GRAVITY RECOVERY ANALYSIS USING GPS FOR STEP AND A LOW-LOW SATELLITE MISSION

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Global Positioning System (GPS) receivers on low earth orbiting satellites can provide three-dimensional global tracking with sub-decimeter orbit accuracy. The precise GPS data can additionally be used to refine the Earth's gravity field. STEP is a proposed European Space Agency mission that may also carry a flight GPS receiver. Using a realistic scenario for processing 5 days of GPS data, a covariance analysis is performed to obtain the expected improvement in the gravity field. Additionally, a similar analysis is then repeated for two low Earth orbiting satellites, each equipped with GPS receivers and with precise ranging measurements between them.

INTRODUCTION

Global Positioning System (GPS) equipped low earth orbiting missions will facilitate an improvement of current Earth gravity models as a result of continuous, three-dimensional global tracking coverage provided by GPS. Comprehensive coverage over the generally inaccessible land masses of eastern Europe and, Asia and the ocean basins will provide enhanced detail of the geoid at these location...

TOPEX/Poseidon, launched in August of 1992, is the first scientific mission to carry a dual-frequency GPS receiver[6]. Sub-decimeter orbit accuracy is expected to be achievable[14,21]. In addition, the precise GPS data from TOPEX/Poseidon can be used to refine the Earth's geopotential, particularly in the low to mid (5-20) degree and order harmonics[3,4,11,18]. Insignificant improvement in shorter wavelengths is due to the 1340 km altitude of the spacecraft. At such relatively high altitudes, the effects of these wavelengths are both small and un-separable so that they do not significantly contribute to current gravity field knowledge.

STEP (Satellite Test of the Equivalence Principle) is a proposed mission to investigate the equivalence principle and the Earth's geopotential. It was proposed to the European Space Agency (ESA) with its main objective to measure the equivalence of gravitational mass and inertial mass to one part in 10^{17} of the total gravitational acceleration[2]. In order to achieve this accuracy, it is necessary to fly STEP as a drag-free low Earth orbiter, Since its orbit is perturbed solely by gravitational effects, and due to its lower 550 km altitude, GPS tracking should further improve the low to mid degree and order harmonics, The first part of this paper demonstrates this improvement.

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Aristoteles, another ESA mission, was proposed to carry a gravity gradiometer that would measure the spatial variation of the Earth's gravitational field. These gradient measurements would then be combined with other field measurements to produced a gravity field that is sensitive to fine details in the Earth's gravitational field[1]. It was expected that the Aristoteles' mission would be able to resolve coefficients to degree and order 250. Due to funding constraints however, the future of Aristoteles is uncertain. In its wake, there are proposals for low-low satellite missions. These missions would also measure gravity gradients but over an extended distance (200-400 km) between the two satellites. Two scenarios for a low-low mission are: 1.) one satellite equipped with a GPS receiver and precise ranging to other passive satellite(s) and 2.) two co-orbiting satellites both equipped with GPS receivers, and with precise ranging between them. The second part of this paper provides a preliminary gravity recovery analysis for the second scenario.

TECHNIQUE

Schrama[12] uses an analytical technique to perform an error analysis of gravity recovery from a low earth orbiting GPS receiver. This technique models the position errors of a low earth orbiting GPS receiver as 3 cm in each component and uncorrelated in time. The analysis presented here accounts for correlations by explicitly modeling the GPS observable and utilizing reasonable data noise assumptions. The results are based on a numerical analysis that simulates one-way range and carrier phase observable from 12 ground receivers and low Earth orbiting satellite(s) observing 24 GPS satellites. Comparisons, based on the results below, indicate that after accounting for data sampling and arc length, Schrama's analytical results are generally optimistic by a factor of three.

The common procedure for gravity recovery is to solve simultaneously for a large number of coefficients of a spherical harmonic expansion, This procedure is computationally demanding and usually requires a powerful supercomputer. The procedure used here is to model the perturbation of the gravity field on the low Earth orbiting satellite(s) as a three dimensional adjustment of position at each measurement time point, Tracking data from multiple short arcs of the orbits are processed separately with a square root information filter and then combined into a long arc solution, This long arc solution can then be converted into spherical harmonics, The resulting field is identical to the conventional method but the computation is generally much faster. This method is generally referred to as the "gravity bin algorithm", after the names given to the position perturbations (gravity bins)[5,15,16,17,19].

