On Intra-Beam Scattering in the proton beam during TeV HEP

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Abstract

A back envelope calculation shows that the relative transverse beam size growh rate $(1/\sigma_x \, d\sigma_x/dt)$ is about 2.8% per hour, for protons, at 980, during good stores. Meanwhile, the lummisosity lifetime of the store is about 9 to 10 hours. When the collider runs optimally, this lifetime can be explained based on the anticipated loss of pbar and protons due to collision and emittance growths. It is shown that Intra Beam Scattering (IBS) is a significant source of this emittance growth. Predictions from the IBS model are compared with data. The predictions is in quantitative agreement with observation. Large uncertainties are due to initial measurement errors, and, we suspect, due to the fact that significant beam losses occurs at large (longitudinal and/or transverse) amplitudes.

1 Motivation and Outline

Peaked Luminosities of about $3.6\,10^{31}$ have been reached. However, our ultimate goal is to produce time-integrated luminosity. Therefore, the luminosity lifetime maters, as the shot setup time (and the time it takes to get a large stack) is not at all neglible compared to the intial luminosity lifetime: around 9 to hours, compared to 3 hours for shot setup time, and   10 to 15 hours to reach a large stack. These numbers do not take into account down-time due to necessary re-tuning of the Tevatron (orbit smoothing, 36x0 stores, Quench recovery, and so forth).

When the ratio of the proton and pbar emittances are of the order of one, the luminosity lifetime can be quantitatively understood as a simple sum of 5 distinct components: 2 terms due to the beam lifetimes (dominated by the pBar lifetime, as we have much fewer $\bar{p}$ and p) and 2 terms coming from the transverse beam size growth rate and, lastly, the change of the HourGlass factor[1]. while there is little one can do about beam losses due to collisions at B0 and D0, we could perhaps control the emittance growth better.

Emittance growth rates, in all 3 planes, are still small, but real. They are measured by the SincLite monitor (transversely) and by the SBD device (longitudinal) (see figure 1). In order to mitigate such emittance dilutions, we need to understand their origin. In this note, we start by describing the IBS formulas for uncoupled beams. We then successfully compare these predictions for an "uncoupled" lattice to recent store data. As the Tevatron lattice is strongly coupled, the emittance growth in the vertical plane is also signficant. We conclude by stating that(i) IBS has been observed, (ii) under current mode of operation, we should not place too much emphasis on decrease the bunch length, in order to keep the transverse emittance growth under control, or conversely, decrease the store duration to omptimize integrated luminosity.

2 Formulas for IBS

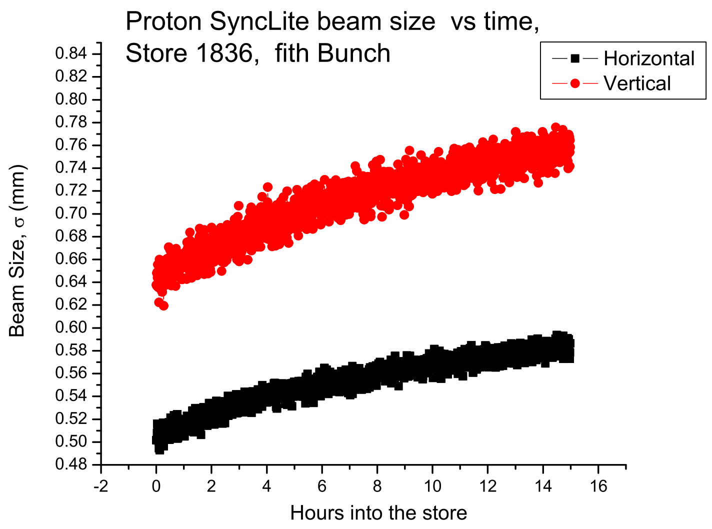
2.1 "Round beams", Crude Estimate

The relative time derivative of the rms (σ) of the horizontal betatron amplitude can be written as [2] and [3] :

$$\frac{d\sigma_x}{\sigma_x\, dt} \approx \frac{36\, L_c\, N_0\, r_0^2\, d\, D_p}{16\, \gamma\, \epsilon_n^2\, \sigma_\tau\, \sigma_p\, \beta_x}$$

where

- L_c is the Coulomb logarithm, taken to be 26.
- N_0 is the number of proton per bunch at low beta, $\approx 1.7\, 10^{11}$.



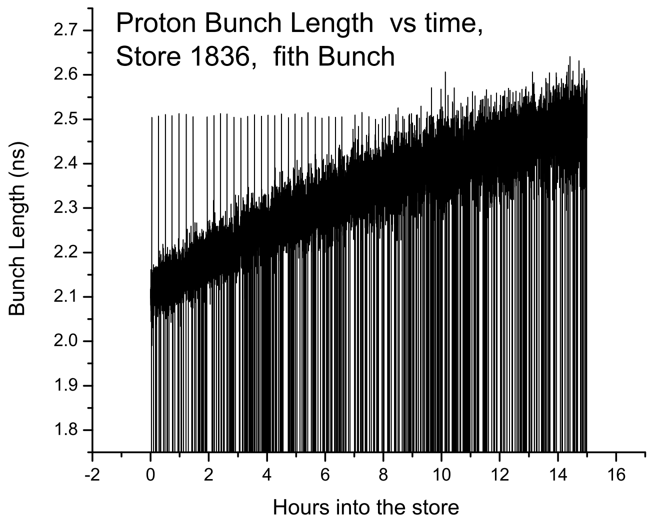


Figure 1: Top Plot: Raw SincLite data: the σ_x, σ_y of the Gaussian fits executed about every $\approx$ 30 seconds on the SincLite front-end computer. Bottom plot: The bunch length reported by the SBD front-end, about every 3 sec. Not all data is shown, for sake of graphical expediency.

- r_0 is the classical radius of the proton, $1.53\,10^{-15}$ mm.
- d is the ratio of the beam horizontal size due to dispersion and the total horizontal size of the beam, $d = D_p\,\sigma_p/\sqrt{\sigma_x^2 + D_p^2\,\sigma_p^2}$. Assume for now $d \approx 0.75$
- D_p is the average dispersion for the entire ring. Let us assume 4 m.
- β_x is the horizontal beta function. Assume 100 m.
- γ is the Lorentz relativistic factor, 1045.
- ϵ_n, the 98%, normalized transverse emittance, expressed in πmm (i.e., don't put the π in the formula above), $\approx 19\,10^{-3}$ mm.
- σ_τ, the rms sigma bunch length, expressed in sec, $\approx 2\,10^{-9}$ seconds.
- σ_p, the fractional momentum spread, $\approx 1.2\,10^{-4}$
- β_x, the average betatron function around the ring, $\approx$ 100m.

