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Beggs Deformeter Stress Analysis of Single-Barrel Conduits

UNITED STATES DEPARTMENT OF THE INTERIOR BUREAU OF RECLAMATION

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Beggs Deformeter Stress Analysis of Single-Barrel Conduits

By H. B. Phillips and I. E. Allen

Experimental Design Analysis Section, Technical Engineering Analysis Branch, Office of Chief Engineer, Denver, Colorado



United States Department of the Interior



BUREAU OF RECLAMATION

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INTRODUCTION

This monograph presents the results of the stress analysis, by means of the Beggs Deformeter apparatus,¹ of nine shapes of single-barrel conduits. A partial analytical check was made using the least work method to determine the redundant reactions for all shapes due to a uniform vertical load and a uniform horizontal load.

All personnel of the Experimental Design Analysis Section, including several rotation engineers who had training assignments in the section, assisted in the experimental work and computations. In particular, the assistance of W. T. Moody in computing the analytical solutions, and the work of H. E. Willmann, who prepared the drawings and also assisted in the experimental work and computations, is gratefully acknowledged.

The nine shapes of conduits studied are those most widely used in Bureau of Reclamation structures. All except shape D and the square shape have semicircular top portions of uniform thickness. They can be further described as follows:

1. Shape A: horseshoe-shaped interior with a horizontal exterior base.

2. Shape B: circular-shaped interior with a horizontal exterior base.

3. Shape C: circular-shaped interior with a curved exterior base.

4. Shape D: circular-shaped interior with a square-shaped exterior.

5. Shape E: uniform thickness with a horizontal base.

6. Shape F: uniform thickness of horseshoe shape.

7. Shape G: transition between shape B and shape E with fillets of $\frac{1}{2}r$ radius in lower interior corners.

8. Circular shape of uniform thickness.

9. Square shape of uniform thickness.

Reaction coefficients for bending moment, thrust, and shear at selected locations along the centroidal axis of the conduits have been determined for 15 different loading conditions. The 15 loading conditions considered are as follows:

- 1. \square top with \square foundation.
- 2. $\Box \Box \Box$ top with $\Box \neg \neg$ foundation.
- 3. \downarrow top with $\square \square$ foundation.
- 4. \downarrow top with \checkmark foundation.
- 5. \checkmark top with \square foundation.
- 6. 4 top with 1 foundation.
- 7. \checkmark top with \blacksquare foundation.
- 8. \swarrow top with \checkmark foundation.
- 9. Dead load with $\square \square$ foundation.
- 10. Uniform horizontal both sides.
- 11. \triangle horizontal both sides.

12. Uniform internal radial.

- 13. \bigtriangleup internal radial with \square foundation.
- 14. \bigtriangleup internal radial with $\checkmark \checkmark$ foundation.

15. A external hydrostatic including dead load.

Figures 1, 2, and 3 show cross sections of each shape, giving the dimensions and the location of points at which the reaction coefficients have been determined.

Each shape was analyzed for three values of crown thickness, t, expressed in terms of the internal crown radius, r. These three values were t=r/2, t=r/3, and t=r/6. A conduit of unit length was considered in the analysis. Bending moment, thrust, and shear coefficients were determined at the various locations shown, and are expressed in terms of unit intensity of loading and unit internal crown radius. Multiplying the reaction coefficient by the proper load factor gives the total bending moment, thrust, or shear at the centroid of the section under consideration.

¹ See Appendix for description of this instrument.

APPLICATION

The reaction coefficients determined in the study are tabulated in figures 4 through 50 for the various shapes and loading conditions. The reaction coefficients are given for points on the right side of the conduits only, since the conduits and loadings are symmetrical about the vertical centerline. The shear reactions on the left side of the vertical centerline will have an opposite sign from those given for the points on the right side.

