Simulations of dynamical friction including spatially-varying magnetic fields

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COOL 05 Workshop
Galena, IL
September 21, 2005

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Work supported by US DOE,
Office of NP, SBIR program,
contract # **DE-FG03-01ER83313**

Motivation

- Parameters for RHIC II cooler are unprecedented
  - see I. Ben-Zvi et al., Proc. COOL 03 Workshop (2003).
  - friction forces must be understood to within a factor of ~2
- There's a need for high-fidelity simulations
  - We are using the VORPAL code
    - C. Nieter and J.R. Cary, Journal of Computational Physics (2004)
    - http://www-beams.colorado.edu/vorpal/
- Goals of the simulations
  - Resolve differences in theory, asymptotics, parametric models
    - Understand magnetization in limit of small Coulomb logarithm
  - Quantify the effect of magnetic field errors
- Numerical approach:
  - use $O(N^2)$ algorithm from astrophysical dynamics community
    - 4^{th}-order predictor-corrector with aggressive variation of time step
    - accurately resolves close binary collisions

4^{th}-Order Predictor/Corrector Hermite Algorithm

- Algorithm developed and used extensively by galactic dynamics community
  - J. Makino, The Astrophysical Journal **369**, 200 (1991)
  - J. Makino & S. Aarseth, Publ. Astron. Soc. Japan **44**, 141 (1992)
- Predictor step:

$$\mathbf{v}_{p,j} = \frac{1}{2}\left(t - t_j\right)^2 \dot{\mathbf{a}}_j + \left(t - t_j\right)\mathbf{a}_j + \mathbf{v}_j$$

$$\mathbf{x}_{p,j} = \frac{1}{6}\left(t - t_j\right)^3 \dot{\mathbf{a}}_j + \frac{1}{2}\left(t - t_j\right)^2 \mathbf{a}_j + \left(t - t_j\right)\mathbf{v}_j + \mathbf{x}_j$$

- where

$$\mathbf{a}_i = \frac{q_i}{m_i}\mathbf{v}_i \times \mathbf{B} + \frac{q_i}{4\pi\varepsilon_0 m_i}\sum_j \frac{q_j \mathbf{r}_{ij}}{\left(r_{ij}^2 + r_c^2\right)^{3/2}}$$

$$\mathbf{r}_{ij} = \mathbf{x}_{p,j} - \mathbf{x}_{p,i}$$

$$\dot{\mathbf{a}}_i = \frac{q_i}{m_i}\mathbf{a}_i \times \mathbf{B} + \frac{q_i}{4\pi\varepsilon_0 m_i}\sum_j q_j \left[\frac{\mathbf{v}_{ij}}{\left(r_{ij}^2 + r_c^2\right)^{3/2}} + \frac{3\left(\mathbf{v}_{ij} \bullet \mathbf{r}_{ij}\right)\mathbf{r}_{ij}}{\left(r_{ij}^2 + r_c^2\right)^{5/2}}\right]$$

$$\mathbf{v}_{ij} = \mathbf{v}_{p,j} - \mathbf{v}_{p,i}$$

r_c ® 0 “cloud” radius

Hermite Algorithm – including a Magnetic Field

- The corrector step: $\mathbf{x}_i\left(t_i+\Delta t_i\right)=\mathbf{x}_{p,i}+\frac{1}{24}\Delta t_i^{\,4}\mathbf{a}_{0,i}^{(2)}+\frac{1}{120}\Delta t_i^{\,5}\mathbf{a}_{0,i}^{(3)}$

$$\mathbf{v}_i\left(t_i+\Delta t_i\right)=\mathbf{v}_{p,i}+\frac{1}{6}\Delta t_i^{\,3}\mathbf{a}_{0,i}^{(2)}+\frac{1}{24}\Delta t_i^{\,4}\mathbf{a}_{0,i}^{(3)}$$

  – where $\mathbf{a}_{0,i}^{(2)}$ and $\mathbf{a}_{0,i}^{(3)}$ are linear functions of $\mathbf{a}(t)$ and $\dot{\mathbf{a}}(t)$ evaluated at times t_i and $t_i+\Delta t_i$

- Adding B-field breaks 4th-order scaling, unless
  – Lab-frame B is purely longitudinal, constant in time
  – **vxB** force is evaluated again at the predicted positions
  – magnetic term in velocity correction (far right term above):
    - $\mathbf{a}_{0,i}^{(3)}$ is split into self-field $\mathbf{a}_{self-field,i}^{(3)}$ & magnetic $\mathbf{a}_{magnetic,i}^{(3)}$ terms
    - the coefficient in front of $\mathbf{a}_{magnetic,i}^{(3)}$ is changed from 1/24 to 5/72

Initial Study of Magnetic Field Errors – Motivation

- The effect of magnetic field errors in a solenoid on the dynamical velocity drag (i.e. friction) of an ion in an electron cooler is not well understood
  - The parametric model of Parkhomchuk treats field errors as an effective transverse rms velocity of the electron Larmor circles
    - Contribution appears in same place as $V_{e,rms,||}$
    - In the absence of an explicit model, field errors have been treated as an effective increase in $V_{e,rms,||}$
- Our primary interest is the cooler for RHIC II
  - We consider the CELSIUS ring here, to take advantage of recent experiments
  - We consider two very different models for the errors

