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Angular momentum fluctuations within the Earth's liquid core and torsional oscillations of the core–mantle system

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SUMMARY

Non-steady differential rotation is one of the main characteristics of the buoyancy-driven flow within the Earth's liquid metallic outer core that generates the main geomagnetic field by magnetohydrodynamic (MHD) dynamo action. Concomitant fluctuations in angular momentum transfer both within the core and between the core and the overlying 'solid' mantle are investigated by assuming that average departures from 'isorotation' on coaxial cylindrical surfaces are negligibly small. The interval of time covered by the analysis is the 15 decades from 1840 to 1990, for which requisite determinations are available of decadal fluctuations of the length of the day (LOD) and also of flow velocities just below the core–mantle boundary (CMB), as inferred from geomagnetic secular variation (GSV) data. Core angular momentum (CAM) fluctuations are most pronounced in the mid-latitudes, where they are generally out of phase with those occurring in equatorial regions. They are roughly in phase with decadal LOD fluctuations, especially after about 1870, with a dominant variability period of approximately 65 years, in keeping with previous analyses based on GSV and/or LOD data. The largest positive correlations (0.8 when data before 1867.5 are excluded) are found in the mid-latitudes, with a maximum at zero lag and with secondary peaks at 67 years and at −64 years, again implying a mode of approximately 65 years. Propagation of CAM anomalies from the equatorial to polar regions is evident in both the time-latitude dependence of CAM and its latitudinal correlation with length of day fluctuations. Future work on excitation mechanisms should establish the connection, if any, between the dominant timescale of approximately 65 years seen in the data and the gravest of the theoretically possible subseismic modes of torsional MHD oscillations of the core.

Key words: Earth's fluid core, Earth rotation fluctuations, geodynamo, magnetohydrodynamics.

INTRODUCTION

Fluctuating motions including strong differential rotation within the Earth's liquid metallic outer core produce the main geomagnetic field by self-exciting MHD dynamo action. Driven by buoyancy forces due to the action of gravity on density inhomogeneities associated with differential heating and cooling, core motions are strongly influenced not only by Coriolis forces due to the Earth's rotation and the geometry of the bounding surfaces, but also by Lorentz forces due to the presence of electric currents and magnetic fields within the core. The main aim of the present study is to shed further light on the dynamical processes within the Earth's deep interior that give rise to decadal fluctuations in the rate of rotation of the solid Earth, and involve angular momentum transfer not only between the core and the overlying mantle but also between different parts of the core.

In realistic dynamical studies of any rotating fluid system, considerations of differential rotation and fluctuations in angular momentum within the system and of the exchange of angular momentum between the fluid and the regions with which it is in contact are of fundamental importance, as exemplified by investigations of planetary-scale motions in atmospheres and oceans (see, for example, Hide & Dickey 1991; Dickey *et al.* 1992; Rosen 1993; Eubanks 1993; Ponte *et al.* 1994). Strong indirect evidence of angular momentum exchange between the core and the overlying mantle stems from general quantitative considerations made nearly 50 years ago during the first realistic attempts to interpret LOD fluctuations on decadal timescales, for the LOD is inversely proportional to the angular momentum

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of the solid Earth (M_S) (see, for example, Munk & MacDonald 1960; Lambeck 1980; Jault *et al.* 1988; Jault & Le Mouël 1993).

It is because of the large moment of inertia of the core, more than 10^5 times that of the atmosphere, that fluctuating relative motions of approximately 3×10^{-4} m s^{-1} in the core are able to produce effects on the rotation of the solid Earth that are somewhat greater in magnitude than those caused by atmospheric winds of approximately 10 m s^{-1}. The amplitude of seasonal atmospheric variations, for example, is about one millisecond (ms), while decadal fluctuations can be as large as 5 ms. Work on the interpretation of fluctuations in the Earth's rotation on subdecadal timescales in terms of dynamical processes in the atmosphere (and oceans) is now quite advanced, owing, in large part, to the abundant meteorological data, which are well-sampled in time and fairly well-sampled in space. The characteristic timescales of the relevant atmospheric phenomena studied are generally much shorter than the data span, and data analyses are correspondingly robust. It is therefore possible to investigate angular momentum transfer between different parts of the atmosphere in some detail, thereby elucidating processes of central importance in theories of the general circulation of the atmosphere and its interaction with the underlying planet. So far as the core is concerned, however, it is unlikely that detailed magnetic observations from much earlier times can be obtained from existing records, so good use has to be made of the limited data sets now available. The 150 years of geomagnetic data available for the present study provides no more than a glimpse of what is happening in the core.

The findings of this paper are in keeping with related studies based on GSV and/or LOD data (see Vestine & Kahle 1968; Braginsky 1970, 1984; Currie 1973; Jault *et al.* 1988; Jackson *et al.* 1993), and they are also consistent with more recent related but not identical studies of torsional oscillations by Le Huy (1995) (see also Jault *et al.* 1996) and by Zatman & Bloxham (1997). The present study was completed several years ago (Hide *et al.* 1995), but the publication of this detailed report was delayed by circumstances beyond our control.

ANGULAR MOMENTUM BUDGET

Denote by M_S the axial component of the angular momentum of the solid Earth and by M that of the liquid core. On decadal timescales, the equation

$$dM_s/dt = -dM/dt \tag{1}$$

expresses angular momentum conservation to better than 10 per cent (the residual being largely associated with atmospheric and oceanic effects). M is given by the axial component of

$$\iiint \rho(\mathbf{r},\, t)\mathbf{r}\times[\boldsymbol{\Omega}\times\mathbf{r}+\mathbf{u}]d\tau\,, \tag{2}$$

where $\rho(\mathbf{r},\, t)$ is the mass density at a general point P in a frame of reference with its origin at the Earth's centre of mass and which rotates with the mantle with angular velocity $\boldsymbol{\Omega}$ relative to an inertial frame, $\mathbf{u}$ is the Eulerian relative flow velocity, and $d\tau$ is an element of volume of the liquid core over which the volume integral is taken.

Thus, to conserve the angular momentum of the whole system, any fluctuations in the total angular momentum of the liquid outer core must be accompanied by fluctuations in the angular momentum not only of the overlying solid mantle but also of the underlying solid inner core which, being a good electrical conductor, should be tightly coupled by Lorentz forces to the liquid core. However, in comparison with the liquid core, the volume of the solid inner core is small, just a few per cent of the total volume of the core (see Fig. 1); the moment of inertia of the inner core is even smaller in comparison, very much less than 1 per cent of that of the outer core. Given the level of accuracy of angular momentum budget analyses, any contributions to dM_S/dt associated with possible fluctuations in the motion of the inner core can be neglected in the calculations presented here.

The LOD data employed here comprise a self-consistent time-series resulting from the analyses of lunar occultations prior to 1955.5, after which a combination of astronomical and modern geodetic techniques are used (Jordi *et al.* 1994). There is evidence based on solar eclipses and other data (Stephenson & Morrison 1995) that over the past 2700 years the LOD has increased at an average rate of 1.70 ± 0.05 ms cy^{-1}. This can be attributed to two main agencies, namely tidal braking of the Earth's spin (2.3 ± 0.1 ms cy^{-1}) and changes in the Earth's polar moment of inertia associated with 'postglacial rebound' (-0.6 ± 0.1 ms cy^{-1}). This trend of 1.7 ms cy^{-1} was removed from the LOD series before comparing it with core angular momentum $M(t)$; results were shown to be insensitive to this trend.

