Phases of Strongly Interacting Matter

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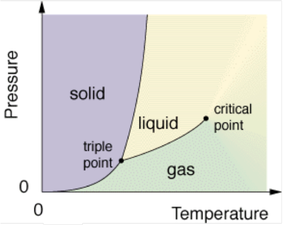

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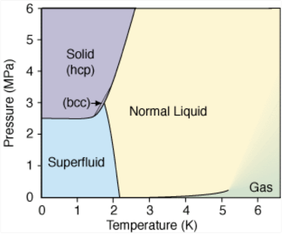

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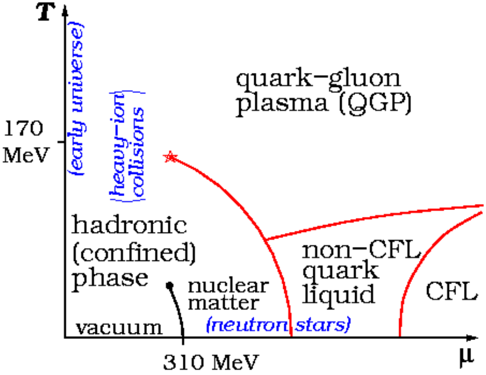
phase diagrams are labeled by intensive variables
in QCD: T, N_f quark masses, N_f chemical potentials

$$\Omega(T, \mu_i; m_i)$$



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chiral symmetry

separation into sectors of 'light' and 'heavy' quarks

$$\mathcal{L}_{QCD} = \mathcal{L}_{light} + \mathcal{L}_{heavy}$$

$$\mathcal{L}_{light} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - \mathcal{M}_q)q; \quad \mathcal{L}_{heavy} = \bar{Q}(i\gamma^\mu D_\mu - \mathcal{M}_Q)Q$$

light quarks $(m_u, m_d, m_s = 0)$

left and right handed quark fields

$$q_{L,R} = \frac{1}{2}(1 \mp \gamma_5)q$$

$\mathcal{L}^0_{light}$ invariant under $U(3)_L \times U(3)_R$

$$q_{L,R} \rightarrow e^{-i\theta_{L,R}\lambda_a} q_{L,R}$$

conserved Noether currents

$$J^\mu_{L,R} = \bar{q}_{L,R}\gamma^\mu \lambda_a q_{L,R}, \quad \partial J^\mu_{L,R} = 0$$

vector and axial-vector current

$$J^\mu_{V,a} = J^\mu_{L,a} + J^\mu_{R,a} = \bar{q}\gamma^\mu \lambda_a q$$

$$J^\mu_{A,a} = J^\mu_{L,a} - J^\mu_{R,a} = \bar{q}\gamma^\mu \gamma_5 \lambda_a q$$

charges

$$Q^a_V = \int d^3x \; q^\dagger(x)\lambda_a q(x)$$

$$Q^a_A = \int d^3x q^\dagger(x)\gamma^5 \lambda_a q(x)$$

commute with H^0_{QCD}

spontaneous chiral symmetry breaking

'Wigner-Weyl' realisation:

$$Q_V^a|0\rangle = Q_A^a|0\rangle = 0$$

$\rightarrow$ parity doublets in the hadron spectrum

not observed!

'Nambu-Goldstone' realisation:

$$U(3)_L \times U(3)_R \sim$$
$$SU(3)_V \times SU(3)_A \times U(1)_B \times U(1)_A$$

$$Q_V^a|0\rangle = 0; \quad Q_A^a|0\rangle \neq 0$$

$$SU(3)_V \times SU(3)_A \rightarrow SU(3)_V$$

$\rightarrow$ massless 'Goldstone' bosons

$$|\pi_a\rangle = Q_A^a|0\rangle$$

$$H^0_{QCD}|\pi_a\rangle = Q_A^a H^0_{QCD}|0\rangle = 0$$

quarks condense into scalar quark-antiqark pairs

$$\langle 0|\bar{q}q|0\rangle \equiv \langle \bar{q}q\rangle \equiv \langle\sigma\rangle \neq 0$$

formal connection:

$$P_a(x) = \bar{q}(x)\gamma^5\lambda_a q(x)$$

$$[Q_A^a, P_b] = -\delta_{ab}\bar{q}q$$

$$\rightarrow Q_A^a|0\rangle \neq 0 \rightarrow \langle\bar{q}q\rangle \neq 0$$

Goldstone's theorem implies

$$\langle 0|J^\mu_{A,a}|\pi_b(p)\rangle = -i\delta_{ab}F_\pi p^\mu e^{-ipx}$$

$$F_\pi = 92.4 \pm 0.3\ \mathrm{MeV}$$

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Z(3) symmetry

QCD partition function:

$$Z(V,T,\mu) = \int \mathcal{D}[q,\bar{q},A^a_\mu] \ \exp\left[-\int_0^{1/T} d\tau \int_V d^3x \left(\mathcal{L}^E_{QCD} - i\mu q^\dagger q\right)\right]$$

boundary conditions:

$$A_\mu(\tau + 1/T, \vec{x}) = A_\mu(\tau, \vec{x}); \quad q(\tau + 1/T, \vec{x}) = -q(\tau, \vec{x}); \quad A_\mu \equiv (\lambda_a/2) A^a_\mu$$

