Simulating Electron Cooling Physics with VORPAL – Recent Results

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Electron Cooling Dynamics working meeting
December 15-16, 2003
Brookhavan National Lab

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Work at Tech-X supported by US DOE:
through the SBIR program, contract #
DE-FG03-01ER83313

Motivation & Goals

- Motivation
  - Support R&D at BNL to help understand and optimize potential performance of the proposed electron cooling section for RHIC
- Primary goal
  - Develop first-principles capability to model electron cooling physics for relativistic ions, especially for RHIC parameters
- General Approach
  - Follow the galactic dynamics community – use direct Coulomb field solve with variable time-stepping to resolve close collisions
  - Accurately calculate friction and diffusion coefficients for the ions
    - » Resolve differences in analytical calculations
      - *Coulomb log >> 1; uniform e- distribution (no space charge)*
    - » Determine validity of Z^2 scaling
    - » Understand the effects of beam space charge on friction
    - » Understand the effects of magnetization
      - *from weak to strong; effect of field errors*
    - » What happens for Coulomb log of order unity or smaller?
    - » For one set of parameters, provide table of coefficients to BetaCool, SimCool

Overview

- Friction coefficients for electron cooling are being simulated at Tech-X and BNL using the VORPAL code
- Some caveats:
  - We are presently neglecting e-/e- interactions
    - Not too bad, because the interaction time is short
    - Initial work with a Poisson solver to correctly capture the e-/e- interactions and the Debye shielding has begun
  - The ion is also influenced by large-scale space charge forces
    - Until recently, this effect was removed from the friction force that we extract from the simulation data as follows:
      - *Run Au+79 ion and "anti" ion (opposite sign)*
      - *Average the velocity changes (space charge effects cancel out, leaving friction)*
  - Recently developed approach to remove bulk space charge forces
    - When calculating forces on any one particle, temporarily shift far away particles from top-to-bottom (or vice-versa), left-to-right, etc. so that each particle is effectively in the center of the distribution

Unmagnetized Simulations of Friction and Diffusion

- These are very preliminary results – work in progress
- Single ion, interacting with 1x10^5 electrons
- Electron distribution is a Gaussian ellipsoid
  - Space charge is removed via ion/anti-ion trick (see below)
  - Electrons are relatively cold, with isotropic temperature

Electron parameters

$$V_{\perp,RMS,e} = 1x10^3 m/s$$

$$V_{\|,RMS,e} = 1x10^3 m/s$$

$$x_{RMS,e} = y_{RMS,e} = 1x10^{-4} m$$

$$z_{RMS,e} = 1x10^{-3} m$$

$$n_e = 6.4x10^{14} m^{-3}$$

$$\omega_{pe} = 1.4x10^9 rad/s$$

Single Au+79 ion

$$V_{\|} = V_z = 1x10^5 m/s$$

$$V_{\perp} = V_y = 0 m/s$$

$$Z = 1 \text{ to } 79$$

System parameters

$$B_{\|} = 0$$

$$L = 30\, m$$

$$\tau = \left(L/\gamma\beta c\right) = 9.35x10^{-10} s$$

Coulomb logarithms

$$\rho_{\max} = 4.0x10^{-5} m$$

$$\rho_{\min} = 2.0x10^{-6}$$

Analytical Friction Force for Cold Electrons, with no B-field

G.I. Budker, At. Energ. **22** (1967), p. 346

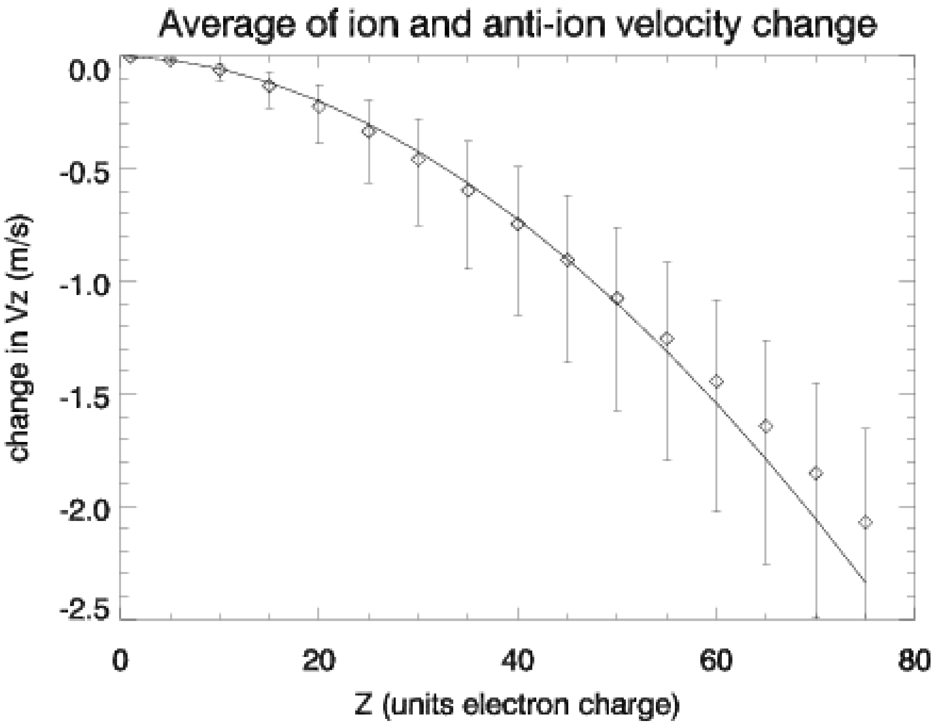
$$\mathbf{F} = -\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{\mathbf{V}_{ion}}{V_{ion}^3}$$

