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# The RCMS dipole aperture and beam pipe

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## 1 Abstract

The RCMS dipole aperture is reduced to 60 × 30 mm (H × V) from the value of 80 × 40 mm used in the Pre-Conceptual Design Report (PCDR) [1].

The cycling frequency is increased to 60 Hz from the PCDR value of 15 Hz.

The chevron dipole is constructed from two straight stacks of laminations, reducing the beam sagitta and providing the same edge focusing in both planes.

The dipole beam pipe is made from four straight sections of circular Inconel beam pipe with a radius of about 15 mm.

The optimum beam pipe thickness is between 0.5 mm and 1.0 mm, balancing the twin desires to keep beam pipe heating less than 300 W/m, and to keep the beam pipe opaque to wake fields at frequencies of around 1 MHz and higher.

The dipole field perturbation from eddy currents induced in the beam pipe is negligible, and the sextupole perturbation is zero (for a circular pipe).

The RCMS is very stable against collective instabilities.

| Parameter | Loma Linda | RCMS (PCDR) | Comment |
|---|---|---|---|
| Protons per pulse | $< 3 \times 10^{10}$ | $3 \times 10^{9}$ | 1993 'typical' |
| Pulse rate [Hz] | .45 | 15 | |
| Patient rate [1/day] | 125 | | 16 hour days |
| Total power [kW] | 370 | $> 182$ | |
| | | | |
| Circumference [m] | 20.053 | 28.6 | |
| Energy range [MeV] | 2 - 250 | 7 - 270 | Inject to top |
| Injection beta gamma | .065 | .122 | |
| B field [T] | 0.1 - 1.5 | .23 - 1.5 | Min - Max |
| | | | |
| Full dipole gap [mm] | 200 x 50 | 80 x 40 | H x V |
| Full vac chamber [mm] | 96 x 50 | | H x V |
| Good field ap. [mm] | 50 x 50 | | measured: sext |
| | | | |
| $\Delta p/p$ RF bucket | $\pm.0044$ | $\pm.004$ | |
| $\Delta p/p$ beam FWHM | .007 | .0046 (total) | at injection |
| Dispersion max [m] | 9.6 | 2.18 | |
| H beam size FWHM [mm] | 60.0 | 10.0 | LL measured |
| | | | |
| RMS emittance [$\mu$m] | 0.11 x 0.10 | 0.3 x 0.3 | HxV normalized |
| Beta max [m] | 6.0 x 3.2 | 3.4 x 4.2 | HxV |
| Beta beam size rms [mm] | 3.2 x 2.2 | 2.9 x 3.2 | HxV injection |
| | | | |
| Tune (H,V) | .600, 1.317 | 3.25, 4.85 | |
| Transition gamma | .583 | 2.39 | |
| Nat. chrom. (H,V) | -.61, -1.25 | -2.24, -2.71 | H, V design |
| Nat. chrom. | -22 | | field calc & msmt |
| | | | |
| RF harmonic | 1 | 1 | |
| RF voltage [kV] | < .3 | 4.5 | |
| RF frequency [MHz] | .974 - 9.174 | 1.27 - 6.61 | |

Table 1: A comparison of parameters between Loma Linda as built and RCMS numbers as originally reported in the Pre Conceptual Design Report.

# 2 Reducing the dipole aperture

Table 1 shows a side-by-side comparison of Loma Linda Synchrotron (LLS) parameters, many of them measured [2, 3, 4, 5], and the Rapid Cycling Medical Synchrotron (RCMS) design, as originally described in the PCDR [1].

A comparison of the parameters shows that it is possible to reduce the dipole aperture below the PCDR value of 80 × 40 mm (H × V):

- **Maximum dispersion** is 9.6 m in the LLS, but only 2.2 m in the RCMS.
- **Momentum width** is approximately the same value, $\Delta p/p \simeq \pm .004$, in both machines.
- **Horizontal beam size** at injection is dominated by momentum spread in both machines.
- **Horizontal beam size** (FWHM) is measured as 60 mm in LLS, but is only 10 mm in RCMS.
- **Normalized emittance** measured in LLS is about $\epsilon \simeq 0.1$ $\mu$m, suggesting that the RCMS value of 0.3 $\mu$m is conservatively large.
- **Vertical betatron beam size** is measured in LLS at about 3 mm, and is less than 4 mm in RCMS.