QCD Lagrangian invariant under local transformations: $g(x) = e^{ig_s \Theta^a(x)\lambda_a/2}$

$$q(x) \to {}^g q(x) = g(x) q(x)$$

$$A_\mu(x) \to {}^g A_\mu(x) = g(x)\left(A_\mu(x) + \frac{i}{g_s}\partial_\mu\right) g^\dagger(x)$$

boundary conditions put constraints on the allowed gauge transformations

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$Z(3)$ symmetry

$$g(\tau + 1/T, \vec{x}) = h g(\tau, \vec{x}); \qquad h \in SU(3) \text{ constant 'twist' matrix}$$

then

$${}^{g}A_\mu(\tau + 1/T, \vec{x}) = h\,{}^{g}A_\mu(\tau, \vec{x}) h^\dagger$$

since ${}^{g}A_\mu$ still has to obey a periodic boundary condition

$$h = z1; \quad z = \exp(2\pi i n/3), \quad n = 1, 2, 3$$

for quarks

$${}^{g}q(\tau + 1/T, \vec{x}) = g(\tau + 1/T, \vec{x}) q(\tau + 1/T, \vec{x}) = -z g(\tau, \vec{x}) q(\tau, \vec{x}) = -z\,{}^{g}q(\tau, \vec{x})$$

antiperiodic boundary condition only allows for $z = 1$
the center symmetry disappears

still useful as an *approximate* symmetry of QCD !

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'Static' quarks

use 'static quarks' to probe the physics of the gauge fields

infinitely heavy 'test' quarks are described by a 'Polyakov loop' (closed Wilson loop around the periodic τ direction)

$$L(\vec{x}) = \mathrm{Tr}_c \left[\mathcal{P} \exp \left(i \int_0^\beta d\tau A_4(\tau, \vec{x}) \right) \right] \quad \text{complex scalar field}$$

transforms non-trivially under $Z(3)$

$$^{g}L(\vec{x}) = zL(\vec{x})$$

Polyakov loop expectation value

$$\langle L(\vec{x}) \rangle = \frac{1}{Z_{YM}} \int \mathcal{D}[A_\mu^a] L(\vec{x}) \; \exp(-S_{YM}^E) = \exp(-\beta F_Q(\vec{x}))$$

measures the free energy of a static test quark at position $\vec{x}$.

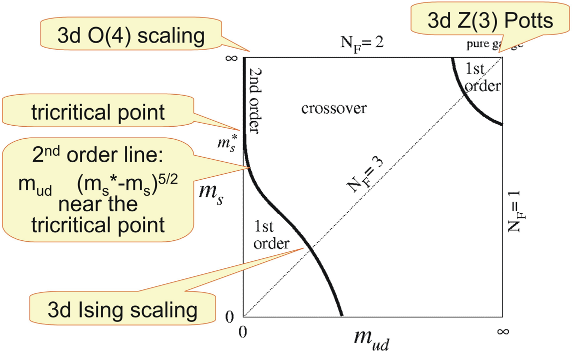
- small T color confined $\rightarrow F_Q = \infty$ and $\langle L \rangle = 0$
- high T quarks and gluons deconfined $\rightarrow F_Q$ finite and $\langle L \rangle = L_0 \neq 0$

at high T the $Z(3)$ symmetry is spontaneously broken!

$Z(3)$ center symmetry of $SU(3)_c$ exact for infinitely heavy quarks

$SU(3)_L \times SU(3)_R$ exact for massless m_u, m_d, m_s

$\longrightarrow$ phase diagram (Pisarski, Wilczek)



The NJL model

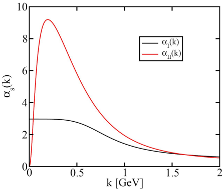
only chiral symmetry aspects

QCD current-current coupling:

$$S_{int} = \frac{1}{2}\int d^4x d^4y \; J_a^\mu(x) g_s^2 D_{\mu\nu}^{ab}(x-y) J_b^\nu(x); \quad J_a^\mu = \bar{q} i\gamma^\mu t_a q$$

at large distances (small momenta $k < \Lambda$)

$$D_{\mu\nu}^{ab}(k) = \delta_{ab}[G_c\theta(\Lambda - k) + \cdots]\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$



$$\mathcal{L}_{int} = G_c J_\mu^a(x) J_a^\mu(x) \rightarrow_{Fierz} G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]\ldots$$

