Magnetic propulsion of a conducting fluid and the theory of a controlled tokamak fusion reactor

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Abstract

Effect of magnetic propulsion has been discovered for conducting fluid in toroidal magnetic field. The theory has been developed for intensive, driven by magnetic propulsion, free-surface streams and for convective cells inside closed volumes. Application of this theory for DT fusion tokamak reactor leads to new solutions for its two most critical elements, i.e., the first wall and the heat/lithium exchanger (breeder). As a result, for the first time, the development of environmentally clean and compatible as well as controlled tokamak DT fusion reactor was put on a realistic path, leading to competitive, commercial tokamak fusion reactors.

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OUTLINE

1. The effect of magnetic propulsion.
2. MHD equations for a thin layer.
3. Free-surface stream in a toroidal field.
4. Convection in a closed volume.
5. Interaction with Hartman layers and equilibrium fields.
6. The theory of a controlled tokamak fusion reactor
   (a) Basic data on Lithium.
   (b) Lithium jet for the tokamak divertor.
   (c) The first wall with a Li stream.
   (d) The heat/lithium mixer (breeder).
   (e) Plasma scenario, stability and confinement.
   (f) Plasma refuelling.
7. Summary. The concept of commercial, environmentally clean and compatible tokamak fusion reactor (LDR).

1 The effect of magnetic propulsion

Both electro-technique and MHD use the Lorentz force

$$\mathrm{j} \times \mathrm{B} \tag{1.1}$$

in order to create a motion.

In electro-technique it creates a motion perpendicular to both j and B.

In MHD it is possible to creates a motion either along j or B.

In plasma physics, the plasma propulsion along B^* is well-known in magnetic reconnection.

$$V_\parallel^2 = \frac{B_X^{*2} - B_Y^{*2}}{4\pi n m_i}. \tag{1.2}$$

Magnetic propulsion is based on nonuniformity of j $\times$ B.

In the simplest case of rectangular aquarium with electrodes,

$$\mathsf{j} = j\mathsf{e}_x, \quad \mathsf{B} = B\mathsf{e}_y \tag{1.3}$$

the vertical Lorentz force

$$\mathsf{j} \times \mathsf{B} = jB\mathsf{e}_z \tag{1.4}$$

is balanced by one component of a pressure gradient

$$p'_z = jB, \quad p = (z-h)jB. \tag{1.5}$$

Then, the horizontal component of the pressure gradient

$$-p'_x = -\left[(z-h)jB\right]'_x \tag{1.6}$$

creates a horizontal propulsive force.

2 MHD equations for a thin layer

We consider a simple configuration of the only toroidal magnetic field

$$B_\varphi \equiv B = \frac{0.2 I_{pol}}{r}, \tag{2.1}$$

The geometry of the guiding plate surface, contacting with the fluid,

is described by parametric equations

$$r = r(s), \quad z = z(s), \tag{2.2}$$

where r, z are the cylindrical coordinates, and s is a coordinate along the plate, measuring the distance from some reference point

$$r = r_i, \tag{2.3}$$

and n is a perpendicular coordinate.

In thin layer approximation

$$n\frac{\partial}{\partial s} \ll 1. \tag{2.4}$$

Kinematics of incompressible fluid is described with use of the stream function $f(s,n)$

$$\mathsf{v} = V\mathsf{s} + w^{-1}\nabla f \times \mathsf{e}_\varphi = V\mathsf{s} - w^{-1}f_n'\mathsf{s} + w^{-1}f_s'\mathsf{n}, \tag{2.5}$$

where w is the width of the flow. In a complete axisymmetric flow it is simply

$$w = 2\pi r(s), \tag{2.6}$$

while it can be intentionally designed by limitting flow by special channels.

