Nonlinear Solution Algorithms for Circuit Simulation

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,
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Collaborators

Xyce™ - Sandia's distributed memory analog circuit simulator

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NOX/LOCA - Sandia's Nonlinear Solver/Bifurcation Analysis Libraries

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Outline

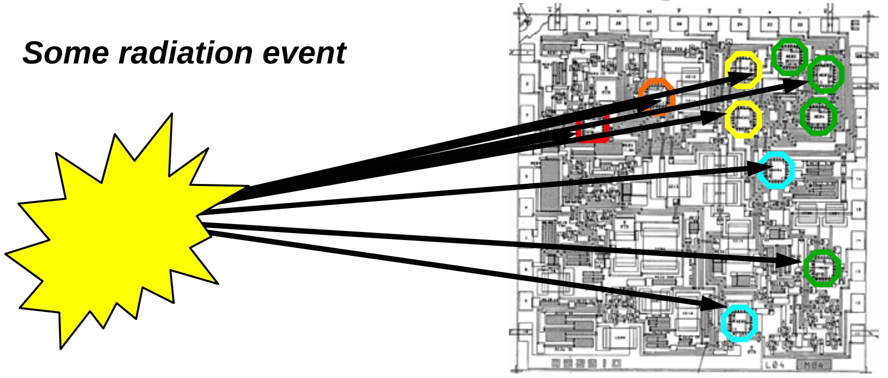
- Motivation
- Newton-based Globalization Techniques
  - Algorithms
  - Results
- MOSFET-based Homotopy
  - ATANSH
  - 3-Pass
  - 2-Pass
- Application of MOSFET-based Homotopy
  - inverter chains
  - 100,000 unknown digital ASIC
  - 300,000 unknown digital SRAM cell.

Motivation: Circuit Simulation at Sandia

- **Sandia is interested in simulating large (>1e7 transistors) integrated circuits.**
- **Special requirements: a circuit's response to a hostile environment:**
  - **Radiation**
  - **Thermal**
  - **Vibrational**
- **To simulate these circuits with radiation, an analog (rather than digital) simulation is required.**
- **Commercial analog (PSPICE, etc.) simulators cannot handle circuits of this size.**
  - **linear solve**
  - **nonlinear solve**

ITS | ***Xyce***

Some radiation event



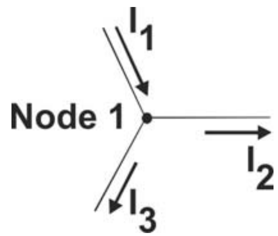
Radiation Effects
- Transient Photocurrent (ϒ dot)
- Neutron Damage

Circuit level response
- Devices
- Printed Circuit Board

Analog Circuit Simulation

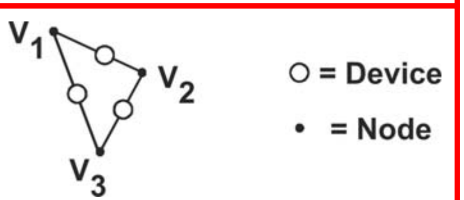
Solve Kirchoff's laws (KCL and KVL).

$$\sum_{i=0}^{n} (I_n) = 0$$



Kirchoff's Current Law (KCL)

$$\sum_{i=0}^{n} (\Delta V) = 0$$



Kirchoff's Voltage Law (KVL)

$$G = \frac{\delta I}{\delta V} = \text{conductance} = \text{Jacobian term}$$

$$\text{Ohm's Law: } I = GV = V/R$$

Difficulties

- **Network topology (not a mesh!)**
- **Very heterogeneous in terms of both the devices and the topology: *There is no "characteristic" circuit!***
- **Stiff, coupled DAEs: Different characteristics than PDEs.**
- **Highly nonlinear (device model discontinuities, hysteresis, etc.).**
- **Poorly scaled systems (unknowns can range over 9-12 orders of magnitude).**
  - **Even direct linear solvers in serial can fail.**
- **Large, ill-conditioned sparse Jacobian matrices present unique ordering and preconditioning challenges.**

Today's Talk

We focus on developing robust algorithms for solving the DC Operating Point (Steady State or DCOP):

- **–DCOP is used as initial condition for transient**
- **–Transient, on the nonlinear solver level, is not as hard.**

$$\boxed{\text{Find } x_* \text{ such that } F(x_*) = 0, \text{ where } F : \mathbb{R}^n \rightarrow \mathbb{R}^n.}$$

Newton's Method: $M_k = F_k + J_k \Delta x$

Until Convergence:

$$J_k \Delta x_k = -F_k$$

$$x_{k+1} = x_k + \Delta x_k$$

If x_o is poor, the nonlinear iteration sequence can diverge!