NJL model ($N_f = 2$)

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m_q)q + G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]$$

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Thermodynamics

partition function:

$$\mathcal{Z}(V,T,\mu) = e^{-\Omega V/T} = \mathrm{Tr}\exp\left(-\beta\int_V d^3x\,(\mathcal{H}_{NJL} - \mu\bar{q}^\dagger q)\right)$$

with:

$$\mathcal{H}_{NJL} = \bar{q}(-\vec{\gamma}\cdot\nabla + m_0)q - G[(\bar{q}q)^2 + (\bar{q}\gamma_5\vec{\tau}q)^2]$$

thermodynamic potential (per volume):

$$\Omega(T,\mu) = -\lim_{V\to\infty}\frac{T}{V}\ln\mathcal{Z}(V,T,\mu); \qquad \Omega = \epsilon - Ts - \mu n$$

EoS:

$$p = -\Omega; \quad s = -\frac{\partial\Omega}{\partial T}; \quad n = -\frac{\partial\Omega}{\partial\mu}; \quad \epsilon = Ts - p + \mu n; \quad \langle\bar{q}q\rangle = -\frac{\partial\Omega}{\partial m_0}$$

(static) susceptibilities:

$$\chi_{\mu\mu} = -\frac{\partial^2\Omega}{\partial\mu^2}; \quad \chi_{TT} = -\frac{\partial^2\Omega}{\partial T^2}; \quad \chi_{\mu T} = -\frac{\partial^2\Omega}{\partial\mu\partial T}; \quad \chi_m = -\frac{\partial^2\Omega}{\partial m_0^2}$$

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Mean field approximation

linearization of the Lagrangian:

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L} \quad \rightarrow \quad \Omega_0(T,\mu) + \delta\Omega(T,\mu)$$

$$\bar{q}q = \langle \bar{q}q \rangle + \delta_\sigma; \quad \delta_\sigma = \bar{q}q - \langle \bar{q}q \rangle$$

$$\mathcal{L}_0 = \bar{q}(i\partial\!\!\!/ - \underbrace{(m_0 - 2G\langle \bar{q}q \rangle)}_{m=m_0+\Sigma(m)})\psi - G\langle \bar{q}q \rangle^2$$

grand potential:

$$\Omega_0(T,\mu;m) = \frac{(m-m_0)^2}{4G} - 12\int \frac{d^3p}{(2\pi)^3} \left[\tilde{E}_p + T\ln\left(1+\exp\left(-\frac{E_p-\mu}{T}\right)\right) + T\ln\left(1+\exp\left(-\frac{E_p+\mu}{T}\right)\right)\right]$$

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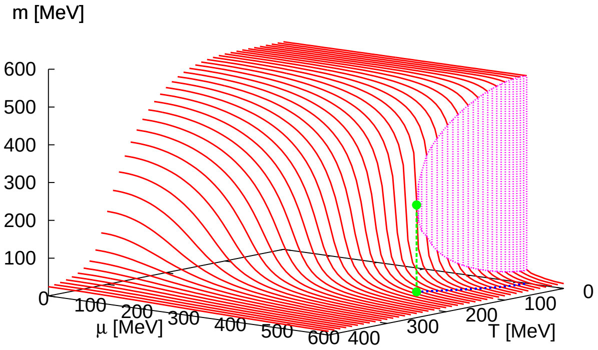
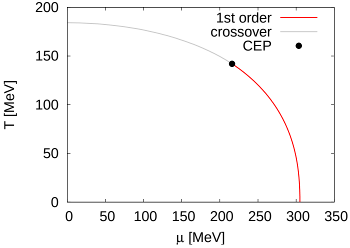
evolution of the chiral order parameter

$$\Omega_0(T, \mu; m) \quad \rightarrow \quad \frac{\partial \Omega_0}{\partial m} = 0$$



critical endpoint (CEP)

second-order phase transition:



at the CEP chiral and number suscept. diverge!

$$\chi_m, \chi_{\mu\mu} \equiv \chi_q \to \infty$$

universality class: 3D Ising model

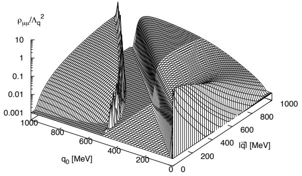
$$\chi_q \sim | g - g_c |^{-\epsilon}; \quad g = T, \mu$$