Putting all this together, it is reasonably prudent to reduce the dipole aperture to 60 × 30 mm, provided beam sagitta can be handled.

## 2.1 Beam sagitta in the dipole and in the beam pipe

Figure 1 shows that the beam sagitta *in the dipole aperture* is reduced by a factor of 4 to 8.2 mm when the dipole is constructed as a "chevron", in two straight rectangular pieces. This permits a dipole aperture width of 60 mm. It has the additional advantage of providing the same edge focusing in both planes – so that focusing and defocusing quadrupoles have approximately the same strength.

A beam sagitta of 8.2 mm *in the beam pipe* is still uncomfortably large, if the beam pipe is circular with a diameter of 30 mm, and the horizontal beam size (FWHM) is approximately 10 mm. The beam pipe sagitta is therefore reduced by another factor of 4, to 2.0 mm, by constructing the dipole beam pipe from four straight sections, instead of two. A circular beam pipe of 30 mm diameter then has an acceptable horizontal physical aperture of 28 mm (minus the thickness of the beam pipe wall).

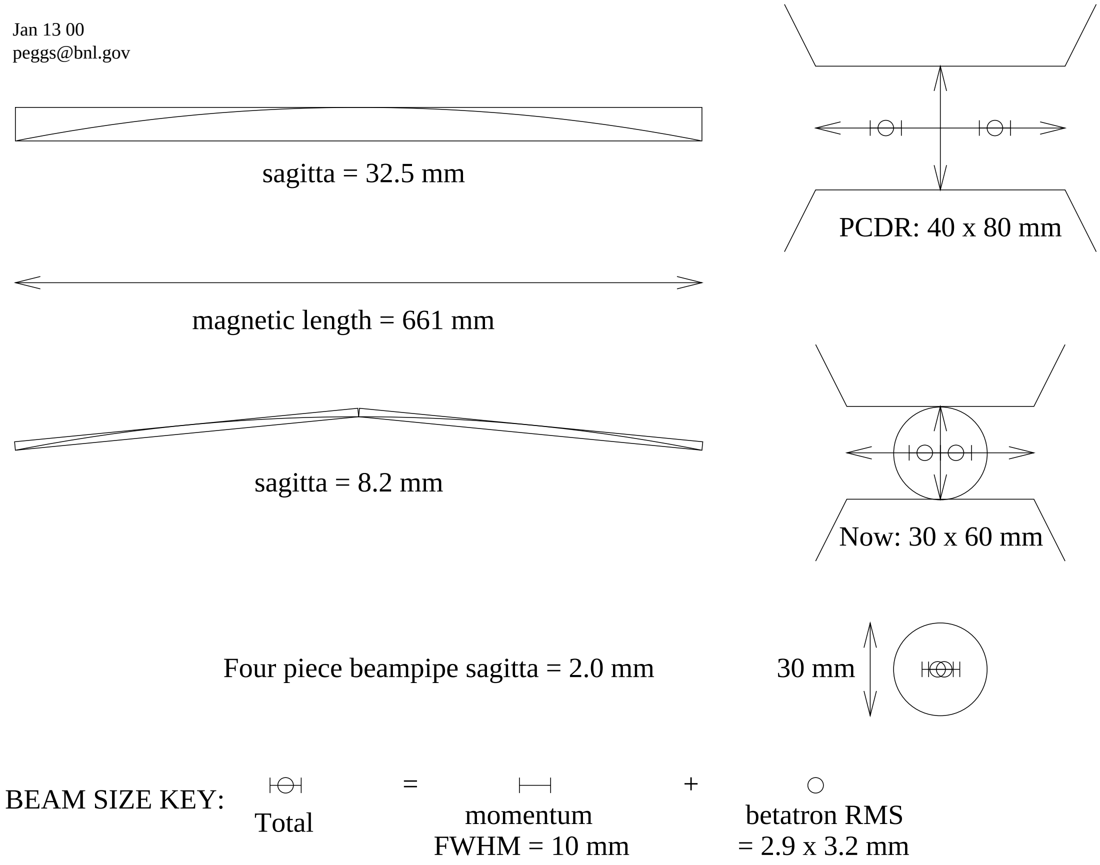

Figure 1: Beam sagitta in the RCMS dipole and in the beam pipe. The laminations of the dipole described in the PCDR are stacked in a rectangular jig, with a horizontal beam sagitta of 32.5 mm inside a straight aperture of 40 x 80 mm (V x H). The beam sagitta in the aperture of a chevron dipole – with two rectangular segments – is reduced to 8.2 mm, enabling the aperture to be reduced to 30 x 60 mm (V x H). The beam sagitta relative to the center of the beam pipe is reduced even further – to only 2.0 mm – if the beam pipe is constructed from four straight segments, instead of two. There is then ample physical aperture for the beam inside a 30 mm diameter pipe.