More Robust Techniques are Required

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A History of Nonlinear Algorithm Capabilities in Xyce

Broyden's Method

$$M_B = f(x_c) + B_c d$$

Newton's Method

$$M_N = f(x_c) + J_c d$$

Tensor Method

$$M_T = f(x_c) + J_c d + \frac{1}{2} T_c dd$$

Globalizations

<u>Voltage Limiting</u>

<u>Line Search</u>
- Interval Halving
- Quadratic
- Cubic
- More'-Thuente
- Curvillinear

<u>Trust Region</u>
- Dogleg
- Inexact Dogleg

<u>Homotopy</u>
- MOSFET Specific (3 variations)
- Artificial Parameter Continuation
- Natural Parameter Continuation

Adaptive Forcing Terms (Eisenstat and Walker)

Voltage Limiting: A Frustratingly Effective Circuit-Specific Globalization

- **Newton Linear System:** $J \triangle x = -F$
- **Voltage Limited Newton Linear System:**

$$J \triangle x = -F + J \triangle x_{corr}$$

- **Limit change in the solution ($\triangle x$) vector in individual devices/unknowns during the load phase.**
- **Each individual device can change the direction away from a Newton direction.**
- **Limits on voltage changes are dependent on the device state/history.**
- **Changes are inconsistent - multiple devices may be adjusting the same unknown.**
- **Devices have a “memory”. Hysteresis is introduced, which breaks other globalizations.**
- **This is the default method in most SPICE-style simulators.**
- **Hack that often works very well, but no real theory backing it up.**

Globalization Methods

$$F(x) = 0$$

- Globalized methods typically force each step to reduce a merit function by a sufficient amount.
- Line Search
  - Newton Direction

$$M_k(s) = F(x_k) + J(x_k)s$$

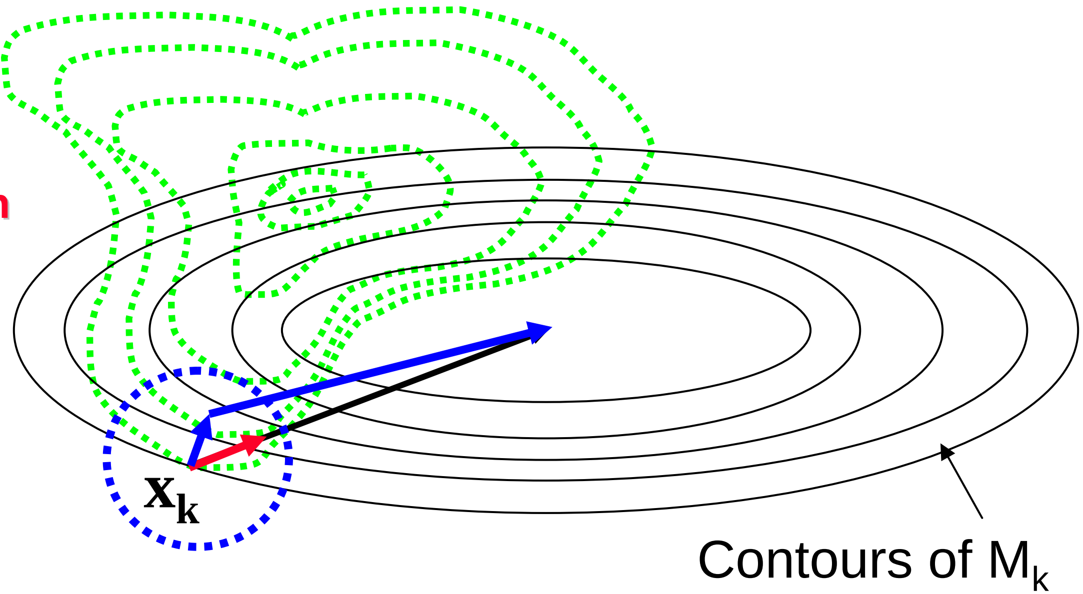
  - Sufficient Decrease Condition

$$\phi(\theta) = \frac{1}{2}\|F(x_k + \theta s)\|^2$$

$$\phi(\theta) \le \phi(0) + \alpha\theta\phi'(0)$$

$$\|F(x_k + \theta s)\| \le \|F(x_k)\|(1 - \alpha(1 - \eta_k))$$

    - » Trapped in local minima
  - How to recover a failed step?
    - » Take a full Newton step
    - » Take the final step length
- Trust Region
  - Changes the direction
  - TR radius is controlled by a condition similar to the sufficient decrease condition