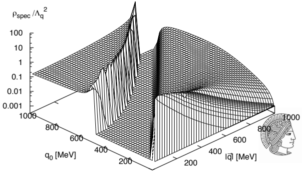
$$\epsilon = 0.78; \quad 2/3 \text{ (mean field)}$$

isothermal compressibility:

$$\kappa_T \equiv \frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_{|T,\mu} = \frac{\chi_q}{n_q^2}$$

$\to \chi_q$ large system easy to compress





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physical nature of the CEP

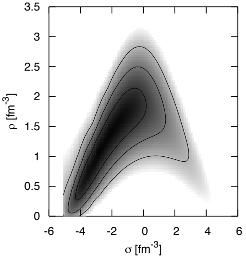
Ginzburg-Landau functional:

$$F(T,n,\sigma) = \Omega(T,\mu,m) - \mu n - m\sigma$$

$$F(T,n,\sigma) = \int d^3x \left[\frac{a}{2}(\partial_i\sigma)^2 + b\partial_i\sigma\partial_i n + \frac{c}{2}(\partial_i n)^2 + V(\sigma,n)\right]$$

$$V(\sigma,n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2 + \cdots$$

CEP: $AC = B^2$



flat direction: $\sigma/n = -B/A = -C/B$

$$\chi_m \sim \frac{TC}{\Delta}, \ \ \chi_q \sim \frac{TA}{\Delta}; \quad \Delta = AC - B^2$$

diverge at the CEP ($\Delta = 0$)

not sufficient: $A < C$ or $A > C$?

for $\mu \to 0$ mixing vanishes, i.e. $B \to 0$

soft mode is σ hence $A < C$

CEP not liquid-gas transition! ($A > C$)

implications for dynamical universality class!

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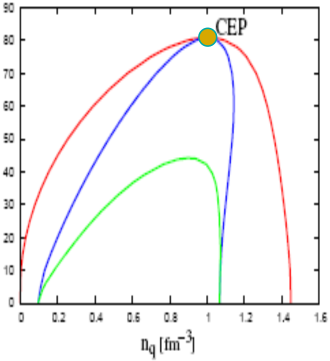
1^{st}-order region and spinodals

$m_q \neq 0$



spinodal lines:

$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \quad : \quad \text{isothermal}$$

$$\left(\frac{\partial P}{\partial V}\right)_S = 0 \quad : \quad \text{isentropic}$$

$$\left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{\partial P}{\partial V}\right)_S + \frac{T}{c_V}\left[\left(\frac{\partial P}{\partial T}\right)_V\right]^2$$

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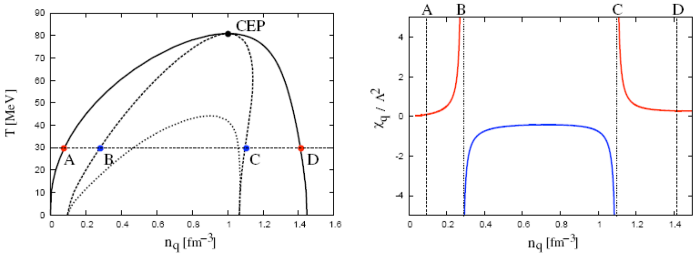
quark number susceptibility

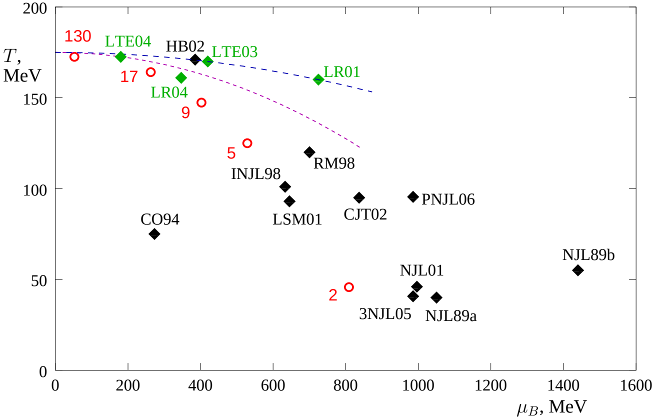
deviation from equilibrium, large fluctuations induced by instabilities



- at first-order point (A,D): $\chi_q \equiv \chi_{\mu\mu}$ finite
- at isothermal spinodal point (B,C): χ_q diverges and changes sign

$$\left(\frac{\partial P}{\partial V}\right)_T = -\frac{n_q^2}{V}\frac{1}{\chi_q}$$

- in the unstable region (B-C): χ_q is finite and negative small large model dependence in the location of the CEP!