In addition to the assumptions made in [3], it is assumed that both twiss parameters and the emittances in both transverse planes are equal. This "back of the envelope" calculation gives a relative growth rate of $\approx 0.8\,10^{-5}$ per seconds, or $\approx$ 2.8% per hour, consistent with observations, as we shall see. In addition, a non-trivial prediction of IBS is :

$$\frac{d\sigma_p}{\sigma_p\,dt} = \frac{d\sigma_\tau}{\sigma_\tau\,dt} \approx \frac{d\;\sigma_x}{\sigma_x\,dt}$$

2.2 Gaussian and Elliptical beams, with correct averaging the Tevatron Lattice

A more accurate calculation has recently been done by Valery Lebedev[4], taking into account the proper averaging of the dispersion and betatron function, including the straight section. For elliptical beams, the emittance growth rate in the horizontal plane is (in the ultra-relativistic regime where $\beta = 1$.)

$$\frac{d\epsilon_x}{dt} = \frac{L_c\, N_0\, r_0^2\, Ax_{av}}{4\,\sqrt{2.}\,\gamma^3\,\sigma_\tau\,\sigma_x\,\sigma_y\,\sqrt{\theta_x^2+\theta_y^2}}$$

where the newly introduced (or modified) parameters are ;

- L_c is the coulomb logarithm, taken in this calculation to be 28.108,
- σ_x is the effective horizontal beam size (Gaussian σ), average over the entire ring, respectively, taken to be:

  $$\sigma_x = \sqrt{(\epsilon_{g,x}\,\beta_{x,av})\ +\ (D_{av}^2\,\sigma_p^2)}$$

  $\epsilon_{g,x}$ is the geoemtrical emittance (one sigma!). Thus, σ_x includes the contribution due to dispersion. The lattice beta $\beta_{x,av}$ and dispersion D_{av}^2 functions are averaged over the entire lattice.
- σ_y is the vertical beam size, average over the entire ring.
- θ_x, θ_y is the angular spread of the beam (one sigma), excluding the contribution from dispersion, once again, average over the entire ring, in the horizontal and vertical planes, respectively.
- Ax_{av} is length-wise average of the following combined Twiss function,

  $$\frac{D_x^2\ +\ (D_x\,\beta_x\ +\ \alpha_x\,D_x)^2}{\beta_x}$$

Likewise, longitudinally, we have,

$$\frac{d\epsilon_s}{dt} = \frac{L_c\, N_0\, r_0^2\, \Gamma_s^2}{4\,\sqrt{2.}\,\gamma^3\,\sigma_\tau\,\sigma_x\,\sigma_y\,\sqrt{\theta_x^2+\theta_y^2}}$$

where Γ_s can be defined as $(\sigma_\tau\, c)/\sigma_p$

3 Measurements

The analysis presented here relies exclusively on data coming from good HEP stores. Although dedicated studies have been conducted with coalesced proton beams, we opted for statistical accuracy here, as the data from such studies is a bit limitedin duration. Therefore, in comparing with IBS, we implictly make the asumption here that the emittance dilution due to the antiproton beam acting on the proton beam is small.

3.1 Data Selection

The dependency of diffusion rate on particle density, and/or on bunch intensity is the signature of IBS. Therefore we will refrain ourself from selecting stores based on good luminosity, or good specific luminosity, such that we include data at high and low particle densities. However, stores with obvious problems (beam abort) are rejected. We required colliding beams at 980 for at least ≈ 2.5 hours. The Fast Bunch Integrator (FBI), the Synchrotron Light monitor (SL), the Flying Wire (FW) and the Sample Bunch Display must be operating properly and reporting data for most of the 36 bunches.[1]. Finally, the Dataloggers (D44) must report data at a fraction of a Hz such that lifetime fits can be performed with reasonable accuracy (one measurement every 15 seconds for SyncLight, every 3 to 5 seconds for SBD). This data must be available in our D44 archives, so that we can chaneck and re-process the data at a later date.

The list of stores is:

1836, 1841, 1843, 1845, 1916, 1918, 1921, 1929, 1935, 1937, 1953, 1971, 1991, 2097, 2099, 2103, 2105, 2105, 2113, 2121, 2123, 2134, 2138, 2153, 2155

3.2 Bunch Length Growth Rates

The bunch length expansion rate, absolute ($d\sigma_z/dt$) and relative, i.e.;

$$r_z = 1/\sigma_z \, d\sigma_z/dt$$

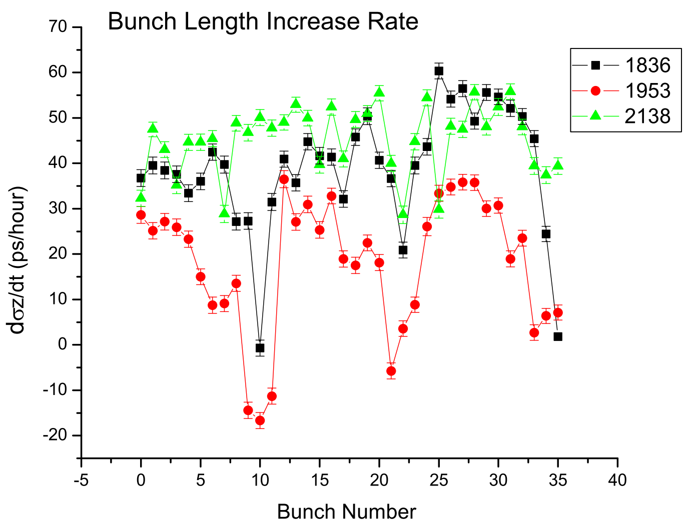
[1]For instance, some crude sanity check on the size of the SL beam spot size (σ_{sl}) are made. We require $0.05 < \sigma_{sl} < 5.0$ mm

for the 36 proton bunches are shown for the "record" stores 1953 and 1836, and for a recent, arbitrarily chosen, store (2138) on figure 2. These rates have been obtained with a linear fit of the SBD bunch length measurements. To reject obvious algorithm deficiencies of the SBD Front-End software, shown as glitches in figure 1, the relative change of one measurement to next ($\approx$ 3 seconds later) can not be greater then 25%. A relative error of 2% on each bunch length measurement is assumed, leading to a quite small error bars on the growth rate with respect to the bunch to bunch, or store to store, fluctuations in the data. These errors are based on the r.m.s. of the deviation with respect to the fit. The determination of the root cause of such fluctuations, or variability is one of our goals.