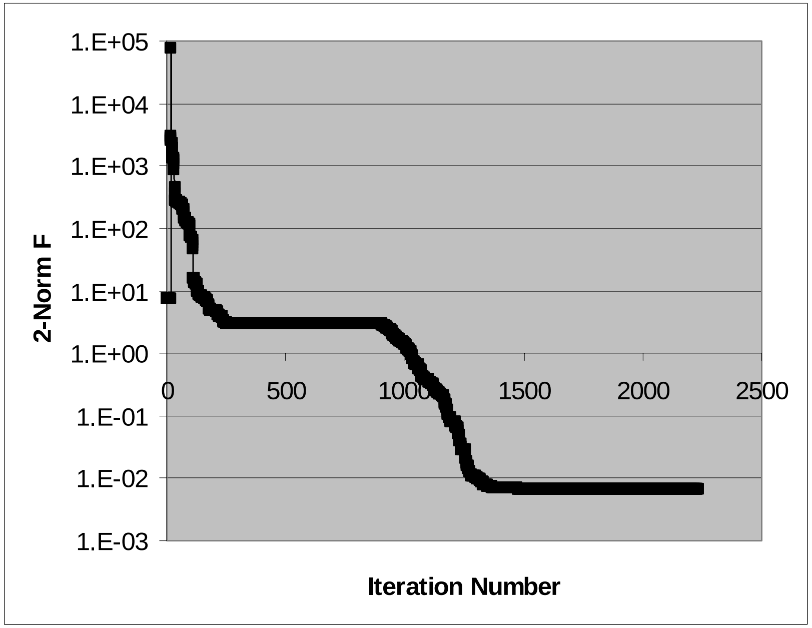
Globalization Comparisons
(Standard Test Circuits)

- **No single globalization algorithm works for everything – transistor devices in particular.**
- **Overall, voltage limiting is much more robust than the other globalizations.**
- **Voltage limiting is the default solver algorithm in virtually all commercially available circuit simulators.**
- **When both converge, efficiency comparisons between Newton and voltage limiting are inconclusive.**
- **Trust Region and Line Search methods trade efficiency for robustness but for circuits the number of iterations is excessive.**

Method	Circuit				
	FPGA	Single Channel	Dual Channel	TX Adder	L1 Inv.
Newton	116	13	436	113	2
Voltage Limiting	57	58	42	122	2
Line Search (2)	133	26	10	4000	2
Line Search (20)	131	500	45	4000	2
Line Search (20 + r=1)	131	13	45	4000	2
Trust Region	500	5	4000	4000	2

Comments on Globalization

- **When voltage limiting fails, there are no parameters available to users to tweak.**
- **One recourse is to hack on the line search options:**
  - **Limit inner iterations to 2 and ignore the failed sufficient decrease condition.**
  - **Use the last computed step length.**
  - **This works on a variety of circuits – prevents divergence!**
- **Poor performance of Line Search and Trust Region is due to the sufficient decrease condition:**
  - **Flip-flopping of device states causes oscillations in ||F||.**
  - **The step length to be severely reduced.**
  - **Many local minima.**



None of these methods were successful on every circuit.

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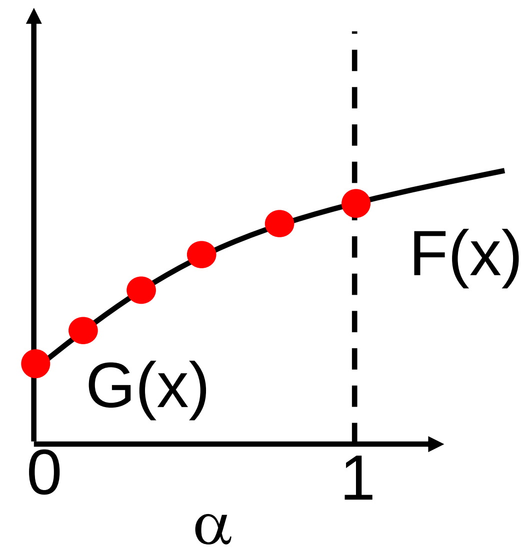
LOCA: Homotopy Development in Xyce

- **General Idea:**
  - **Start with easy problem, related, by a single parameter to the hard problem.**
  - **Perform repeated solves while incrementing parameter until the hard problem solution is obtained.**

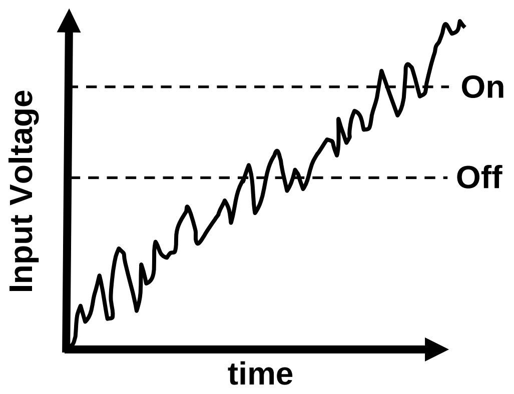
Difficult Problem:	**F(x) = 0**
Easy Problem:	**G(x) = 0**