CORE MOTIONS AND MAGNETIC FIELDS

Within the core, the geomagnetic field possesses a toroidal part, $\mathbf{B}_T$ (say), which, unlike the poloidal part, $\mathbf{B}$ (say), has lines of force that are confined to the core (Elsasser 1947), and may be so much stronger on average than $\mathbf{B}$ that it accounts for most

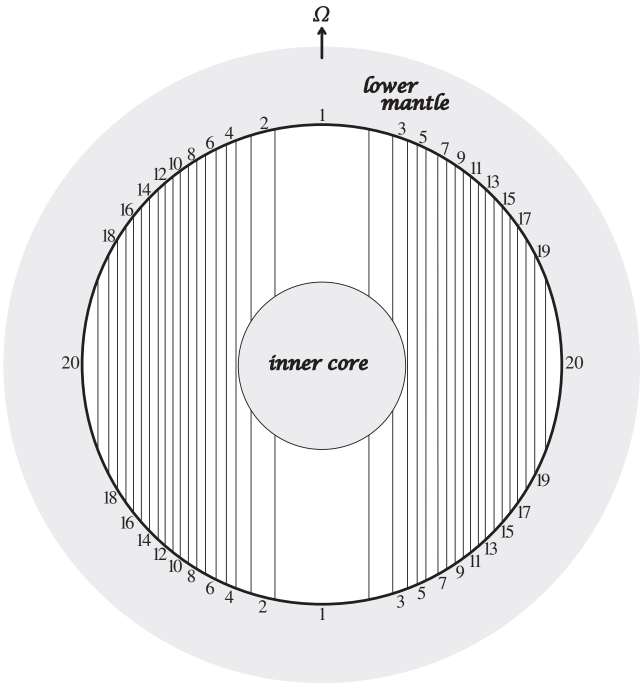

Figure 1. Illustrating the division of the Earth's liquid outer core into $Q=20$ annular regions of equal volume numbered $q=0, 1, 2, 3\ldots20$ and bounded on the equator-ward side by the colatitude θ_q in the northern hemisphere and $\pi-\theta_q$ in the southern hemisphere, where values of θ_q are shown in Table 1.

of the magnetic energy of the Earth. Denote by $\mathbf{B}(r, \theta, \phi, t)$ the main (poloidal) geomagnetic field at a general point r with spherical polar coordinates (r, θ, ϕ) at time t, and by $\dot{\mathbf{B}}$ the GSV, $\partial\mathbf{B}/\partial t$.

Determinations of $\mathbf{B}$ made at and near the Earth's surface at various epochs have been used by various groups to infer $\mathbf{u}_s$, the Eulerian flow velocity just below the CMB (see Bloxham & Jackson 1991 for a review). The first of the three geophysically reasonable key assumptions that underlie the method used is that the electrical conductivity of the mantle and magnetic permeability gradients there are negligibly small, so that $\mathbf{B}$ satisfies $\nabla\times\mathbf{B}=0$ as well as $\nabla\cdot\mathbf{B}=0$ and can therefore be expressed as the gradient of a potential U satisfying Laplace's equation $\nabla^2 U=0$. This facilitates the downward extrapolation of the observed field to obtain $\mathbf{B}$ and $\dot{\mathbf{B}}$ at the CMB.

The second assumption is that the electrical conductivity of the core is so high that, when dealing with fluctuations in $\mathbf{B}$ on GSV timescales that are very much less than that of the ohmic decay of magnetic fields in the core (which is several thousand years for global-scale features), $\mathbf{B}$ satisfies Alfvén's 'frozen flux' theorem expressed by the equation

$$\partial\mathbf{B}/\partial t = \nabla\times(\mathbf{u}\times B)\,. \tag{3}$$

To this approximation, the lines of magnetic force emerging from the core are advected by the horizontal flow $\mathbf{u}_s=(v_s, w_s)$ just below the CMB, and B_r, the radial component of $\mathbf{B}$ at the CMB, satisfies

$$\frac{\partial B_r}{\partial t}+\frac{v_s}{c}\frac{\partial B_r}{\partial\theta}+\frac{w_s}{c\sin\theta}\frac{\partial B_r}{\partial\phi}=B_r\left[\frac{\partial u}{\partial r}\right]_{r=c} \tag{4}$$

(Roberts & Scott 1965; Backus 1968).

A third assumption is needed to secure uniqueness, and one physically plausible suggestion is that, to a first approximation, the flow in the upper reaches of the core is in geostrophic balance with the pressure field there (Le Mouël 1984; Gire & Le Mouël 1990); this implies that

$$\frac{\partial}{\partial\theta}(v_s\sin\theta\cos\theta)+\cos\theta\frac{\partial w_s}{\partial\phi}=0 \tag{5}$$

in the case of an incompressible fluid, for which $\nabla\cdot\mathbf{u}=0$.

The hypothetical flow fields $\mathbf{u}_s$ thus determined are all similar in their general appearance but they exhibit some discrepancies. Most determinations of $\mathbf{u}_s$ make use of spherical harmonic expansions of the variables involved, as explained in detail in a review by Bloxham & Jackson 1991). Significant reductions can be expected in these discrepancies in the near future from improved GSV determinations based on recently acquired additional data from historical records (Jackson *et al.* 2000).

DETERMINATIONS OF CORE ANGULAR MOMENTUM

Just as it is convenient to divide the atmosphere into the troposphere, stratosphere and higher regions, and the oceans into the thermocline and lower regions, the liquid metallic core can be divided into the 'torosphere', where $\mathbf{B}_{\mathrm{T}}$ is so strong that Lorentz forces are comparable in magnitude with Coriolis forces (Hide 1995), and the overlying 'polosphere', where $\mathbf{B}_{\mathrm{T}}$ is typically no stronger than $\mathbf{B}$ and the associated Lorentz forces are correspondingly much weaker than Coriolis forces (Le Mouël 1984).

Owing to the presence of the solid inner core, Coriolis forces inhibit flow across the imaginary cylindrical surface that is tangential to the inner core at the equator and intersects the outer core at latitudes of 69.5° (see Fig. 1). It is therefore convenient to subdivide the liquid core further into 'polar' regions lying within the tangent cylinder and 'extrapolar' regions lying outside the cylinder. This scheme proves useful not only in work on the dynamics of the Earth's core (Hide 1966; Aurnou *et al.* 1996), but also in studies of other geophysical and astrophysical fluids, such as the various fluid layers of Jupiter and Saturn and the convective outer layers of the Sun. Some justification for the scheme is provided by the laboratory experiments on thermal convection in an electrically insulating rotating fluid upon which the scheme was originally based (Hide 1953, 2000), and further justification comes from the flow fields produced in numerical models of buoyancy-driven MHD flows in the Earth's core (Glatzmaier & Roberts 1995; Kuang & Bloxham 1997; Busse 2000; Jones 2000).

We suppose here that the Earth's liquid metallic outer core is bounded by concentric spherical surfaces of radii $c=3485$ km and $b=1222$ km (see Fig. 1). The volume and moment of inertia of the solid inner core are, respectively, much less than 10^{-1} and 10^{-2} times the volume and moment of inertia of the liquid outer core. An imaginary cylinder that is tangent to the inner sphere at the equator intersects the outer sphere at colatitude $\theta=\theta^*$ in the northern hemisphere and $\pi-\theta^*$ in the southern hemisphere, where

$$\theta^* = \sin^{-1}(b/c)\,. \tag{6}$$

The colatitude angle θ^* is about 20.5° for the Earth.