Magnetic field errors – "Model 1"

- A sum of sinusoidal terms (lab frame)

$$B_x = \sum_i b_i \frac{k_{x,i}}{k_{z,i}} \exp(k_{x,i}x)\exp(k_{y,i}y)\sin(k_{z,i}z + \varphi_{z,i})$$

$$B_y = \sum_i b_i \frac{k_{y,i}}{k_{z,i}} \exp(k_{x,i}x)\exp(k_{y,i}y)\sin(k_{z,i}z + \varphi_{z,i})$$

$$B_z = B_0 + \sum_i b_i \exp(k_{x,i}x)\exp(k_{y,i}y)\cos(k_{z,i}z + \varphi_{z,i})$$

$$k_{z,i}^2 = k_{x,i}^2 + k_{y,i}^2 \qquad \lambda_i = 2\pi / k_{z,i}$$

  – a more general form of the equations is allowed
  – we assume $b_i \ll B_0$ for all i
  – appropriate choices for b_i, λ_i, etc. are not yet clear
  – here, we consider a single component

Magnetic field errors – "Model 2"

- A sum of piece-wise constant "tilts" (lab frame)

$$B_x = \sum_i b_{x,i} H(z - z_{x,i}) H(z_{x,i+1} - z)$$

$$B_y = \sum_i b_{y,i} H(z - z_{y,i}) H(z_{y,i+1} - z)$$

$$B_z = B_0$$

  - *H(x)* is the unit Heaviside function
  - we assume $b_{ix}, b_{iy} << B_0$ for all *i*
  - small abuse of Maxwell's eqn.'s at discontinuities
  - parameters taken from design report
    - M. Sedlacek et al., "Design and Construction of the CELSIUS Electron Cooler," http://preprints.cern.ch/cgi-bin/setlink?base=cernrep&categ=Yellow_Report&id=94-03
    - amplitude of tilts (highly variable) is ~1.e-03
    - length of segments (highly variable) is ~20 cm

Fields are Lorentz-transformed to beam frame

- VORPAL cooling sim.'s are in the beam frame

$$B_z' = B_z\left(x', y', \gamma\beta c t'\right)$$

$$B_x' = \gamma B_x\left(x', y', \gamma\beta c t'\right) \quad B_y' = \gamma B_y\left(x', y', \gamma\beta c t'\right)$$

$$E_x' = -\beta c B_y' \qquad E_y' = \beta c B_x' \qquad E_z' = 0$$

  - **E** fields are dominant for "relativistic" coolers
    - because electrons are non-relativistic in the beam frame
    - not strictly true for CELSIUS, for which β~0.3

Basic Parameter set with 5 variations

Symbol	Meaning	Value	Units
B_0	solenoid field	0.1	T
L_{sol}	solenoid length	2.5	m
β	proton bunch velocity / c	0.308	
τ_{lab}	interaction time (lab frame)	2.7×10^{-8}	s
τ_{beam}	interact. time (beam frame)	2.6×10^{-8}	s
Δt	largest time step	2.6×10^{-12}	s
dt_{min}	smallest time step	8.0×10^{-14}	s
ω_{pe}	e- plasma frequency	4.1×10^{8}	rad/s
Ω_L	e- Larmor frequency	1.8×10^{10}	rad/s
r_L	e- Larmor (gyro-) radius	7.9×10^{-6}	m
$L_{x,y,z}$	sim. domain dimensions	6.0×10^{-4}	m
n_e	e- number density	5.4×10^{13}	m^{-3}
N_e	# of simulated e-'s	1.2×10^{3}	
$\Delta_{e,\perp}$	transverse rms e- velocity	1.4×10^{5}	m/s
$\Delta_{e,\parallel}$	long. rms e- velocity	3.0×10^{3}	m/s
$\Delta_{eff,\parallel}$	effective long. rms e- vel.	9.0×10^{3}	m/s

We consider 5 separate cases – 2 with field errors & 3 without

$V_{ion,\parallel} = 10{,}000$ m/s

$V_{ion,\perp} = 10{,}000$ m/s

"cld" – $\Delta_{e,\parallel} = 3{,}000$ (no errors)