As shown later, the rate decreases as the bunches are getting more diffused. (This is already obvious looking at a randomly chosen bunch, as shown on figure 1.) Therefore, unless specified otherwise, we measure the expansion rate over a finite time, less than 2.5 hours, starting at the beginning of the store. The rate $d\sigma_z/dt$ is about 45 picoseconds/hour, or $\approx 2.5\%$ per hour, as shown on figure 3

3.3 Transverse Size Expansion Rates.

We define the transverse expansion rates, absolute and relative as we did for the longitudinal plane. The absolute expansion rates deduced from Synchrotron Light Detector fits are shown on figure 4. The overall pattern of variability in bunch/store is similar to the one observed for the longitudinal plane. This correlation is discussed more systematically below. As shown on figure 5, the transverse and longitudinal expansion rate are about equal.



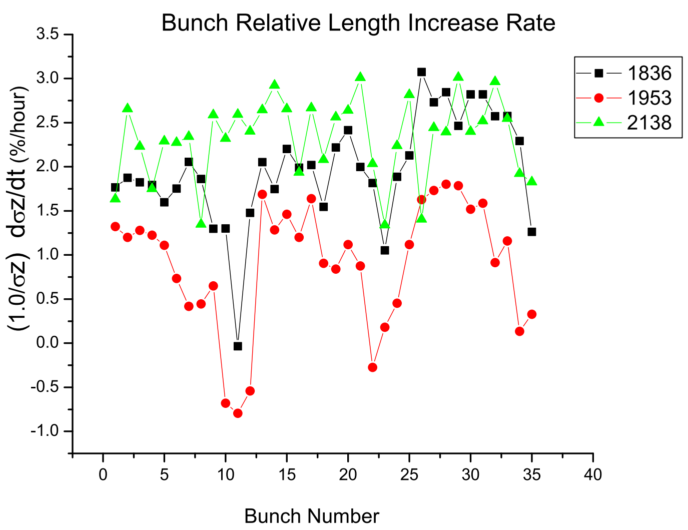


Figure 2: Top plot: The bunch length expansion rate for the two best stores (so far) and a recent pre-January shut-down store. Possible correlation with the bunch number is discussed below. Bottom plot: The relative bunch length increase rate r_z for these three stores.

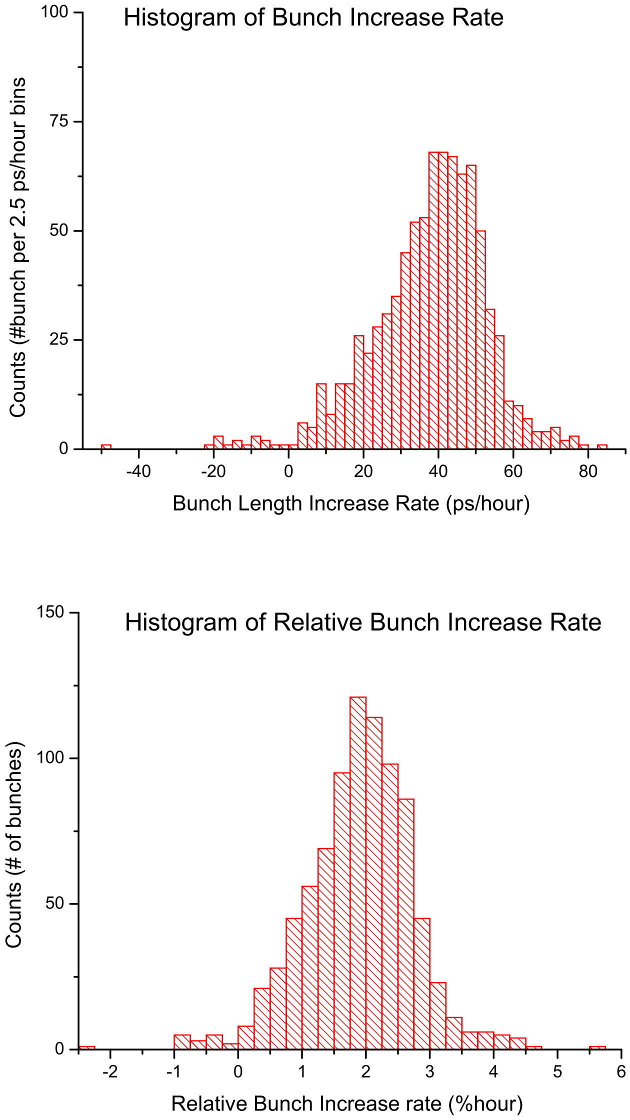


Figure 3: Histogram of the bunch length expansion rate (top), and the relative rate (bottom plot) for all proton bunches and all stores listed in the text.

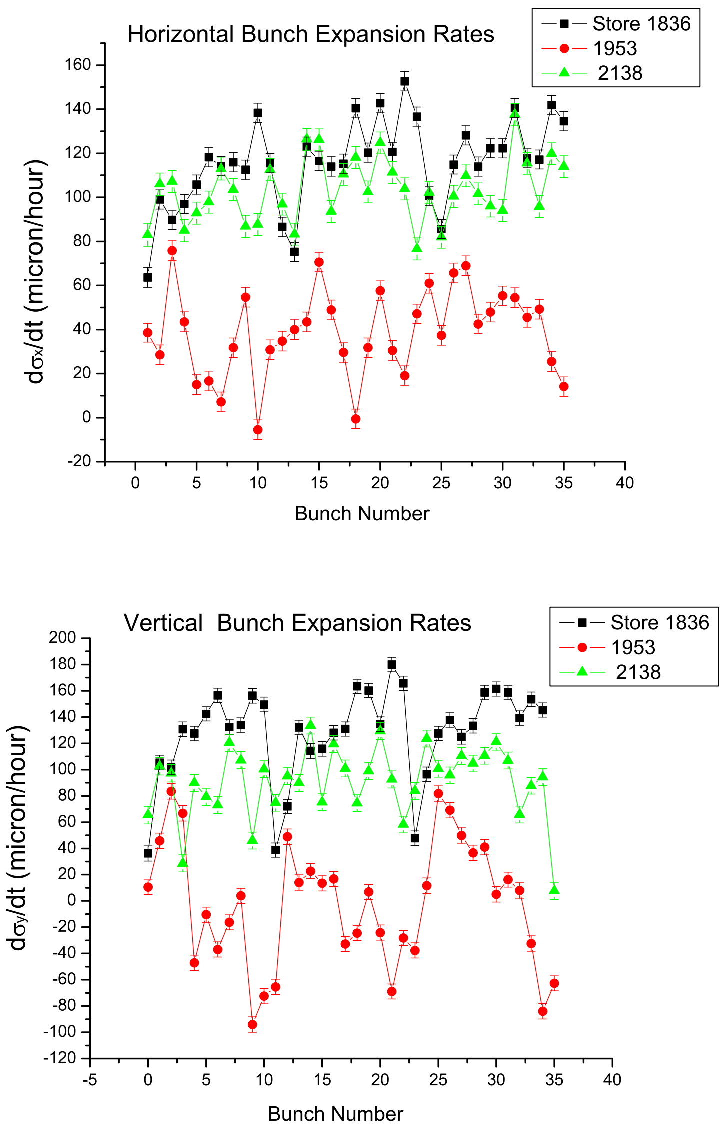


Figure 4: Top plot: The absolute transverse size (σ) expansion rate for the two best stores (so far) and a recent pre-January shut-down store. Possible correlation with the bunch number is discussed below. Top plot refers to the horizontal plane, bottom to the vertical plane.