Figs. 1 and 2 demonstrate the advantage in using gravity bins over a conventional approach. Fig. 1 demonstrates that as the degree and order increases, the gravity bin algorithm becomes more computationally advantageous, Fig. 2 demonstrates that the gravity bin advantage is additionally a function of the data sampling interval. As the measurement rate increases, more gravity bin parameters are required in the short filter arcs, and hence the efficiency of these short filter arcs is reduced, Likewise, if the length of the short filter arcs is increased, more gravity bin parameters are required in the short filter arcs, and the efficiency is again reduced. Generally, short data arcs of one hour duration prove to be sufficient,

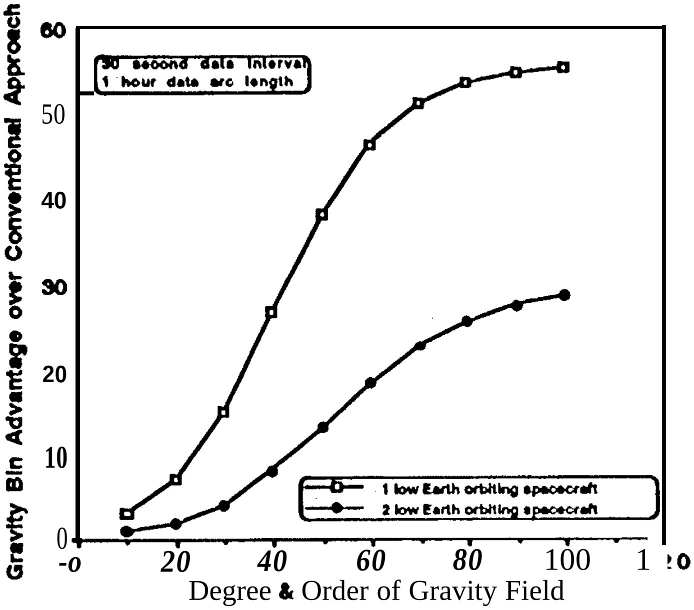

Fig. 1) Gravity Bin Advantage vs. Degree & Order of Gravity Field

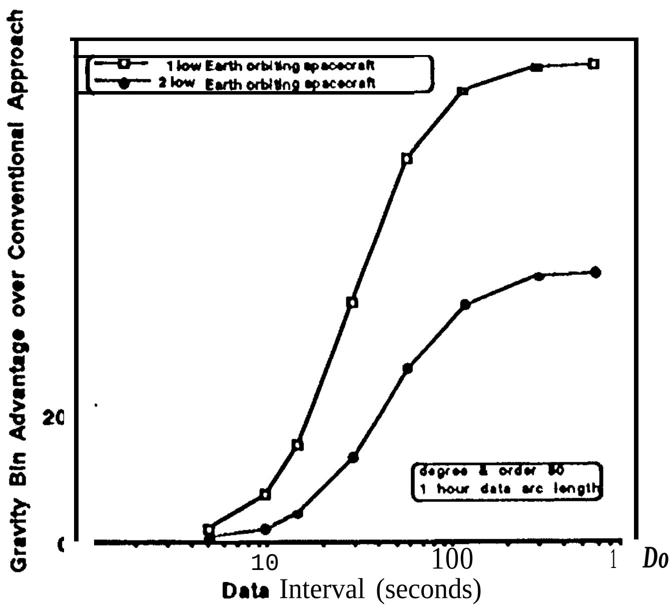

Fig. 2) Gravity Bin Advantage vs. Increasing Data Interval

One particular advantage of the bin technique is that the analyst can decide the degree and order of the gravity field to be generated from the long arc solution.

STEP ANALYSIS

For the STEP covariance analysis, the STEP orbit is modeled as sun synchronous with a semi-major axis of 7068 km. A six orbit plane, 24-satellite constellation is assumed for GPS. GPS measurements are simulated for STEP and 12 globally distributed ground stations. The ground network included the three NASA Deep Space Network (DSN) tracking sites at Goldstone, California; Madrid, Spain; and Canberra, Australia. The remaining sites are placed in Usuda, Japan; Santiago, Chile; Hartebeesthoek, South Africa; Bangalore, India; Tahiti; Kerguelen Island; Ny Allesund, Norway; Hawaii; and Alaska. Carrier phase and P code pseudorange data are simulated every 30 seconds. Only GPS data observed by STEP above zero degree elevation are processed. This typically results in STEP observing between 8-10 GPS spacecraft, Furthermore, only GPS data observed by the ground network above 15 degree elevation are processed. 'I'his ensures that low elevation multi path does not enter into the processing. The 15 degree elevation cutoff results in each ground station observing between 6-8 GPS spacecraft,

All carrier phase data are assumed to have 1 cm data noise, For a TOPEX/Poseidon type receiver the observed data noise is about 1 to 2 cm; for Rogue ground receivers the observed data noise is about a factor of 2 better. All P code pseudorange data are assumed to have a 30 cm data noise. This is about right for Rogue ground receivers but optimistic by a factor of 2 for a TOPEX/Poseidon type receiver,