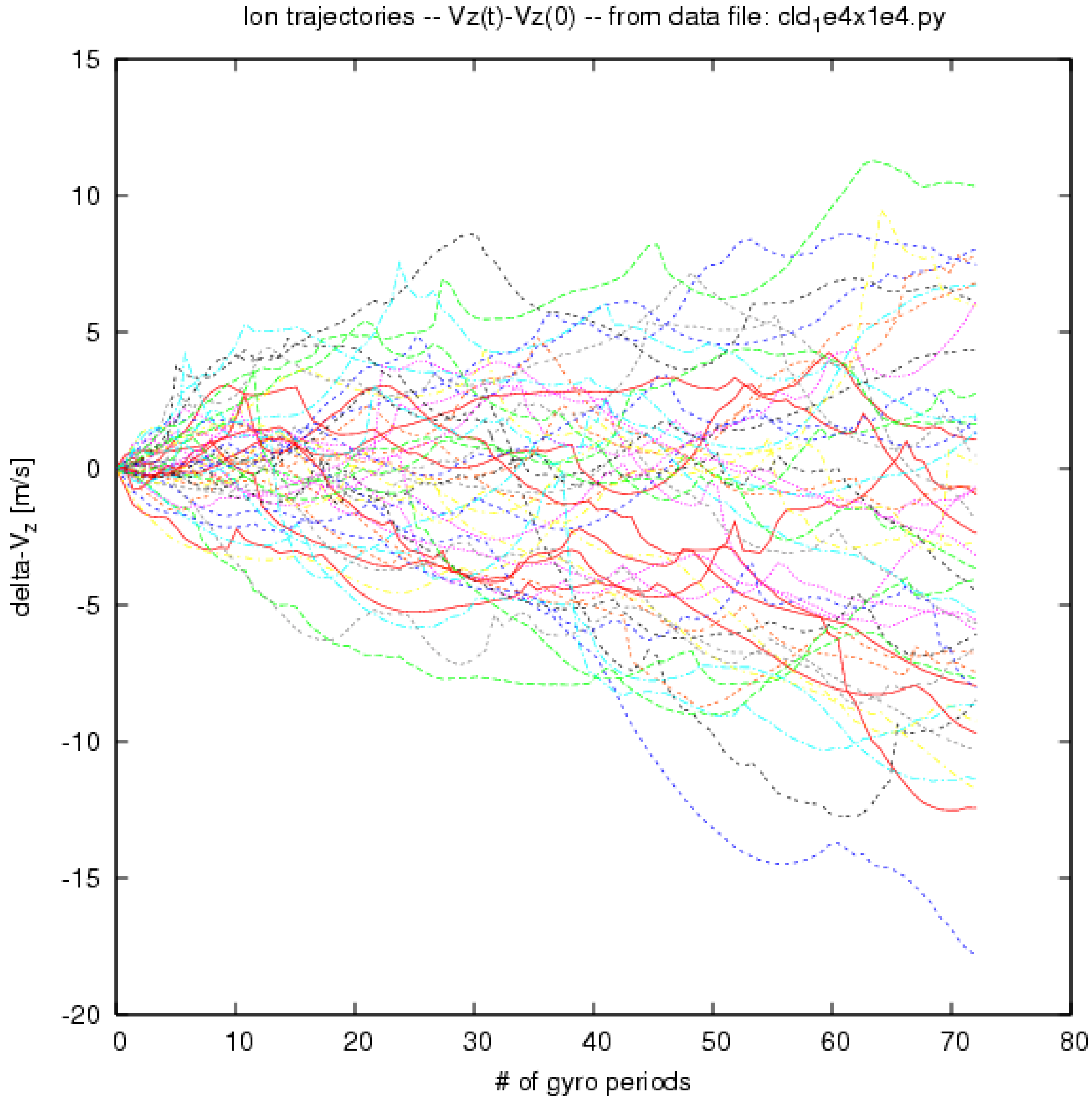
"wrm" – $\Delta_{e,\parallel} = 9{,}000$ (no errors)

"hot" – $\Delta_{e,\parallel} = 18{,}000$ (no errors)

"sin" – $\Delta_{e,\parallel} = 3{,}000$ (Model 1)

