Session 8
Cruise Performance

1.0 Definitions

Endurance - A measure of how long an aircraft is able to remain airborne on a given amount of fuel.

Fuel flow - The number of gallons (or pounds) of fuel used per hour of flight time.

Maximum endurance airspeed - The airspeed (for a given weight and altitude), where the fuel flow is the minimum. The low fuel flow permits the aircraft to remain aloft longest.

Range - A measure of how far an aircraft can go with a given amount of fuel.

Maximum range airspeed - The airspeed that results in the best ratio of fuel flow to airspeed. This airspeed results in the maximum distance for a given amount of fuel.

2.0 Introduction

Many performance parameters tested on a car or an airplane are ancillary to the overall purpose of the vehicle. Evaluating the horsepower available, the takeoff distance, or the acceleration rate are all secondary factors for the real purpose of a motorized vehicle; the primary being how efficiently does it get from point ***A*** to point ***B***. That efficiency is usually measured in miles per gallon because it directly relates to miles per dollar. If the fuel mileage of the vehicle is low, it costs more dollars per mile to operate. So even though an aircraft may be able to achieve Mach 2, it cannot remain at Mach 2 for very long because it uses a lot of fuel when flying at high speeds.

The cruise performance of an aircraft is measured in two specific areas; 1) how **long** can the aircraft remain airborne on a specific amount of fuel (commonly referred to as its *endurance*) and 2) how **far** can the plane travel on a given amount of fuel (referred to as the aircraft's *range*). This session investigates the factors influencing an aircraft's cruise performance and describes how to determine the best cruise speed for both endurance and range.

3.0 What is Range Performance?

Range performance can be presented in the form of a ratio between distance travelled and fuel used. A ratio is found by dividing one term by another. Ratios can also be graphically represented by plotting the numerator on the ordinate and the denominator on the abscissa. From this graph, the slope of the line drawn is the ratio. In other words, the ratio gives the rate of change of one parameter with reference to another. For example, to find a car's fuel mileage experimentally, you might proceed in the following manner:

a) fill the car with fuel
b) drive for 100 miles
c) refill the car with fuel
d) divide the number of miles travelled (100) by the amount of fuel you just put in (possibly 4 gallons)
e) the result would be 25 miles per gallon

To look at this problem graphically, plot miles on the vertical axis and gallons on the horizontal axis. Then place a mark at the point which corresponds to 100 miles and 4 gallons. Drawing a line from the origin to this point graphically shows the ratio between miles driven and fuel used, as shown in Figure 8.1.

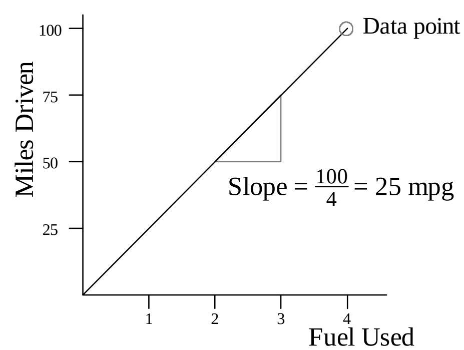


Figure 8.1 Ratio of Miles Driven vs. Fuel Used

The slope of the line is the "miles-per-gallon." The usefulness of a graph like this can be seen when considering a trip of less than 100 miles. To determine the amount of gas need for a 58 mile trip, enter the graph on the vertical axis at "58," go across until it meets the slope line, then drop down to the horizontal axis. The value will be 2.32 gallons (Figure 8.2).

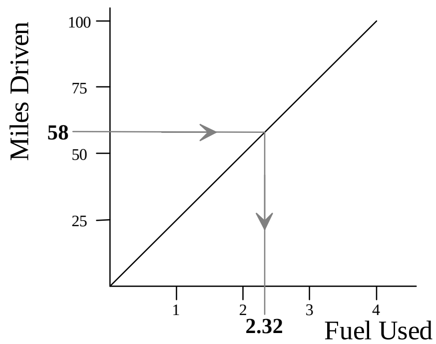


Figure 8.2 Determining Fuel for Trip

However, the above test was performed at only one speed. To find the *best* speed, a similar test would have to be accomplished for several speeds resulting in a series of graphs. Determine the slope of the graph for each speed tested as in Figure 8.1.

To determine the best speed to travel for the maximum gas mileage, make another graph. For each of the speeds tested plot the slope versus the speed from which it came. This procedure yields a curve similar to that shown in Figure 8.3.