120 short data arcs, each of 1 hour long duration, are processed. As Fig. 3 indicates, this 5 day data arc is sufficient to uniformly sample the Earth's potential with at most 5 degree gaps in longitude and 20 degree gaps in latitude. In each arc, all 24 six-component GPS states are estimated with an *a priori* error of 2 meters in each component of position and 0.2 mm/sec in each component of velocity. This is somewhat conservative when compared to recent results of routine GPS data processing.[9,20]. Additionally, zenith troposphere delays are modeled as random walks with an *a priori* error of 50 cm and a cumulative error growth of 1 cm/hour. After each 1 hour long data arcs, along with the GPS phase biases, the GPS states and troposphere parameters are reset. These resets are necessary to preserve the sparse block structure needed for the combining portion of the gravity bin algorithm. Also estimated throughout the processing is STEP's six-component state with a conservative *a priori* error of 1 km in each component of position and 1 meter/sec in each component of velocity, and 9 of the 12 ground station locations with an *a priori* error of 5 cm. The remaining 3 Deep Space Network sites in California, Spain, and Australia are considered with an error of 5 cm. Only information associated with STEP's state and the estimated station locations are passed from one short arc to the next. Finally, white noise clocks are estimated and reset after every data batch with very loose constraints.

The longest continuous track of a GPS spacecraft observed by STEP is about 35 minutes and a typical track is about 25 minutes. Ground tracks of GPS spacecraft are generally greater than 1 hour. As mentioned above, after each 1 hour long arc solution, all GPS phase biases are reset. These artificial phase breaks every hour lead to a slight (5-10%) degradation of the solution[3].

After all the 120 short arcs are processed, they are combined into a single long arc (5-day) solution. This long, arc solution is then converted to a 50x50 degree and order gravity field.

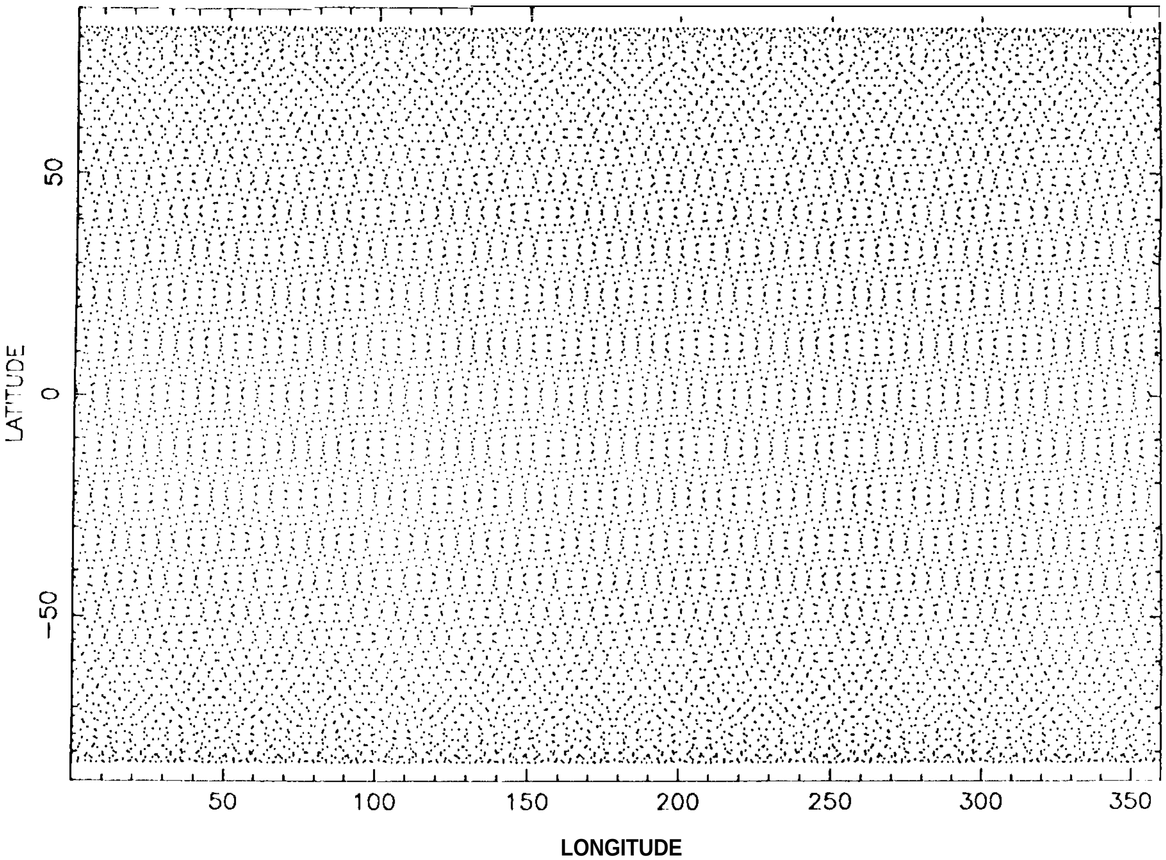

Fig. **3) Data Sampling for STEP Analysis**

STEP **RESULTS**

Fig. 4 compares the root-mean-square (rms) of the sigmas of the normalized gravity coefficients over order for several gravity representation.. vs. increasing degree. Both GEMT2 and GEMT3 (Goddard Earth Models) are indicated in this figure[10]. Also indicated is the results of the above STEP covariance analysis. The curve indicates that a 5 day data arc can improve the GEMT3 low to mid (2-23) degree and order harmonics. The figure also shows the spectrum of the gravity field that would result upon 1.) merging the a 5 day data arc with GEMT3, and 2.) merging a 6 month data arc with GEMT3. Finally, Figs, 5 and 6 indicates the improvement in the geoid height error after combining GEMT3 with the 6 month data arc. Over the oceans the geoid height error reduces from about 40 cm to 16 cm: and over the land masses the geoid height error reduces from 60-1 00 cm to 30-40 cm.