Consistent units should be employed when using these data. Thus, if loads are expressed in pounds per square inch, all dimensions of the conduit must be expressed in inches. The bending moment will then be in inch-pounds per inch of conduit length and the thrust and shear in pounds per inch of conduit length. If the load is expressed in terms of pounds per square foot, the dimensions of the conduit must be expressed in feet, and the bending moment will be in foot-pounds per foot of conduit length and the thrust and shear in pounds per foot of conduit length. It will be noted that the bending moment in inch-pounds per inch is numerically equal to the bending moment in foot-pounds per foot.

One should bear in mind that this analysis assumes no restraint to the deformation of the conduit.

In some cases this restraint, or passive pressure, may be important. Some work on passive pressures on tunnel linings through rock has been done by R. S. Sandhu.² By using his method for determining the intensity of the passive pressure, and using the moment, thrust, and shear coefficients for a circular conduit given by figure 50, the effect of restraint may be approximated.

The foundation load distribution due to a vertical load on the conduit must be assumed, and is influenced by the modulus of elasticity of the foundation material. As the foundation modulus increases, the foundation load distribution approaches a concentration at the outside corners of the conduit, and as it decreases the load approaches a uniform distribution. For all vertical loading conditions except three, two distributions were assumed, viz., uniform, and triangular with zero at the center and maximum at the outside corners.

For the dead load the assumed foundation reaction is minimum at the center varying linearly to a maximum at the outside corners, with the intensity at the center equal to the intensity of the weight of the conduit at the center of the base.

For the triangular internal radial load the assumed foundation reactions were uniform, and triangular with zero at the outsides and maximum at the center.

For the triangular external hydrostatic load, including dead load, the unit weight of the conduit material and the unit weight of water were assumed to be 150 and 62.4 pounds per cubic foot, respectively. With these assumptions the weight of the conduit for the t=r/6 case, except shape D, is less than the uplift, causing the conduit to float. The reaction is assumed to be uniformly distributed across the top. The coefficients for this assumption (conduit floating) are given in figure 49. In the other figures of this loading condition, tension is assumed to develop uniformly along the foundation.

² Sandhu, R. S., "Design of Concrete Linings for Large Underground Conduits," *Journal of the American Concrete Institute*, December 1961, pp. 737-750.

DETERMINATION OF NORMAL STRESS DISTRIBUTION

In a curved beam the neutral axis will not be coincident with the centroidal axis, and the normal stress distribution on radial lines, due to moment, will not be linear. However, the radius to the neutral axis and the normal stress distribution may be determined by the following equations, derived from the Winkler-Bach theory for curved beams: ³



$$r_n = \frac{t}{\ln (r_o/r)} \tag{1}$$

where

- r_n is the radius to the neutral axis
- r is the internal radius
- r_o is the external radius
- t is the wall thickness $(r_o r)$
- ln is the log to the base e,

$$\sigma_{\theta} = \frac{T}{t} + \frac{My_n}{(r_n + y_n)te} \tag{2}$$

where

- σ_{θ} is the normal stress in the tangential direction
- M is the bending moment at the centroidal axis

- T is the thrust at the centroidal axis
- y_n is the distance from the neutral axis to the point of interest (positive outward)
- is the distance from the centroidal axis to the neutral axis.

As t decreases e approaches zero, and the σ_{θ} distribution approaches linearity.

 σ_{θ} , as computed by equation (2), is only for a constant thickness section. Where the section thickness is not constant, the distribution of stresses must be determined by some other method, such as photoelasticity.

The extreme fiber stress in a constant thickness curved beam due to bending moment may be determined by the equation:

$$\sigma_b = K \frac{Mt}{2I} \tag{3}$$

where

 σ_b is the extreme fiber stress

- M is the bending moment at the section
- t is the width of the section
- *I* is the moment of inertia of the section
- K is the factor by which the extreme fiber stress, assuming linear distribution, is modified to correct for curvature.