$$\rho_{\min} = \left(Ze^2 / 4\pi\varepsilon_0\right) / m_e V_{ion}^2$$

$$\rho_{\max} = V_{ion} / \left(\omega_{pe} + 1/\tau\right)$$

$$\omega_{pe} = \sqrt{n_e e^2 / \varepsilon_0 m_e}$$

- VORPAL simulations agree very nicely with Budker
  - Electron density, ion velocity are small compared to RHIC par.'s
  - Three initial seeds were used in each case, to get error bars
  - Scatter (diffusion) is strong, especially for large Z
  - Indications that simulated force is dropping for large Z



Diffusive Dynamics, with no B-field

$$\frac{d}{dt}\left\langle \left(V_\perp - \langle V_\perp \rangle\right)^2 \right\rangle = \omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \frac{m_e}{m_i^2} \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{1}{V_{ion}}$$

Perpendicular diffusion

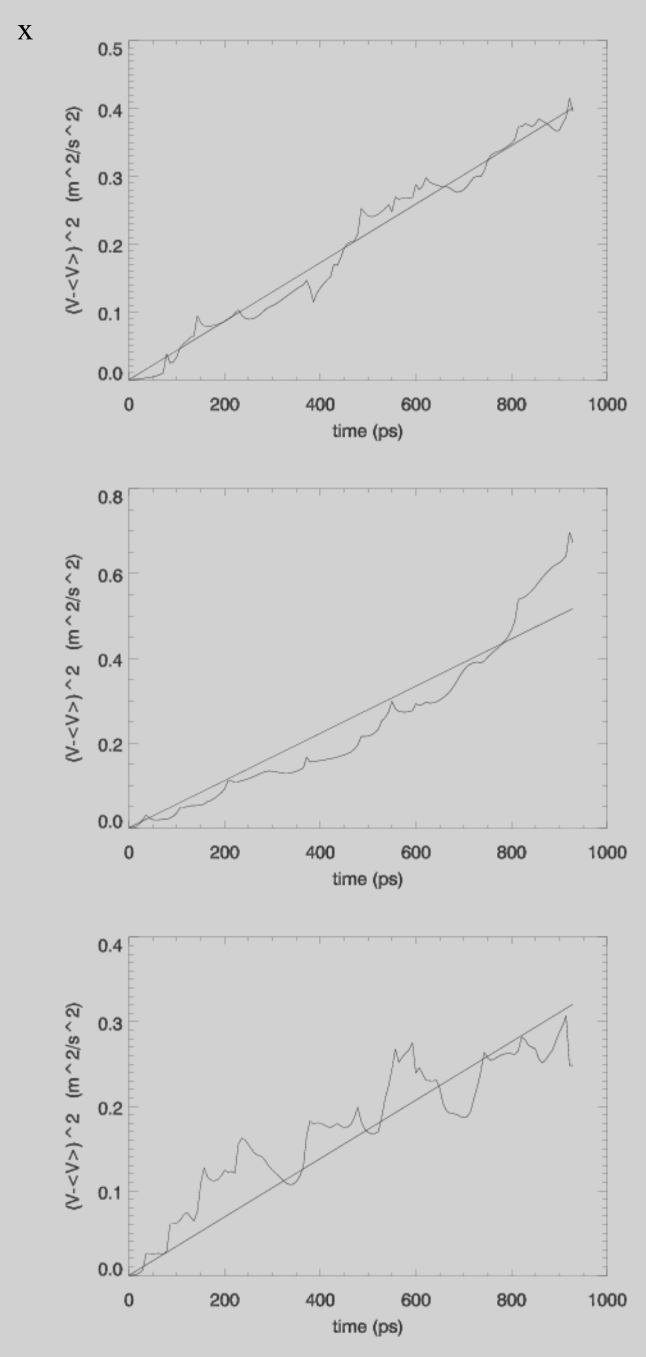
D&S: 7.0×10^8 m^2/s^3
VORPAL: 4.3×10^8 m^2/s^3 5.5×10^8 m^2/s^3

Parallel diffusion

D&S: 0 m^2/s^3 (in the limit of cold e-)
VORPAL: 3.5×10^8 m^2/s^3

Good agreement with theory for transverse diffusion.

Larger numerical diffusion in parallel direction (direction of ion velocity) is not yet understood.