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The PNJL model

Polyakov loop variable:

$$\Phi(x) \equiv \frac{1}{N_c} \langle L(x) \rangle \to \phi = \frac{1}{3} \mathrm{Tr}_c \exp \left[\frac{iA_4}{T} \right]$$

Lagrangian:

$$\mathcal{L}_{PNJL} = \bar{q}(i\not{D}(\phi) - m_q)q + G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] - U(\phi, \phi^*)$$

covariant derivative:

$$\not{D}(\phi) = \not{\partial} - i\gamma_4 A_4(\phi)$$

effective potential:

$$U(\phi, \phi^*)/T^4 = -\frac{b_2(T)}{2}\phi^*\phi - \frac{b_3}{6}\left(\phi^3 + \phi^{*3}\right) + \frac{b_4}{4}(\phi^*\phi)^2$$

with

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

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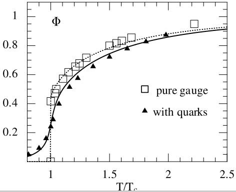
$T_0 = 270 MeV$

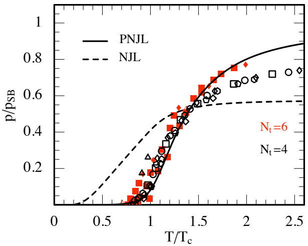
a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

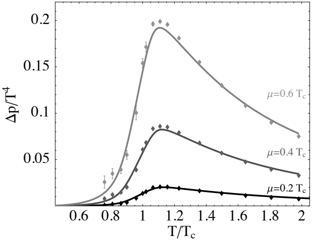


Results

thermodynamic potential:

$$\Omega_0(T,\mu;m) = U(\phi,\phi^*) + \frac{(m-m_0)^2}{4G} - 2N_f T \int \frac{d^3p}{(2\pi)^3}$$
$$\left[\ln\left(1 + 3(\phi + \phi^* e^{-(E_p-\mu)/T})e^{-(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}\right) + \ln\left(1 + 3(\phi^* + \phi e^{-(E_p+\mu)/T})e^{-(E_p+\mu)/T} + e^{-3(E_p+\mu)/T}\right)\right]$$





The PQM model

quarks coupled to $(\sigma, \vec{\pi})$-fields and the Polyakov loop

Lagrangian:

$$\mathcal{L}_{PQM} = \bar{q}\left[i\not{D}(\phi) - G(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\right] q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma, \vec{\pi}) - U(\phi, \phi^*)$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - c\sigma$$

thermodynamic potential:

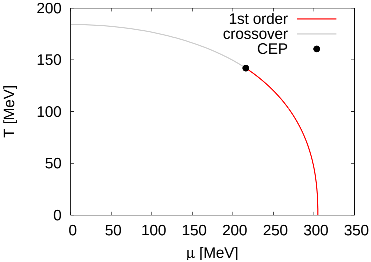
$$\Omega_0(T, \mu; m) = U(\phi, \phi^*) + U(\sigma) + \Omega_0^{\bar{q}q}$$

N_f and μ-dependence of T_0

$$T_0(N_f, \mu) = T_\tau e^{-1/\alpha_\tau b(\mu)}$$

$$b(\mu) = \frac{1}{6\pi}(11N_c - 2N_f) - \frac{16}{\pi}N_f\frac{\mu^2}{T_\tau^2}$$

N_f	0	1	2	2+1	3
T_0 [MeV]	270	240	208	187	178



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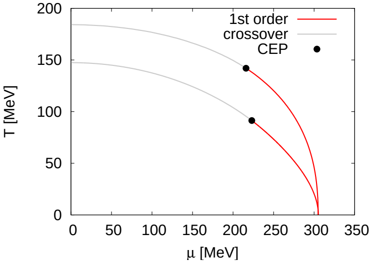

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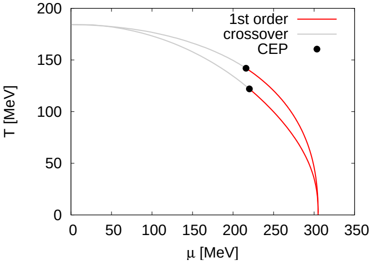

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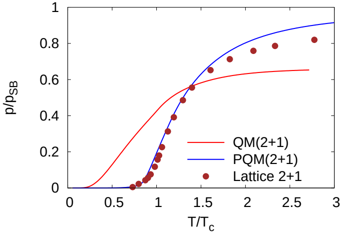
$$\Omega_0(T, \mu; m) = U(\phi, \phi^*) + U(\sigma) + \Omega_0^{\bar{q}q}$$

N_f and μ-dependence of T_0

$$T_0(N_f, \mu) = T_\tau e^{-1/\alpha_\tau b(\mu)}$$

$$b(\mu) = \frac{1}{6\pi}(11N_c - 2N_f) - \frac{16}{\pi} N_f \frac{\mu^2}{T_\tau^2}$$