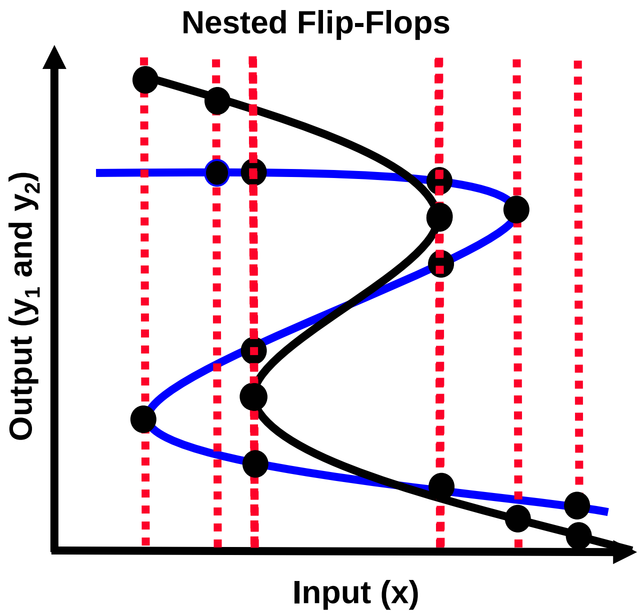


- **Circuit homotopy failures:**
  - **Natural Parameters such as voltage sources (Power nodes)**
  - **Artificial parameter homotopy:**
    $F^H(x) = \alpha F(x) + (1 - \alpha)(x - a)$

Multi-pass Homotopy

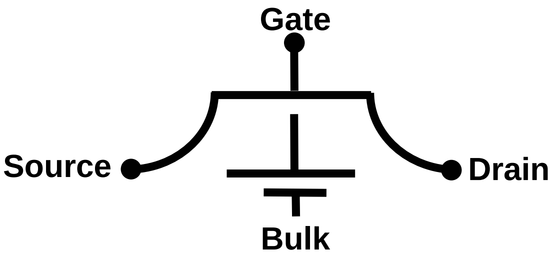
- Jaijeet Roychowdhury (U. Minnesota) - Circuit homotopy methods fail due to:
  - Ill conditioned numerics lead to path failing in the homotopy algorithm.
  - Exponentially (infinitely) long homotopy branches occur due to nested loops. (Some circuits are intentionally designed with multiplicity - Schmidt triggers)
- Device specific, multi-pass homotopies are required using specially constructed parameters:
  - » MOSFET
  - » BJT





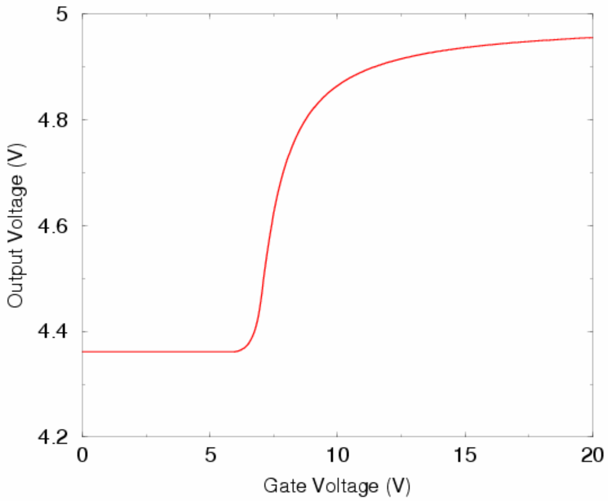
MOSFET Circuits

- MOSFET = Metal Oxide Semiconductor Field Effect Transistor.
- Very common integrated circuit component.
- Behaves as an on/off switch. (2 Distinct states, even if simulated in analog)



- MOSFET circuits pose problems for nonlinear solvers, because they will often repeatedly change state (on-off-on, etc) over the course of the nonlinear solve.