Friction for Magnetized Electrons – Derbenev & Skrinsky

Ya. S. Derbenev and A.N. Skrinsky, “The Effect of an Accompanying Magnetic Field on Electron Cooling,” Part. Accel. **8** (1978), p. 235.

$$\mathrm{F}_{\parallel}^{A} = -\frac{3}{2}\omega_{pe}^{2}\frac{(Ze)^{2}}{4\pi\varepsilon_{0}}\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\left(\frac{V_{\perp}}{V_{ion}}\right)^{2}\frac{V_{\parallel}}{V_{ion}^{3}}$$

$$\mathrm{F}_{\perp}^{A} = -\frac{1}{2}\omega_{pe}^{2}\frac{(Ze)^{2}}{4\pi\varepsilon_{0}}\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\frac{\left(V_{\perp}^{2}-2V_{\parallel}^{2}\right)}{V_{ion}^{2}}\frac{V_{\perp}}{V_{ion}^{3}}$$

$$\rho_{\min}^{A} = \max\left(r_{L},\rho_{\min}\right)$$

$$\rho_{\max}^{A} = \min\left(r_{beam},\rho_{\max}\right)$$

$$r_{L} = V_{\perp,RMS,e}/\Omega_{L}\left(B_{\parallel}\right)$$

- Magnitude of forces is comparable to Budker’s prediction
- Realm of applicability is quite different
  - Electrons are assumed to be strongly magnetized
  - Parallel electron temperature must be cold
- Complicated dependence on ion’s velocity components
  - Possibility for “anti-cooling” when $V_{\perp}$ is relatively small

Friction for Magnetized Electrons – Derbenev, Skrinsky & Meshkov

- The perpendicular force is the same as on previous slide
- The parallel force has a slightly different form:
  - The factor of 2/3 offsets the “defect” of adiabatic collisions by contributions with large impact parameters, so the parallel friction force is no longer zero when the transverse ion velocity is zero

$$\mathrm{F}_{\parallel}^{A} = -\frac{3}{2}\omega_{pe}^{2}\frac{(Ze)^{2}}{4\pi\varepsilon_{0}}\left(\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\left(\frac{V_{\perp}}{V_{ion}}\right)^{2} + 2/3\right)\frac{V_{\parallel}}{V_{ion}^{3}}$$

Friction for Magnetized Electrons – Parkhomchuk

V.V. Parkhomchuk, "New insights in the theory of electron cooling," Nucl. Instr. Meth. in Phys. Res. **A 441** (2000), p. 9.

Missing factor of π from Eq. (4) has been added below.

In this presentation, we always choose $V_{eff} = 0$.

$$\mathbf{F} = -\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \ln\left(\frac{\rho_{\max} + \rho_{\min} + r_L}{\rho_{\min} + r_L}\right) \frac{\mathbf{V}_{ion}}{\left(V_{ion}^2 + V_{eff}^2\right)^{3/2}}$$

- This result differs only slightly from Budker's equation
  - Logarithm has a different form and will tend to be smaller
  - An effective velocity has been introduced into the denominator
- Predicted to work reasonably well for arbitrary electron temperatures and arbitrary magnetic fields

Simulations of the Friction Force with near-RHIC parameters

- These are very preliminary results – work in progress
- Single ion, interacting with $7x10^5$ electrons
- Parameters are reasonably close to RHIC parameters
  - Goal is to compare numerics with analytical models
- Electrons uniformly fill a box (dimensions specified below)
  - Space charge is removed via ion/anti-ion trick
  - Ion remains far away (at least ρ_{max}) from all edges of the box

