# Transverse Momentum Spectra of Hadrons and Clusters in Central Au+Au Collisions at RHIC from Hydrodynamics*

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We employ hydrodynamics with longitudinal scaling flow to calculate the transverse momentum spectra of $\pi$, $K$, $N$, $\overline{N}$, $\Phi$, $\Lambda$, $\overline{\Lambda}$, $\Omega$, $\Lambda\Lambda$, $d$, $He$ in central $Au+Au$ collisions at BNL-RHIC energy up to $p_T = 4$ GeV. We discuss the sensitivity of those spectra to the hypersurface on which they are calculated as well as to the initial density profile. We show that the average transverse flow velocity, is rather similar to central $Pb+Pb$ collisions at SPS energy. This is due to "stall" of the flow within the long-lived mixed phase at RHIC. We also discuss that in case of thermalization and hydrodynamic expansion $\langle p_T \rangle$ increases linearly with the hadron mass. In contrast, the string model FRITIOF 7.02 predicts a strong dependence of $\langle p_T \rangle$ on the quark composition of the hadron. On average, hadrons with charm acquire significantly more transverse momentum than hadrons without $c$-quarks.

## I. INTRODUCTION

A variety of experimental data [1–4] on relativistic heavy-ion collisions at the BNL-AGS and CERN-SPS, as e.g. rapidity and transverse momentum spectra of various hadrons and clusters, indicate significant collective (longitudinal and transverse) expansion of the produced hot and dense fireball [5–10]. Also at the much higher (center of mass) energy that will be reached with the BNL-RHIC, $\sqrt{s} = 200A$ GeV, it is commonly expected that the parton density at midrapidity in central collisions is high enough to lead to rapid thermalization [11–13].

In the present paper we discuss the collective evolution of the system following thermalization within a hydrodynamical model. We focus on the transverse momentum spectra, and mean transverse momenta and velocities of various hadron species. Finally, we also calculate the $\langle p_T \rangle$ of various hadrons in $Au+Au$ at RHIC within a string model without rescattering of produced particles (FRITIOF 7.02 [14]) and compare its predictions to those of hydrodynamics. A more detailed presentation of the model, as well as further results, e.g. a comparison of the space-time evolution at RHIC and at SPS, can be found in ref. [15].

## II. MODEL DESCRIPTION

Hydrodynamics is defined by (local) energy-momentum and net charge conservation [16],

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad \partial_\mu N_i^\mu = 0 \quad . \tag{1}$$

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$T^{\mu\nu}$ denotes the energy-momentum tensor, and $N_i^\mu$ the net four-current of the $i$-th conserved charge. We will explicitly consider only one such conserved charge (i.e. baryon number). We implicitly assume that all other charges that are conserved on strong-interaction time scales vanish locally. The corresponding four-currents are therefore identically zero.

We employ the ideal fluid approximation [16], $T^{\mu\nu} = (\epsilon + p)\, u^\mu u^\nu - p g^{\mu\nu}$, $N_B^\mu = \rho_B u^\mu$, where $\epsilon$, $p$, $\rho_B$ are energy density, pressure, and net baryon density in the local rest frame of the fluid, and $u^\mu = \gamma(1, \vec{v})$ is its four-velocity. For simplicity, we assume cylindrically symmetric expansion with a longitudinal scaling flow profile, $v_z = z/t$ [17]. This considerably simplifies the set of eqs. (1), cf. [15,17,18].

To close the system of coupled equations of hydrodynamics, an equation of state (EoS) has to be specified. Lattice QCD predicts a phase transition from ordinary nuclear matter to a so-called Quark-Gluon Plasma (QGP) at a critical temperature of $T_C = 140 - 160$ MeV [19] (for $\rho_B = 0$). We model the high-temperature phase as an ideal gas of $u$-, $d$-, $s$-quarks (with masses $m_u = m_d = 0$, $m_s = 150$ MeV), and gluons, employing the well-known MIT bag model EoS [20]. In the low-temperature region we assume an ideal hadron gas that includes the well-established (strange and non-strange) hadrons up to masses of 2 GeV. The phase coexistence region at low $\rho_B$ (as required in our calculation) is constructed employing Gibbs' conditions of phase equilibrium. The bag parameter of $B = 380$ MeV/fm$^3$ is chosen to yield the critical temperature $T_C \approx 160$ MeV.