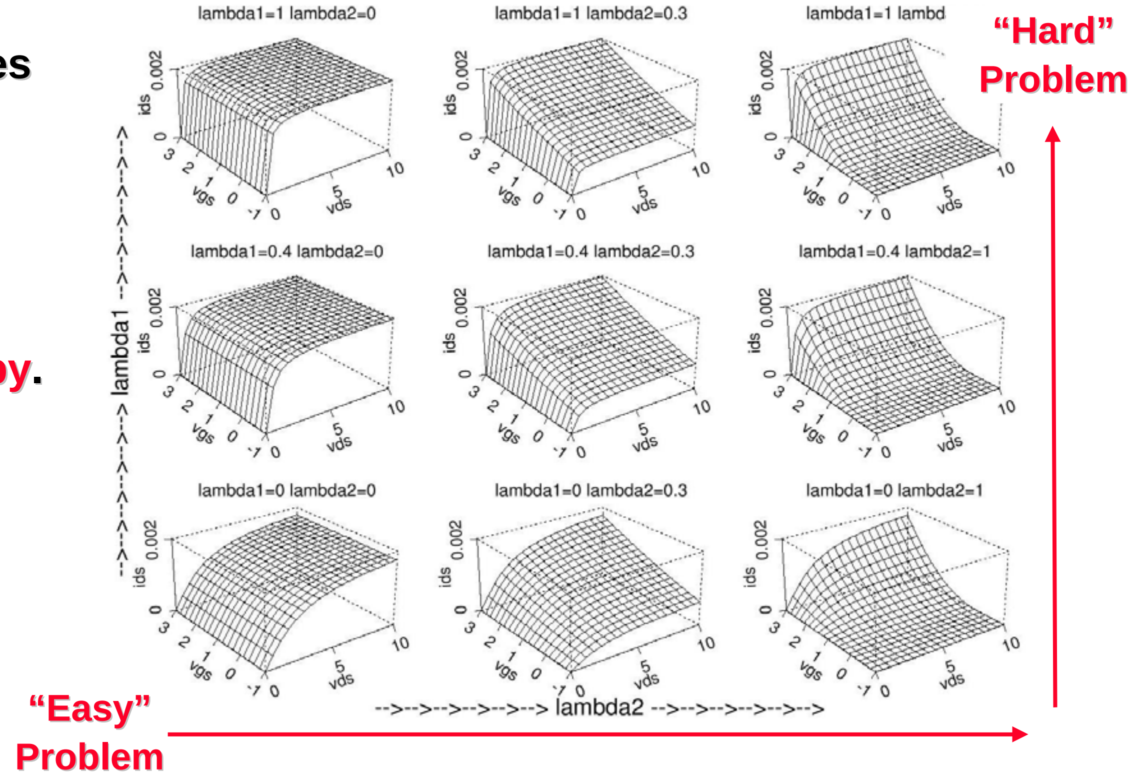
- Critical Variables (voltage differences):
  - Gate to source voltage: Vgs = Vg - Vs
  - Drain to source voltage: Vds = Vd – Vs
  - Output Current: Ids = f(Vgs, Vds)



- Extremely complex models:
  - Many different models (MOS1, MOS3, BSIM3 etc…)
  - BSIM3 has 12,000 lines of code, 400+ parameters
  - Models are industry standards: have to be “bug” compatible.

Specialized MOSFET Homotopy

- **MOSFET-based homotopy does not appear in the simulation literature, but is crucial.**
- **Based on conversations with Jaijeet Roychowdhury (U. Minnesota), ATANSH homotopy.**
  - **Pass 1: gain**
  - **Pass 2: nonlinearity**
- **When both parameters are 0 ("Easy") all MOSFETs act as nearly linear resistors.**



Gain (horizontal path): **Controls the transfer characteristic – the influence of the gate on the drain. Fixes the on/off states.**

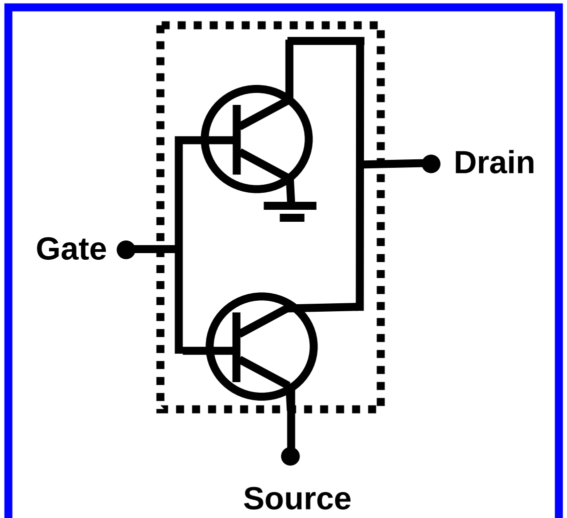
$$V_{gs_{model}} = \lambda_2 V_{gs_{input}} + (1 - \lambda_2) V_{gs_{const}}$$

Nonlinearity (vertical path): **Sharpens the driving point characteristic – the degree of nonlinearity in the drain-source driving point.**

$$V_{ds_{model}} = \lambda_1 V_{ds_{input}} + (1 - \lambda_1) V_{ds_{const}}$$

Scalable Problem: Inverter Chain

- Test problem where one can adjust problem difficulty via the chain length.
- Constructed from two MOSFETs (one n-type, one p-type).
- Has lots of conditioning problems, which get worse with size.



Inverter Unit Cell

Level 1 Inverter Chains (MOS1)

Method	Number of Inverters						
	1	10	25	50	75	100	1000
Newton	2	200	60	2	2	2	2
Volt Limiting	7	13	55	200	2	2	2
Line Search (2 steps)	2	200	200	2	2	2	2
Line Search (20 steps)	2	200	50	2	2	2	2
Trust Region	2	41	155	200	2	2	2

- Shows number of Newton steps until Convergence/Failure
- Failures are RED

- Not perfect test:
  - Does not contain multiplicity
  - Not much cross-linking - topological straight line.