Electron parameters

$$V_{\perp,RMS,e} = 5x10^5 \, m/s$$

$$V_{\|,RMS,e} = 1x10^3 \, m/s$$

$$L_{x,sim} = 2.5x10^{-4} \, m$$

$$L_{y,sim} = 7.5x10^{-4} \, m$$

$$L_{z,sim} = 7.5x10^{-4} \, m$$

$$n_e = 5.5x10^{15} \, m^{-3}$$

$$\omega_{pe} = 4x10^9 \, rad/s$$

Single Au+79 ion

$$V_{\|} = V_z = 3x10^5 \, m/s$$

$$V_{\perp} = V_y = 5x10^5 \, m/s$$

$$V_{ion} = \sqrt{V_y^2 + V_z^2} = 5.83x10^5 \, m/s$$

$$Z = 79$$

System parameters

$$B_{\|} = 1 \, Tesla$$

$$L = 30 \, m$$

$$\tau = (L/\gamma\beta c) = 9.35x10^{-10} \, s$$

Coulomb logarithms

$$\rho_{\max}^A = \rho_{\max} = 1.1x10^{-4} \, m$$

$$\rho_{\min} = 5.9x10^{-8}$$

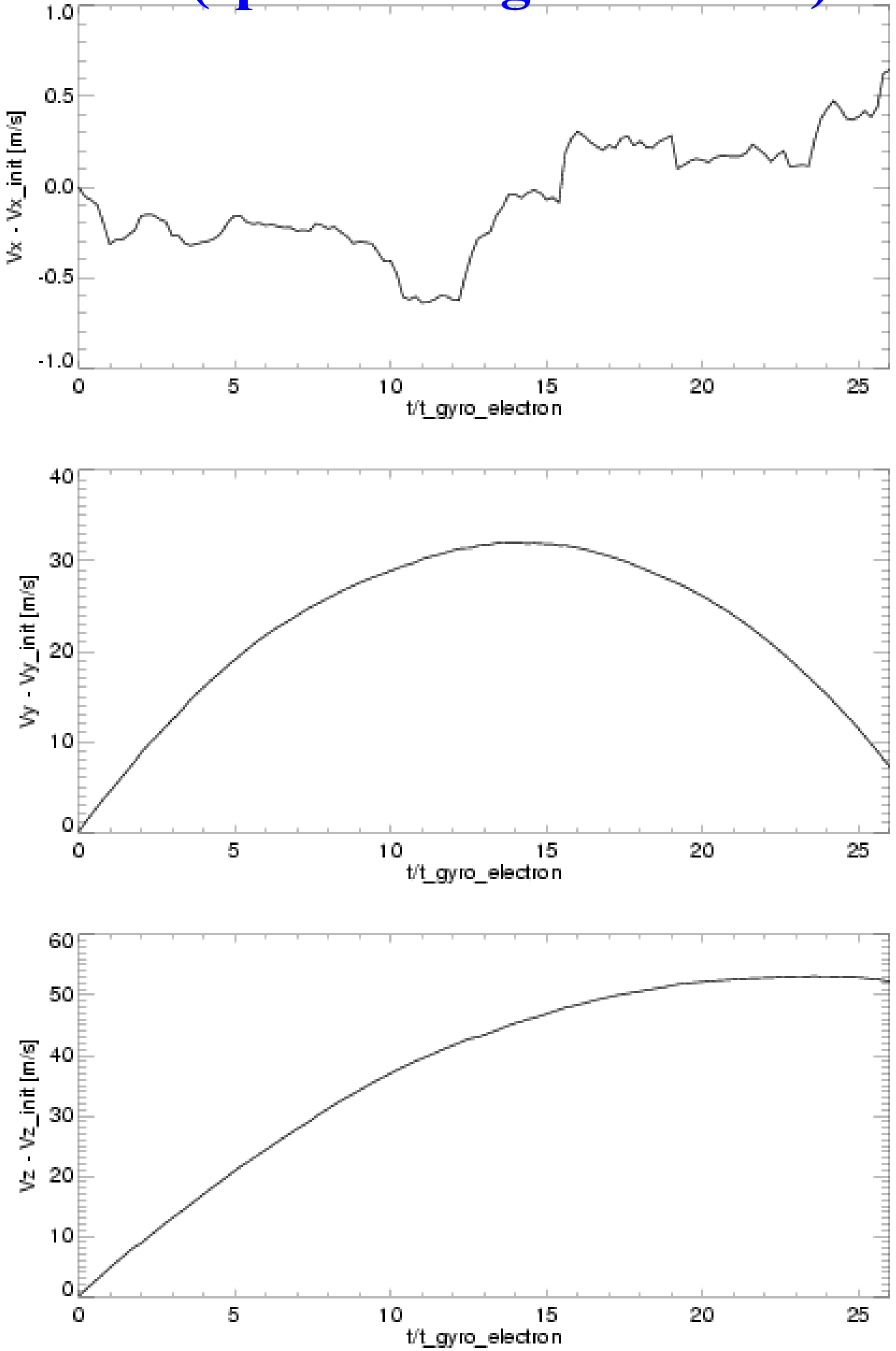
$$\rho_{\min}^A = r_L = 2.8x10^{-6} \, m$$

$$\ln\left(\frac{\rho_{\max} + \rho_{\min} + r_L}{\rho_{\min} + r_L}\right) \approx 3.6$$

$$\ln\left(\frac{\rho_{\max}^A}{\rho_{\min}^A}\right) \approx 3.6$$

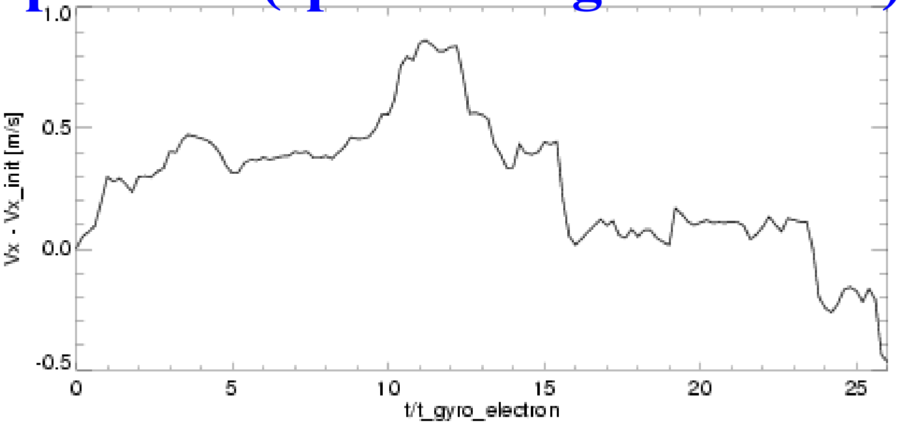
Ion Dynamics over 26 Gyro-periods (space charge included)