It is convenient to imagine the liquid core divided into an 'extrapolar' liquid region E where $\theta^*<\theta<\pi-\theta^*$, and two 'polar' liquid regions P where $0<\theta<\theta^*$ in the northern hemisphere (NH) and $\pi-\theta^*<\theta<\pi$ in the southern hemisphere (SH) (Hide 1966). Consider a cylindrical shell in region E with bounding surfaces that intersect the outer spherical surface at colatitudes θ_{q-1} and θ_q in the NH (and $\pi-\theta_{q-1}$ and $\pi-\theta_q$ in the SH), where $\theta^*\leq\theta_{q-1}<\theta_q\leq\pi/2$ (see Fig. 1). The volume V_q of this cylindrical shell (in region E) is given by

$$V_q=\frac{4}{3}\pi c^3[\cos^3\theta_{q-1}-\cos^3\theta_q]\,, \tag{7}$$

which follows from the so-called 'apple core theorem', that the volume of a spherical apple that remains when an axisymmetric cylindrical core has been removed depends only on the length of the cut (Hide & James 1983). The corresponding combined volume of the two identical cylindrical shells in polar regions P (where $0<\theta<\theta^*$ and $\pi-\theta^*<\theta<\pi$) is given by

$$V_q=\frac{4}{3}\pi c^3[\cos^3\theta_{q-1}-\cos^3\theta_q]-\frac{4}{3}\pi b^3[\cos^3\bar{\theta}_{q-1}-\cos^3\bar{\theta}_q]\,. \tag{8}$$

Here $\bar{\theta}_{q-1}$ and $\bar{\theta}_q$ are the colatitudes at which the inner and outer surfaces of the cylindrical shell intersect the surface of the inner sphere of radius b, so that

$$b\sin\bar{\theta}_q=c\sin\theta_q\,,\qquad b\sin\bar{\theta}_{q-1}=c\sin\theta_{q-1}\,. \tag{9}$$

Eqs (6) to (9) are expressions needed for the purpose of dividing up the liquid core into Q (say) imaginary coaxial cylindrical shells of equal volume. In the case of a full sphere ($b=0$) (when the P regions shrink to zero volume), the volume of each cylindrical shell is equal to $4\pi c^3/3Q$, so that (by eq. 7) we have

$$\theta_q = \cos^{-1}(1 - q/Q)^{1/3}, \tag{10}$$

where $q=1, 2\ldots, Q$, and $\theta_Q = \pi/2$. The innermost shell, which has zero inner radius, extends from $\theta=0$ to $\theta=\cos^{-1}(1-Q^{-1})^{1/3}$ The outermost shell extends from $\theta=\theta_{Q-1}=\cos^{-1}Q^{-1/3}$ to $\theta=\theta_Q=\pi/2$. In the case when $b\neq 0$, it is necessary to use more complicated expressions based on eqs (8) and (9) when calculating the latitudinal extent of the qth cylindrical shell. Values of θ_q, $1\leq q\leq Q$, in degrees, are given in Table 1.

The choice of equivolume annuli is arbitrary but convenient, and so is $Q=20$ for the total number of annuli (see Fig. 1 and Table 1). With core velocity fields based on geomagnetic fields expressed as spherical harmonic series up to degree 14, values of Q closer to 10 than 20 would suffice, but taking $Q=20$ facilitates calculations without affecting the main findings of the investigation, which largely refer to fluctuations in groups of annuli.

The total axial angular momentum $\mu_q(t; Q)$ associated with relative core motions with Eulerian flow velocity $\mathbf{u}(\mathbf{r}, t)=(u, v, w)$ at a general point P with spherical polar coordinates (r, θ, ϕ) is given by

$$\mu_q(t; Q) \equiv \frac{4\pi}{3Q}(c^3 - b^3)\{\rho w r \sin\theta\}_q, \tag{11}$$

$4\pi(c^3-b^3)/3Q$ being the total volume of each annulus. Here $\rho=\rho(r, \theta, \phi, t)$ is the density at P, $w=w(r, \theta, \phi, t)$, and the symbol $\{\}_q$ denotes the spatial average over the volume occupied by the qth annulus (see Fig. 1). One of the main objectives of the present study is to determine temporal fluctuations in $\mu_q(t; Q)$ for all 20 values of q using available data and to examine the fluctuations for evidence of torsional oscillations.

Owing to the inaccessibility of the core, direct determinations of $\mathbf{u}(\mathbf{r}, t)$ and $\rho(\mathbf{r}, t)$ are impossible, but, as discussed above, methods have now been developed for making indirect estimates of the (θ, ϕ) components (v_s, w_s) of $\mathbf{u}_s$, the Eulerian flow velocity just below the CMB from GSV data. The data used in the present study cover the interval from 1840 to 1990 and were generously made available by Dr Andrew Jackson. By introducing the additional assumption that variations in ρw in the direction parallel to the rotation axis are negligibly small (see, for example, Hide 1966; Jault *et al.* 1988; Jault & Le Mouël 1991; Jackson *et al.* 1993), ρw can be replaced in eq. (11) by the known quantity

$$\frac{1}{2}\rho_s[w_s(\theta, \phi, t) + w_s(\pi - \theta, \phi, t)], \tag{12}$$

Table 1. Values of θ_q, in degrees, where $q=0, 1, 2, 3\ldots, n$ $(=20)$, based on eqs (8) to (10).

q	θ_q	q	θ_q	q	θ_q	q	θ_q
1	12.56	6	28.94	11	40.95	16	54.80
2	17.52	7	31.38	12	43.43	17	58.42
3	20.99	8	33.77	13	46.01	18	62.78
4	23.80	9	36.15	14	48.72	19	68.71
5	26.43	10	38.53	15	51.62	20	90.00

where ρ_s is the average density of the core just below the CMB. This gives for $\mu_q(t; Q)$ the approximate relationship

$$\mu_q(t; Q) = \frac{2\pi\rho_s c(c^3 - b^3)}{3Q}\langle[w_s(\theta, \phi, t) + w_s(\pi - \theta, \phi, t)]\sin\theta\rangle_q, \tag{13}$$

where $\langle\rangle_q$ signifies the spatial average over the surface area of the qth annulus, covering the ranges $0\leq\phi\leq 2\pi$ and $\theta_{q-1}\leq\theta\leq\theta_q$ (see eq. 7).

The assumption that average departures from isorotation on cylindrical surfaces are negligible is difficult to justify on straightforward theoretical grounds, owing largely to departures from the Proudman–Taylor theorem associated with baroclinicity due to horizontal density gradients and Lorentz forces (Hide 1956). However, the assumption enables progress to be made and it has already been used with notable success in important recent studies by Jault & Le Mouël *et al.* (1991) and Jackson *et al.* (1993) of the angular momentum budget of the core–mantle system. Combining eqs (10) and (2) gives

$$M(t) = \sum_{q=1}^{Q} \mu_q(t; Q) \tag{14}$$

as a rough measure of the total axial angular momentum associated with relative motions in the core (*cf.* eq. 1 and Jault & Le Mouël 1991).