It is convenient to specify the initial conditions in scaling hydrodynamics on a proper-time hyperbola $\tau = \tau_i$, where $\tau = \sqrt{t^2 - z^2}$. Except for $\tau_i$, we also have to specify the entropy per net baryon, $s/\rho_B$, and the net baryon rapidity density at midrapidity, $dN_B/dy$. By virtue of the EoS, the initial energy and (net) baryon densities as well as the temperature and the chemical potentials are also determined. Finally, we assume some initial transverse density distribution, and the solution of relativistic ideal hydrodynamics itself is uniquely determined in the forward light-cone.

Thermalization of the dense parton gas in the central region might occur within roughly half a fermi [11–13]. Following [18] we will assume $\tau_i = R_T/10 = 0.6$ fm for $Au$ nuclei.

According to various microscopic models, e.g. PCM [12], RQMD 1.07 [21], FRITIOF 7.02 [22], and HIJING/B [23], a net baryon rapidity density of $dN_B/dy \approx 20 - 35$ and specific entropy of $s/\rho_B \approx 150 - 250$ might be expected in central $Au + Au$ at $\sqrt{s} = 200A$ GeV at midrapidity. Perturbative QCD calculations of minijet production in leading logarithm approximation with a transverse momentum cutoff of $p_0 = 1$ GeV yield $s/\rho_B = 150$, if a soft component with $s/\rho_B = 50$ is assumed [13].

We will employ $dN_B/dy = 25$ and $s/\rho_B = 200$. For our choice of $\tau_i$ and equation of state, the resulting energy and baryon densities at midrapidity are $\epsilon_i = 17$ GeV/fm$^3$ and $\rho_i = 2.3\rho_0$. The initial temperature and quark-chemical potentials follow as $T_i = 300$ MeV, $\mu_q = 47$ MeV, $\mu_s = 0$, respectively. For such high specific entropy and relatively low net baryon number a pion multiplicity of $dN_\pi/dy \sim 850$ and antibaryon to baryon ratios of $\simeq 0.5$ and higher can be expected at midrapidity [24]. The Bjorken formula, $dE_T/dy = \pi R_T^2 \epsilon_i \tau_i$, multiplied by 1/2 to roughly account for the work performed by transverse expansion, predicts a collective transverse energy at midrapidity (in central collisions) of $dE_T/dy \approx 600$ GeV.

The radial energy and net baryon densities at $\tau = \tau_i$ are taken to be proportional to $\Theta(R_T - r_T)$. $R_T = 6$ fm denotes the nuclear radius. To show the effect of the initial pressure gradients on collective particle production at high $p_T$ we will also employ a so-called "wounded nucleon" distribution $\epsilon(\tau_i, r_T) = \epsilon_i f(r_T)$, $\rho_B(\tau_i, r_T) = \rho_i f(r_T)$, with $f(r_T) = 3\sqrt{1 - r_T^2/R_T^2}/2$.

We will employ the description of Cooper and Frye [25] to calculate the momentum distributions of the particles on some given hypersurface,

$$\frac{dN}{d^2p_T dy} = \int_0^1 d\zeta \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\eta \; r_T \, \tau \left( p_T \cos\phi \frac{d\tau}{d\zeta} - m_T \cosh(y - \eta) \frac{dr_T}{d\zeta} \right) f\left(p_\mu u^\mu\right) \quad , \tag{2}$$

where $p^\mu = (m_T \cosh y, p_T \sin\chi, p_T \cos\chi, m_T \sinh y)$, and $u^\mu = \gamma_T(\cosh\eta, v_T \sin(\phi - \chi), v_T \cos(\phi - \chi), \sinh\eta)$ denote the particle four-momentum and the fluid four-velocity, respectively. $\zeta \in [0, 1]$ parametrizes the hypersurface (counter clock-wise) on which the momentum distribution of particles is to be calculated, such that $r_T(\zeta)$ and $\tau(\zeta)$ specify the space-time points on the hypersurface. Since we focus on particles with $m_T \gg T_{fo}$, we approximate the phase-space distribution function $f$ by a Boltzmann distribution.

We will calculate the particle momentum distributions at RHIC on the hypersurface where the fluid enters the hadronic phase[1], $\lambda = 0$, as well as on the $T = 130$ MeV isotherm.

## III. RESULTS

Fig. 1 shows the $p_T$-spectra as calculated on the $\lambda = 0$ and $T = 130$ MeV hypersurfaces. The $\Lambda\Lambda$ is a hypothetical bound state of two $\Lambda$-baryons for which we assume a binding energy of 30 MeV. If freeze-out occured deeper in the hadronic phase, e.g. on the $T = 130$ MeV isotherm, the resulting spectra are of course "stiffer" since collective flow is significantly stronger than on the $\lambda = 0$ hypersurface. This leads to much broader $p_T$-distributions of the heavy hadrons. Note that the spectra of $\pi$, $K$, $N$, $\Lambda$ at $p_T \lesssim 1$ GeV will be modified considerably due to post-freeze-out decays of heavier resonances [9].