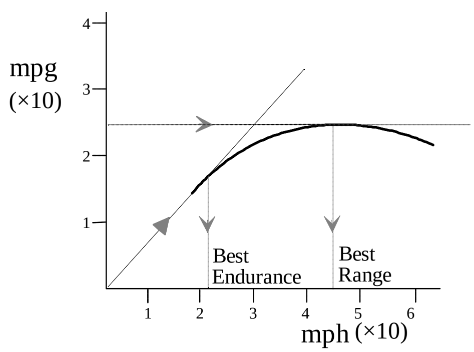


Figure 8.3 Miles per Gallon
vs. Miles per Hour

Two valuable pieces of information are available from this curve: a tanget from the origin to the curve (that is to a point where the line just touches the curve) shows the speed and fuel mileage which will result in the car's best *endurance*. In other words, for a given amount of fuel, traveling at this speed will result in the *longest time* between fuel stops.

The point at top of the curve is the speed and fuel mileage for the car's *best range*. This allows the *farthest* distance between fuel stops.

You've just completed a test to find the cruise performance of your car and presented the data in a manner which is useful to the owner.

Caution:

Each of the tests must be conducted on the same section of road. Conducting one test on a flat road and another on a hill will obviously interfere with the results. Additionally, if the tests are conducted on the same day, atmospheric effects (wind and density changes) and road surface conditions (wet, icy, etc.) can be minimized.

Although the terminology is a little different, the cruise performance of an aircraft is determined in much the same manner.

4.0 Determining the Maximum Endurance Airspeed

When an aircraft is in level, unaccelerated flight, it is usually thought of as being in a cruise condition. Since the plane is not accelerating in any given direction, Newton says the forces acting on the airplane are balanced. Recall from earlier sessions this means the lift equals the weight and the thrust equals the drag. Then the force equations which describe cruise flight are written as:

$$L = w = {}^{1}\!/_{2}\,\rho\, V^2\, SC_L \qquad (8.1)$$

And

$$D = T = {}^{1}\!/_{2}\,\rho\, V^2\, SC_D$$

The way to determine how efficient the aircraft is in the cruise configuration is look at the amount of drag at some weight. This is logical because more thrust required means more fuel burned, which in turn costs more money. To accomplish this, a ratio of lift to drag (or weight to thrust) is created. Equation 8.1 can be used to show the similarity between the lift-to-drag ratio and the C_L to C_D ratio:

$$\frac{L}{D} = \frac{\frac{1}{2}pV^2SC_L}{\frac{1}{2}pV^2SC_D} = \frac{C_L}{C_D}$$

The values are put into the coefficient form because it is a more general reflection of the relationship between lift and drag. Rearranging Equations 8.1 to solve for the coefficients of lift and drag yields the following relationships:

$$C_L = \frac{2w}{pV^2S} \qquad (8.2)$$

and

$$C_D = \frac{2T}{pV^2S}$$

By putting these measurements into Equation 8.2, the lift and drag coefficients can be determined. Plotting the values of C_L and C_D for a plane results in a curve which takes on the shape of a parabola, as shown below. From this drag curve we can obtain the same information for endurance that we did for the car.

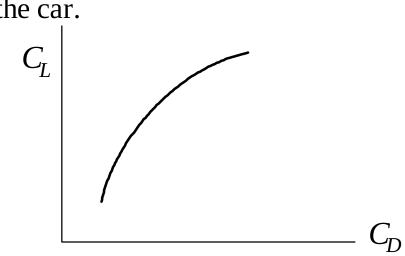


Figure 8.4 Drag Curve

Drawing a line from the origin to the tangent, the point of intersection occurs where the ratio of C_L to C_D is the maximum. This is illustrated below:

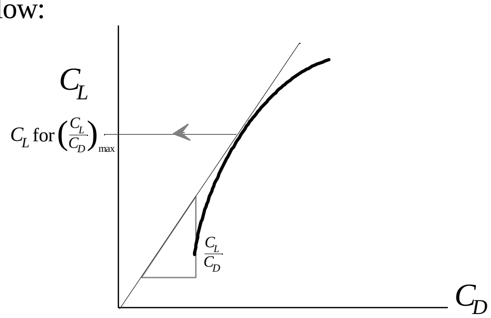


Figure 8.5 Determining Tangent

The slope of the tangent line is the maximum C_L to C_D ratio. By drawing a line from the tangent point to the C_L axis, the *optimum* lift coefficient is determined. Inserting this value of C_L into Equation 8.2, the optimum velocity is found. This speed yields the maximum C_L to C_D ratio.

$$V = \sqrt{\frac{2w}{pSC_L}} \qquad (8.3)$$

This velocity is the "maximum endurance airspeed" and gives the pilot the greatest amount of time airborne for a given amount of fuel. This is the speed the pilot would fly if stuck in a holding pattern.