In the same way, the electric current density can be presented in terms of the current stream function $i(s,n)$

$$\mathsf{j} = J\mathsf{s} + w^{-1}\nabla i \times \mathsf{e}_\varphi = J\mathsf{s} - w^{-1}i'_n\mathsf{s} + w^{-1}i'_s\mathsf{n} \qquad (2.7)$$

and the net longitudinal current density J, which for simplicity is assumed to be uniform across the stream $J = J(s)$. It is determined by the height $h(s)$ of the fluid and the total electric current I as

$$J = \frac{I}{w(s)h(s)}, \qquad (2.8)$$

which reduces simply to

$$J = \frac{I}{2\pi r h} \qquad (2.9)$$

for a complete flow covering full axisymmetric plate.

The Ohm's law

$$E\mathsf{s} - \nabla\varphi + \mathsf{v}\times\mathsf{B} = \frac{\mathsf{j}}{\sigma}, \quad E \equiv \frac{J}{\sigma} \tag{2.10}$$

determines the current distribution in the fluid, while the equation of motion

$$\begin{aligned} \rho\frac{\partial\mathsf{v}}{\partial t} + \rho v_s v_s'\mathsf{s} + \nabla p &= \mathsf{j}\times\mathsf{B} + \nu v_{s,nn}''\mathsf{s}, \\ \rho\frac{\partial\mathsf{v}}{\partial t} + \rho v_s v_s'\mathsf{s} + \nabla p &= \mathsf{j}\times\mathsf{B} + \nu V_{nn}''\mathsf{s} - \nu w^{-1} f_{nnn}'''\mathsf{s} \end{aligned} \tag{2.11}$$

determines the velocity profile with ν being viscosity of the fluid.

3 Free-surface stream

For the thin stream, assuming fluid supply such that

$$V >> |\nabla f|, \tag{3.1}$$

we can neglect the convective motion of the fluid

$$\mathbf{v} = V\mathbf{s}. \tag{3.2}$$

The perpendicular component of the Ohm's law

$$-\varphi'_n + VB = \frac{j_n}{\sigma}, \tag{3.3}$$

gives an expression for electric potential

$$\varphi'_n = VB, \quad \varphi = nVB + \varphi_0(s). \tag{3.4}$$

The longitudinal component of the Ohm law

$$E-\varphi'_s=\frac{j_s}{\sigma}=\frac{J}{\sigma}-\frac{i'_n}{w\sigma},\quad E\equiv\frac{J}{\sigma},\quad n(VB)'_s+\varphi'_0=\frac{i'_n}{w\sigma} \tag{3.5}$$

determines the stream function i of electric current

$$i=w\sigma\frac{n^2-nh}{2}(VB)'_s,\quad \varphi'_0=-\frac{nh}{2}(VB)'_s. \tag{3.6}$$

It gives the expression for the perpendicular current density

$$j_n=w^{-1}i'_s=w^{-1}\left[\frac{n^2-nh}{2}\sigma(VB)'_s\right]'_s. \tag{3.7}$$

This current density creates a “diamagnetic drag” force for the longitudinal motion of the fluid (possibly locally negative).

Now, it is possible to consider the equation of motion.

The perpendicular force balance

$$p'_n = -JB, \quad p = (h-n)JB = (1-\overline{n})hJB, \quad \overline{n} \equiv \frac{n}{h} \tag{3.8}$$

determines the pressure profile in the fluid. We introduced here the locally normalized perpendicular coordinate $\overline{n}$.

The longitudinal equation of motion gives the longitudinal velocity profile

$$\begin{aligned} \rho V V'_s + p'_s &= -j_n B + \nu V''_{nn}, \\ \left(\frac{\rho V^2}{2} + p\right)'_s &= -j_n B + \nu V''_{nn}. \end{aligned} \tag{3.9}$$

Neglecting fluid viscosity, we have for the stream velocity the expression

$$\begin{aligned}\frac{1}{2}\rho(V^2 - V_i^2) + p - p_i &= -\int_0^s j_n B ds \\ &= -\int_0^s w^{-1}\left[\frac{n^2 - nh}{2}\sigma(VB)'_s\right]'_s B ds,\end{aligned} \tag{3.10}$$

where V_i and p_i are the incoming velocity and the pressure in the reference point.