CORE ANGULAR MOMENTUM FLUCTUATIONS

The total core angular momentum $M(t)$ as well as the contributions from the individual cylinders $\mu_q(t; 20)$ are displayed in Fig. 2(a). Here we introduce the 'equivalent millisecond unit' (emsu), defined as that amount of axial angular momentum, namely 0.60×10^{26} kg m^2 s^{-1}, which, if transferred to the overlying solid earth would, if the solid earth were perfectly rigid, reduce the length of the day (LOD) by 1 ms. Two broad maxima occur in $M(t)$, with the highest value attained around 1900 with a 'full-width half-max' (FWHM) of approximately 25 years (Fig. 2a). The second smaller maximum has its peak near 1970 with a FWHM of 15 years. There is a considerable range of variability of the individual bands (Fig. 2b), with the time-averaged total relative core angular momentum (CAM) $\bar{M}(t)$ being negative (-0.237 emsu). The equatorial band ($q=20$) with the largest lever arm produces the largest contribution, with time-averaged values being largely negative [i.e. $\mu_q(t; 20)\approx -0.4$ emsu]. The two large peaks are clearly seen in the equatorial annuli (Fig. 2a), with the first peak near 1885 corresponding to the plateau region near 1885 in the total CAM, and the second peak occurring near the 1975 maximum in total CAM. The other bands are also highly bimodal, with cylinders 16–19 having their maxima near 1885 and 1950, and cylinders 3–15 having maxima near 1910 and 1970. The contribution from bands 1–3 with a short lever arm are small, with the largest contribution near 1910.

The 3-D diagram (Fig. 3) of the contributions from the individual cylinders (M_i) given as a function of time permits a unique insight into core dynamics (*cf.* Fig. 1 of Zatman & Bloxham 1997). The dominant feature is a strong, approximately 65 yr, oscillation that is particularly evident in the mid-latitude

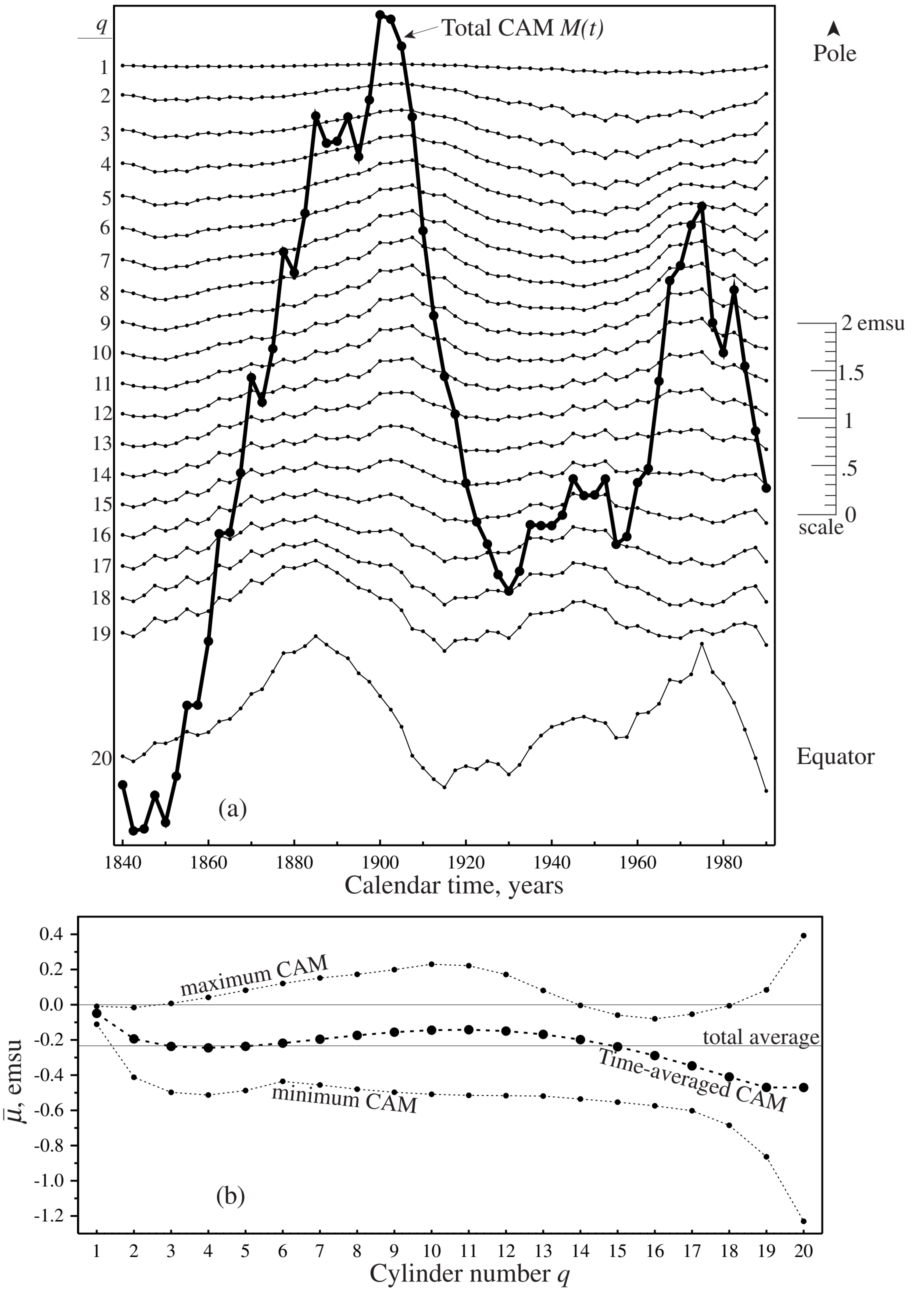

Figure 2. (a) Temporal fluctuations in $\mu_q(t;\ Q)$, $q=0, 1, 2, 3 \ldots Q=20$, the angular momentum of each annulus, from 1840 to 1990 (thin lines) and of $M(t)$, the total angular momentum (thick line) (see eqs 13 and 14 and part b). (b) The dependence on q of the temporal mean value of $\mu_q(t;\ Q)$, the core angular momentum, from 1840 to 1980 (thick dashed line) and of the range of temporal fluctuations (thin dotted lines). For each value of q, the fluctuation shown in (a) is about the corresponding mean value given in (b).

bands (3–15). Note that the maxima in the mid-latitudes coincide in time with the largest geomagnetic 'jerk' events over the time period considered, namely 1912 and 1969 (Malin & Hodder 1982). Variability in cylinders 16–20 generally precedes that in the mid-latitudes, with results suggestive of angular momentum propagation from the equatorial towards mid-latitude regions. On the basis of Figs 1 to 3 we divide the core into three regions: polar (P), mid-latitude (ML) and equatorial (EQ). The polar region (where $q=1, 2, 3$) is a natural division given the dimension of the solid inner core (Fig. 1), whereas the division into mid-latitude ($q=4$–15) and equatorial ($q=16$–20) is motivated by the characteristic behaviour of these two regions.