"err" – $\Delta_{e,\parallel} = 3{,}000$ (Model 2)

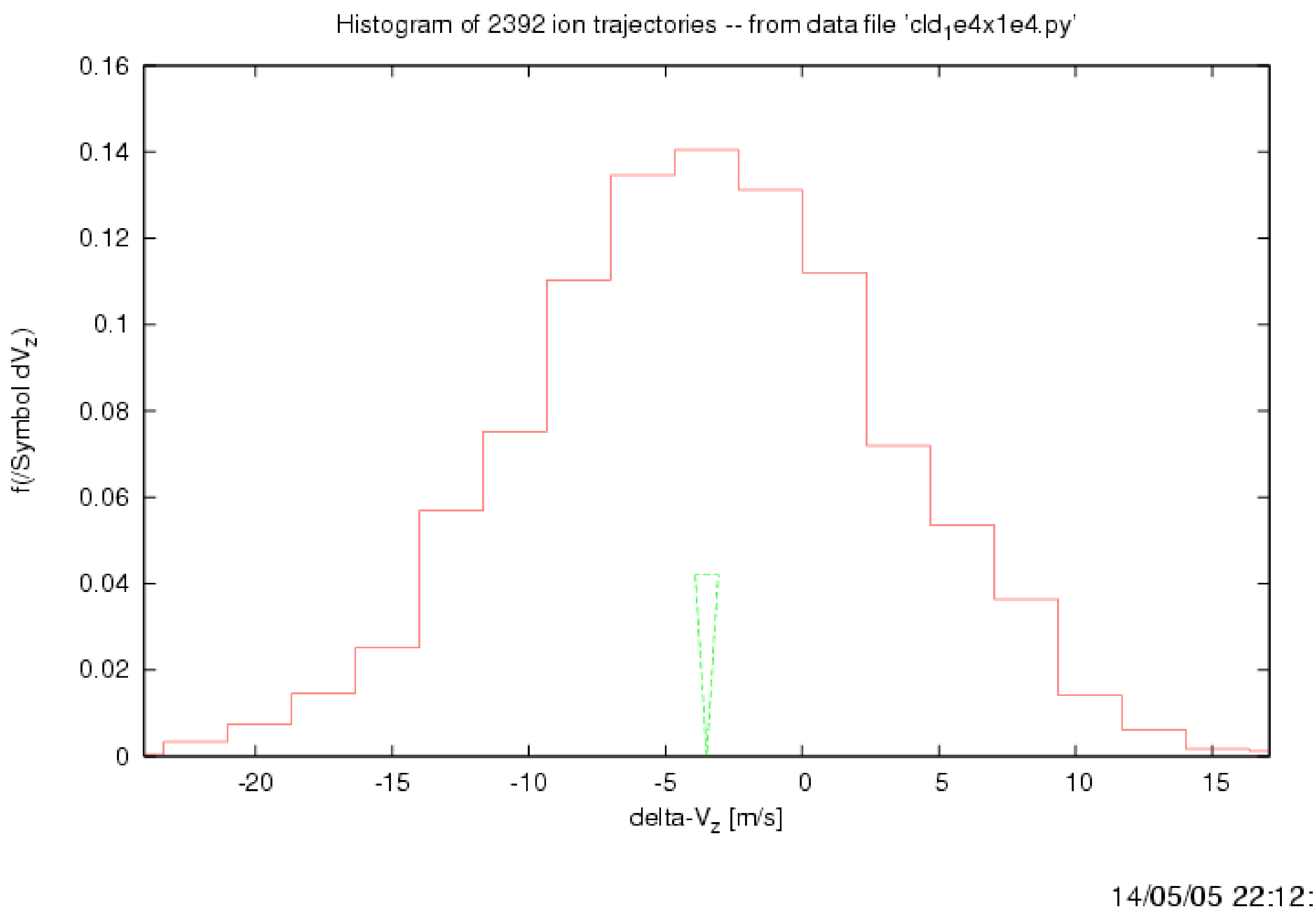
"cld" parameters – $\Delta_{rms,||}$ = 3000 (no field errors)