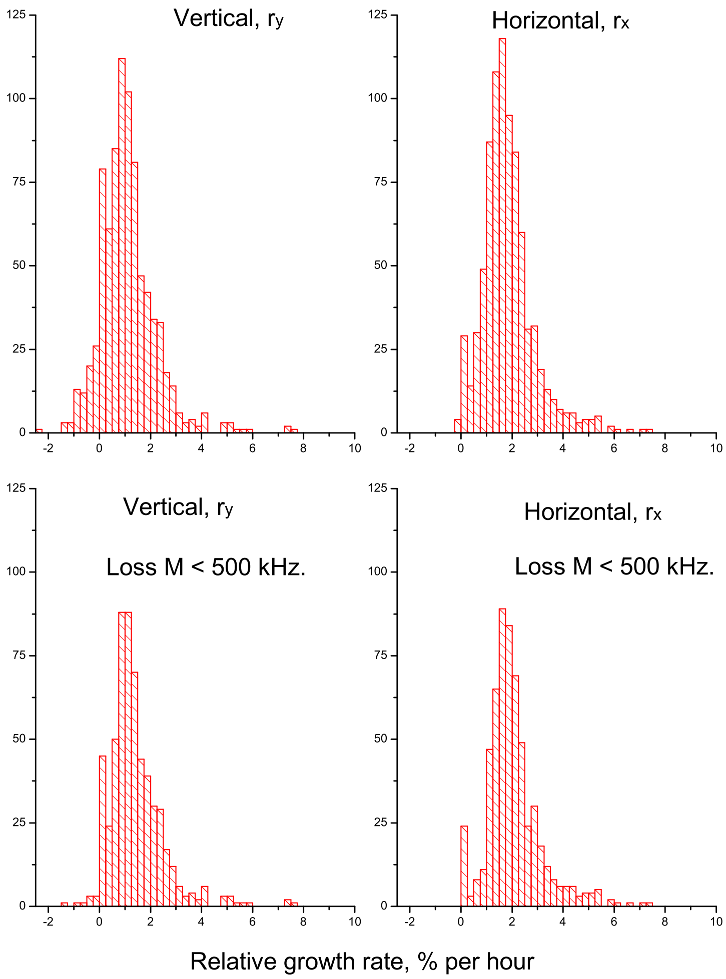


Figure 5: Histograms of the transverse, relative, expansion rates for all proton bunches and all stores listed in the text. The cut imposed in the two bottom plots is discussed next section.

4 Analysis

4.1 Second derivative of the bunch size vs time

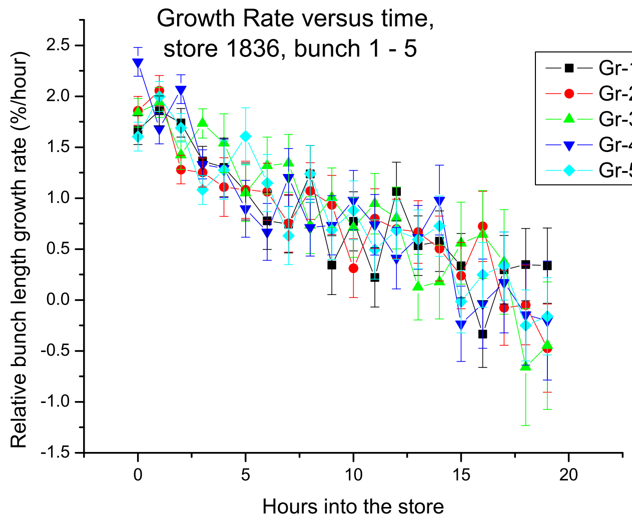
To the exception of store 1929[2], the bunch length growth rate tends to decrease with time. This is shown for a few bunches of the "golden store" 1836 on figure 6, and, more systematically for all stores mentioned above. The horizontal growth rate also does decrease with time, in average, but not as significantly as the bunch length. The growth rate of the vertical emittance remains more or less constant. This is shown on figure 7.

4.2 Correlation of bunch length growth rate and bunch intensity

This correlation has now been firmly established.[3]. The correlation coefficient between the bunch intensity (measured after scraping, just before the HEP store begins), and the bunch length increase rates are statistically significant, both for the relative and absolute increase rates. This result is based on the evolution of hundreds of distinct bunch is statistically significative, as shown on fig 8.

[2]During this store, the proton lifetime at 980 was very bad through the squeezing and the scraping. The betatron tunes were not optimum. After re-scraping, a normal HEP store started. It is unclear if this store should be counted as a good one for this analysis, due to the anomalous initial conditions

[3]There still might be some confusion here. In his recent talk on the status of the Tevatron, Vladimir Shiltsev mentioned that such a correlation is not clearly established. His statement is based on an analysis of the store 1028 done in March 2002[6]. A dedicated beam study confirmed this result apparently few month later. However, this last data set has been re-analysed, and showed that the level of correlation is consistent with the correlation seen in stores taken at that time[5]. Results presented here are consistent with this latter analysis. Note that the value of $d\sigma_z/dt$ quoted in Vladimir's note are based on only a few SBD measurements per hour, resulting in a loss of statistical accuracy in rate measurement. There was also some confusion about relative vs absolute rates: the dependency of these two quantities on bunch intensity are different. In addition, no cuts on beam losses have been placed (see below)



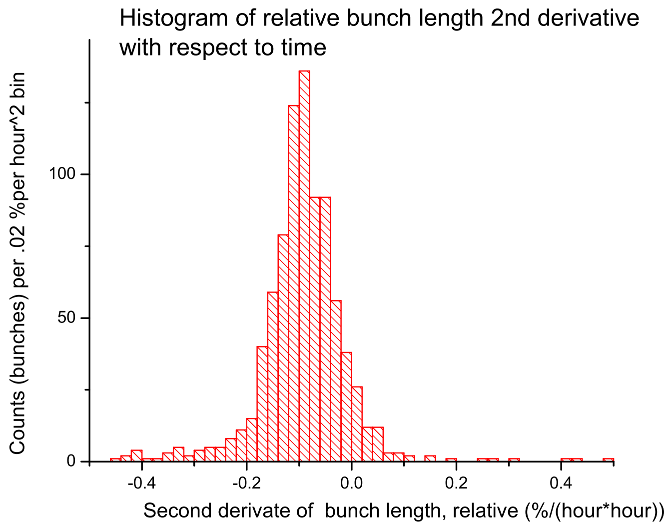


Figure 6: Top: the relative bunch length growth rate for bunch 1 through 5 (bunch 1 being the second bunch in the train) vs time, for store 1836. Linear fits on such plots have been performed systematically for all stores, yielding an estimate of the 2nd order derivative of the relative bunch length with respect to time (growth acceleration) Bottom Plot: Histogram of this "acceleration".