The comparison of LOD fluctuations with total and regional $M(t)$ (Fig. 4) shows that the decadal LOD variability is well matched with $M(t)$, especially after 1870. Data prior to 1870 are not as robust as the more modern data, with a mismatch occurring during the series between 1840 and 1870. The maximum correlation between M and LOD is 0.58 at a lag of approximately 17.5 yr when the full series (1840–1990) is considered, and is 0.64 with a lag of approximately 5 yr with the shorter series (1870–1990). The $M(t)$ data spacing is 2.5 yr; hence, care must be taken in the interpretation of the lag of the latter correlation. The contribution from mid-latitudes (M_{ML}) (Figs 3 and 4) dominates $\dot{M}$ and accounts for a major portion

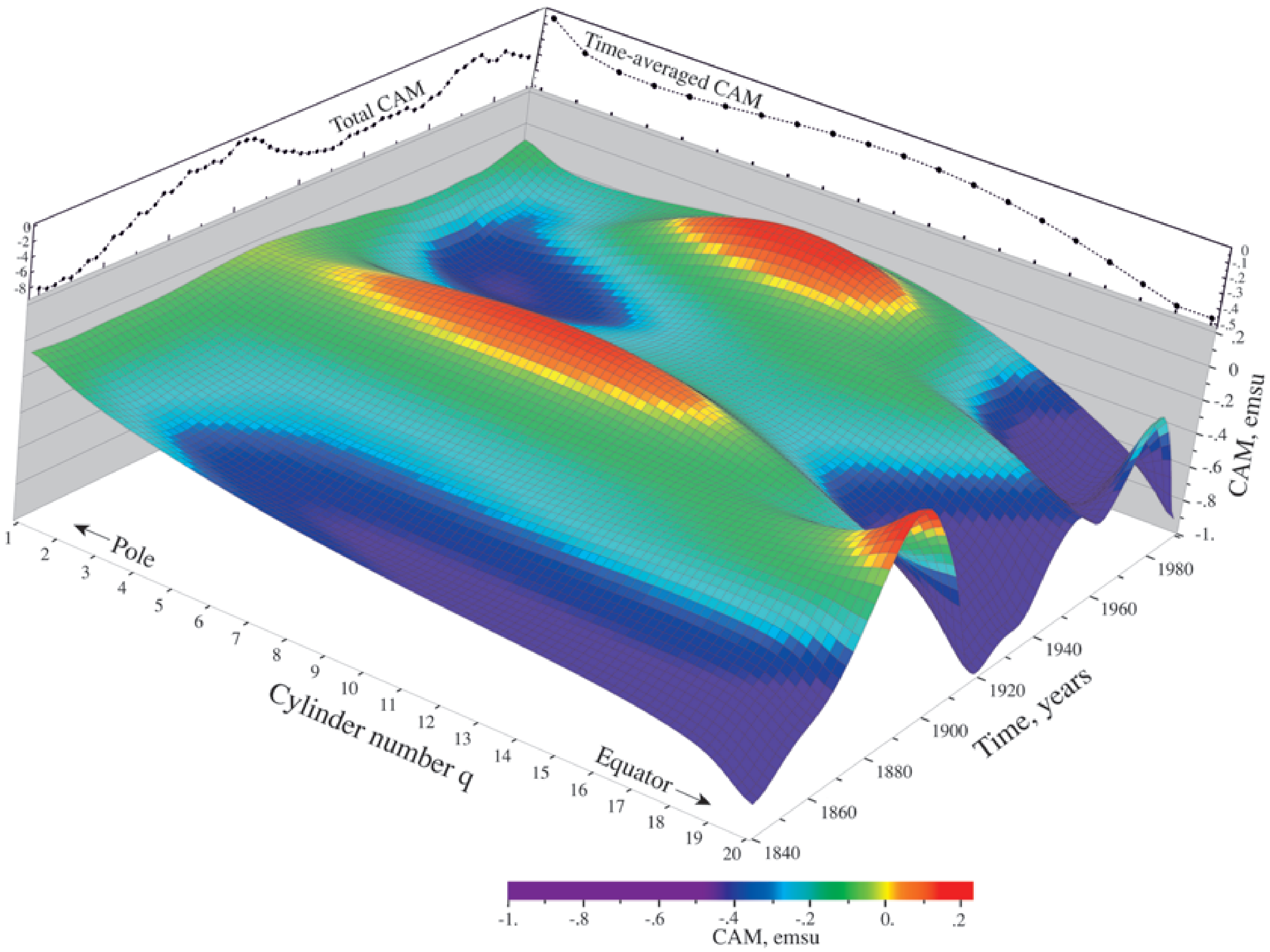

Figure 3. 3-D representation of the core angular momentum of 20 coaxial equivolume cylindrical shells as a function of cylinder number and time (Hovmöller diagram). Note the dominant approximately 65 yr fluctuation and its dependence on latitude.

of the LOD decadal variability (56.9 per cent for the full series and 74.9 per cent for the short series). M_{ML} is in phase with LOD, having a maximum correlation of 0.8 for the shorter series and 0.5 for the full series (Fig. 5); secondary maxima occur at 67 and -64 yr, consistent with the approximately 65 yr periodicity.

The equatorial contribution to the CAM time-series (M_{EQ}; cylinders 16–20) is bimodal, with maxima in 1885 and 1950, and leads the M_{ML} by approximately 20 yr (Fig. 4). The correlation of M_{EQ} with LOD has a principal maximum at a lead of approximately 25 yr, with secondary maxima near 44 and 90 yr, again consistent with a 65 yr periodicity. It is the superposition of these two groups of cylinders (*ML* and *EQ*) that gives rise to the broad LOD maximum near 1900.

The correlation of individual cylinders with LOD (Fig. 6) indicates that angular momentum anomalies propagate from the equatorial to the polar cylinders. The total CAM (shown in red) results from the summation of the individual cylinders, with a maximum at a 15 yr lead with respect to LOD and secondaries at an 80 yr lead and at a 60 yr lag (note the $\sim$65 yr periodicity). The colour diagram vividly displays the approximately 65 yr period, with four maxima visible (two strong, two weak) and four minima. The strong propagation pattern indicates that a period of approximately 60 yr is required for a signal to be transmitted from the equatorial region to the polar cylinder. Similarly, a propagation pattern is evident in the top line plots as the peaks and valleys are traced from one cylinder to another.

The torques responsible for angular momentum transfer between the Earth's atmosphere and the underlying planet are due to tractions produced by turbulent viscosity in the oceanic and continental boundary layers and also to topographic tractions due to normal pressure forces acting on orography. Topographic torques produced by atmospheric motions are typically comparable in magnitude with boundary layer torques, but they have somewhat different temporal characteristics (see, for example, Ponte *et al.* 1994). Less is known about the torques that couple the core to the mantle, but electrostatic and magnetostatic torques are unlikely to be significant (Hide 1998a). It is generally considered that torques associated with viscous stresses at the CMB are also much less important than those associated with Lorentz forces due to electric currents flowing in the weakly conducting lower reaches of the mantle (see Rochester 1960; Holme 1998), topographic torques associated with a bumpy core–mantle boundary (Hide 1969a, 1995, 1998b; Jault & Le Mouël *et al.* 1999), and gravitational effects (Jault &