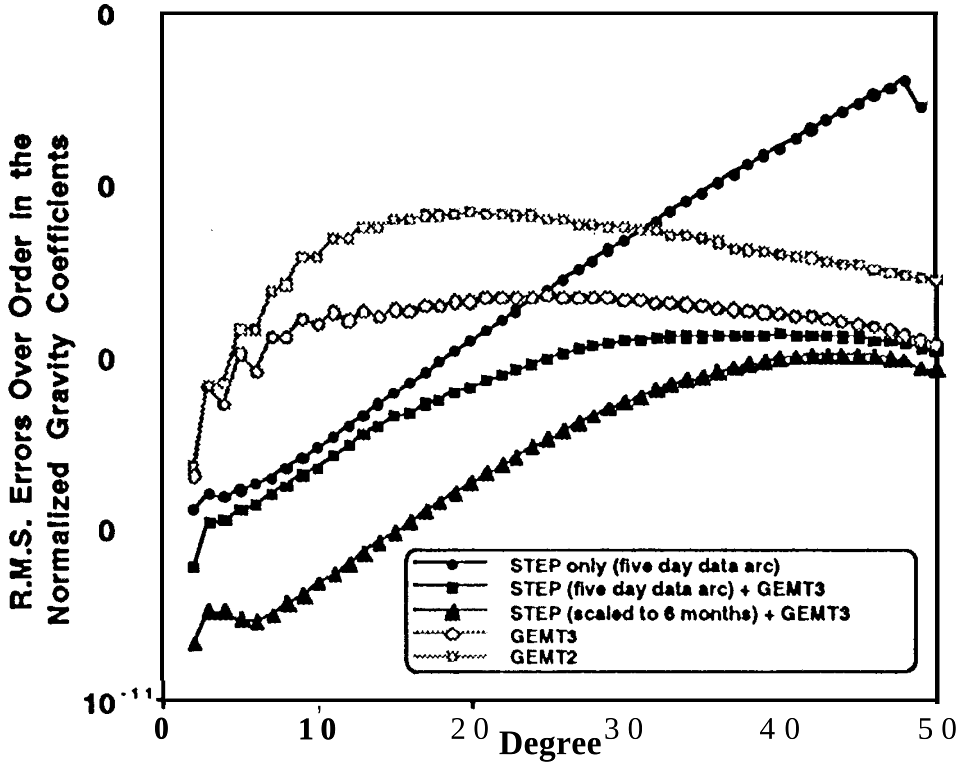

Fig 4.) RMS of Sigmas Over Order vs. Degree for STEP Analysis

LOW-LOW SATELLITE ANALYSIS

For the low-low satellite covariance analysis, the orbits are modeled as sun synchronous with semi-major axes of 6628 km. The separation between the two spacecraft decreases from 1000 to 700 km over the 5 day data arc. Again, a six orbit plane, 24-satellite constellation is used for GPS, and the same ground network is assumed, Carrier phase and P code pseudorange data are simulated every 30 seconds. Only GPS data observed by the low satellites above 5 degrees elevation is processed, This typically results in the satellites observing between 6-8 GPS spacecraft. Again, only GPS data observed by the ground network above 1 5 degrees elevation is processed.

The carrier phase data observed by the ground stations are assumed to have a 1 cm data noise. The carrier phase data observed by the low satellites are assumed to have a 5 mm data noise, All P code pseudorange data are assumed to have a 50 cm data noise. Finally, the precise range measurements between the two low satellites are assumed to have a data noise of 0.07 mm for 30 second normal points.