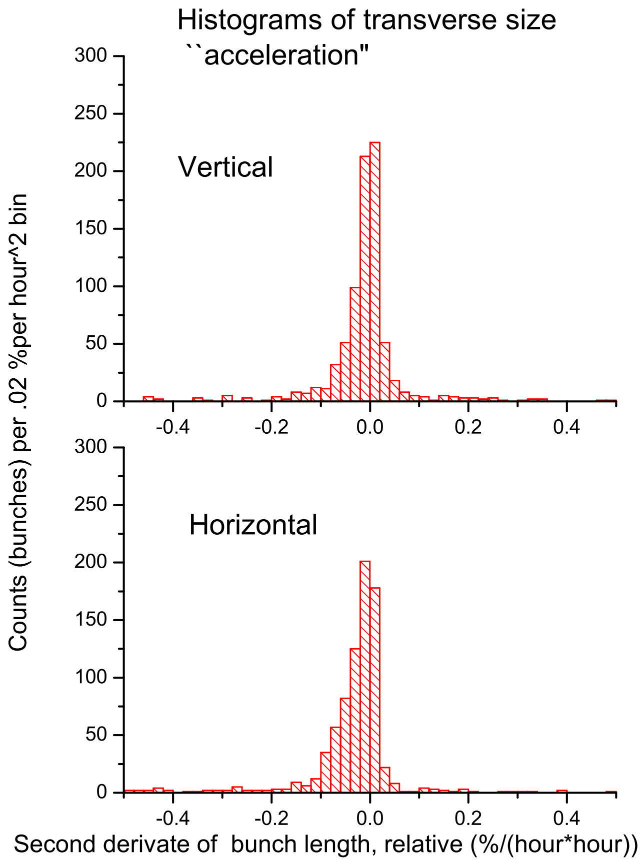


Figure 7: Histogram of the second derivative of the bunch size (σ) with respect to time, divided by this σ

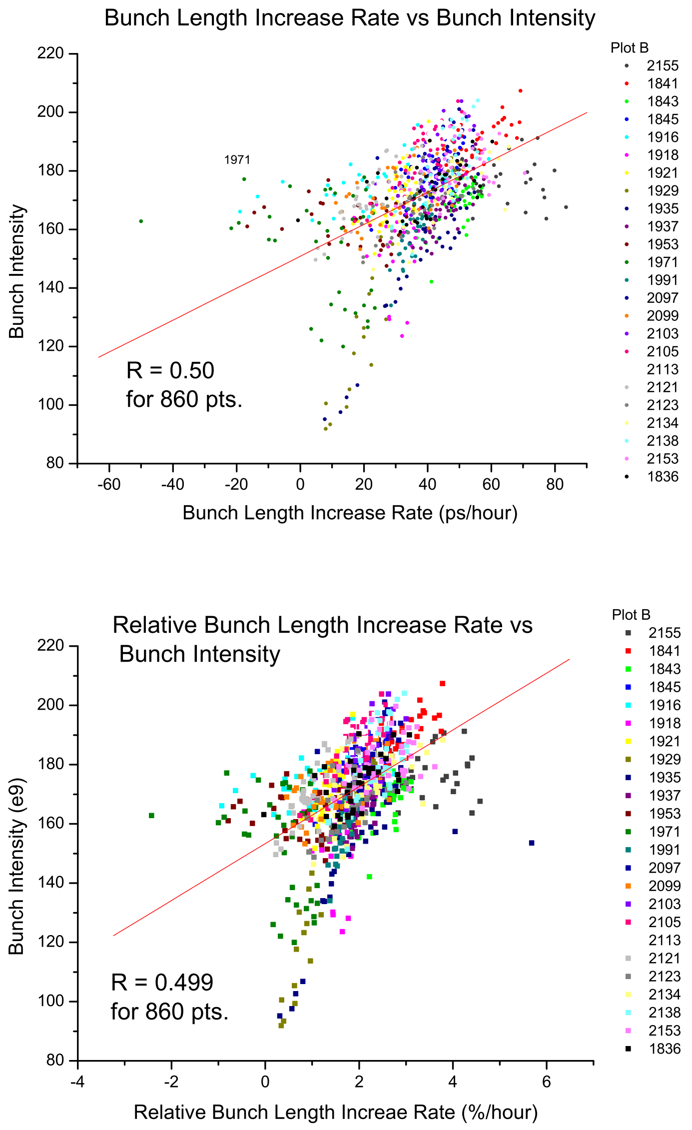


Figure 8: Correlation plot between the bunch intensity (e9) and the bunch length expansion rate (top plot), and the relative rate (bottom plot).

4.3 Rejection of Leaky Bunches

Large fluctuations in growth rate are observed. In fact, a few bunches are shrinking instead of expanding, presumably because they loose particles at predominantly high longitudinal amplitude. If we reject bunch leaking more than 500,000 particles per second (corresponding to a relative loss of 1% per hour, for a bunch of 1.810^{11} protons), bunches with a negative growth rate disappear (see figure 9). These particle losses due to any effect except those due to HEP collisions at B0 or D0 that are predominantly high amplitude losses, transervse or longitudinal, can be computed based on the instantaneous luminosity and the bunch intensity lifetime[4]. These "machine" losses (as opposed to Luminosity losses) are well correlated with the proton loss counter at B0. Therefore, we will reject any bunch with a machine loss rate greater 500 Khz.

As shown in the bottom plots of figure 5, the bunches whose transverse size decreases are also rejected by the same cut. It is however a bit surprising that this "bunch shaving" process occurs in the vertical plane and not in the horizontal plane.