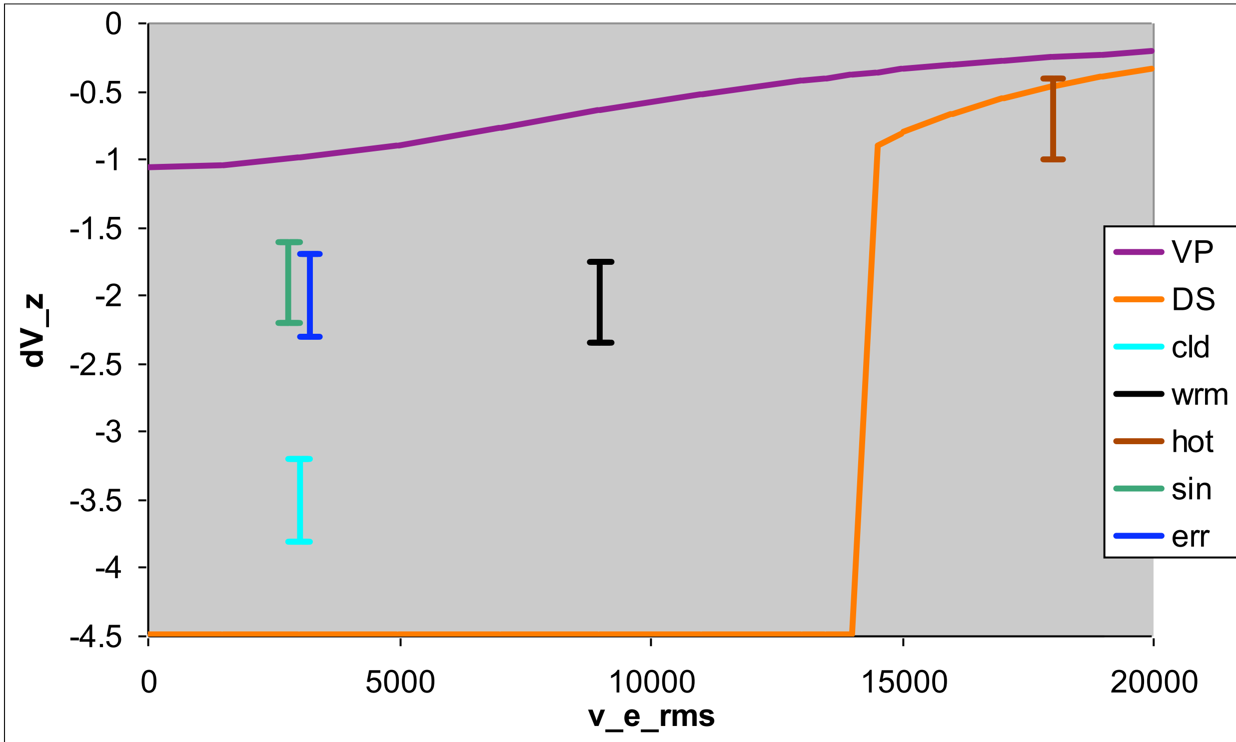


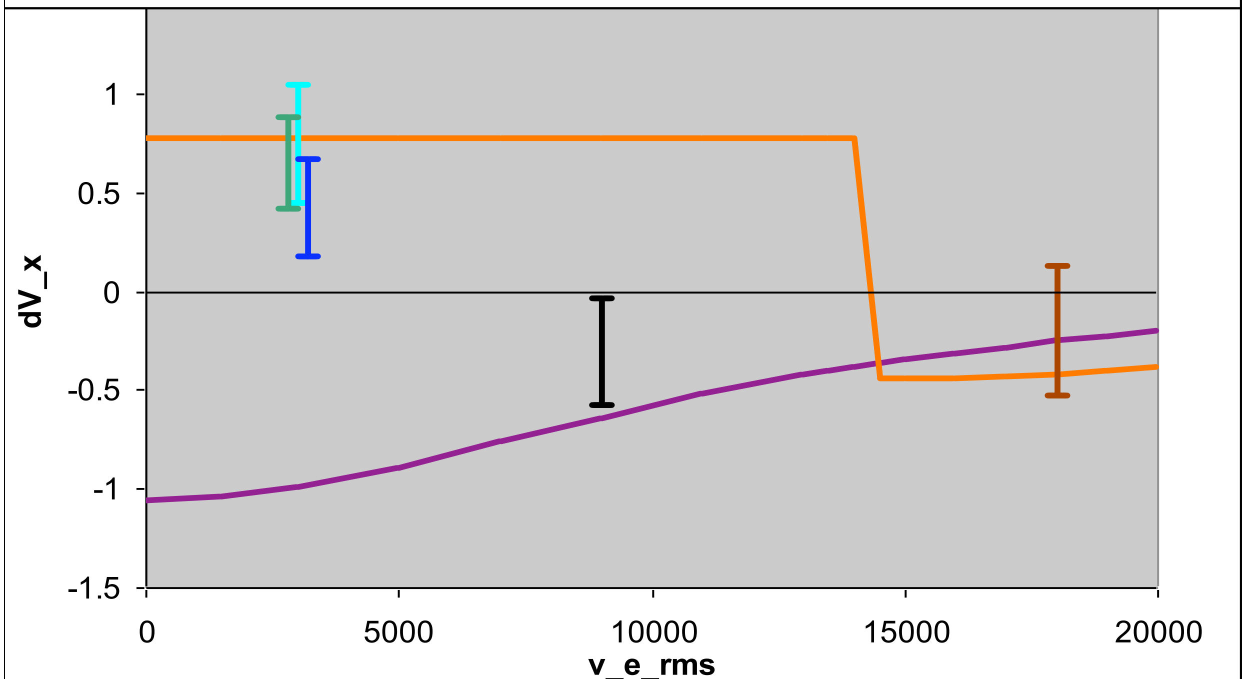
Diffusive dynamics can obscure friction/drag

- For a single pass through the cooler
  - Diffusive velocity kicks are larger than velocity drag
  - Consistent with theory
- For sufficiently large $\Delta_{e,||}$
  - numerical trick of e-/e+ pairs can suppress diffusion
  - not valid for CELSIUS parameters
- Only remaining tactic is to generate 100's of trajectories
  - Central Limit Theorem states that mean velocity drag is drawn from a Gaussian distribution, with rms reduced by $N_{traj}^{1/2}$ as compared to the rms spread of the original distribution
  - Hence, error bars are +/- 1 rms / $N_{traj}^{1/2}$
- Not practical to routinely generate 100's or 1000's of trajectories "by hand"
  - run 8 trajectories simultaneously
  - use "task farming" approach to automate many runs

“cld” parameters – $\Delta_{rms,||}$ = 3000 (no field errors)