In an explicit form

$$\begin{aligned}\frac{1}{2}\rho(V^2 - V_i^2) =&(1 - \overline{n})(h_i J_i B_i - hJB) \\ &- \int_0^s w^{-1}\left[\frac{n^2 - nh}{2}\sigma(VB)'_s\right]'_s B ds,\end{aligned} \tag{3.11}$$

where perpendicular coordinate acts as a parameter.

Neglecting diamagnetic dragging term

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_i^2 + (1-\overline{n})h_i J_i B_i \left(1 - \frac{w_i^2}{w^2}\right), \tag{3.12}$$

or for a complete flow

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_i^2 + (1-\overline{n})h_i J_i B_i \left(1 - \frac{r_i^2}{r^2}\right). \tag{3.13}$$

Propulsion factor P_M (for referencing to the middle point)

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_i^2 + (1-\overline{n})P_P \frac{IB_{tor}}{2\pi R}, \tag{3.14}$$

where

$$P_P \equiv \frac{4aR^3}{(R^2-a^2)^2}, \quad R \equiv \frac{r_i + r_e}{2}, \quad a \equiv \frac{r_e - r_i}{2}. \tag{3.15}$$

Comparison of the averaged propulsion effect

$$\frac{1}{2}h_iJ_iB_i\left(1-\frac{r_i^2}{r^2}\right)=\frac{\delta BB}{2\mu_0}\left(1-\frac{r_i^2}{r^2}\right)=\frac{0.1II_{pol}}{2\pi r_i^2}\left(1-\frac{R^2}{r^2}\right) \quad (3.16)$$

with the averaged drag force term

$$\frac{1}{h_i}\int_0^s w^{-1}\left[\frac{h^3}{12}\sigma(VB)'_s\right]'_s Bds \simeq \frac{1}{h_i}\int_{r_i}^r w^{-1}\left[\frac{h^3}{12}\sigma(VB)'_s\right]'_r Bdr \quad (3.17)$$

leads to a resistive diamagnetic flow parameter P_D

$$P_D \equiv \frac{\mu_0\sigma h^2 V}{12r}\frac{I_{pol}}{I} \quad (3.18)$$

relevant for magnetic propulsion.

P_D is controlled by the inflow velocity and the total electric current in fluid.

For thin layers it can be made much less than 1.

4 Convection in a closed volume

Closed volume has the contour prescribed by equation

$$r = r(s), \quad z = z(s), \tag{4.1}$$

with cross-section given by

$$h = h(s), \quad w = w(s), \tag{4.2}$$

where h is the total height (size along n) and w is the total width (size along φ). In this case there is no net velocity and only convection described by

$$\mathsf{v} = w^{-1}\nabla f \times \mathsf{e}_\varphi = -w^{-1} f'_n \mathsf{s} + w^{-1} f'_s \mathsf{n}. \tag{4.3}$$

Its vorticity $\mathsf{U} \equiv \nabla \times \mathsf{v}$ is given by

$$\mathsf{U} = [(w^{-1} f'_s)'_s + w^{-1} f''_{nn}]\mathsf{e}_\varphi \simeq f''_{nn}\mathsf{e}_\varphi = f''_{nn}\mathsf{e}_\varphi, \quad U \equiv f''_{nn}. \tag{4.4}$$

The equation of motion, rewritten in the form

$$\rho\frac{\partial \mathsf{v}}{\partial t}+\frac{1}{2}\rho\nabla \mathsf{v}^2-\rho \mathsf{v}\times U\mathsf{e}_\varphi+\nabla p=\mathsf{j}\times\mathsf{B}, \tag{4.5}$$

gives for vorticity

$$\rho\frac{\partial U}{\partial t}+\rho\mathsf{v}\nabla U=-(\mathsf{j}\nabla)B\simeq -j_r B_r'. \tag{4.6}$$

The second term in the LHS can be written as

$$\rho\mathsf{v}\nabla U=\nabla(\rho\mathsf{v}U), \tag{4.7}$$

which indicates that the equation for vorticity has no stationary solution, if the current is prescribed.