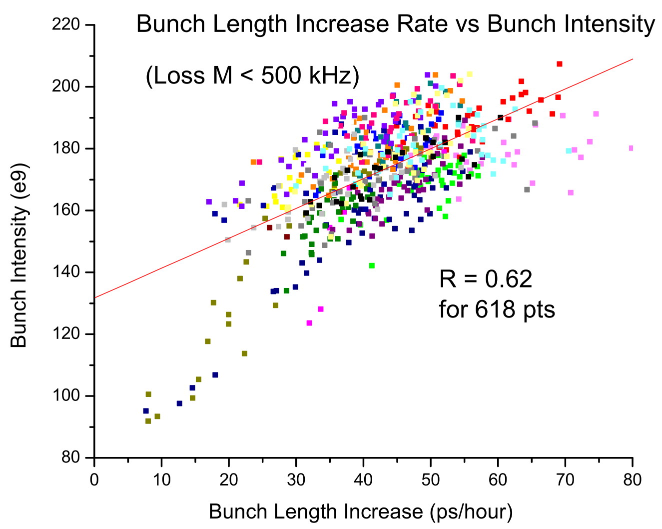
4.4 Directional Correlations

Bunches are expanding in all 3 spatial directions. These growth rate are definitely correlated to each others, as shown on figure 10. Once again, there are clearly multiple populations in these samples, for which the correlation factor are different from one an other. These do fluctuate on a store by store basis.

4.5 Comparison with IBS Calculation

Instead of establishing a correlation between emittance and growth rate, let us simply compare these growth rates, on a bunch by bunch basis, the predicted growth rate assuming IBS is the only diffusion source. These growth rate are measured at the beginning of each store, based on, at most, the first 2.5 hours of the HEP phase of the store. The predicted growth rates

[4]There is a caveat here, due to slopiness in our SDA Data Acquisition system: we assume that, for a given proton bunch, the loss due to luminosity is the same at CDF and D0, and is taken from the CDF luminosity counter.



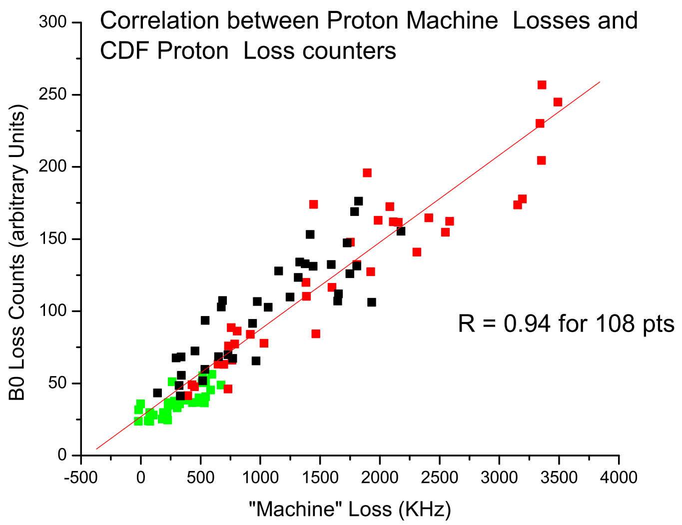


Figure 9: Correlation plot between the "machine" losses and the bunch length expansion rate (top plot). Also shown below is the expected strong correlation between these "machine" losses and the proton loss counters located on the CDF detector. Only a few store are shown, for sake of expediency.

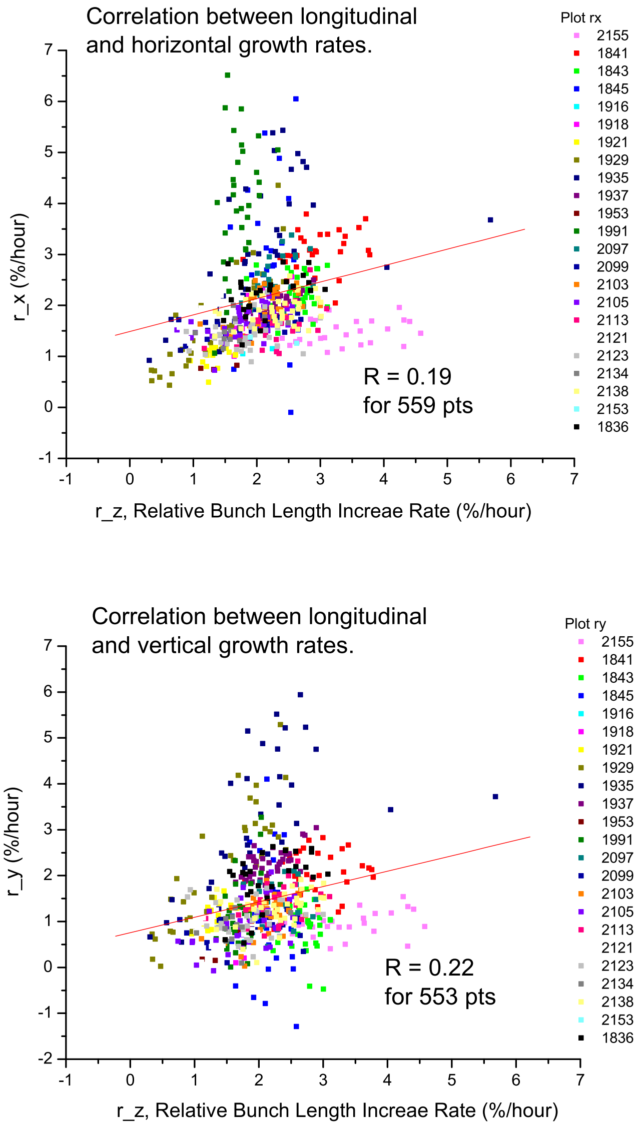


Figure 10: Correlation plot between the relative longitudinal growth rate and the relative horizontal (top plot), or vertical (bottom plot) growth rate. Both correlation coefficient are statistically significative.

are computed using the transverse beam sizes reported by the Synchrotron Light (SL) detector, the lattice functions at that location, and the averages (over the entire ring) of the lattice function described above. The beta functions at (SL) are taken to be 50 m and 107 m, horizontally and vertically, respectively. The horizontal dispersion is 1.517 m. The momentum spread δ_p is deduced from SBD bunch length measurements using the numerical fits from Mike Church[7].

Leaky bunches are rejected (Machine losses $<$ 500 Khz). The direct comparison IBS theory vs data for the longitudinal plane is shown on figure 11. Excluding store 1929, 1935, 1991 and 2097, a nice correlation is observed. We reject these stores because the exceedingly large predicted growth rates predicted are suspicious. The beam sizes reported by SL are most likely the culprit, as, for these stores, the $\sqrt{\epsilon_y}$, as reported by the Flying wires after scraping, are not well correlated with the vertical size of the beam reported by SL (see figure 12).

The comparison between the horizontal growth rate, predicted vs observed is shown on figure 13. Stores with poor SL measurements have been rejected. Also shown is the correlation between the radial expansion rate of the bunches and the IBS-predicted horizontal growth rate.

