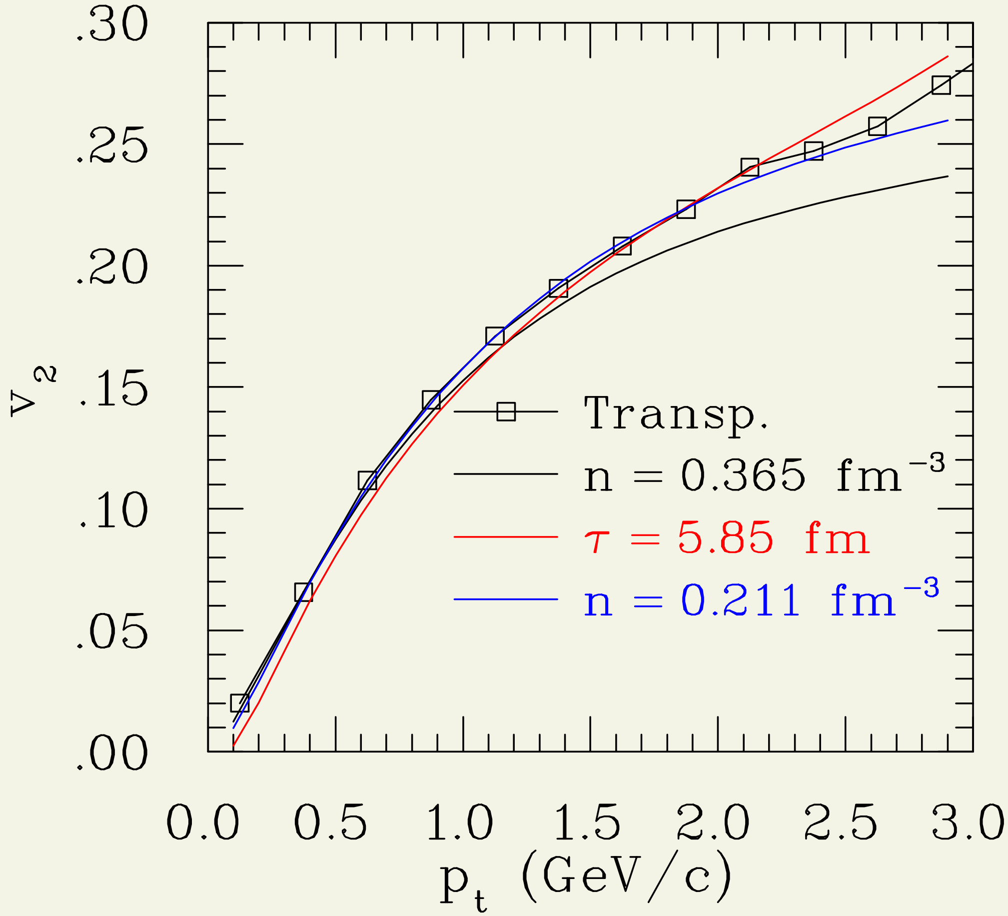


- **excellent agreement when $\sigma =$ const ~ 47 mb**
- **good agreement fo $\eta/s \approx 1/(4\pi)$, i.e. $\sigma \propto \tau^{2/3}$**

- **BUT results sensitive to freeze-out criterion, especially at high p_T**

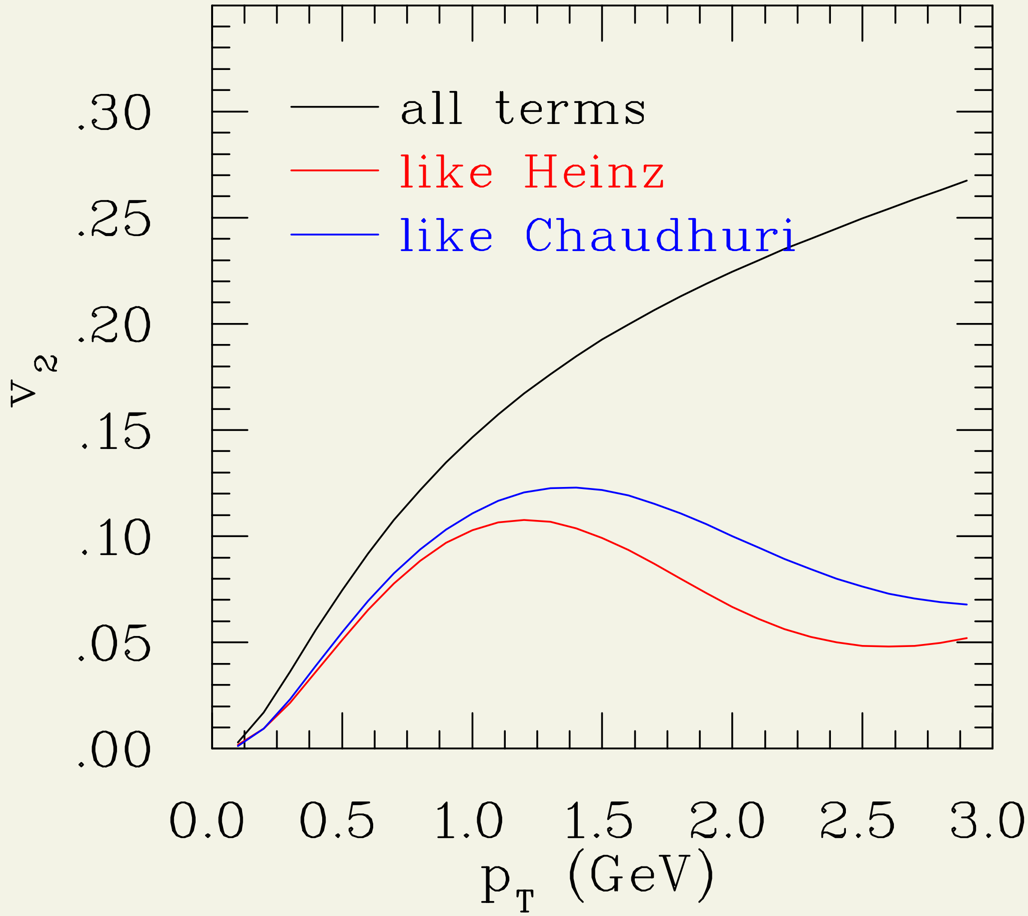
Effect of freeze-out criterion

$\eta/s \approx 1/(4\pi)$ $(\sigma_{\mathrm{tr}} \propto \tau^{2/3})$



- **some sensitivity to the freeze-out criterion**
- **not crucial for the results**

Which terms to keep?



- **Important to keep all terms!**

Conclusions

Prospects for applicability of Israel-Stewart causal hydrodynamics at RHIC look promising, based on comparisons with covariant $2 \to 2$ transport in 2+1D Bjorken scenario

Dissipative effects change both flow and distributions

Dissipation reduces $v_2(p_T)$ by $20 - 30$% for $\eta/s = 1/(4\pi)$ and conditions expected at RHIC

Hydrodynamical results are sensitive to the freeze-out procedure

$\longrightarrow$ being investigated