NOTE:

After considering the problem, it should seem logical that the best endurance occurs at the plane's best lift-to-drag ratio (same as C_L/C_D ratio):

1. The best endurance occurs when the fuel flow is as low as possible.
2. Since fuel flow is directly related to thrust, the best endurance should come at the condition for minimum thrust.
3. Since thrust equals drag in cruising flight, the thrust (and fuel flow) will be lowest when the drag is lowest.
4. For any given weight, the lowest drag occurs when the lift-to-drag ratio is highest.
5. Since $L/D = C_L/C_D$, then the highest C_L/C_D ratio (tangent point) yields the best endurance.

Consider the following example:

***Example 1*:**
During a test flight, the following data is collected:

Velocity (ft/sec)	**Weight** (lbs)	**Thrust** (lbs)
591	10,000	1,350
513	9,850	925
440	9,700	600
366	9,550	400
293	9,400	250
257	9,250	215
220	9,100	200
205	8,950	205
190	8,800	220
184	8,650	240

If the aircraft has a wing area of 205.33 square feet and is flying at an altitude where the density is 0.002 slugs per cubic foot, what is the maximum endurance lift coefficient and airspeed for a 9000 pound aircraft?

***Answer*:**
***Step 1*:** Compute the lift coefficient (C_L) and drag coefficient (C_D) for each point in the table above using Equation 8.2.

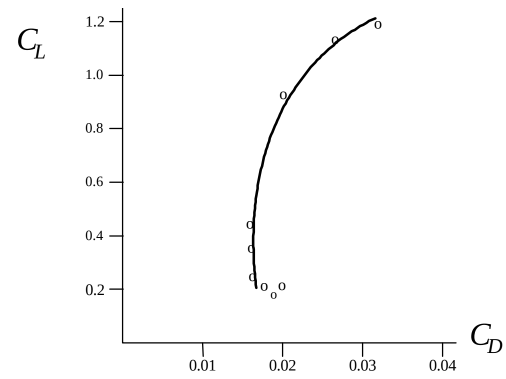
$$C_L = \frac{2w}{\rho V^2 S} \qquad (8.2)$$

and

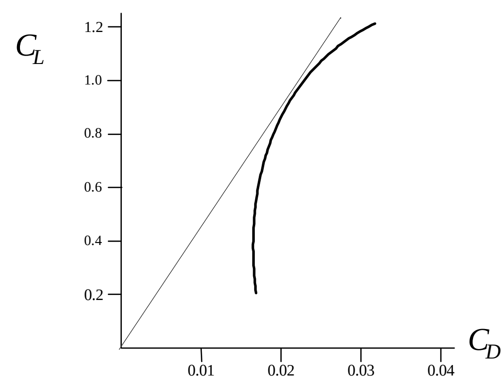
$$C_D = \frac{2T}{\rho V^2 S}$$

Velocity (ft/sec)	**Weight** (lbs)	**Thrust** (lbs)	C_L	C_D
591	10,000	1,350	0.14	0.019
513	9,850	925	0.18	0.017
440	9,700	600	0.24	0.015
366	9,550	400	0.35	0.015
293	9,400	250	0.53	0.014
257	9,250	215	0.68	0.016
220	9,100	200	0.92	0.02
205	8,950	205	1.04	0.024
190	8,800	220	1.19	0.03
184	8,650	240	1.24	0.035

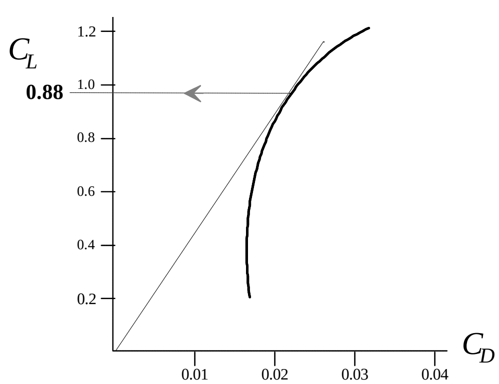
Step 2: Plot a graph of C_L vs. C_D



Step 3: Draw a tangent line from the origin to the curve.



Step 4: From the tangent point, determine the optimum C_L by drawing a line to the vertical axis.