The following equation for K was obtained by equating equations (2) and (3):

$$K = \frac{ty_n}{6(r_n + y_n)e} \tag{4}$$

The values of K and e for the t/r ratios used in this study are tabulated below:

TABLE 1.—Correction factors for different radii of curvature

	i	K	_
t	Inside fiber	Outside fiber	e.
r/2	1. 153	0. 880	0. 0168r
r/3	1. 105	0. 912	0.0080r
r/6	1.054	0. 951	$0.\ 0021r$

³ Murphy, Glenn, Advanced Mechanics of Materials, McGraw-Hill Book Co., Inc., New York, 1946, pp. 217-219.







9

11

		1	$t = \frac{r}{2}$		$t = \frac{r}{3}$	<u>.</u>	,	t = [
SHAPE A	2 v(r + t)	POINT M 1 +1.0 2 +0.5 3 +0.1 4 -0.1 5 -0.4 6 -0.4 7 -0.4 8 -0.3 9 -0.1 10 -0.0 11 +0.2 13 +0.3 14 +0.3	$\begin{array}{c c} \hline r \\ r \\$	S. M. vr vr +1.500 +0.83 +1.361 +0.44 +1.129 +0.10 +0.820 -0.16 +0.426 -0.33 +0.060 -0.34 -0.340 -0.37 -0.534 -0.27 -0.534 -0.12 -0.628 +0.00 -0.668 +0.14 -0.311 +0.27 -0.089 +0.33 0 +0.34	T vr 5 +0.297 +0.632 4 +0.924 +0.924 3 +1.153 +1.333 1 +1.365 +1.333 4 +1.333 +1.282 3 +1.282 3 3 +1.282 3 3 +1.282 3 1 -0.161 1 1 -0.256 3 2 -0.291 2	S vr +1.333 +1.211 +1.006 +0.733 +0.410 +0.410 +0.410 +0.297 -0.297 -0.386 -0.473 -0.558 -0.691 -0.591 -0.590 0 0	M vr ² t0.683 t0.365 t0.088 -0.130 -0.274 -0.333 -0.305 -0.219 -0.112 t0.018 t0.126 t0.265 t0.319 t0.329	T vr +0.256 +0.549 +1.0805 +1.138 +1.193 +1.146 +1.121 +1.146 +1.121 +1.090 -0.110 -0.212 -0.250 -0.256	S. vr +1.167 +0.061 +0.083 +00644 +0.055 -0256 -0.256 -0.355 -0.413 -0.488 -0.724 -0.332 -0.092 0
SHAPE B	2 v(r + t)	POINT M vr 1 ±0.9 2 ±0.5 3 ±0.1 4 -0.2 5 -0.4 6 -0.4 7 -0.4 8 -0.3 9 -0.1 10 ±0.0 11 ±0.2 12 ±0.3 13 ±0.3 14 ±0.3	$\begin{array}{c c} & T \\ \hline & vr \\ \hline 94 & +0.362 \\ 24 & +0.738 \\ 17 & +1.063 \\ 17 & +1.063 \\ 17 & +1.063 \\ 11 & -1.163 \\ 11 & -1.163 \\ 12 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 29 & +1.542 \\ 20 &$	S M vr vr2 1.500 t0.82 1.355 t0.43 1.118 t0.100 t0.805 -0.36 0.033 -0.40 0.362 -0.36 0.039 -0.40 0.352 -0.36 0.736 t0.19 0.352 +0.30 0.736 t0.19 0.352 +0.30 0 +0.36	T Vr	S vr +1.333 +1.208 +0.999 +0.723 +0.399 +0.045 -0.311 -0.465 -0.612 -0.752 -0.726 -0.726 -0.726 -0.334 -0.104 0	M vr ² +0.669 +0.353 +0.079 -0.271 -0.223 -0.287 -0.191 -0.058 +0.190 +0.190 +0.315 +0.367 +0.378	T Vr +0.284 +0.576 +0.829 +1.026 +1.152 +1.200 +1.167 +1.167 +1.064 +0.989 +0.282 -0.228	S vr +1.167 +1.053 +0.624 +0.624 +0.624 +0.337 +0.028 -0.284 -0.423 -0.566 -0.681 -0.725 -0.342 -0.102 0
SHAPE C	2 v(r + t)	POINT M vrl 1 ±0.9 2 ±0.53 3 ±0.10 4 -0.2 5 -0.4 6 -0.5 7 -0.4 8 -0.3 9 -0.1 10 -0.0 11 ±0.0 12 ±0.2 13 ±0.2 14 ±0.3	$\begin{array}{c c} & T \\ \hline vr \\ 92 \\ +0.324 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 1$	S M vr vr2 1.350 +0.84 1.355 +0.45 1.137 +0.10 0.6831 -0.45 0.0833 -0.43 0.324 -0.40 0.505 -0.30 0.678 -0.14 1.048 -0.08 0.706 +0.099 0.413 +0.27 0 +0.27	T vr 3 ±0.265 1 ±0.896 2 ±1.130 1 ±1.356 4 ±1.333 0 ±1.287 1 ±1.355 4 ±0.1356 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.289 0 ±1.285 0 ±1.285 0 ±1.285 0 ±1.285 0 ±1.285 0 ±1.285 0 ±1.285	S yr +1.333 +1.219 +1.022 +0.756 +0.437 +0.089 -0.265 -0.431 -0.590 -0.959 -0.357 -0.129 0	M vrt 10.692 10.372 10.090 -0.363 -0.347 -0.266 -0.142 -0.098 10.070 10.178 +0.224 +0.234	T vr 10.208 10.502 10.763 10.972 11.14 11.167 11.130 11.073 10.193 10.128 -0.081 -0.088 -0.208	<u>S</u> vr + 1.167 + 1.073 + 0.907 + 0.404 + 0.101 -0.208 -0.358 -0.358 -0.502 -0.869 -0.559 -0.302 -0.108 0
+ Sign convention		BEGGS COE FFIC CONCENTRATE	SINGL 5 DEFO IENTS FO 10 VERTICAN SI	E BARRI RMETER R MOMEN L LOAD-TRI HAPES A, I	EL CO STRE T, THR angular 3, and	NDUI ESS A UST, A FOUND C	T ANAL ND S DATION	YSIS HEAR REAGTI	ON K-PEL - 375