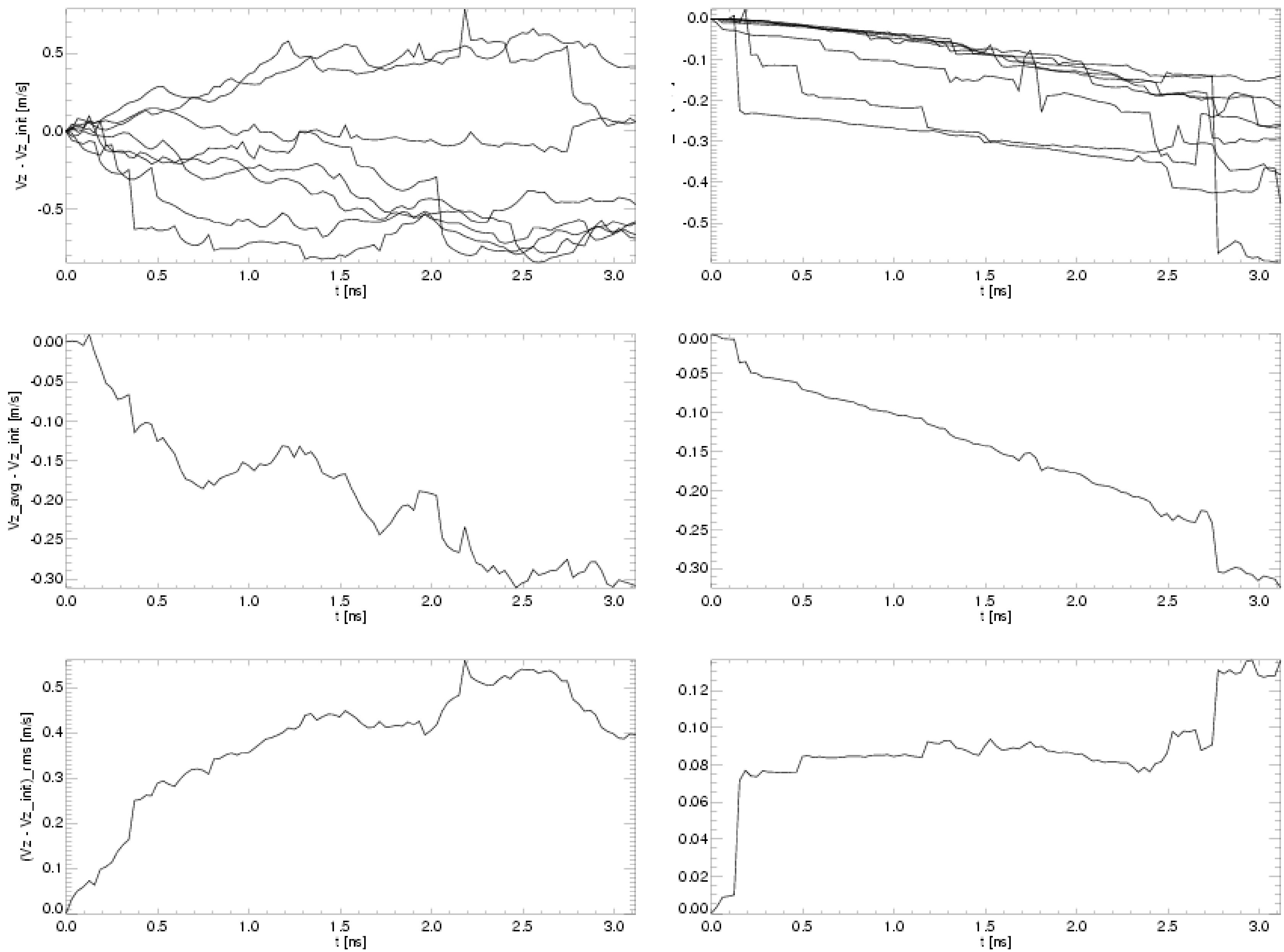
Error models yield similar results –

- Longitudinal velocity drag is significantly reduced
  - in agreement with parametric increase of $\Delta_{rms,||}$
- Transverse velocity drag is less affected
  - NOT consistent with parametric increase of Δ

Wiggler approach to RHIC cooler – Motivation

- Why look for alternatives to solenoid design?
  - solenoid design & beam requirements are challenging
  - accelerator physics group of the RHIC electron cooling project is now considering a wiggler-based approach
- Advantages of a wiggler
  - provides focussing & suppresses recombination
    - Modest fields (~10 Gauss) effectively reduce recombination via 'wiggle' motion of electrons:
    - $$\rho_w = \frac{\Omega_{gyro}}{k_w^2 \gamma_{beam}} \sim 1.4x10^{-3} \lambda_w^2[m] B_w[G] / \gamma$$
  - e- bunch is easier: less charge and un-magnetized
- What's the effect of 'wiggle' motion on cooling?
  - increase minimum impact parameter of Coulomb logarithm: $\rho_{\min}$ ® ρ_w …needs to be simulated

Unmagnetized simulations for “wiggler” param.’s



Wiggler fields in the beam frame

- Tests in absence of wiggler field look promising
  - unmagnetized dynamical friction agrees with theory
  - numerical e-/e+ trick suppresses diffusion by 4x
    - D. Bruhwiler et al., Proc. 33rd ICFA Beam Dynamics Workshop (2004)
- Wiggler field has following form (lab frame)

$$B_x = B_w \cosh(k_w x)\cos(k_w z)$$

$$B_y = B_w \cosh(k_w y)\sin(k_w z)$$

$$B_z = B_w \sinh(k_w y)\cos(k_w z) - B_w \sinh(k_w x)\sin(k_w z)$$

$$k_w = 2\pi / z_w$$

- Lorentz transformation to beam frame
  - yields circularly-polarized EM wave
  - relatively strong, rapidly-oscillating external fields
    - not well-suited for Hermite algorithm; need operator splitting

Operator Splitting Approach

- Numerical technique used for ODE's & PDE's
- Consider Lorentz force equations

$$\dot{\mathbf{x}} = \mathbf{v} \qquad \dot{\mathbf{v}} = \frac{q}{m}\mathbf{E}_{Coulomb} + \frac{q}{m}\left(\mathbf{E}_{ext} + \mathbf{v}\times\mathbf{B}_{ext}\right)$$

- Robust 2nd-order 'Boris' uses operator splitting
  - J. Boris, Proc. Conf. Num. Sim. Plasmas, (1970), p. 3.

$$\Delta\mathbf{x}(\Delta t/2);\ \Delta\mathbf{v}_{\mathbf{E}}(\Delta t/2);\ \Delta\mathbf{v}_{\mathbf{B}}(\Delta t);\ \Delta\mathbf{v}_{\mathbf{E}}(\Delta t/2);\ \Delta\mathbf{x}(\Delta t/2)$$