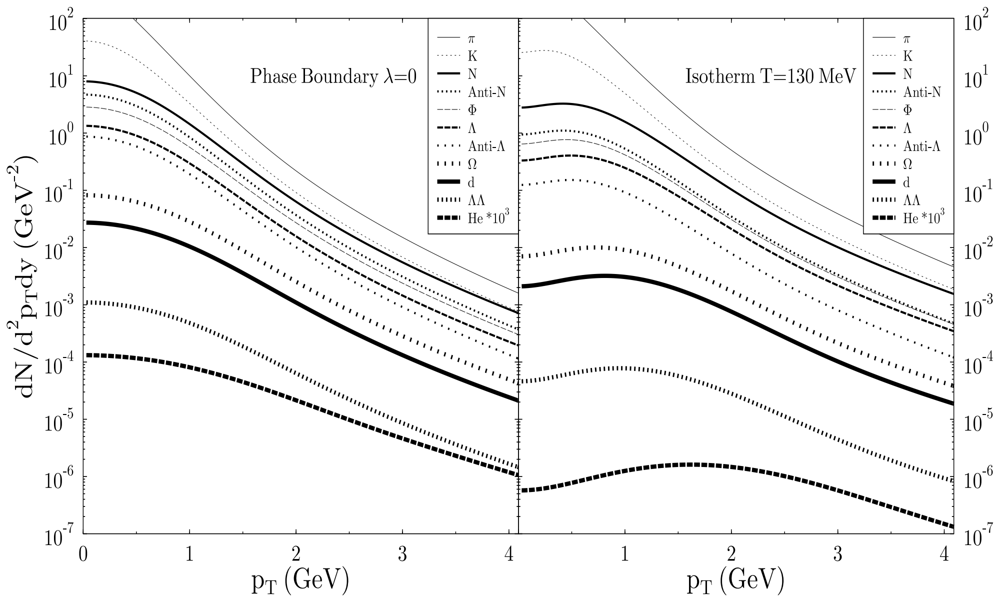


FIG. 1. Transverse momentum spectra of direct thermal hadrons at midrapidity (central $Au + Au$ collisions at RHIC), as calculated on the $\lambda = 0$ and $T = 130$ MeV hypersurfaces, respectively. A step function has been employed for the initial transverse energy and baryon density profiles.

[1] $\lambda \equiv V^{QGP}/V^{tot}$ denotes the fraction of quark-gluon plasma within the mixed phase.

For the "wounded nucleon" density profile the very fast tails in the velocity distribution on the hypersurface are absent, thus reducing the number of hadrons at $p_T > 2$ GeV. Figures 1 and 2 show the possible range of collective particle production at high transverse momenta. On the other hand, we have found that the two initial density distributions lead to *average* transverse momenta that differ by $\leq 10\%$. Therefore, we present results for $\langle p_T \rangle$ only for the sharp density profile.

Figure 3 depicts the average transverse momenta and velocities of various hadrons at midrapidity, as calculated from the $p_T$-distributions:

$$(\langle p_T \rangle_i, \langle v_T \rangle_i) = \frac{\int d^2 p_T \, (p_T, p_T/p^0) \frac{dN_i}{d^2 p_T dy}}{\int d^2 p_T \, \frac{dN_i}{d^2 p_T dy}} \quad . \tag{3}$$

One observes that due to the collective transverse expansion, $\langle p_T \rangle = m \langle \gamma_T v_T \rangle$ increases linearly with the particle mass[2].