Step 5: For this value of C_L, use Equation 8.3 to determine the velocity for an aircraft which weighs 9000 pounds.

$$V = \sqrt{\frac{2w}{\rho S C_L}} \qquad (8.3)$$

$$V = \sqrt{\frac{2(9000\ lbs)}{(0.002\ slugs/ft^3)(205.33\ ft^3)(0.88)}}$$

$$V = 223\ \text{ft/sec}\ (152\ \text{miles/hr})$$

However, this speed only applies to the weight entered into Equation 8.3. As fuel is burned, the weight will decrease, therefore the lift required also goes down. The result is the speed associated with the optimum C_L is lower. Therefore, to achieve the maximum endurance, the aircraft should fly at a slower speed.

While traveling large distances, instead of maximizing the *time* spent airborne, the mileage is the main area of interest. To optimize the mileage, an aircraft will fly at the optimum airspeed for range.

5.0 Determining the Maximum Range Airspeed

When an aircraft flies at its maximum range airspeed, it travels the maximum distance for a given amount of fuel. In other words, it yields the best fuel mileage and is therefore most cost efficient. Through experience, engineers have determined that the C_L for maximum range airspeed is approximately equal to 70% of the C_L for maximum endurance. By inserting this value into the lift equation the maximum range airspeed is calculated.

NOTE:

The details of this approximation are outside the scope of this course. There may be some question as to why the point of best C_L/C_D is not also the best range condition. The qualitative explanation is: The condition of best C_L/C_D is the absolute lowest fuel flow possible. Flying at this condition turns out to be a fairly slow speed.

If the pilot adds more thrust, then both the fuel flow and speed increase. The key is that the fuel flow increases only a little, but the speed increases a lot. This means an increase in mileage. If, however, the pilot adds **too** much more thrust, then just the opposite happens and the milage goes down. Experience and analysis shows that the proper amount of extra thrust occurs when the C_L is only 70% of the C_L for best endurance

The following example will highlight the relationship between maximum endurance and maximum range airspeeds.

Example 2: Using the same aircraft and test data from Example 1, what is the maximum range airspeed for the 9000 pound aircraft?

Answer:

Step 1: Since the C_L for maximum endurance has already been determined then simply multiply this number by 0.70.

$$(0.88)\ (0.70) = 0.62$$

Step 2: Place this value of C_L into Equation 8.3.

$$V = \sqrt{\frac{2w}{\rho S C_L}} \qquad (8.3)$$

$$V = \sqrt{\frac{2(9000\ lbs)}{(0.002\ slugs/ft^3)(205.33\ ft^3)(0.62)}}$$

$$V = 265\ \text{ft/sec}\ (181\ \text{miles/hr})$$

Notice how this speed is higher than that for maximum endurance. Because the lift coefficient is smaller, the speed must be higher.

Here again, as fuel is used and the weight decreases, so airspeed for maximum range also decreases. Then whether the flight is made at maximum endurance airspeed or maximum range airspeed, as fuel is burned off, the plane's speed should decrease.

The whole reason we fly in aircraft from point **A** to point **B** is to be there quicker. For this reason, its not really desirable to fly *slower* as the plane lightens. In order to keep the speed high, a reexamination of the lift equation is in order.

6.0 How to Keep the Cruise Speed High

As the weight (and subsequently the lift required) decreases and the velocity is kept the same, what other items can be changed to maintain the optimum C_L? To answer this, look again at equation 8.1.

$$L = W = {}^{1}/_{2}\,\rho\, V^2\, SC_L \qquad (8.1)$$

Obviously, it's difficult to change the wing area, S, and the goal is to still fly at the optimum C_L. Therefore, to keep the velocity the same, the only factor left to decrease is the air density. The pilot can decrease the density quite easily by flying at a higher altitude. To illustrate this, look at the following example.

***Example 3*:**
Again using the aircraft in Example 1, assuming the aircraft maintains the optimum C_L and airspeed for maximum range, what should the flying altitude be if 2000 pounds of fuel is burned?

Answer:
Step 1: Rearrange Equation 8.1 to solve for the density.

$$L = w = {}^{1}/_{2}\,\rho\, V^2\, SC_L \qquad (8.1)$$

$$q = \frac{2w}{V^2 SC_L}$$

Step 2: Insert the appropriate values from Example 2.

$$q = \frac{2(7000\ lbs)}{(265\ \frac{ft}{\sec})^2(205.33 ft^2)(0.62)}$$

$$\rho = 0.0016\ slugs/ft^3$$

Step 3: The engineers would then use the "Standard Atmosphere Chart" to determine what altitude corresponds to this density. Recall the original density was 0.002 slugs/ft^3. From the table this corresponds to an altitude of 5,000 feet. Similarly, a density of 0.0016 slugs/ft^3, corresponds to an altitude of 12,000 feet. As a result, the pilot would climb to 12,000 feet in order to keep the optimum C_L and airspeed for the maximum range.