By integrating it over volume of the fluid, we get the rate of accumulation of vorticity in the fluid

$$\rho\frac{d}{dt}\int U r ds dh d\varphi=-\int j_r B_r' r ds dh d\varphi. \tag{4.8}$$

- Stationary solution for a given Voltage

$$\mathsf{v} = w^{-1}\nabla f \times \mathsf{e}_\varphi = -w^{-1} f'_n \mathsf{s} + w^{-1} f'_s \mathsf{n}. \tag{4.9}$$

From the Ohm's law

$$\varphi'_n = -w^{-1} f'_n B, \quad \varphi = -w^{-1} f B. \tag{4.10}$$

Relation between the stream function and Voltage

$$\varphi'_n = -w^{-1} f'_n B, \quad \varphi_e - \varphi_i = \left(\frac{B_i}{w_i} - \frac{B_e}{w_e}\right) \int_0^h v_s dn. \tag{4.11}$$

- Viscous stationary solution

$$-(nJB)'_s = \nu v''_{nn}, \tag{4.12}$$

which gives the velocity profile in the form

$$v = \frac{h^3}{\nu} \frac{\overline{n} - \overline{n}^3}{6} (JB)'_s. \tag{4.13}$$

5 Inter. with Hartman lrs and eq. flds

BEING NOT DISCLOSED

Problems	International Fusion Program	Post TFTR PPPL* *no*Fusion/*no*Science Program**
1. Plasma Containment	Huge progress since 70s	All major machines are in idle state
2. Plasma Control	No solutions, no realistic hopes for solutions	No ideas for tokamaks. Decision to desmantle major tokamaks.
3. The first wall	Traditional approach. No hopes for reactor.	Ignorance of the problem. Successful atack on ITER
4. Heat Exchanger	Traditional approach. No hopes for reactor.	Ignorance of the problem. Successful atack on ITER
5. Tritium reproduction	Traditional approach. No hopes for reactor.	Ignorance of the problem. Successful atack on ITER
6. Activation & environment	No solution.	Ignorance of the problem. No lesson has been learned (CIT,BPX,TPX)
7. Funding	Big problem	Complete disaster. Reliance only on personality of Rob and political games.
8. Interaction with the society	Only weakly justified promises.	Totally outdated.

* ***Fusion part of PPPL (excluding NSTX)***

** ***Italics made by L.Z.***

1. Mass density	$500\ \frac{kg}{m^3}$	half of water
2. Melting temperature	$\simeq 200\ C^o$	
3. Boiling temperature	$\simeq 1300\ C^o$	
4. c_p	$4200\ \frac{J}{kg \cdot K}$	like a water
5. Conductivity σ	$2.2 \cdot 10^6\ \frac{1}{Om \cdot m}$	20 times less than copper
6. Viscosity ν	$0.5 \cdot 10{-}3\ Pa \cdot s$	like a water

Essentially everybody, especialyy households, have experience with liquid Lithium when working in their backyards.

Magic numbers.

- In order to absorb the power $P_{th}$of 1 GW with increase in T^o of 300 C^o, it is necessary to propel 1 T/sec of Li (2 m^3).
- For fusion reactor with R=5 m, and Li layer of 1 cm thick the propulsion velocity is 7 m/sec.

$$V = \frac{10^{-3}\frac{dM}{dt}}{\pi Rh} \tag{6.1}$$

- Magnetic Reinolds number

$$R_m \equiv 3hV = 3\frac{10^{-3}\frac{dM}{dt}}{\pi R} = \frac{P_{th}[GW]}{R[m]} = 0.2 \tag{6.2}$$

No effect on equilibrium fields.

- Dynamic pressure of the fluid

$$
\begin{aligned}
P_d =& \rho \frac{V^2}{2} = 250 V^2 \ [Pa] = 2.5 \cdot 10^{-4} V^2 \ [MPa] \\
&= 25 \cdot 10^{-4} V^2 \ [Atm], \\
V =& 10, \quad P_d = 0.025 \ [MPa] = 0.25 \ [Atm], \\
V =& 100, \quad P_d = 2.5 \ [MPa] = 25 \ [Atm]
\end{aligned} \tag{6.3}
$$

Solution to the tokamak divertor problem:

1. Ordinary high-speed liquid Lithium jet ($V = 100$, $h = 1\ mm$)
2. Magnetically propulsed free-surface liquid Lithium stream ($V = 10$, $h = 1\ cm$) for transportation of absorbed power outside Vacuum Vessel and TFC.a

Completely resolves the problem of creation of stationary conditions for the plasma in the best way possible.