# 3 Beam pipe eddy currents

Eddy currents in the dipole beam pipe are driven by the oscillating part of dipole field, given by

$$B_{drive} \;=\; B_0 \sin(\omega t) \tag{1}$$

where $B_0 \approx 0.75$ T if the peak field is 1.5 T. At a horizontal distance $x$ from the pipe centerline, and when the skin depth is much greater than the pipe thickness, the eddy current density is

$$j \;=\; j(x) \;=\; \sigma \dot{B} x \tag{2}$$

where $\sigma$ is the conductivity of the pipe, and $\dot{B}$ is the rate of change of dipole field. These eddy currents distort the dipole magnetic field (in time and space), and also cause beam pipe heating. With a 30 mm diameter Inconel beam pipe the cycling frequency $f_{AC}$ is increased to 60 Hz from the 15 Hz value quoted in the PCDR.

## 3.1 Dipole field distortions

A thin circular beam pipe has a $\cos\theta$ eddy current distribution which only distorts the dipole field – there is no sextupole component, et cetera. According to Chao and Tigner [6] (p. 264) a beam pipe of radius $b$ and thickness $t$ causes the net dipole field to become

$$B \;=\; \frac{B_0}{\sqrt{1+\omega^2\tau^2}} \sin\left(\omega t - \tan^{-1}(\omega\tau)\right) \tag{3}$$

where

$$\tau \;=\; \mu_0 \sigma b t / 2 \tag{4}$$

Assuming an Inconel 625 or X750 beam pipe, with $b = 15$ mm and a resistivity of $\rho = 1/\sigma \approx 1.25\ \mu\Omega{-}\mathrm{m}$, then

$$\tau \;=\; 7.5\ \ t\,[\mathrm{mm}]\ 10^{-6}\ [\mathrm{s}] \tag{5}$$

Combining this with an angular frequency of $\omega = 2\pi f_{AC}$ gives

$$\omega\tau \;=\; 0.0028\ \frac{f_{AC}\,[\mathrm{Hz}]}{60}\ \ t\,[\mathrm{mm}] \tag{6}$$

The dipole field is barely perturbed by a 1 mm thick beam pipe.

## 3.2 Beam pipe heating

The instantaneous heating power per unit volume of beam pipe is

$$\frac{dP}{dV} \;=\; \frac{j^2}{\sigma} \;=\; \sigma \dot{B}^2 x^2 \tag{7}$$

so that the instantaneous power per unit length is

$$\frac{dP}{ds} \;=\; \pi\sigma\dot{B}^2 b^3 t \tag{8}$$

and the average power per unit length is

$$\left\langle \frac{dP}{ds} \right\rangle \;=\; 2\pi^3 f_{AC}^2 \sigma B_0^2 b^3 t \tag{9}$$

With an Inconel pipe of radius $b = 15$ mm, and an amplitude of $B_0 = 0.75$ T, this power becomes

$$\left\langle \frac{dP}{ds} \right\rangle \;=\; 339\;[\mathrm{W/m}] \;\; \left(\frac{f_{AC}\;[\mathrm{Hz}]}{60}\right)^2 \;\; t\;[\mathrm{mm}] \tag{10}$$

Chao and Tigner [6] (p. 315) state that the temperature rise above an ambient temperature of 300 K due to the free convection of air over a vertical panel of height $h$ is

$$\Delta T \;=\; 0.454 \left\langle \frac{dP}{ds} \right\rangle^{4/5} h^{-3/5} \;=\; 300 \left( \left\langle \frac{dP}{ds} \right\rangle /339 \right)^{4/5} \;\; [\mathrm{K}] \tag{11}$$

where an effective vertical height of $h = \pi b = 47$ mm has been assumed. In practice the nearby magnet poles impede the free flow of air over the beam pipe, but also act as substantial heat sinks.