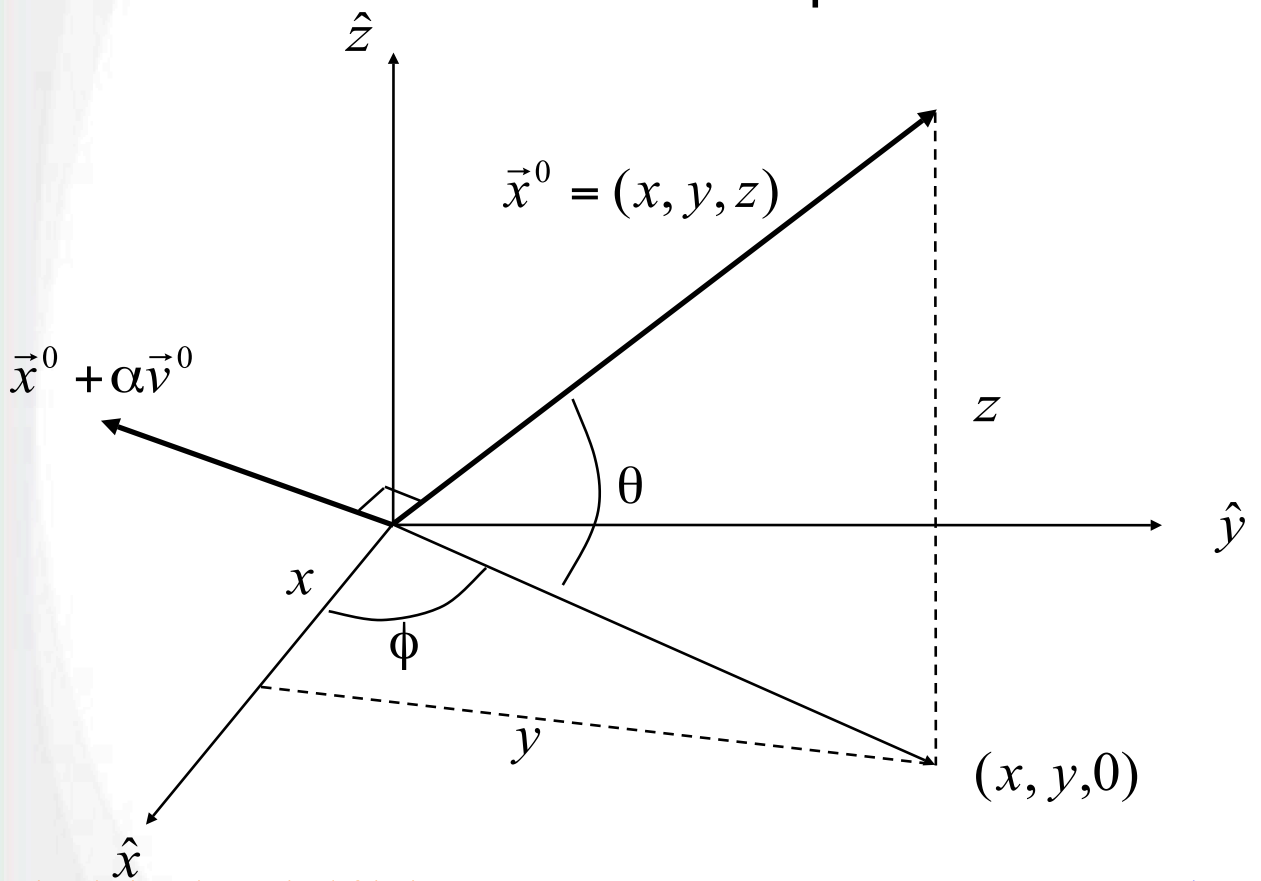
- Add external **E**, **B** fields via operator splitting
  - Hermite algorithm: drift + coulomb fields
  - Boris 'kick': all external **E**, **B** fields
- Benchmark w/ pure Hermite alg. for constant $B_{||}$

Operator splitting is implemented for 'reduced' model

- Use analytical two-body theory for ion/e- pairs
  - handle each pair separately in center-of-mass frame
  - calculate initial orbit parameters in relevant plane
  - advance dynamics for a fixed time step
    - electron's new position and velocity are known
    - changes to ion position/velocity are small perturbations
  - total ion shift is sum of individual changes
- Initial algorithm is in place and partially tested
  - Speed and stability need to be improved
  - Initial comparisons with Hermite algorithm look good
  - Value of operator-splitting approach is verified
    - Hermite implementation in VORPAL will be modified so it can also be used with operator splitting