N_f	0	1	2	2+1	3
T_0 [MeV]	270	240	208	187	178



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The PQM model

quarks coupled to $(\sigma, \vec{\pi})$-fields and the Polyakov loop

Lagrangian:

$$\mathcal{L}_{PQM} = \bar{q}\left[i\not{D}(\phi) - G(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\right] q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma,\vec{\pi}) - U(\phi,\phi^*)$$

$$U(\sigma,\vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - c\sigma$$

thermodynamic potential:

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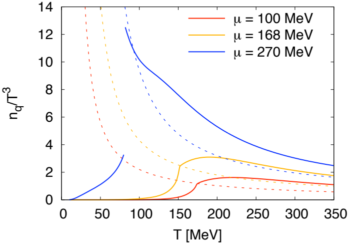

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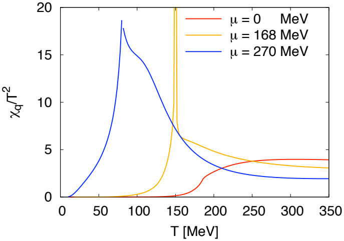
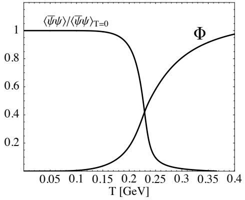
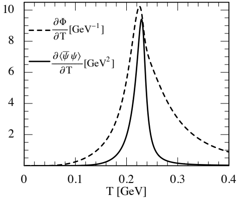


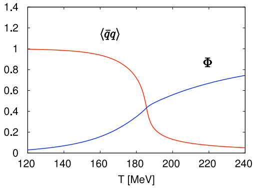
PNJL vrs PQM

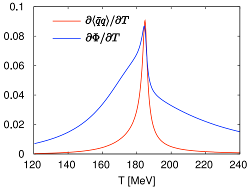
PNJL ($\mu = 0$)





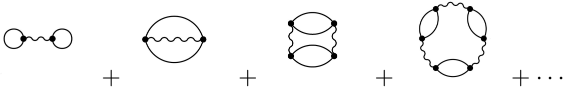
PQM ($\mu = 0$)





$1/N_c$-expansion

grand potential beyond mean field → mesonic fluctuations:



$1/N_c$-diagrams can be summed to all order ('ring sum')

$$\delta\Omega = \sum_M \Omega_M; \qquad \Omega_M = \int \frac{d^3q}{(2\pi)^3} \frac{T}{2} \sum_{i\omega_q} \ln(1 - 2G\Pi_M(i\omega_q, \vec{q}))$$

$$\Omega_M = -\int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \left(1 + 2n_B(\omega)\right) \phi_M; \quad \phi_M = \frac{1}{2i} \ln \frac{1 - 2G\Pi_M(\omega - i\eta, \vec{q})}{1 - 2G\Pi_M(\omega + i\eta, \vec{q})}$$

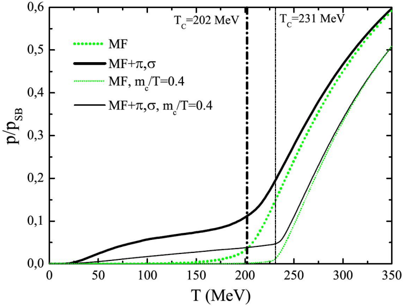
Grand potential

$$\Omega_M = \underbrace{-\int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{\pi} n_B(\omega)\phi_M}_{NSR\ \ finite} + \underbrace{\int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \Delta\phi}_{qfl}$$

$$\Delta\phi = \phi_{T,\mu} - \phi_0$$

$$\Delta\phi^{(\pi)} \to \frac{36}{\pi}\Lambda_q^4(m_{vac}^2 - m^2)\frac{1}{(\omega^2 - \vec{q}^2)}$$

$$\Delta\phi^{(\sigma)} \to \frac{12}{\pi}\Lambda_q^4(m_{vac}^2 - m^2)\frac{1}{(\omega^2 - \vec{q}^2)}$$



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Grand potential

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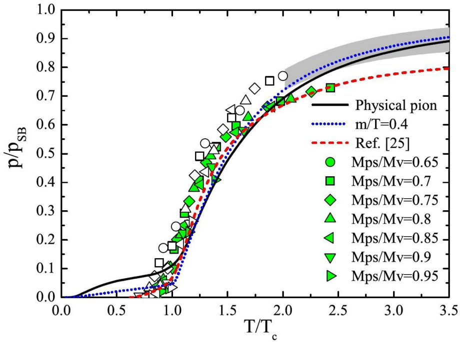