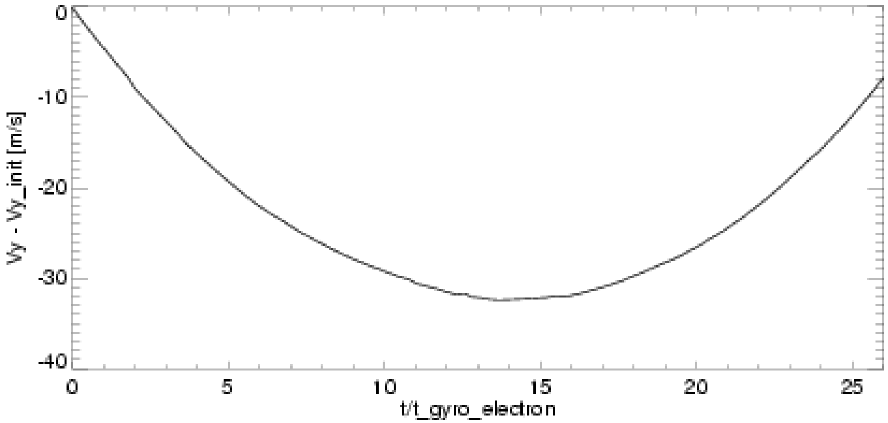
- Perpendicular dynamics:
  - dv_x shows random walk
    - » initial velocity was zero
    - » diffusion, no friction
  - dv_y dominated by space charge
    - » sign of F_{sc} changes as ion crosses center of e- slab
- Parallel dynamics:
  - dv_z dominated by space charge
    - » F_{sc} vanishes as ion reaches center of e- slab

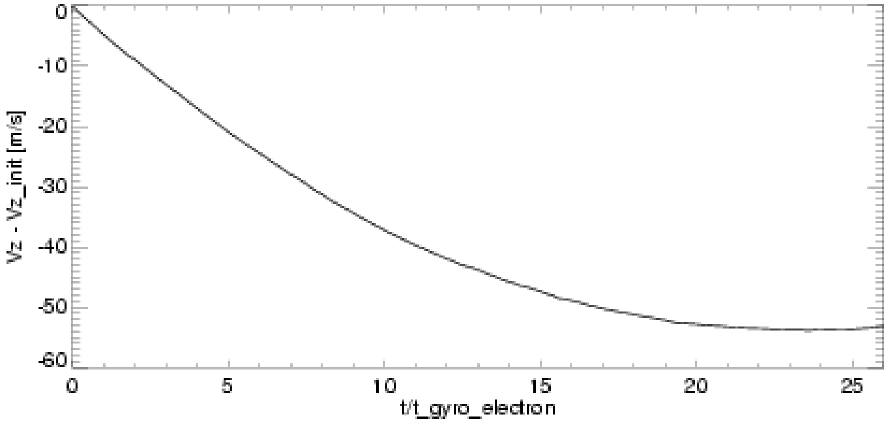


Anti-Ion Dynamics over 26 Gyro-periods (space charge included)

- Perpendicular dynamics:
  - dv_x shows random walk
    - initial velocity was zero
    - diffusion, no friction
  - dv_y dominated by space charge
    - sign of F_{sc} changes as ion crosses center of e- slab
    - sign of F_{sc} is opposite that on previous slide
- Parallel dynamics:
  - dv_z dominated by space charge
    - F_{sc} vanishes as ion reaches center of e- slab
    - sign of F_{sc} is opposite that on previous slide