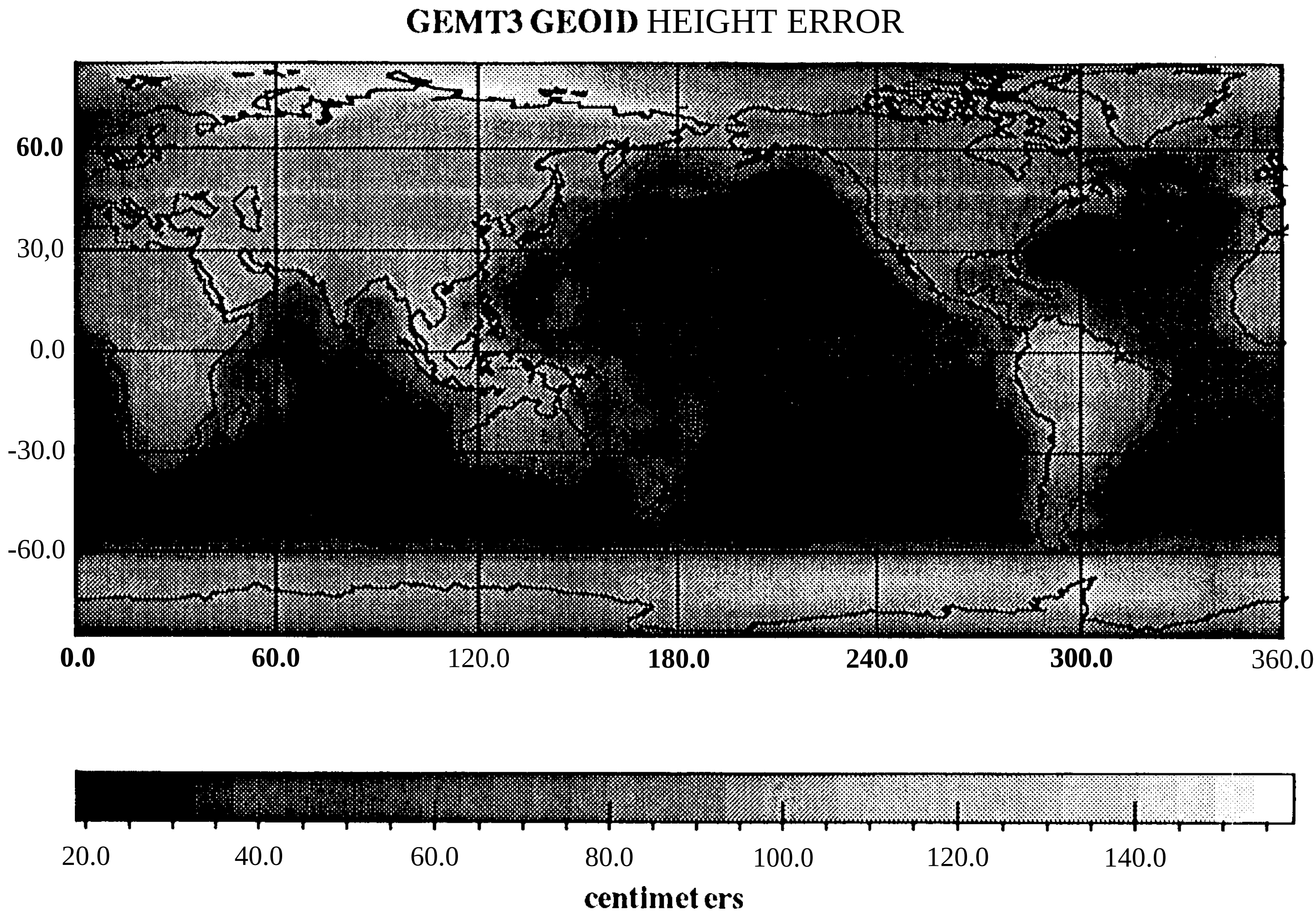

Fig. S) GEMT3 Geoid Height Error

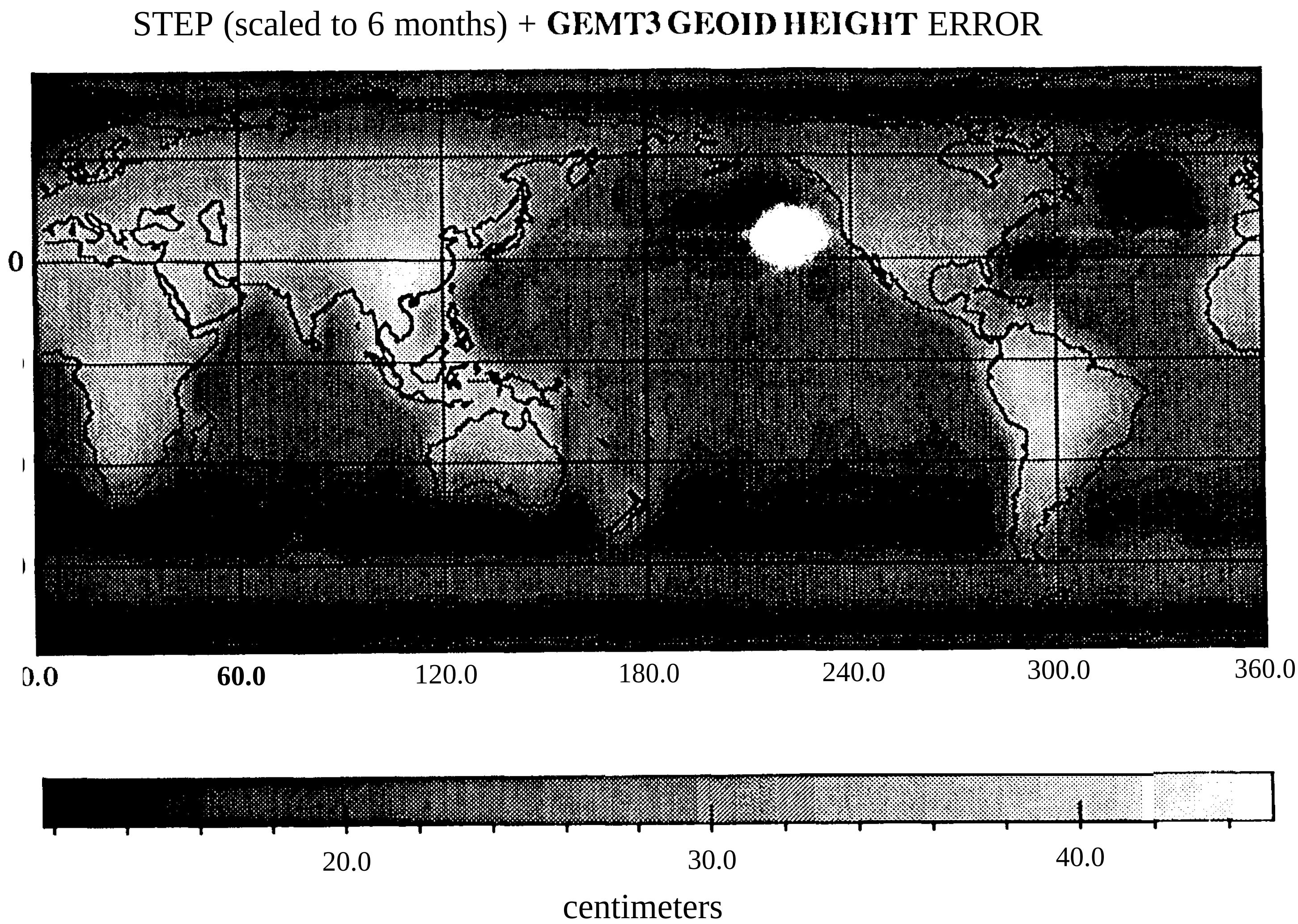

Fig. 6) Improvement in GEMT3 Due to a 6 Month STEP Data Arc

The same scenario of processing 120 short data arcs, each of 1 hour long duration, and then combining the results into a long arc is employed. The error assumptions and filtering strategies used in the above STEP analysis are applicable. Information associated with the station locations and now both satellites' states are passed from one short arc to the next. After all the 120 short arcs are processed, they are combined into a single long arc (5-day) solution. This long arc solution is then converted to a 40x40 degree and order gravity field. Ongoing analysis is extending this to degree and order 50.

Unlike the prior processing, the 5 day arc is not sufficient to uniformly sample the Earth's geopotential. Fig. 7 shows several 10 by 50 degree gaps in the northern and southern hemispheres. These gaps will later become evident in the geoid height errors.