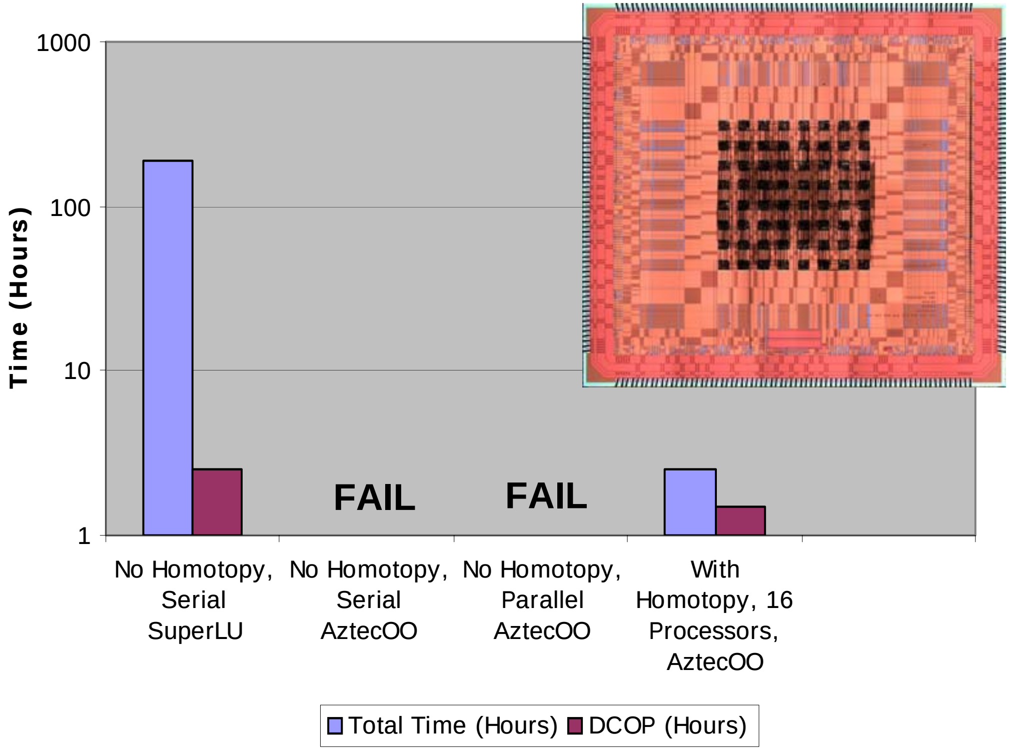
Specialized MOSFET Homotopy (ATANSH)

Method	Number of Inverters					
	1	10	25	50	75	100
Newton	2	200	60	2	2	2
Volt Limiting	7	13	55	200	2	2
Line Search (2 steps)	2	200	200	2	2	2
Line Search (20 steps)	2	200	50	2	2	2
Trust Region	2	41	155	200	2	2
MOSFET Homotopy (ATANSH)	242	274	286	302	318	334

- In use internally at AT&T/Lucent Microelectronics since 1995.
- We have found that ATANSH Homotopy works well, but it can not solve all of our circuits (we go to much larger sizes):
  - Level 1 (MOS1) <= 150
  - Level 9 (BSIM3) <= 1000

MOSFET Homotopy Impact on a Real Problem
(Oct 2003 results)

- **ASIC with over 233,000 MOSFET devices in the model.**
- **Without homotopy, this ASIC could run, but only in serial, and it had a lot of trouble. It only worked with a direct solver.**
- **With ATANSH homotopy, Xyce was robust enough to hand off to users to experiment with the circuit, and it was possible to run in parallel.**

