Phases of QCD: lattice thermodynamics versus PNJL model [1]

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Abstract. We investigate QCD-based thermodynamics by means of an extended Nambu–Jona-Lasinio model in which quarks couple simultaneously to the chiral condensate and to a background temporal gauge field representing Polyakov loop dynamics. The model thus incorporates features of both deconfinement and chiral symmetry restoration. Thermodynamic quantities at finite quark chemical potential are presented, and comparison with lattice QCD results is discussed.

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INTRODUCTION

In our work [1] we have studied the thermodynamics of two-flavour QCD at finite quark chemical potential. Our investigation is based on a synthesis of a Nambu Jona-Lasinio (NJL) model and the non-linear dynamics involving the Polyakov loop [2, 3, 4, 5]. In this Polyakov-loop-extended (PNJL) model, quarks develop quasiparticle masses by propagating in the chiral condensate, while they couple at the same time to a homogeneous background (temporal) gauge field representing Polyakov loop dynamics.

The main aim of our work is to answer the basic question, whether results coming from lattice QCD thermodynamics can be understood and interpreted in terms of quasiparticle degrees of freedom. The NJL model, with just the simplest possible one-parameter colour current-current interaction between quarks, is remarkably successful in reproducing the thermodynamics of $N_c = 2$ Lattice QCD at finite μ [6]. Encouraged by this result, the NJL quasiparticle concept does suggest itself as a useful starting point. However, whereas aspects of deconfinement are less significant in the $N_c = 2$ case, they figure prominently for $N_c = 3$. This motivates our extension towards the PNJL Lagrangian as a minimal approach incorporating both chiral symmetry restoration and deconfinement.

RESULTS

The thermodynamic potential of the system has the following form:

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$$
\begin{aligned}
\Omega &= \mathscr{U}\left(\Phi,\bar{\Phi},T\right)+\frac{\sigma^2}{2G}-2N_f T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{\mathrm{Tr}_c\ln\left[1+L\,\mathrm{e}^{-(E_p-\mu)/T}\right]\right.\\
&+ \left.\mathrm{Tr}_c\ln\left[1+L^\dagger\,\mathrm{e}^{-(E_p+\mu)/T}\right]\right\}-6N_f\int\frac{\mathrm{d}^3p}{(2\pi)^3}E_p\,\theta\left(\Lambda^2-\vec{p}^{\,2}\right) \qquad (1)
\end{aligned}
$$

where we have introduced the quark quasiparticle energy $E_p=\sqrt{\vec{p}^{\,2}+m^2}$, with a constituent quark mass given by their interaction with the chiral condensate (related to the σ scalar field): $m=m_0-\langle\sigma\rangle=m_0-G\langle\bar{\psi}\psi\rangle$. Φ and $\bar{\Phi}$ are the traced Polyakov loop and its conjugate: $\Phi=(\mathrm{Tr}_c\,L)/N_c,\ \bar{\Phi}=(\mathrm{Tr}_c\,L^\dagger)/N_c$. The Polyakov loop L is a matrix in colour space explicitly given by

$$
L(\vec{x})=\mathscr{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right], \qquad (2)
$$

with $\beta=1/T$ the inverse temperature and $A_4=iA^0$. $\mathscr{U}(\Phi,\bar{\Phi},T)$ is a phenomenological potential for the Polyakov loop, having a polynomial form, which is chosen following the symmetry pattern of the three-colour QCD Lagrangian.

All the parameters of the model are fixed a priori: the three parameters related to the NJL part of the Lagrangian (m_0, Λ, G) are chosen to reproduce some known physical quantities in the hadronic sector. The parameters contained in the Polyakov loop potential are constrained in the pure gauge sector of QCD. Therefore, the results that we obtain are predictions of the model, which we will compare to the available lattice data. By minimizing the thermodynamic potential of the system, we obtain the behaviour of Polyakov loop and chiral condensate as functions of temperature and chemical potential. We obtain the non-trivial result that chiral and deconfinement phase transitions take place at the same critical temperature, around 200 MeV. We show these results in Fig. 1.