4.6 Deviation from IBS, Bunch Number Dependency

At this point, we have rejected perturbed bunches, or those for which we have poor emittance measurements. It is therefore justified to ask if any other correlation that would *not* be related to IBS could appear in the sample. For instance bunch to bunch induced instabilities, or if bunch loading is a significant factor, than a signficant correlation between the bunch number in the 12-bunch trains could appear. As shown on figure 14, this is not the case (The correlation factor R is this plot is -0.049 for 486 points, which barely one sigma effect.).

5 Discussion

The longitudinal growth rate varies greatly from bunch to bunch as well as store to store. Sunch fluctuations can not be entirely explained by Intra Beam Scattering. In order to reject bunches that are unstable, or "leak-

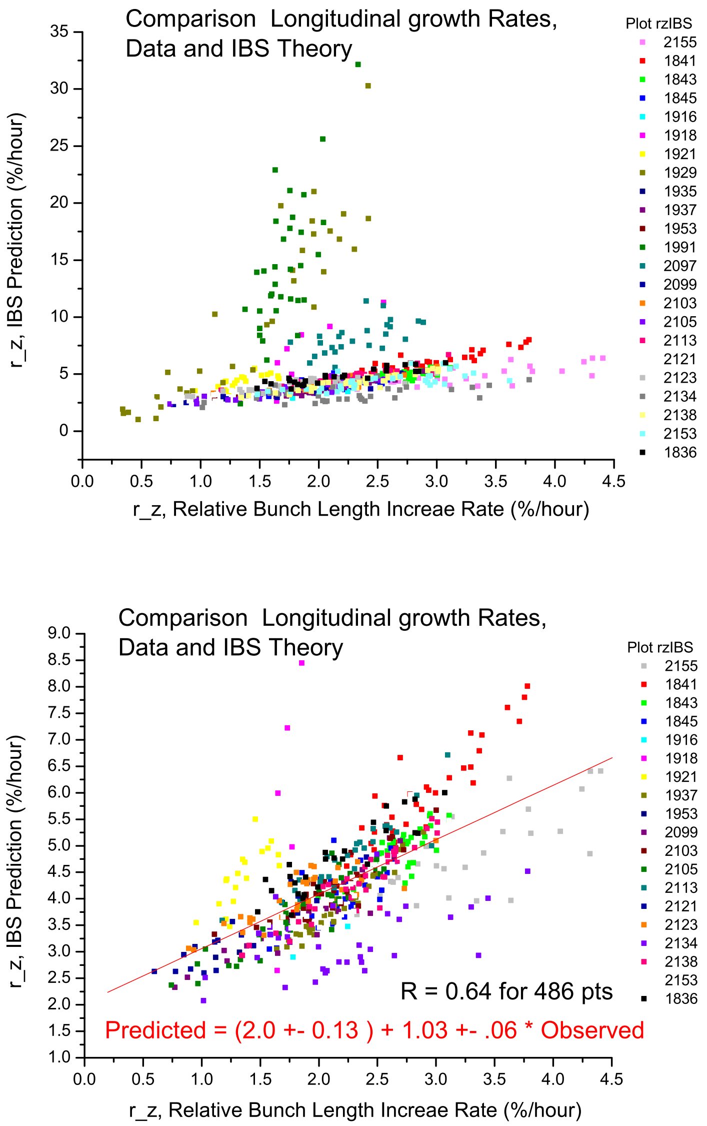


Figure 11: Correlation plot between the relative longitudinal growth rates, observed and predicted from the IBS theory. On the bottom plot, 4 additional "bad stores" have been rejected.

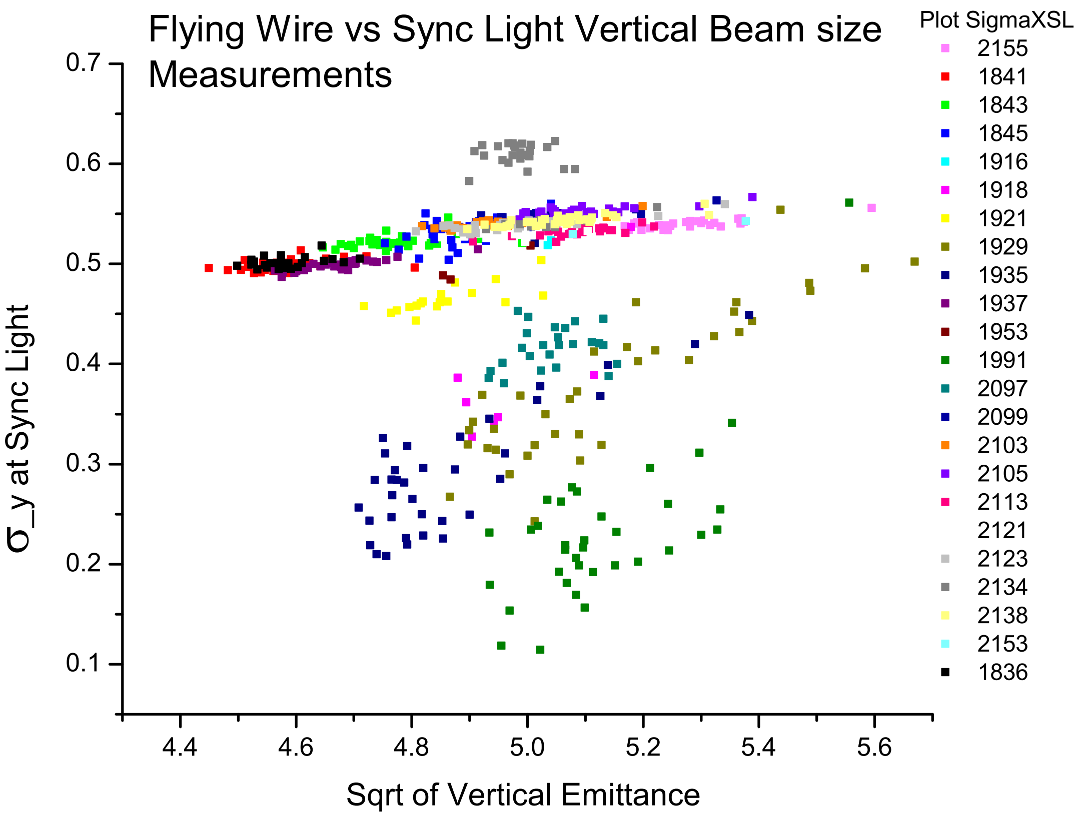
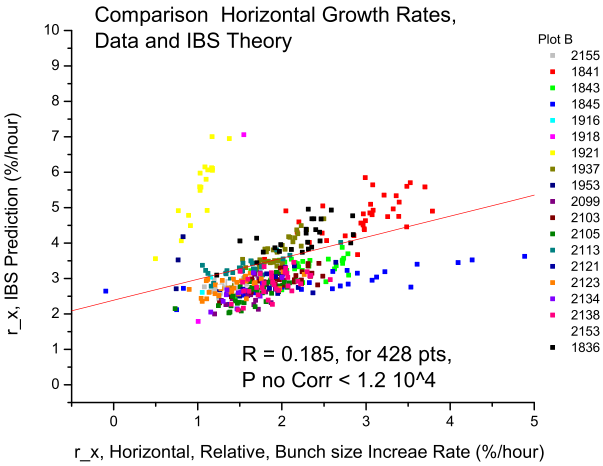