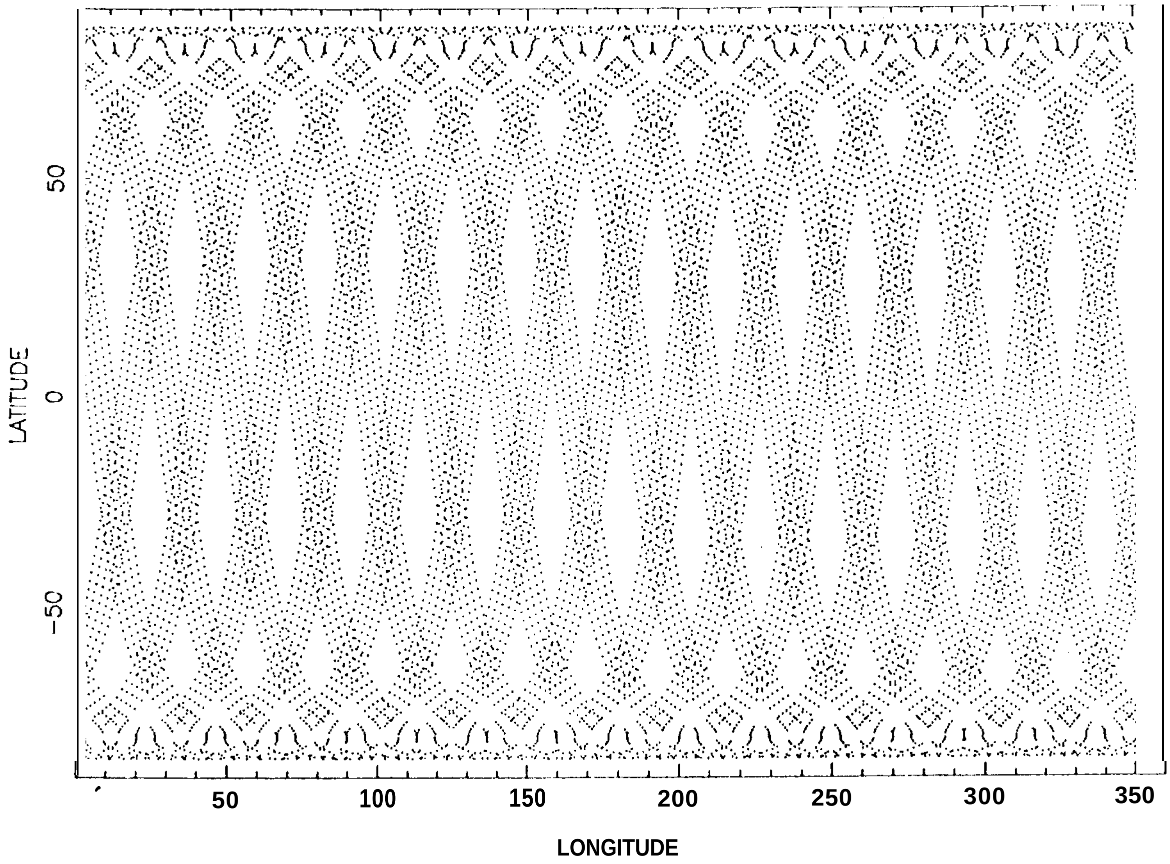

Fig. 7) Data Sampling for Low-Low Satellite Analysis

LOW-LOW SATELLITE RESULTS

Fig. 8 compares the root-mean-square (rms) of the sigmas of the normalized gravity coefficients over order for several gravity representations vs. increasing degree. Both GEMT2 and GEMT3 (Goddard Earth Models) are indicated in this figure. Also indicated are the results of the above satellite-to-satellite covariance analysis. The analysis shows an anticipated 2 orders of magnitude improvement (at least upto degree 40) over GEMT3 in all but the very low degree terms, where the improvement is only 1 order of magnitude, These results should be considered optimistic and represent a best case scenario. Fig. 9 represents the geoid height error manifested by this 40x40 degree and order gravity field. The poorly determined gaps in the geoid height error are the result of not completely sampling the Earth's geopotential as indicated in Fig. 7. Note that these sub-centimeter geoid height errors include error contribution.. from only the first 40 degree terms in the gravity field, Ongoing analysis will include terms up to degree and order 50.