Semi-analytic ‘Reduced’ Model for Binary Collisions

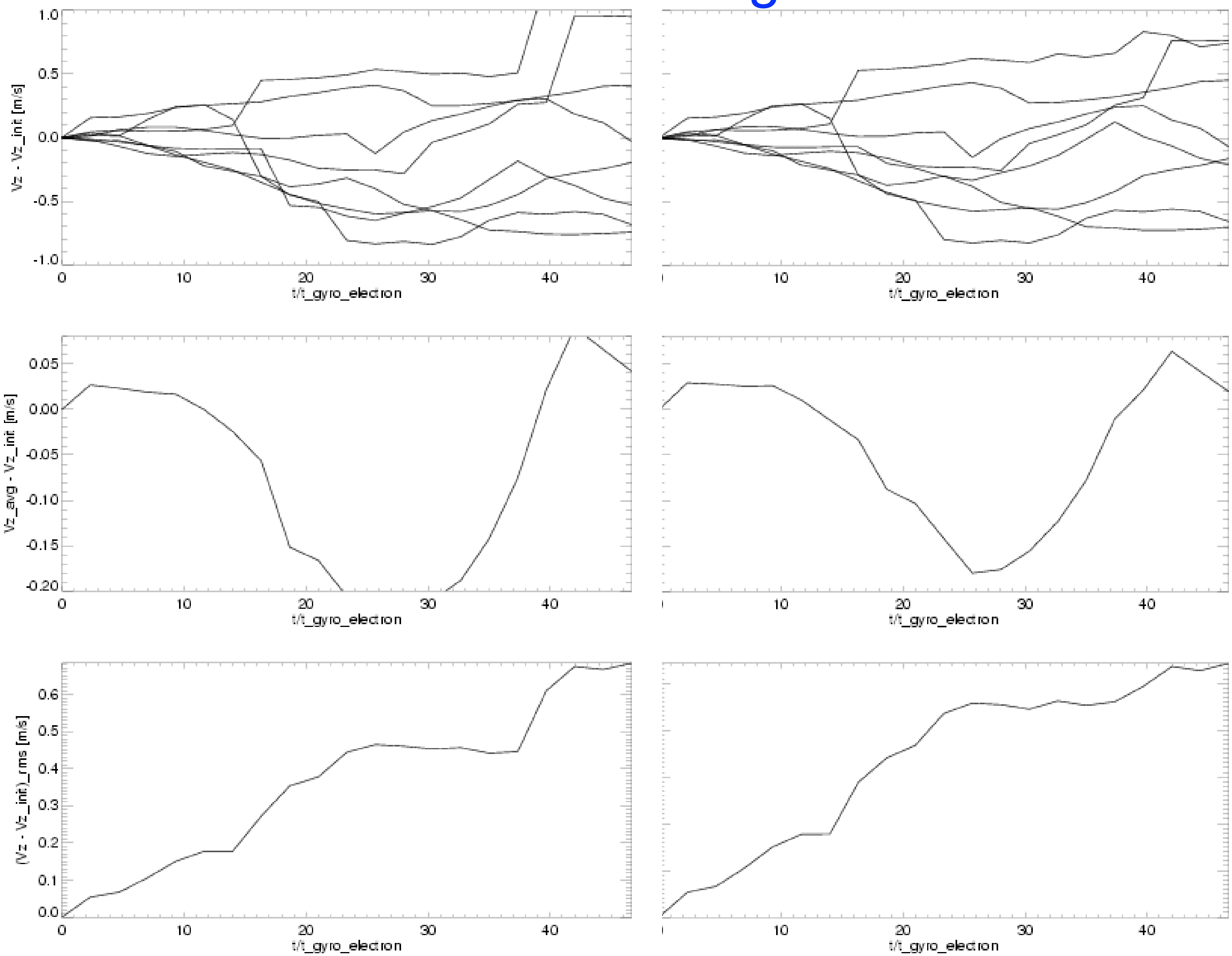
- Must find the plane in which partial orbit occurs
  - necessary rotations (yaw, pitch, roll) are complete
  - transformations are messy, but straightforward
  - “initial” positions & velocities obtained in this plane
- Then standard orbital parameters are calculated



$$R\vec{x}^{\,0} = \begin{pmatrix} \left|\vec{x}^{\,0}\right| \\ 0 \\ 0 \end{pmatrix}$$

$$R^T(\Delta x)$$

Hermite & Reduced Model agree well for B=0



Hermite Algorithm

Reduced Model

Hermite algorithm & Reduced Model compare well for RHIC parameters w/ 5 Tesla B-field

- Agreement validates reduced model & operator splitting approach

Results for initial ion speeds: Vx=0.0 m/s; Vz=3.0E+05 m/s; 800 trajectories

	Hermite	Binary Collision
<delta_Vz_ion> [m/s]	-0.067	-0.066
dVz_rms/sqrt(# traj)	0.007	0.005
Time steps / TSPG*	3978 / 70	576 / 10
Processor - min/run	145	25

Results for initial ion speeds: Vx=2.83E+05 m/s; Vz=1.0E+05 m/s; 800 trajectories

	Hermite	Binary Collision
<delta_Vx_ion> [m/s]	-0.033	-0.043
dVx_rms/sqrt(# traj)	0.008	0.003
<delta_Vz_ion> [m/s]	-0.068	-0.062
dVz_rms/sqrt(# traj)	0.007	0.004
Time steps / TSPG*	3978 / 70	1017 / 20
Processor - min/run	144	44

* TSPG = Time steps per gyroperiod

Acknowledgements

We thank A. Burov, P. Schoessow, P. Stoltz, V. Yakimenko & the Accelerator Physics Group of the RHIC Electron Cooling Project for many helpful discussions. Discussions with A.K. Jain regarding solenoid field errors were especially helpful.

Work at Tech-X Corp. was supported by the U.S. Dept. of Energy under grants DE-FG03-01ER83313 and DE-FG02-04ER84094. We used resources of NERSC.