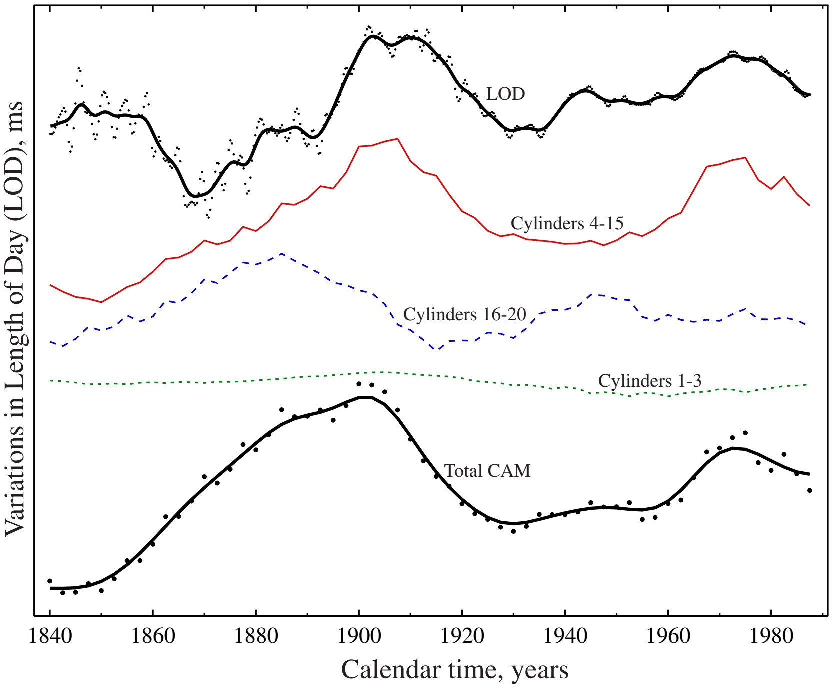

Figure 4. Comparison of LOD and CAM. A slope of 1.7 ms century^{-1} has been removed from the LOD data to allow for the effects of postglacial rebound and the secular acceleration of the moon and a 10 yr smoothed series are shown from each of the unsmoothed series. The total CAM (*M*) is shown as well as three regions (cylinders 1–3, 4–15, 16–20). Note the large correlation between LOD fluctuations and CAM fluctuations in mid-latitude regions (belts 4–15).

Le Mouël *et al.* 1989; Buffett 1996). Uncertainties about the electrical conductivity of the lower mantle and about the shape of the CMB and horizontal density variations in the mantle and core make it difficult at present to establish the relative importance of those agencies. The topographic torque is likely to be of sufficient magnitude to explain the observed decadal LOD variations, and dominant contributions to this torque arise in mid-latitudes (Hide *et al.* 1993). This may bear on our

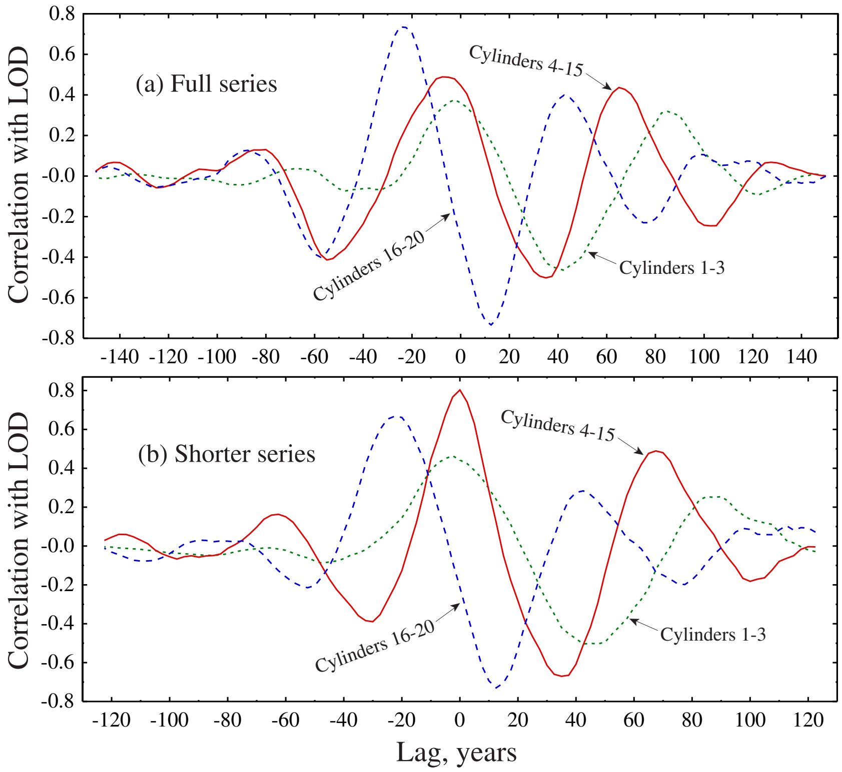

Figure 5. The correlation of grouped cylinders (1–3, 4–15, 16–20) of CAM with LOD as a function of lag of the CAM with respect to the LOD. The top diagram (a) is based on the full series (1840–1990), and the bottom diagram (b) on the shorter series (1867.5–1990), where the correlation is higher.

finding here that mid-latitude CAM fluctuations are in phase with LOD fluctuations, and therefore in antiphase with fluctuations in the angular momentum of the mantle.

CONCLUDING REMARKS

A rotating electrically conducting fluid can in principle support a wide range of transverse ('shear') oscillations at 'subacoustic' frequencies (see Lehnert 1954; Hide 1969b; Acheson & Hide 1973), with periods in the case of the Earth's core ranging from days to centuries. Generated by internal instabilities and/or external forcing, the oscillations are modified by background flows and non-linear interactions of various kinds.

One particularly important class of slow *non-axisymmetric* oscillations, characterized by near 'magnetostrophic balance' between Coriolis and Lorentz restoring forces associated largely with $\mathbf{B}_T$, is probably manifested in the main features of the GSV on timescales of centuries (Braginsky 1964, 1967; Hide 1966; see also Malkus 1967; Hide & Stewartson 1972). The restoring forces associated with any *axisymmetric* torsional oscillations about the rotation axis would be provided solely by Lorentz forces associated with azimuthal displacements (Braginsky 1970, 1984; Zatman & Bloxham 1997), giving much shorter oscillation periods, namely decades rather than centuries. A period of 65 yr corresponds to about 2×10^{-4} T (2 gauss) for the effective value of the average strength of the component of $\mathbf{B}$ perpendicular to the axis of rotation. This is no more than about 0.02 times the likely average strength of the toroidal magnetic field, $\mathbf{B}_T$ within the core and about 0.5 times the average strength of $\mathbf{B}$ in the lower mantle.