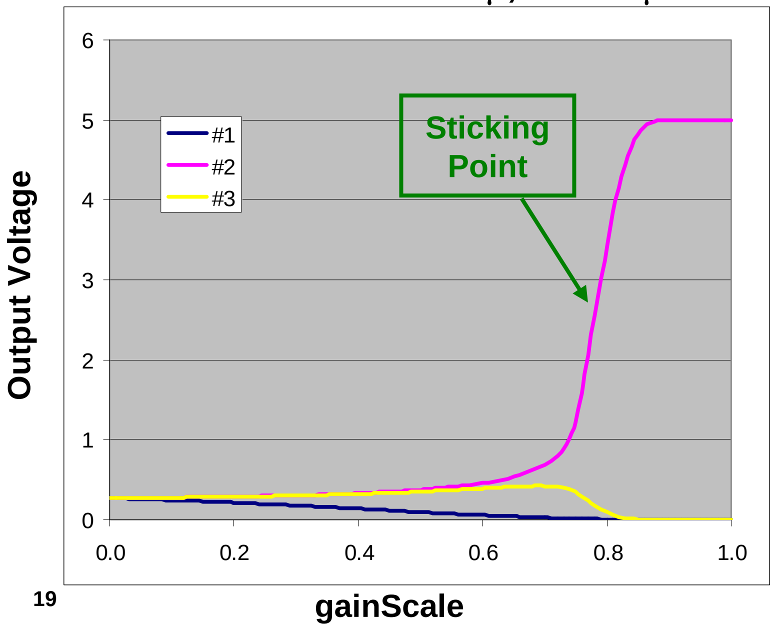


SuperLU = Trilinos direct solver
AztecOO = Trilinos iterative solver

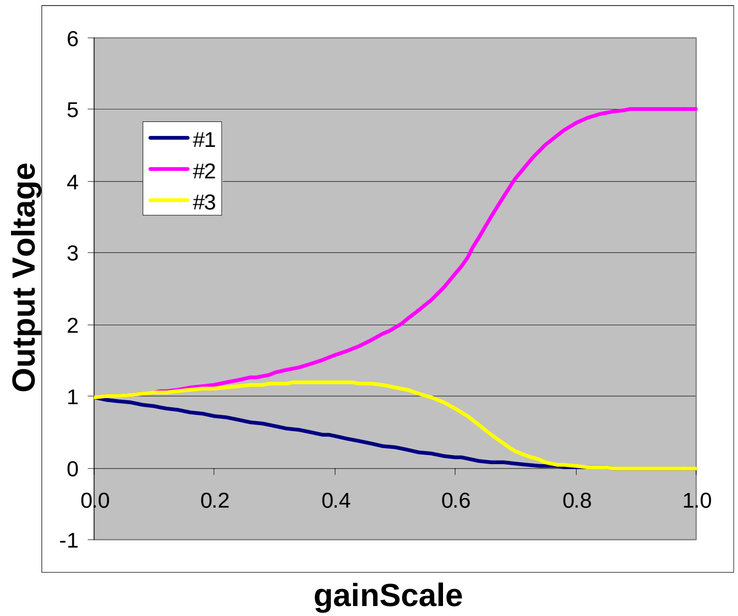
ATANSH Failure Mode
Suggests new 3-pass homotopy

- **Failures of ATANSH Homotopy were during continuation in the “gain” pass. (sticking point near 0.7, when the devices “turn on”)**
- **Sticking point appeared in a lot of MOSFET circuits, not just inverters.**
- **This effect was less pronounced if the MOSFETs were physically large.**
- **Thus, a modification is proposed for ATANSH, in which an addition SIZE parameter is used: 3 parameters: gainScale, nltermScale, sizeScale.**

SMALL SIZE: W = 5μ, l = 175μ



BIG SIZE: W = 1000μ, L = 1000μ



New (3-pass) HOMOTOPY applied to Inverter Chains

- ATANSH: gain, nonlinearity
- 3 Pass: gain, nonlinearity, size
- 2 Pass: nonlinearity, size (why is this best!?)

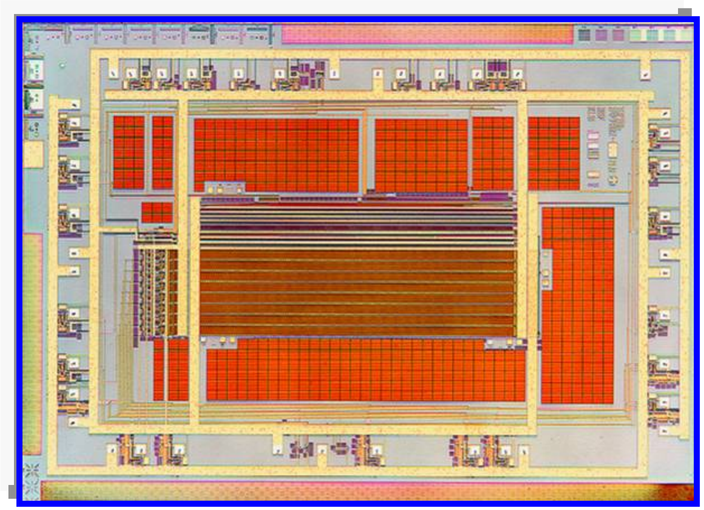
Method	Number of Inverters				
	100	200	500	1000	5000
Newton	F	F	F	F	F
Voltage Limiting	122	F	F	F	F
ATANSH	178	184	400	480	F
3 Pass (L=5, W=175)	387	386	F	F	F
3 Pass (L=1000, W=1000)	193	214	F	F	F
2 Pass (L=1000, W=1000)	129	144	195	195	195
gain	57	105	151	F	F
nonlinearity	113	115	F	F	F
size	F	F	F	F	F

- 2 Pass in nonlinearity and size works extremely well.
- This no longer decouples nested turning points so we are susceptible to infinitely long homotopy branches.

3-Pass MOSFET Homotopy Impact (March 2004 results)

- SRAM Cell. Much harder than the ASIC.

- The number of devices is ~250k.
- The size of the nonlinear system is ~278k.