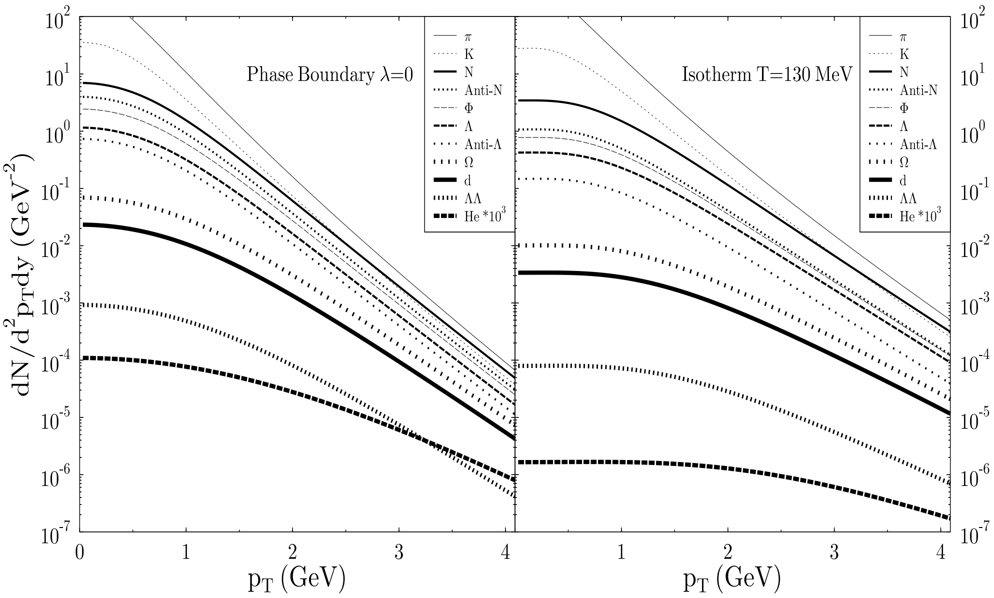


FIG. 2. Same as Fig. 1 but for a "wounded nucleon" initial distribution.

Also, $\langle v_T \rangle$ saturates with increasing $m$, since the thermal velocities $\sim \sqrt{T_{fo}/m}$ become negligible and $\langle v \rangle \approx v_{flow}$. Thus, clusters ($d$, $He$, ...) provide the opportunity to determine the transverse flow velocity *directly* [6,26], while the thermal averaging can not be neglected for nucleons and especially for pions and kaons. To extract the collective flow from their $\langle p_T \rangle$ or $\langle v_T \rangle$ requires the knowledge of the freeze-out

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[2] Note, however, that $\langle \gamma_T v_T \rangle$ does in principle depend on the particle mass since it includes flow as well as thermal velocity components, cf. left panel of Fig. 3. Thus, the result $\langle p_T \rangle \sim m$ is not trivial.

temperature, and moreover might be distorted by decays of resonances [4,27]. The average transverse flow velocity in $Pb + Pb$ at SPS ($\sqrt{s} = 18A$ GeV), as extracted from fits of the $p_T$-dependent multiplicity ratios and $p_T$-spectra, is $\approx 0.3$ on the boundary between mixed and hadronic phase (i.e. $\lambda = 0$) [3,15], and increases to $\approx 0.4$ on the $T = 130$ MeV isotherm [4,8,9,15,27].

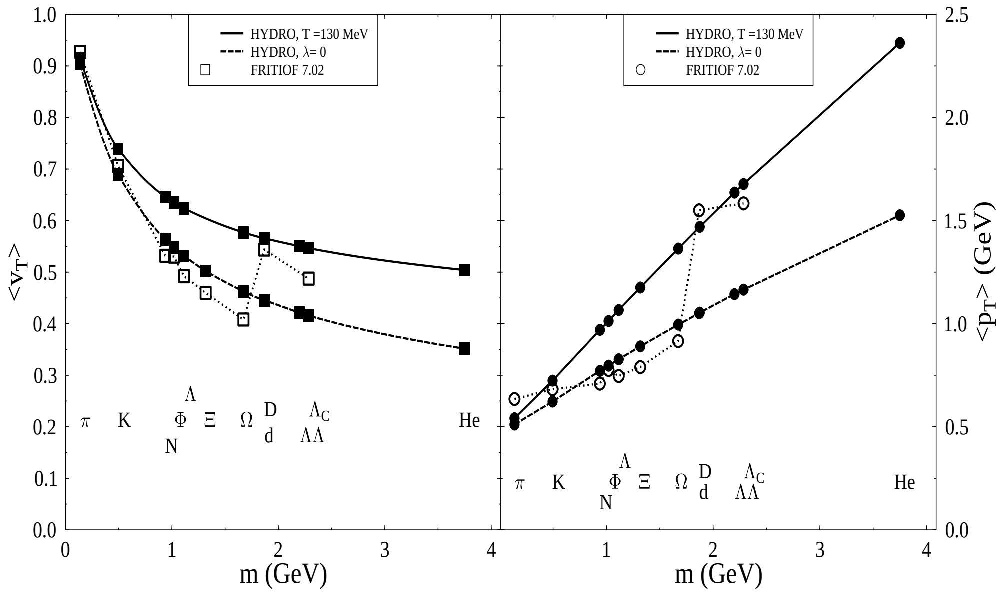


FIG. 3. Average transverse velocities and momenta (at midrapidity) of direct thermal hadrons and clusters on the hadronization hypersurface ($\lambda = 0$) and the $T = 130$ MeV isotherm (for central $Au + Au$ collisions at RHIC). Open symbols depict the predictions of the string model FRITIOF 7.02.