From the perspective of beam pipe heating alone, it is desirable to use an Inconel beam pipe thinner than $t = 1$ mm. This is not a challenge to mechanical stability, since pipes thinner than 0.5 mm are strong enough to withstand atmospheric pressure, even without the use of reinforcing ribs.

# 4 Beam pipe impedance

The skin depth for Inconel is

$$\delta_s \;=\; \frac{1}{\sqrt{\sigma f \pi \mu}} \;=\; \frac{0.56}{\sqrt{f\;[\mathrm{MHz}]}} \;\; [\mathrm{mm}] \tag{12}$$

where it is assumed that the relative permeability $\mu_r = \mu/\mu_0 = 1$. The critical frequency above which wake fields will not penetrate a pipe of thickness $t$ is

$$f_c \;=\; \frac{1}{\sigma\pi\mu t^2} \;=\; \frac{0.314}{(t\;[\mathrm{mm}])^2} \;\; [\mathrm{MHz}] \tag{13}$$

This frequency is to be compared with that of the RF system, which is an approximate lower bound on the power spectrum of the wake fields of the bunch.

The radio frequency is the same as the revolution frequency $f_{rev}$, since the harmonic number $h = 1$. It increases from 1.27 MHz at injection to 6.61 MHz at top energy. Thus, an Inconel beam pipe must be thicker than about $t = 0.5$ mm for almost no wake fields to penetrate during injection.

If the same Inconel beam pipe of radius $b = 15$ mm is used around the entire circumference of the RCMS, then the longitudinal resistive wall impedance at a frequency $f$ well above $f_c$ is given by

$$\frac{Z_\parallel}{n}(f) \;=\; \frac{\mu_r Z_0}{2b}\,\delta_s \;=\; \frac{7.0}{\sqrt{f\,[\mathrm{MHz}]}}\;\;[\Omega] \tag{14}$$

where $Z_0 = 377\ \Omega$ is the impedance of free space, and $n$ is the mode number (so $f = n f_{rev}$). Similarly the transverse resistive wall impedance is

$$Z_\perp(f) \;=\; \frac{\mu_r Z_0 R}{b^3}\,\delta_s \;=\; \frac{0.51}{\sqrt{f\,[\mathrm{MHz}]}}\;\;[\mathrm{M}\Omega/\mathrm{m}] \tag{15}$$

where $R = C/2\pi = 4.55$ m is the average radius of the RCMS.

### 4.1 The longitudinal microwave instability

Following Chao and Tigner [6] (p. 118), the Boussard criterion for stability against the longitudinal microwave instability is written

$$\frac{Z_\parallel}{n} \;\le\; F'\frac{m_p c^2}{e}\frac{|\eta|\gamma}{I_b}\left(\frac{\Delta p_{FWHM}}{p}\right)^2 \tag{16}$$

where $F' \approx 1$ is a form factor depending on the details of the bunch distribution, and $\eta = 1/\gamma_T^2 - 1/\gamma^2$ is the slip factor. The peak gaussian bunch current is

$$I_b \;=\; N_B e f_{rev}\frac{C}{\sqrt{2\pi}\sigma_s} \tag{17}$$

where $C$ is the circumference, and the rms length $\sigma_s$ is always at least 3 m in the RCMS. Table 2 shows that the threshold impedances are orders of magnitude larger than that of the beam pipe when a nominal proton bunch of $N_B = 3.3 \times 10^9$ is injected at a current of 2.1 mA.

The low intensity strong focusing RCMS is very stable against the longitudinal microwave instability in particular, and against collective instabilities in general.

| Parameter | Injection | Top energy |
|---|---|---|
| Slip factor $\eta$ | $-0.81$ | $-0.43$ |
| Lorentz $\gamma$ | 1.008 | 1.288 |
| Bunch peak current $I_b$ [mA] | 2.1 | $\sim 1$ |
| Relative momentum spread, $\Delta p_{FWHM}/p$ | 0.006 | 0.001 |
| Threshold impedance $Z_{\|}/n$ [$\Omega$] | $1 \times 10^7$ | $5 \times 10^5$ |

Table 2: Parameters in the calculation of the maximum impedance for stability against the longitudinal microwave instability.

# References

[1] *Pre-Conceptual Design of a Rapid Cycling Medical Synchrotron*, S. Peggs editor, BNL C-A/AP/6, October 1999.

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