Method	Status
Newton	F
Voltage Limiting	2 days
ATANSH	F
3-Pass	4 hours

Nonlinear Solver Capabilities

NOX

LOCA

Library of Continuation Algorithms

Model

Tensor Method

$M_T = F(x_c) + J_c d + 1/2 T_c dd$

Newton's Method

$M_N = F(x_c) + J_c d$

Broyden's Method

$M_N = F(x_c) + B_c d$

Globalizations

Line Search

Interval Halving

Quadratic

Cubic

More'-Thuente

Curvilinear

Trust Region

Dogleg

Inexact Dogleg

Homotopy

Parameter Continuation

zero-order

first-order

arc-length

Predictors

Tangent

Secant

Random

Automated Step Size Control

Matrix-Free Newton-Krylov

Jacobian Estimation via colored finite diff.

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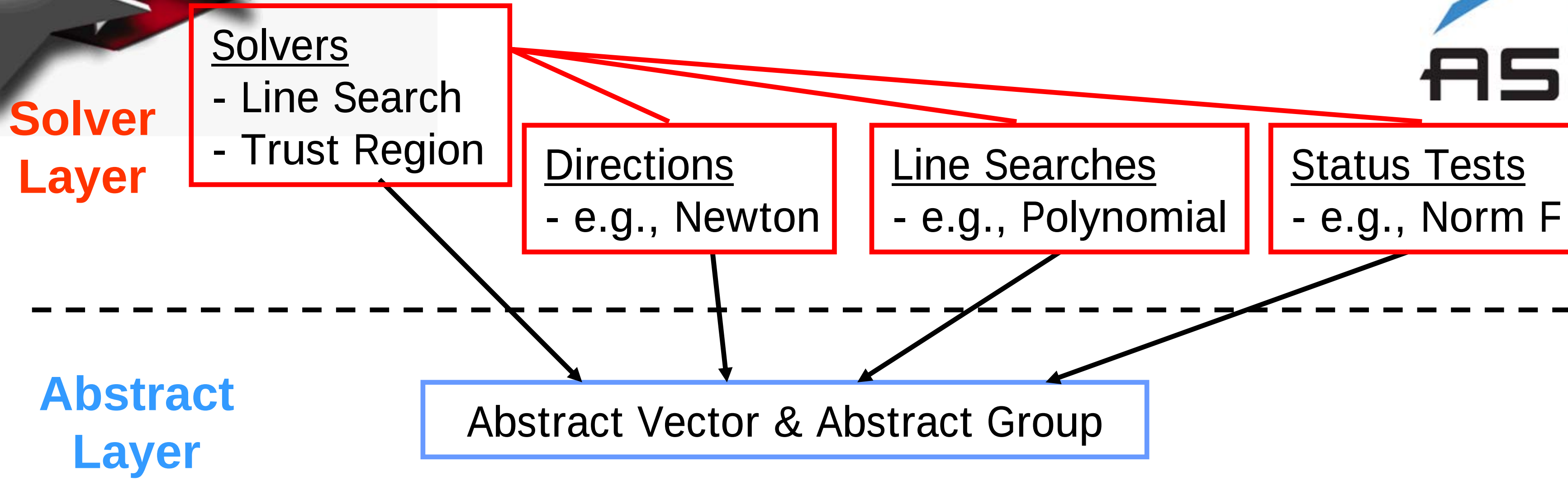
NOX Framework



- Don't need to directly access the vector or matrix entries, only manipulate the objects!

Wraps user interface
Stores: x, F, J

Abstract Vector		**Abstract Group**	
– Init	– Norm	– Compute F	– apply J
– Abs	– Dot	– Compute J	– apply J^T
– Scale	– Clone	– Compute Grad F	– apply J^{-1}
– Update		– Clone	

- Implementation is independent of the linear algebra storage format and parallel services.
- We provide implementations for LAPACK, Epetra (Trilinos) and PETSc.

Conclusions

- Large-scale systems with many transistors require divice specific homotopy for the DC Operating point calculation.
- MOSFET homotopy has yielded a HUGE benefit in robustness over standard globalization techniques.
- Artificial homotopy and continuation on obvious natural parameters, such as the power node does not work well.
- We expect the 3-Pass and the 2 Pass (nonlinear, size) to be even better.

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Nonlinear Solver Software in Trilinos

NOX: Nonlinear Object-oriented Solutions

- Provides a common solver framework for nonlinear methods.
- Algorithmic construction via parameter lists
- Robust algorithms: Newton, Line Search, Trust Region, Tensor, Broyden solvers.
- User Defined Quantities: stopping criteria, algorithmic norms, merit functions.
- Independent of linear algebra storage format/ solver algorithm!

LOCA: Library of Continuation Algorithms

- Provides robust continuation and bifurcation analysis algorithms
- Provides zero-order, first-order, and arc-length continuation routines.
- Automatic step size control and predictor/corrector algorithms.
- Allows for direct tracking of bifurcation points (turning point, pitchfork and Hopf).

Download
http://software.sandia.gov/trilinos
http://software.sandia.gov/nox

Contacts

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