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Flow equation

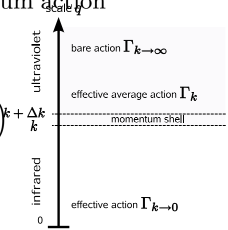
scale-dependent effective action:

$$\Gamma_k[\phi] \equiv \frac{V}{T}\Omega_k(T,\mu;\phi); \qquad \lim_{k\longrightarrow 0}\Gamma_k = \Gamma \quad \text{full quantum action}$$

flow equation: (PTRG) $\qquad t = \ln(k/\Lambda)$

$$\partial_t \Gamma_k[\phi] = -\frac{1}{2}\int_0^\infty \frac{d\tau}{\tau}[\partial_t R_k(\tau k^2)]\mathrm{Tr}\exp\left(-\tau\Gamma_k^{(2)}[\phi]\right)$$

$$\Gamma_k^{(2)}[\phi] = \frac{\delta\Gamma_k}{\delta\phi\delta\phi}$$



quark-meson model: $(\vec{\Phi} = (\sigma,\vec{\pi}); \quad \phi^2 = \sigma^2 + \vec{\pi}^2)$

$$\Gamma_k[\phi] = \int_0^{1/T} d\tau \int d^3x\ \bar{q}\,[i\partial\!\!\!/ - G(\sigma + i\gamma_5\vec{\tau}\vec{\pi})]\,q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U_k(\phi^2) + \mu q^\dagger q$$

at the cutoff schale Λ

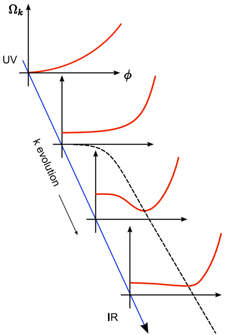
$$U_\Lambda(\phi) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - c\sigma$$

grand potential

flow of Ω:

$$\partial_t \Omega_k(T,\mu) = \frac{k^4}{12\pi^2}\left[\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right)+\frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right) - \frac{2N_cN_f}{E_q}\left\{\tanh\left(\frac{E_q-\mu_q}{2T}\right)+\tanh\left(\frac{E_q+\mu_q}{2T}\right)\right\}\right]$$

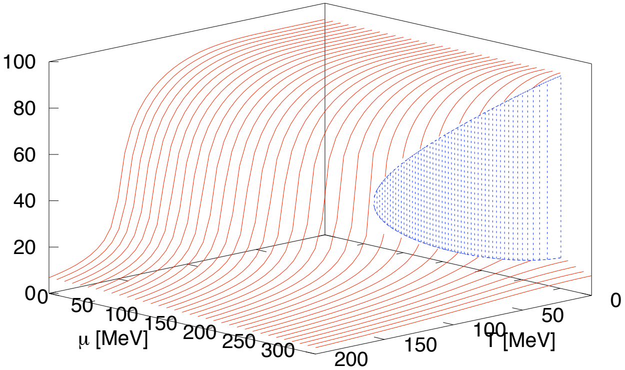
$$E_\pi^2 = 1 + 2\Omega_k'/k^2$$
$$E_\sigma^2 = 1 + 2\Omega_k'/k^2 + 4\phi^2\Omega_k''/k^2$$
$$E_q^2 = 1 + G\phi^2/k^2$$
$$\Omega_k' = \partial\Omega_k/\partial\phi \quad etc \quad \phi = \langle\sigma\rangle$$



mean field phase diagram

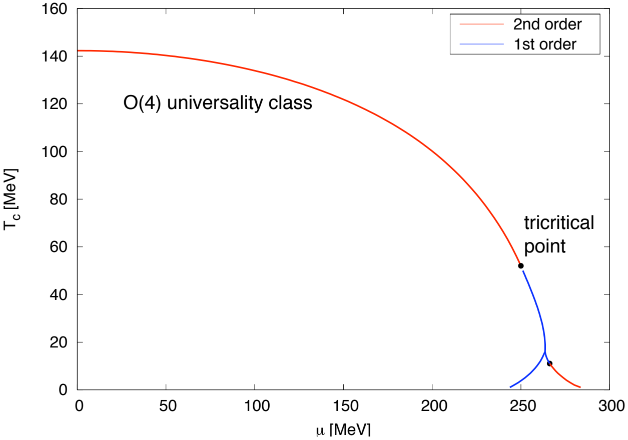
mean field:

$$\partial_t \Omega_k(T,\mu) = \frac{k^4}{12\pi^2}\left[-\frac{2N_c N_f}{E_q}\left\{\tanh\left(\frac{E_q-\mu_q}{2T}\right)+\tanh\left(\frac{E_q+\mu_q}{2T}\right)\right\}\right]$$