1. Thin (about 1 mm) toroidal chamber.
2. Magnetically propulsed free-surface liquid Lithium stream ($V = 10$, $h = 1\ cm$) which
   (a) withstand the atmospheric pressure;
   (b) absorbs the heat/particle flux;
   (c) transports it outside the chamber.

This is the first (since 50s) realistic solution of the first wall problem for the tokamak fusion reactor

1. No vacuum Vessel anymore. Full access to any point behine 2-5 cm from the plasma boundary;
2. Practacally undamagable wall, which is insensitive to thermal deformations, mechanical deformations or possible disruptions, and,
3. I believe, can be made completely undamagable by any neutron flux;
4. Solution, insensitive to cracks, holes etc.;

1. Thin (about 1 mm) toroidal chamber with C-shaped cross-section.
2. Magnetically propulsed convection of the liquid Lithium ($V = 1$, $h = 10 - 20\ cm$) which
   (a) provides most of the force only on the outer surface;
   (b) absorbs full neutron flux;
   (c) mixes temperature uniformly;
   (d) provides heat exchange outside TFC;
   (e) reproduces tritium;

The first (since 50s) realistic solution of the shield/breeder problem fro toroidal geometry, manifasting the BIRTH of concept of ecologically clean and compatible DT fusion reactors:

1. Ratio of weights of non-Lithium (activated) materials to Lithium (non-activated) to can be esealy made less than 10 % in the neutron zone.
2. No necessity in aneutronic fusion (totally unrealistic at present).
3. Practacally undamagable breeder, which is insensitive to thermal deformations, mechanical deformations;
4. I believe, can be made completely undamagable by any neutron flux;
5. Solution, insensitive to cracks, holes etc.;
6. No needs in stationary regimes for DT reactor. Pulsed tokamak regime with midnight recharging is perfect and compatible with the machine.
7. “Water” bag design is completely insensitive to un-probable plasma disruption of machine control failure.

Plasma scenarion IS NOT BEING DISCLOSED.

Plasma profiles (Partially DISCLOSED)

$$T_e = T_i = const, \quad n_e = n_e(r), \quad n_e(a) \simeq 0. \tag{6.4}$$

Plasma stability (Partially DISCLOSED). Skin penetration time of the Lithium

$$\tau_\eta = 3h^2. \tag{6.5}$$

For disruptions time

$$\tau_I = 0.0001 \ sec \tag{6.6}$$

the effective distance of the conductive wall is

$$1.7h = 0.01/\sqrt{3} = 1 \ cm. \tag{6.7}$$

Practically, no limits on q_a or β from stability considerations.

NOT BEING DISCLOSED.

7 Summary. The concept of LDR

In fact, a better name, reflecting the essence of the approach, is Lithium/Deuterium Reactor, or LDR

8 Summary. The concept of LDR

No one issue should be ignored in developng LDR

Problem	State	Approach for solving
1. Interaction with the society	Decline in belief in fusion.	a. Clear demonstration of lithium MHD. b. Full contact with neuclear engineers. c. Intensive interaction with environmentalists
2. Funding	No funds	Private/public company. Market approach.
3. Activation & environment	Conceptial solution	Well focused tests of appropriate samples at appropriate temperatures
4. Tritium reproduction	Conceptial solution	Studies of handling big quantities of tritium
5. Heat Exchanger	Conceptial solution	a. First reactor as a “samovar” with closed Lithium circles b. Development of heat/electricity conversion
6. The first wall	Conceptial solution	a. Studies with no plasmas b. Studies on real tokamaks (PBX, TFTR)
7. Plasma Control	Conceptial solution	Intensive disruption experimental studies on big machines in a real LDR environment
1. Plasma Containment	Conceptial solution	Experiments on big machines in a real LDR environment