Such an eigenmode of MHD torsional oscillation will be readily excited by the fluctuating background of 3-D flow in the core if, in the power spectrum of the fluctuation, there is sufficient energy to overcome attenuation due (in this case) largely to ohmic dissipation associated with electric currents induced in the weakly conducting lower mantle (Hide & Roberts 1962; Gubbins & Roberts 1987). It is also possible of course that the approximately 65+ yr period is not at all associated with MHD torsional oscillations in the core, but simply reflects the timescale of some dominant instability or

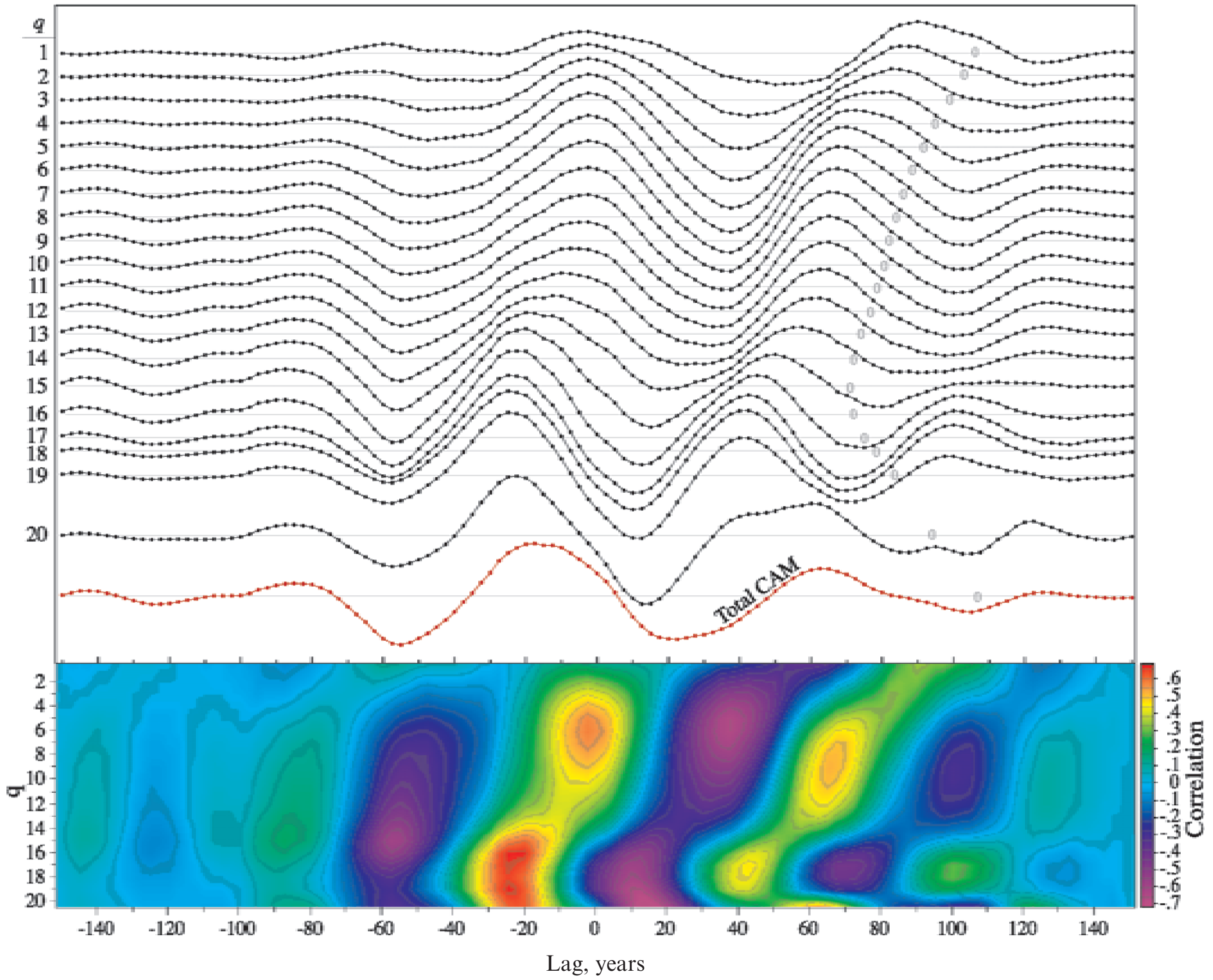

Figure 6. Correlation of LOD with the CAM as a function of the lag of the CAM with respect to LOD. The effect of the individual bands is shown in black and by the colour graphics; the total CAM effect is depicted by the red line.

non-linear mode interactions responsible for angular momentum advection within the core.

Beyond the scope of the present paper is any detailed discussion of excitation and attenuation mechanisms and the role of advection and other non-linear processes in the dynamics of torsional oscillations. The numerical models of core flow and the geodynamo now being developed by various groups (for references see Busse 2000; Jones 2000) could be used for such purposes in future research. Indeed, a stringent test of any such model would be its ability to simulate the geophysical phenomena revealed by this investigation of torsional oscillations of the Earth's core, based on observations of the geomagnetic secular variation and of fluctuations in the Earth's rotation.

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REFERENCES

Acheson, D.J. & Hide, R., 1973. Hydromagnetics of rotating fluids, *Rept Prog. Phys.,* **36,** 159–221.

Aurnou, J.M., Brito, D. & Olson, P.L., 1996. Mechanics of inner core super-rotation, *Geophys. Res. Lett.,* **23,** 3401–3404.

Backus, G.E., 1968. Kinematics of geomagnetic secular variation in a perfectly conducting core, *Phil. Trans. R. Soc. Lond., A,* **263,** 239–266.

Bloxham, J. & Jackson, A., 1991. Fluid flow near the surface of the Earth's outer core, *Rev. Geophys.,* **29,** 97–120.

Braginsky, S.I., 1964. Magnetohydrodynamics of the Earth's core, *Geomag. Aeron.,* **4,** 698–712.

Braginsky, S.I., 1967. Magnetic waves in the Earth's core, *Geomag. Aeron.,* **7,** 1050–1060 (English translation pp. 851–859).

Braginsky, S.I., 1970. Torsional magnetohydrodynamic vibrations of the Earth's core and variations in the length of the day, *Geomag. Aeron.,* **10,** 3–12 (English translation pp. 1–8).

Braginsky, S.I., 1984. Short period geomagnetic secular variation, *Geophys. astrophys. Fluid Dyn.,* **30,** 1–78.

Buffett, B.A., 1996. Mechanism for decade fluctuations in the length of day, *Geophys. Res. Lett.,* **23,** 3803–3806.

Busse, F.H., 2000. Homogeneous dynamos in planetary cores and in the laboratory, *Ann. Rev. Fluid Mech.,* **32,** 383–408.

Currie, R.G., 1973. Geomagnetic line spectra—20 to 70 years, *Astrophys. Space Sci.,* **21,** 425–438.

Dickey, J.O., Marcus, S.L. & Hide, R., 1992. Global propagation of interannual fluctuations in atmospheric angular momentum, *Nature,* **357,** 484–488.

Elsasser, W.M., 1947. Induction effects in terrestrial magnetism: Part III. Electric modes, *Phys. Rev.,* **72,** 821–833.

Eubanks, T.M., 1993. Variations in the orientation of the Earth, in *Contributions of Space Geodesy to Geodynamics,* p. 1, eds Smith, D.E. & Turcotte, D.L., Geodynamics Series, Vol. 24, Am. Geophys. Un., Washington, DC.

Gire, C. & Le Mouël, J.-L., 1990. Tangentially geostrophic flow at the core–mantle boundary compatible with observed geomagnetic secular variation: The large scale component of the flow, *Phys. Earth planet. Inter.,* **59,** 259–287.

Glatzmaier, G.A. & Roberts, P.H., 1995. A three-dimensional convective dynamo solution with rotating and finitely conducting inner core and outer mantle, *Phys. Earth planet. Inter.,* **91,** 63–75.

Gubbins, D. & Roberts, P.H., 1987. in *Geomagnetism,* Vol. 2, pp. 1–183, ed. Jacobs, J.A., Academic, London.

Hide, R., 1953. Some experiments on thermal convection in a rotating liquid, *PhD thesis,* University of Cambridge, Cambridge.