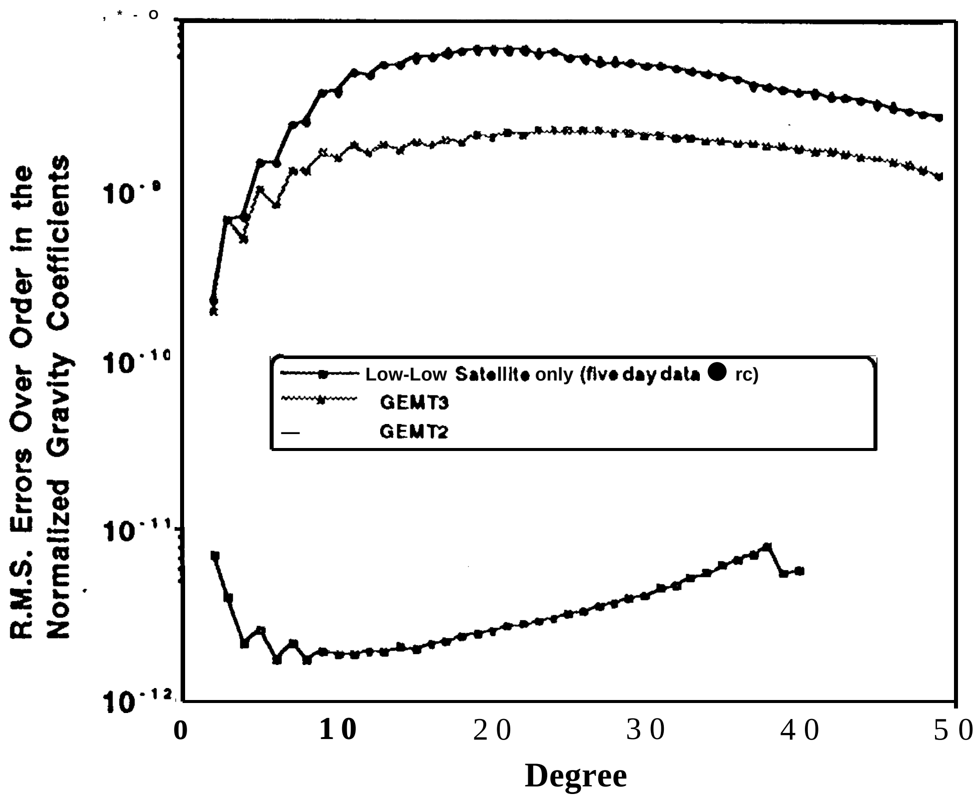

Fig 8.) RMS of Sigmas Over Order vs. Degree for Low-Low Satellite Analysis

CONCLUSION

Using a realistic scenario for processing 5 days of GPS data, a covariance analysis is performed to obtain the expected improvement in the gravity field for STEP. The results indicate that a 5 day data arc can improve the GEMT3 low to mid (2-23) degree and order harmonicas, Moreover, the geoid height error reduces from about 40 cm to 16 cm over the oceans; and from 60-100 cm to 30-40 cm over the land masses. Preliminary results for a low-low satellite mission with two active low orbiting satellites, and a spacecraft to spacecraft separation of 1000 700 km, suggest 2 orders of magnitude improvement (at least upto degree 40) over GEMT3 in all but the very low degree terms, where the improvement is only 1 order of magnitude.

Two Active Spacecraft (5 Day Data **Arc) Geoid** Height Error

40X 40 Field

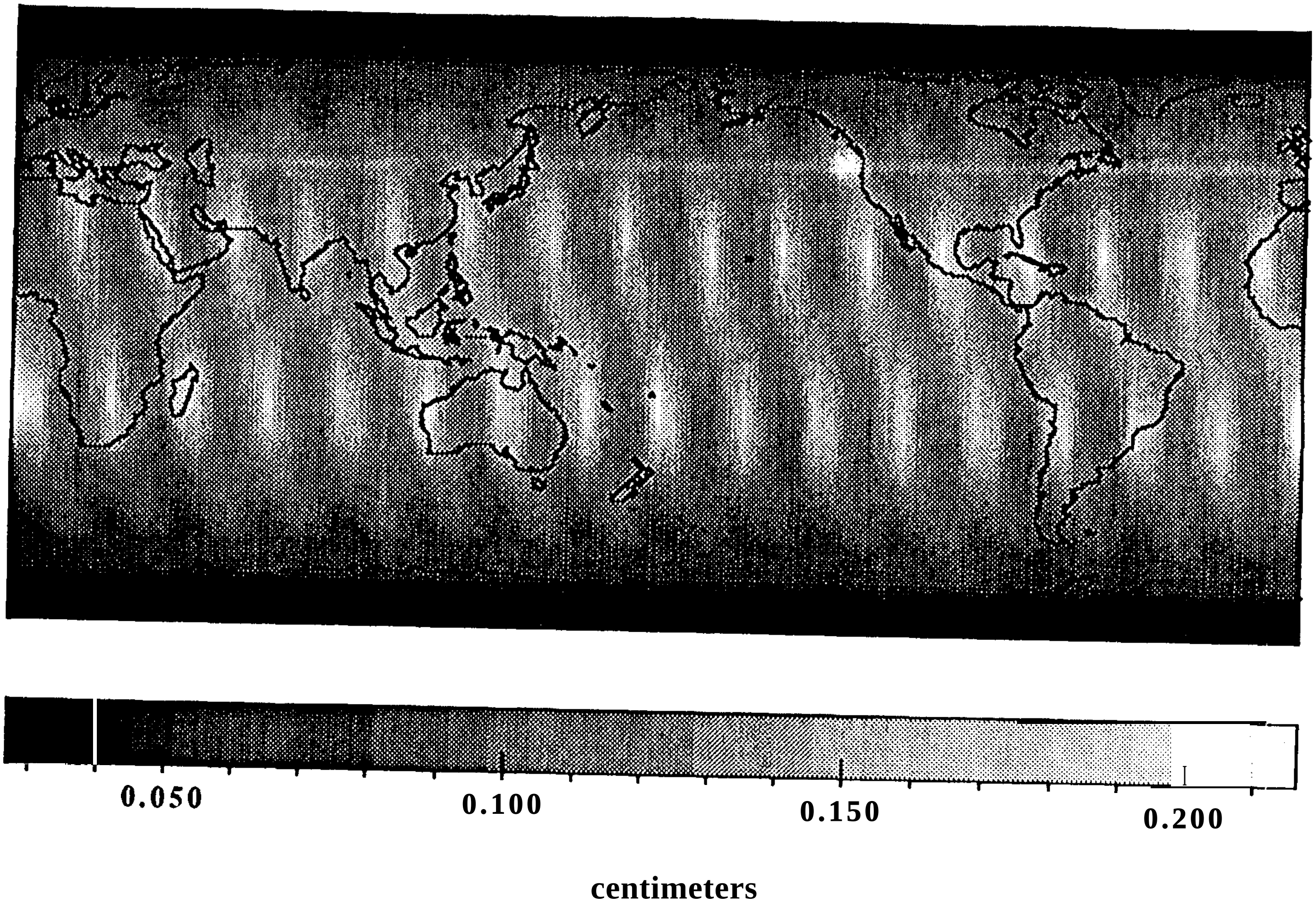

Fig. 9) **Low-Low** Satellite **Geoid Height Errors**

ACKNOWLEDGEMENT

The work described in this paper was performed by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautic and Space Administration.

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