This is the procedure aircraft use when flying long distances. The pilot will cruise at a specific altitude until he burns a certain amount of fuel. Then he will climb to another altitude until more fuel is burned, then repeat the process until the point is reached where the descent for landing should begin. This procedure is called a "step climb" profile and works especially well for jet aircraft which burn large amounts of fuel.

The descriptions in Sections 3, 4, and 5 describe how engineers predict the cruise performance of an aircraft. Flight testers then verify these predictions using a slightly different technique.

7.0 Flight Testing Cruise Performance

The test instrumentation used on a test flight greatly assists in verifying the engineering predictions. A test aircraft will be outfitted with a fuel flow meter which measures the amount of fuel the engine (or engines) use per hour.

Figure 8.6 Fuel Flow Meters on Test Aircraft

At each altitude, the aircraft is flown at various airspeeds and the fuel flow at each of those speeds is recorded. This data is then plotted to create a graph similar to the one shown below.

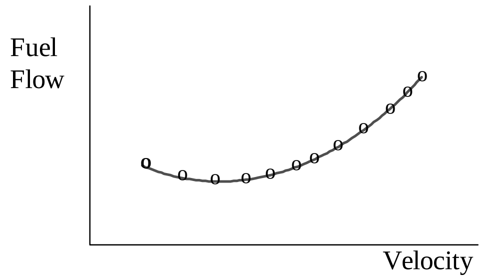


Figure 8.7 Plot of Flight Test Data

This single curve will verify the predicted speeds for both maximum endurance and maximum range. Draw a tangent from the origin tangent to the curve. At the tangent point, a line is draw straight down to the "airspeed" axis and another line is drawn over to the "fuel flow" axis. The axis values are the maximum range airspeed and the fuel flow associated with that airspeed. Figure 8.8 illustrates this procedure:

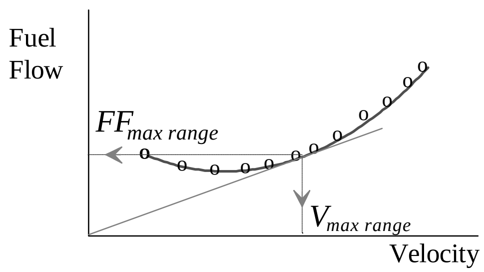


Figure 8.8 Determining Maximum Range Airspeed and Fuel Flow

The bottom of the curve shows the maximum endurance airspeed and fuel flow, as shown in Figure 8.9.

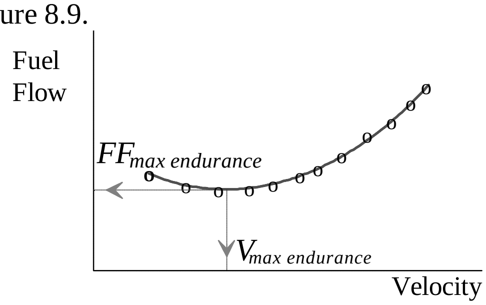


Figure 8.9 Determining Maximum Endurance Airspeed and Fuel Flow

Each altitude will have a curve constructed so the pilot can determine how much fuel he will need and what airspeed he should fly at that altitude.

8.0 Summary

Predicting cruise performance is really straight forward once you realize that the critical fuel flow value is related directly to thrust and drag. Since the forces are balanced in cruise flight, thrust equals drag. The lowest fuel flow occurs at the speed for lowest drag. The best range occurs at the lowest ratio of drag to velocity (Figure 8.8). During the course of a flight test, by measuring the velocity and fuel flow, a graph can be quickly generated to verify the predicted results. Once it has been determined how to efficiently cruise to your destination, the next step is to land. Determining landing performance is the topic of the next session.

9.0 Measures of Performance

1 What is the definition of endurance?

2 What is the definition of range?

3 As fuel is burned and the aircraft's weight decreases, what is the best course of action a pilot can take?

10.0 Problems

1. A jet airplane yields the following flight test data. The aircraft weighs 3600 pounds, has a wing area of 125 square feet, and is flown at an altitude where the density is 0.0019 slugs/ft^3. What are the predicted maximum range and endurance airspeeds? Neglect any changes in weight.

Velocity (ft/sec)	**Thrust** (lbs)
132	230
147	220
161	200
176	205
190	240
205	260
220	300