Hide, R., 1956. The hydrodynamics of the Earth's core, *Phys. Chem. Earth,* **1,** 94–137.

Hide, R., 1966. Free hydromagnetic oscillations of the Earth's core and the theory of the geomagnetic secular variation, *Phil. Trans. R. Soc. Lond., A,* **259,** 615–647.

Hide, R., 1969a. Interaction between the Earth's liquid core and solid mantle, *Nature,* **222,** 1055–1056.

Hide, R., 1969b. On hydromagnetic waves in a stratified rotating incompressible fluid, *J. Fluid Mech.,* **39,** 283–287.

Hide, R., 1995. The topographic torque on the rigid bounding surface of a rotating gravitating fluid and the excitation of decadal fluctuations in the Earth's rotation, *Geophys. Res. Lett.,* **22,** 961–964.

Hide, R., 1998a. Magnetostatic and electrostatic torques within a planet, *Geophys. J. Int.,* **135,** 1150–1152.

Hide, R., 1998b. A note on topographic core–mantle coupling, *Phys. Earth planet. Inter.,* **109,** 91–92.

Hide, R., 2000. Generic nonlinear processes in self-exciting dynamos and the long-term behaviour of the main geomagnetic field including polarity reversal superchrons, *Phil. Trans. R. Soc. Lond., A,* **358,** 943–955.

Hide, R. & Dickey, J.O., 1991. Earth's variable rotation, *Science,* **253,** 629–637.

Hide, R. & James, I.N., 1983. Differential rotation produced by large-scale potential vorticity mixing in a rapidly-rotating fluid, *Geophys. J. R. astr. Soc.,* **74,** 301–312.

Hide, R. & Roberts, P.H., 1962. Some elementary problems in magnetohydrodynamics, *Adv. appl. Mech.,* **4,** 215–316.

Hide, R. & Stewartson, K., 1972. Hydromagnetic oscillations of the Earth's core, *Rev. Geophys. Space Phys.,* **10,** 579–598.

Hide, R., Clayton, R.W., Hager, B.H., Spieth, M.A. & Voorhies, C.V., 1993. Topographic core–mantle coupling and fluctuations in the Earth's rotation, in *Relating Geophysical Structures and Processes: The Jeffreys Volume,* eds Aki, K. & Dmowska, R., Geophys. Monog. Am. geophys. Un., 76, 107–120.

Hide, R., Boggs, D.H. & Dickey, J.O., 1995. Torsional magneto-hydrodynamic oscillations of the Earth's core (Abstract), *EOS, Trans. Am. geophys. Un.,* **76,** F60.

Holme, R., 1998. Electromagnetic core–mantle coupling—1. Explaining decadal variations in the length of day, *Geophys. J. Int.,* **132,** 167–180.

Jackson, A., Bloxham, J. & Gubbins, D., 1993. Time-dependent flow at the core surface and conservation of angular momentum of the coupled core–mantle system, in *Dynamics of Earth's Deep Interior and Earth Rotation,* eds Le Mouël, J.-L., Smylie, D.E. & Herring, T.A., Geophys. Monog. Am. geophys. Un., 72, 97–107.

Jackson, A., Jonkers, A.R.T. & Walker, M.R., 2000. Four centuries of geomagnetic secular variation from historical records, *Phil. Trans. R. Soc. Lond., A,* **358,** 957–990.

Jault, D. & Le Mouël, J.-L., 1989. The topographic torque associated with a tangentially geostrophic motion at the core surface and inferences on flow inside the core, *Geophys. astrophys. Fluid. Dyn.,* **48,** 273–296.

Jault, D. & Le Mouël, J.-L., 1991. Exchange of angular momentum between the core and mantle, *J. Geomag. Geoelectr.,* **43,** 111–129.

Jault, D. & Le Mouël, J.-L., 1993. Circulation of the liquid core and coupling with the mantle, *Adv. Space Res.,* **13,** 221–223.

Jault, D. & Le Mouël, J.-L., 1999. Comment on 'On the dynamics of topographical core–mantle coupling' by Weijia Kuang and Jeremy Bloxham, *Phys. Earth planet. Inter.,* **114,** 211–215.

Jault, D., Gire, C. & Le Mouël, J.-L., 1988. Westward drift, core motions and exchanges of angular momentum between core and mantle, *Nature,* **333,** 353–356.

Jault, D., Hulot, G. & Le Mouël, J.-L., 1996. Mechanical core–mantle coupling and dynamo modelling, *Phys. Earth planet Inter.,* **98,** 187–191.

Jones, C.A., 2000. Convection driven geodynamo models, *Phil. Trans. R. Soc. Lond., A,* **358,** 873–897.

Jordi, C., Morrison, L.V., Rosen, R.D., Salstein, D.A. & Rossello, G., 1994. Fluctuations in the Earth's rotation since 1983 from high-resolution astronomical data, *Geophys. J. Int.,* **117,** 811–818.

Kuang, W. & Bloxham, J., 1997. An Earth-like numerical dynamo model, *Nature,* **389,** 371–374.

Lambeck, K., 1980. *The Earth's Variable Rotation,* Cambridge University Press, Cambridge.

Le Huy, M., 1995. Le change geomagnetique, les mouvements du fluide à la surface de noyau, et les variations decennales de la rotation de la Terre, *PhD thesis,* Université de Paris 7, Paris.

Le Mouël, J.-L., 1984. Outer core geostrophic flow and secular variation of Earth's magnetic field, *Nature,* **311,** 734–735.

Lehnert, B., 1954. Magnetohydrodynamic waves under the action of the Coriolis force, *Astrophys. J.,* **119,** 647–654.

Malin, S.R.C. & Hodder, B.M., 1982. Was the 1970 geomagnetic jerk of internal or external origin? *Nature,* **296,** 726–728.

Malkus, W.V.R., 1967. Hydromagnetic planetary waves, *J. Fluid Mech.,* **28,** 793–802.

Munk, W.H. & MacDonald, G.J.F., 1960. *The Rotation of the Earth,* Cambridge University Press, Cambridge.

Ponte, R.M., Rosen, R.D. & Boer, G.J., 1994. Angular momentum and torques in a simulation of the atmospheres response to the 1982–83 El Niño, *J. Climate,* **7,** 538–550.

Roberts, P.H. & Scott, S., 1965. On analysis of secular variation: 1: A hydromagnetic constraint, *J. Geomag. Geoelectr.,* **17,** 137–151.

Rochester, M.G., 1960. Geomagnetic westward drift and irregularities in the Earth's rotation, *Phil. Trans. R. Soc. Lond. A,* **252,** 531–555.

Rosen, R.D., 1993. The axial momentum balance of Earth and its fluid envelope, *Surv. Geophys.,* **14,** 1–29.

Stephenson, F.R. & Morrison, L.V., 1995. Long-term fluctuation in the Earth's rotation—700 B.C. to A.D. 1990, *Phil. Trans. R. Soc. Lond., A,* **351,** 165–202.

Vestine, E.H. & Kahle, A.B., 1968. The westward drift and geomagnetic secular change, *Geophys. J. R. astr. Soc.,* **15,** 29–37.

Zatman, S. & Bloxham, J., 1997. Torsional oscillations and the magnetic field within the Earth's core, *Nature,* **388,** 760–763.