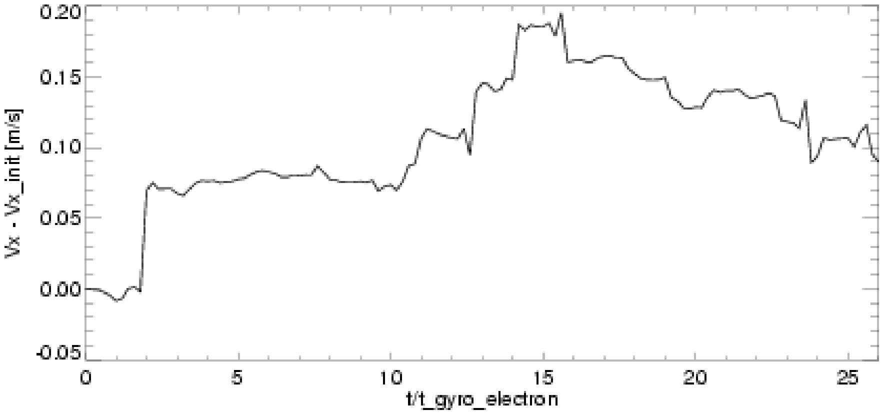


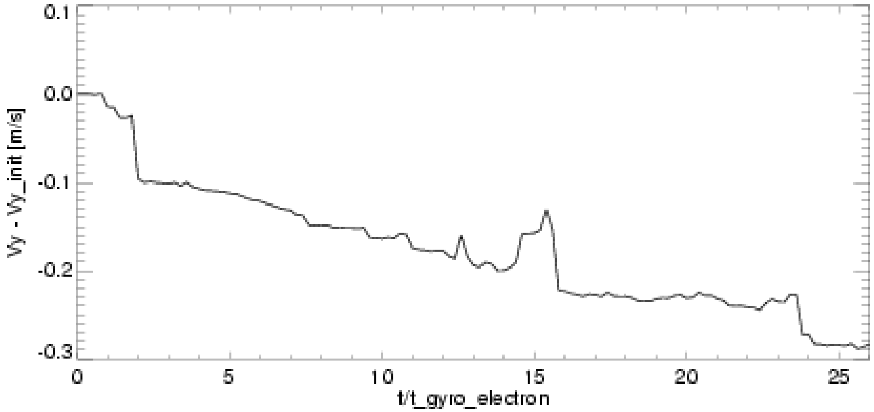


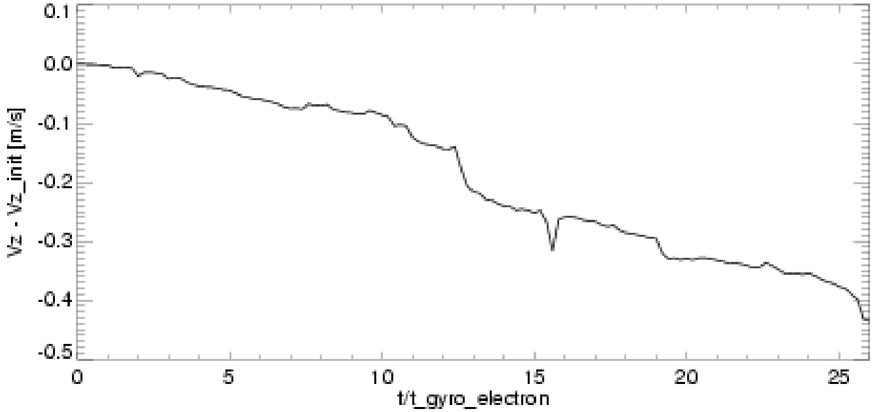


Ion Dynamics over 26 Gyro-periods (space charge removed)

- These plots are simple averages of the previous two data sets
- Perpendicular dynamics:
  - dv_x shows random walk
    - » initial velocity was zero
    - » diffusion, no friction
  - dv_y shows friction force
    - » diffusive effects are strong
- Parallel dynamics:
  - dv_z shows friction force
    - » diffusive effects are strong
- $F_{\perp} = m_{Au} * dv_y / \tau$
- $F_{\parallel} = m_{Au} * dv_z / \tau$







Simulations agree partially with analytical predictions

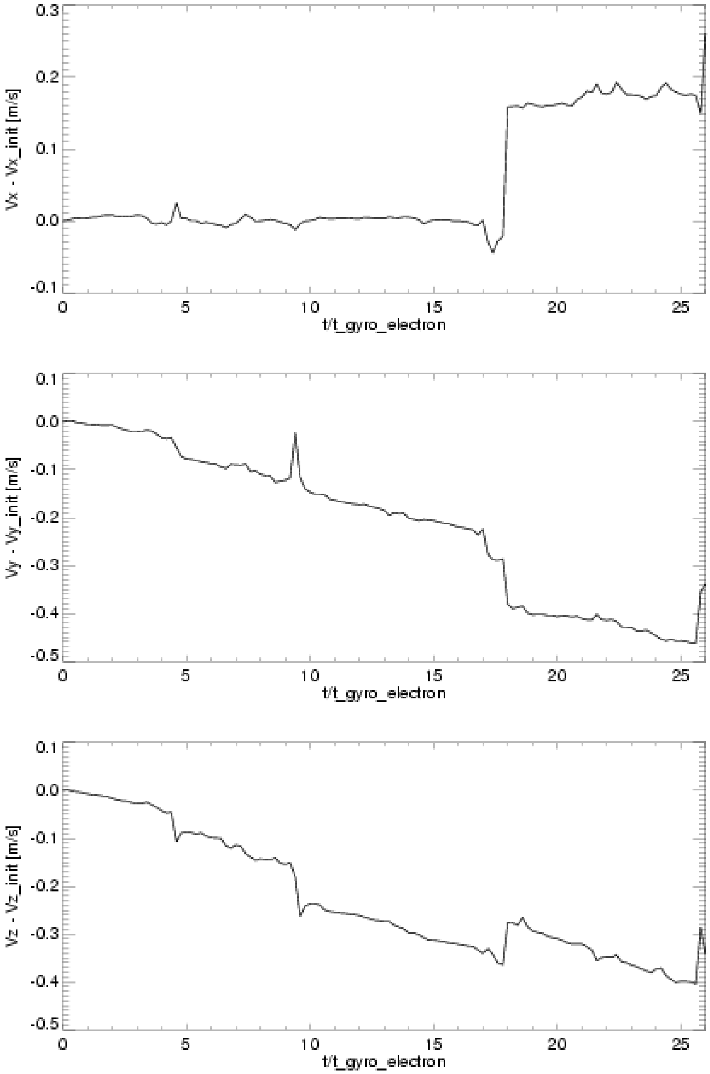
	Derbenev-Skrinsky (DS)	D-S-Meshkov (DSM)	Parkhomchuk (VP)	VORPAL
$F_\perp$	$-2.2\text{x}10^{-17}$	$-2.2\text{x}10^{-17}$	$-2.2\text{x}10^{-16}$	-1.0 +/- 0.4 x10^{-16}
$F_\parallel$	$-1.4\text{x}10^{-16}$	$-1.8\text{x}10^{-16}$	$-1.3\text{x}10^{-16}$	-1.5 +/- 0.4 x10^{-16}