RG phase diagram

$O(4) \sim SU(2)_L \times SU(2)_R$ chiral limit



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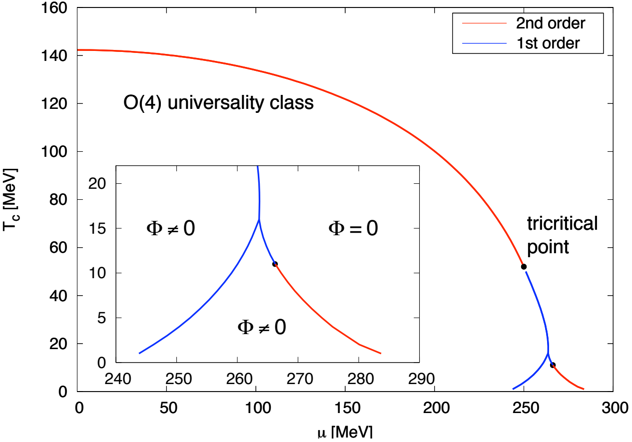
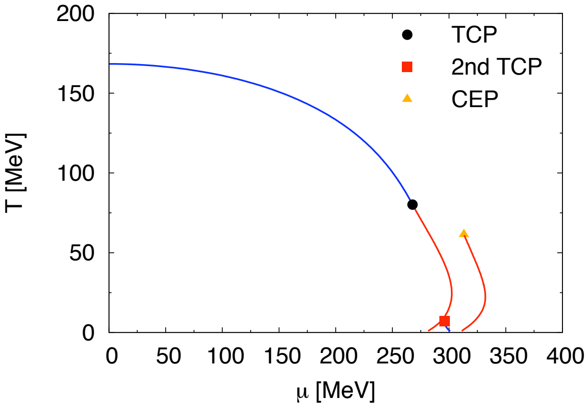


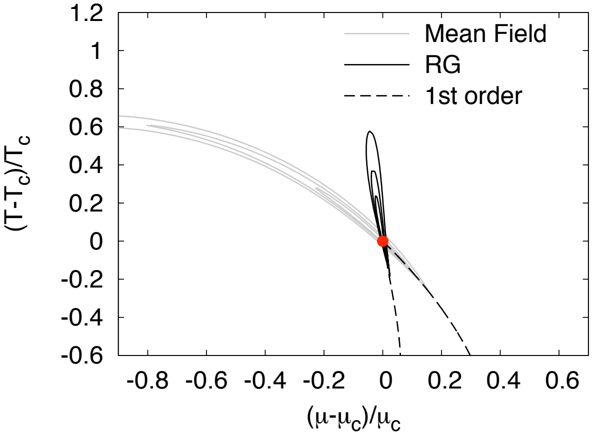
RG phase diagram

finite quark mass



RG phase diagram

finite quark mass



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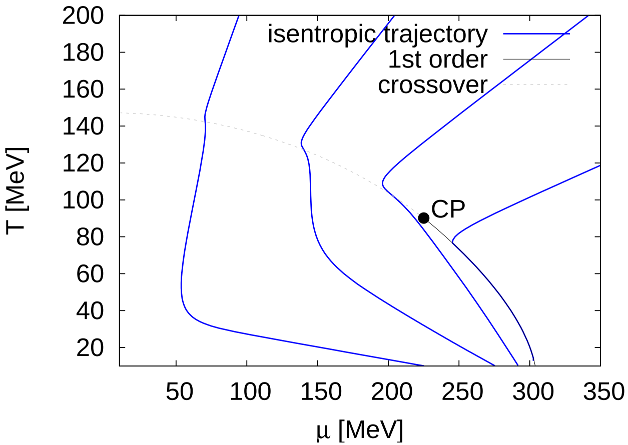
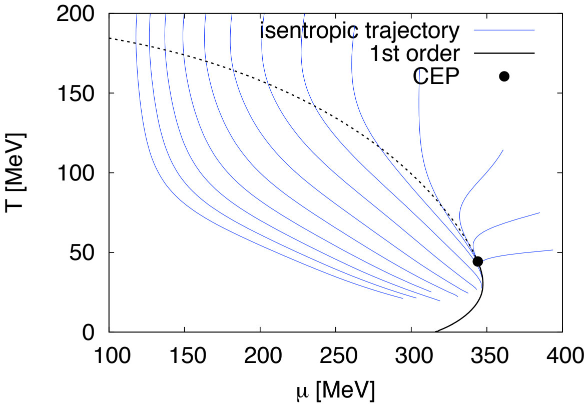
RG phase diagram

isentropes



RG phase diagram

isoentropes



Summary

- symmetries of QCD and the phase diagram
  - chiral symmetry
  - center symmetry
- phase diagram in QCD inspired models
  - location of the CEP
  - fluctuation properties at mean field
- beyond mean field
  - $1/N_c$ contributions to EoS (confined phase!)
  - size of the critical region is small
  - second CEP (liquid gas-transition?)

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