Figure 12: Correlation plot between the square root of the vertical emittance reported by the flying wire and the vertical size of the proton bunches



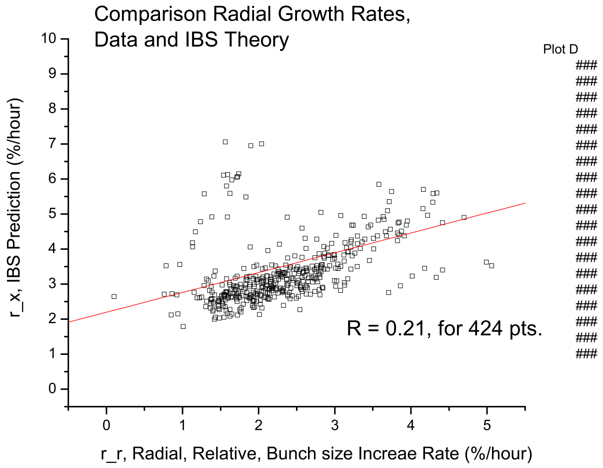


Figure 13: Correlation plot between the relative horizontal growth rates, observed and predicted from the IBS theory (top). On the bottom plot, we compare the observed radial expansion rate to the IBS-predicted horizontal expansion rate

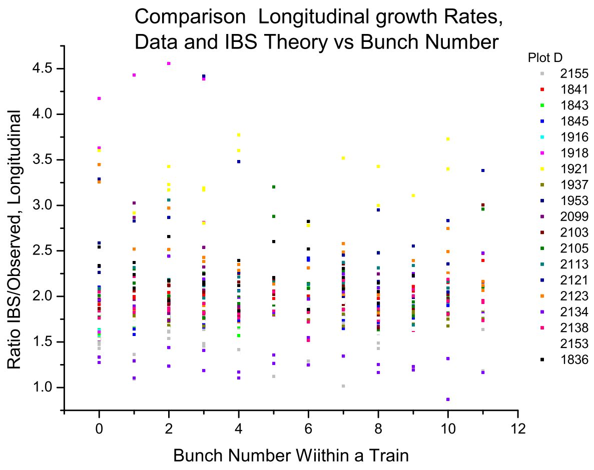


Figure 14: Correlation (or lack thereof) plot between the bunch number and the ratio between the IBS-predicted relative longitudinal growth rate, and the observed one.

ing", a cut on the non-luminoisity particle loss has been imposed. For the remaining sample for which we have relatively good (10 to 20%) consistency between the vertical Flying Wires and Synchrotron Light detectors, both the transverse and longitudinal relative beam size growth rates agree with the IBS prediction, wtihin a factor two or better. In addition, a nice linear correlation is observed between measured and predicted values of r_z in the longitudinal plane, for which the measurements are a bit more accurate. Note that this correlation, shown on figure 11, is definitly more convincing that the correlation between growth rate and intensity (figure 8), because longitudinal emittance and bunch intensities are themself correlated in the Main Injector, thereby confusing the intensity correlation.

The predicted longitudinal grow rate is systematically higher than the one observed[5]. This is either coming a residual shaving effect, where the beam size can't grow because of dynamical - or otherwise - aperture restriction, or conversely, from a systematic underestimation of the tranverse beam size. The latter conjecture is unlikely, given our specific luminosity measurements. When a more systematic study of the emittance measurement errors becomes available, we will perhaps understand these discrepancy, and the significant spread shown on figure 13 will hopefully be smaller.

IBS predicts very small growth in the vertical plane. However, if the betatron X-Y coupling is large, and as the tunes are not that far apart, an emittance growth in the horizontal plane will show-up soon or later in the vertical plane. In fact, the observed growth rate in the radial plane (X and Y growth rate added in quadrature) agrees well with the horizontal growth rate predicted by IBS. A more thorough comparison between IBS for a strongly coupled lattice and this data remains to be done.

Despite these uncertainties, it is very likely that we observed IBS during HEP, for numerous stores taken during the first part of RunIIa. Getting the right figures here is far from being strictly academical: as we hope to increase the density by at least a factor two (by rasing the bunch intensity by a factor $\approx 300/180$, and, at the same time, significantly reducing the bunch length, which is currently a factor two larger than designed), it is clear that severe emittance dilution will occur in the first few hours of the store. In addition, the growth rate at 150 GeV is also non negligible. This

[5]The factor two between the relative emittance growth and the relative beam size growth rate has been properly taken into account (I think)

will force us to rethink luminosity leveling, and/or reducing the store turn around time to a few hours. An other possibility is to voluntarily diffuse the bunch longitudinally, so that the transverse emittance growth, to which the luminosity is quite sensitive, can be mitigated.

That is, if IBS at 150 Gev does not dilute the emittance right away. The predicted rate at 150 GeV are about twice as large as those at 980 GeV. Note that we need to reduce the longitudinal emittance at injection, to avoid stringent dynamical aperture restriction. Thus, the optimum running conditions are not obvious...

References

[1] P. Lebrun *On Tevatron Lifetimes, Beam and Luminosity* talk presented the at the RunII commissioning meetings, see http://www-cpd.fnal.gov/personal/lebrun/TeVMeeting/Runii- Nov14/ProtLife150-ProtIBS1-Nov02_v2.pdf

[2] J. Wei and G. Parzen *Intra-Beam Scattering scaling for very Large Hadron Colliders*. PAC 2001 conf, Chicago.

[3] G. Parzen, *Intra-Beam Scattering at High Energies* Nucl. Instr. Meth **A256** 231(1987).

[4] Valery A. Lebedev *Tevatron Beam Heating*, private Mathematica notebook, October 2002.

[5] P. Lebrun, *On Longitudinal Emittance Dilution, July 9th study, Revisited* Talk presented early August at the Tevatron meeting, added in the Beam document data base along with this note.

[6] V. Shitlsev *First look into the SBD operation and bunch length growth analysis* Beam Document # 218

[7] M. Church, private communication. See also note *A study of the Emittances of P1 during Proton Injections for Store 2070* December 18 2002