- Perpendicular friction force:
  - Simulated result is of same order as VP calculation (but smaller)
  - D-S and D-S-M calculations are much smaller
- Parallel friction force:
  - Simulated result agrees well with D-S, D-S-M & VP calculations
- Caveats regarding simulations
  - Strong variations with random number generator (strong diffusive term)
    - » error bars are roughly inferred from plot on previous page
  - $\omega_{pe}*\tau=3.7$, which implies that e-/e- dynamics may be important

Unmagnetized case shows B-field has little effect

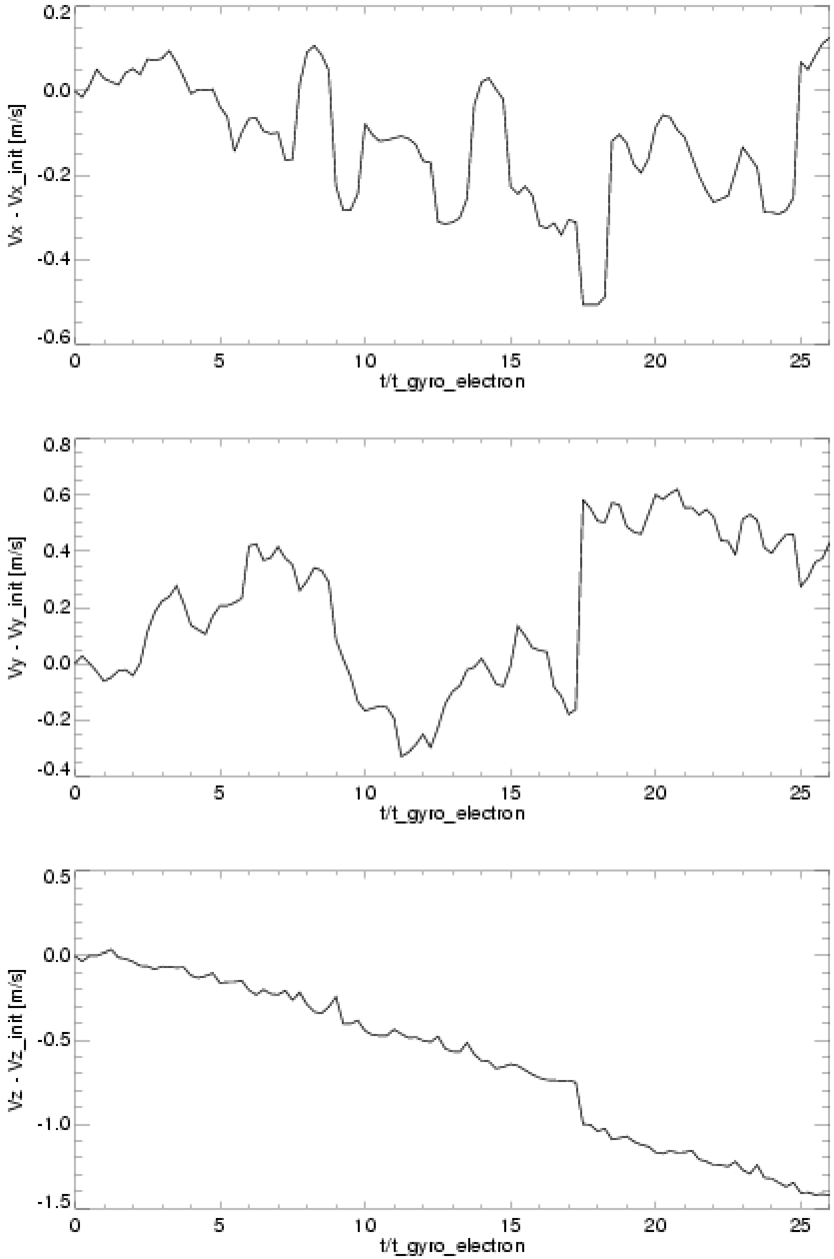
	Budker	VORPAL
$F_\perp$	-4.5×10^{-16}	$-1.2 \pm 0.4 \times 10^{-16}$
$F_\parallel$	-2.7×10^{-16}	$-1.2 \pm 0.4 \times 10^{-16}$

- Budker forces larger than D-S, VP!
  - Due in part to small value of ρ_{min}
  - Not valid because e- are not cold
- VORPAL results differ from Budker
  - smaller (effect of hot electrons)
  - isotropic (transv. temp. dominates)
- Simulated forces are comparable to results with B = 1 T
  - Magnetic field has marginal effect here
  - Non-magnetized parallel friction force contributes to B=1T runs above…?