Among the things that we have evaluated in our model, the most interesting ones are the results for the scaled pressure difference $\Delta p/T^4=(p(T,\mu)-p(T,\mu=0))/T^4$ and quark number density $n_q/T^3=-1/T^3\partial\Omega(T,\mu)/\partial\mu$. We show our results in Fig. 2 in comparison with the available lattice data [7]. These quantities are plotted as functions of the temperature, for different values of the quark chemical potential. As we can see, the predictions of our model are in perfect agreement with the corresponding results coming from lattice simulations.

CONCLUSIONS

We have studied a Polyakov-loop-extended Nambu and Jona-Lasinio (PNJL) model with the aim of exploring whether such an approach can catch essential features of QCD thermodynamics when confronted with results of lattice computations at finite temperature and non-zero quark chemical potential. This PNJL model represents a minimal synthesis of the two basic principles that govern QCD at low temperatures: spontaneous chiral symmetry breaking and confinement.

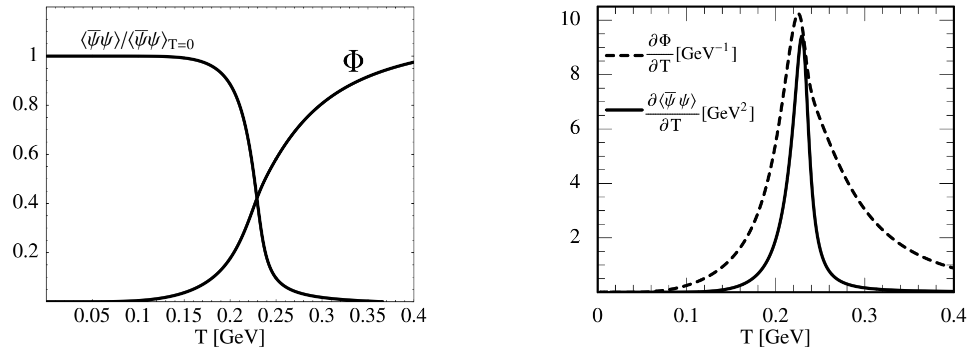

FIGURE 1. Left: scaled chiral condensate and Polyakov loop $\Phi(T)$ as functions of temperature at zero chemical potential. Right: plots of $\partial\langle\bar{\psi}\psi\rangle/\partial T$ and $\partial\Phi/\partial T$.

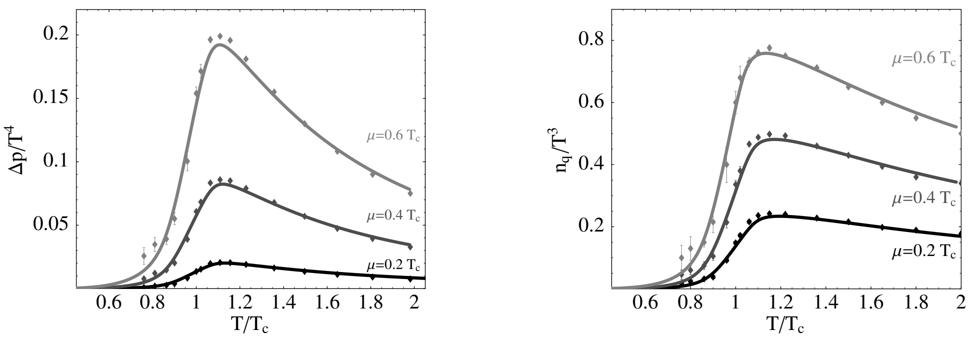

FIGURE 2. Left: scaled pressure difference as a function of temperature at different values of the quark chemical potential, compared to lattice data. Right: scaled quark number densities as a function of temperature at different values of the chemical potential, compared to lattice data. In both figures, the lattice data are taken from Ref. [7].

Considering the simplicity of the PNJL model, the conclusion that can be drawn at this point is promising: it appears that a relatively straightforward quasiparticle approach, with its dynamics rooted in spontaneous chiral symmetry breaking and confinement and with parameters controlled by a few known properties of the gluonic and hadronic sectors of the QCD phase diagram, can account for essential observations from two-flavour $N_c = 3$ Lattice QCD thermodynamics.

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