Although the entropy per net baryon at RHIC is much larger than at SPS, the transverse flow velocity increases only slightly. This is due to the fact that the transverse flow can not increase strongly in the expansion of the mixed phase, since the velocity of sound is very small [18,28]. Thus, hydrodynamics predicts that the transverse flow velocity at RHIC is comparable to that at SPS, if the EoS exhibits a first-order phase transition.

It is also interesting to note that (for the assumed fast thermalization of the partons in the central region) a significant part of the transverse flow is already produced within the QGP and (to a smaller part also within) the mixed phase, compare the dashed curves in fig. 3 with the solid lines.

The flow of the $\phi$-meson is closely related to this point. Even if $\phi$-mesons do not rescatter with surrounding hadronic or mixed phase matter, their $\langle p_T \rangle$, $\langle v_T \rangle$ or inverse slope should be affected by collective flow if a QGP with $s$-quarks in local thermal equilibrium is created. This is due to the fact that the transverse momenta of $s$-quarks are subject to a Doppler-shift according to the collective flow velocity built up in the QGP phase. According to Fig. 3, $\langle p_T \rangle_\phi \approx 800$ MeV and $\langle v_T \rangle_\phi \approx 0.55$ at RHIC, even if the $\phi$'s decouple from collective flow immediately at the boundary between mixed and hadronic phase. However, the string model FRITIOF 7.02 also predicts $\langle p_T \rangle_\phi \sim \langle p_T \rangle_N \sim \langle p_T \rangle_\Lambda$ at RHIC. It is thus not straightforward to deduce early onset of flow due to QGP formation from this observation.

For comparison, we have also calculated the average transverse momentum and velocity of direct hadrons within FRITIOF 7.02 [14], cf. fig. 3. In this string model, secondary hadrons do not rescatter. Thus, collective transverse flow does not develop. One observes that even though hard scatterings are taken into account, the $\langle p_T \rangle$ of $N$, $\phi$, $\Lambda$, and $\Omega$ is not higher than in case of hydrodynamic expansion with freeze-out on the $\lambda = 0$ hypersurface.

Only the $\langle p_T \rangle$ of charmed hadrons, $D$-meson and $\Lambda_C(2285)$-baryon is strongly enhanced by semi-hard scatterings, and is close to the solid line in fig. 3. These very heavy hadrons emerge as the leading hadrons in the fragmentation of semi-hard jets and acquire high transverse momentum. Thus, in FRITIOF 7.02 the $\langle p_T \rangle$ of various hadrons does not increase linearly with mass but depends strongly on the sensitivity to semi-hard scatterings, and thus to the flavor content. In contrast, if $D$-mesons thermalize [29] and follow the collective transverse flow, their $\langle p_T \rangle$ is "pulled" back onto the linear $\langle p_T \rangle(m)$ systematics.

## IV. SUMMARY

We have investigated the collective evolution of the central rapidity region with initial conditions as expected for heavy-ion collisons at BNL-RHIC energies. Due to "stall" of transverse acceleration within the mixed phase, the average transverse flow velocity is $\leq 0.5$, thus not much above that extracted for $Pb+Pb$ collisions at SPS.

We have calculated the transverse momentum distributions of various hadrons up to $p_T = 4$ GeV on the $T = 130$ MeV isotherm and on the $\lambda = 0$ hypersurface. Large gradients in the initial transverse density distributions (e.g. a sharp step function) can strongly enhance particle production at high $p_T$. However, the high-$p_T$ tails of the hadron distributions (above $p_T = 2$ GeV) are sensitive to the initial pressure gradients.

Within hydrodynamics the $\langle p_T \rangle$ of the various hadrons from $\pi$ to $\Lambda_C$ practically is proportional to the particle mass and is independent of other quantum numbers (provided the corresponding hadron is in local thermal equilibrium). Heavy hadrons and clusters, if identified, allow for a direct measurement of the collective transverse expansion velocity, without requesting knowledge of freeze-out temperatures.

Finally, we have compared to the results obtained within the string model FRITIOF 7.02. Due to the increasing "sensibility" on semi-hard processes, the $\langle p_T \rangle$ of hadrons with $c$-quarks increases more strongly than $\langle p_T \rangle$ of hadrons with only light (anti-)quarks. Consequently, there is a non-linear relation between the mean transverse momentum and the particle mass within this model. In particular, there is a strong step in $\langle p_T \rangle$ as one goes from the $\Xi$ or $\Omega$-baryons to the $D$-meson.

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