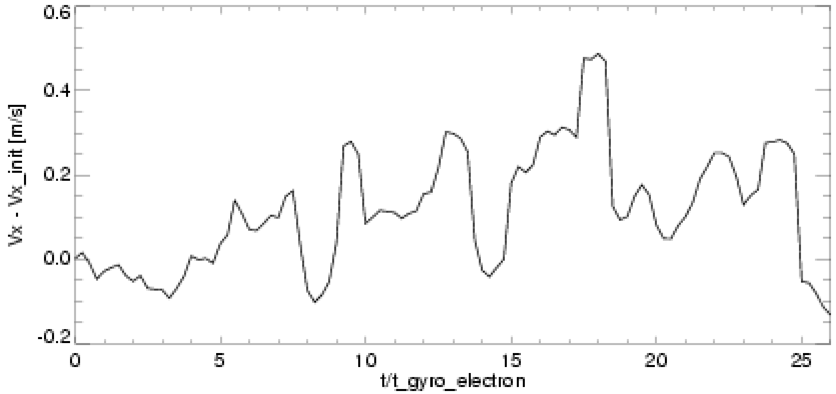
Unmagnetized case, with space charge removed

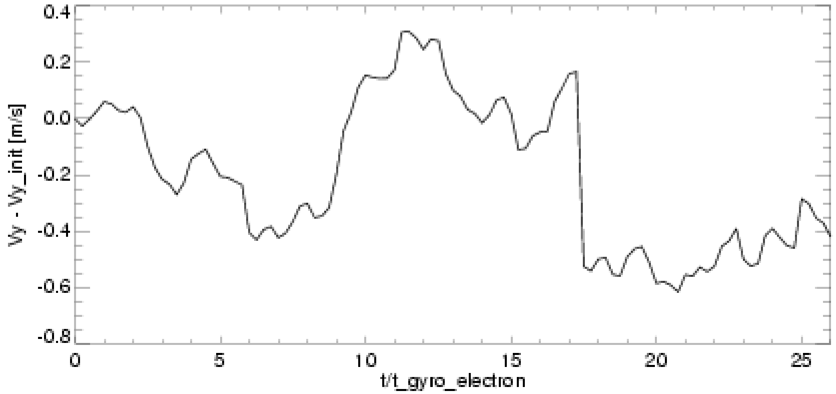
- Friction force is seen along z
- Transverse dynamics is purely diffusive
- Diffusive component of longitudinal dynamics is very small
  - Consistent with theory

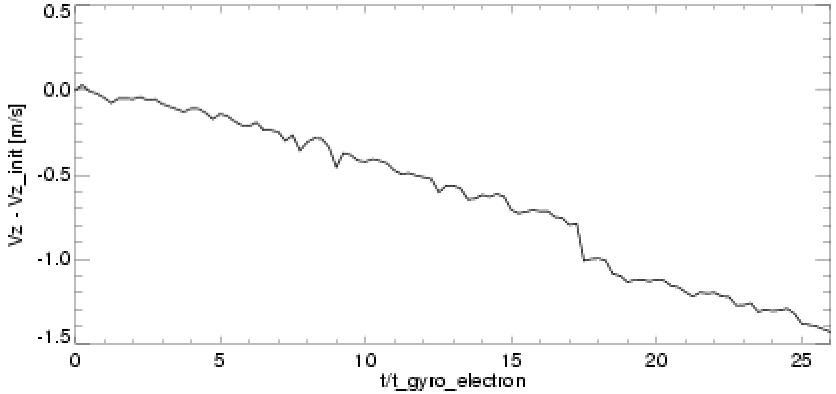


Unmagnetized case, space charge removed, anti-ion

- Friction force is seen along z
  - Same as for case of +79 ion
- Transverse dynamics is purely diffusive
  - Velocity changes are the negative of those for the anti-ion
- Diffusive component of longitudinal dynamics is very small
  - Consistent with theory
  - Same as for case of +79 ion







Conclusions

- Unmagnetized VORPAL friction forces agree with Budker
  - When the electrons are cold!
  - Approximate scaling with Z^2 is seen, with correct magnitude
- Unmagnetized VORPAL diffusion coefficients are also reasonable
  - Perpendicular values agree well with Derbenev & Skrinsky
  - Parallel values much larger than D&S (comparable to perpendicular)
- Magnetized friction simulations agree partially with analytic models
  - Simulations and all analytical predicitons agree for $F_{\parallel}$
  - Derbenev, Skrinsky & Meshkov prediction for $F_{\perp}$ is too low
    - » Adding in the unmagnetized friction force yields reasonable agreement
  - Parkhomchuk prediction for $F_{\perp}$ is 2x larger than simulations
- These conclusions hold only for a few cases of $V_{\perp}$ and $V_{\parallel}$
  - We must systematically study properties of the simulated results:
    - » scaling with Z
    - » scaling with $V_{\perp}$ and $V_{\parallel}$
    - » scaling with $B_{\parallel}$
  - Must increase e- velocities (RHIC parameters) so that magnetized case differs more strongly from the B=0 case.
- We must stop using the ion/anti-ion trick
  - Repeating many of our previous runs and then moving forward