(

	t = [2	t = [3	$t = \frac{r}{6}$
Pressure distribution along vertical 🐑 of conduit-	POINT $\frac{M}{wr^3}$ $\frac{T}{wr^2}$ $\frac{S}{wr^2}$	M T S wr ³ wr ²	$\frac{M}{Wr^3} \frac{T}{Wr^2} \frac{S}{Wr^2}$
	I +0.281 -0.687 0 2 +0.252 -0.664 +0.175	+0.266 -0.691 0 +0.239 -0.667 +0.176	+0.239 -0.693 0 +0.214 -0.669 +0.176
	3 +0.169 -0.598 +0.320 4 +0.048 -0.501 +0.410	+0.162 -0.601 +0.322	+0.142 -0.603 +0.323
	5 -0.090 -0.390 +0.424	-0.083 -0.392 +0.427	-0.065 -0.393 +0.429
	7 -0.310 -0.215 +0.187	-0.290 -0.215 +0.191	-0.278 -0.215 +0.193
	9 -0.245 -0.174 -0.508	-0.224 -0.175 -0.503	-0.210 -0.176 -0.501
	10 -0.055 -0.152 -0.967	+0.149 -0.970 -0.103	+0.114 -0.954 -0.169
- <u>3317</u> wr ²	12 +0.218 -1.147 -0.158 13 +0.271 -1.266 -0.148	+0.212 -1.133 -0.229	+0.195 -1.120 -0.311 +0.282 -1.253 -0.257
¥	14 +0.297 -1.313 0	+0.314 -1.309 0	+0.324 -1.307 0
Pressure distribution along vertical & of conduit			M T S
	i +0.259 -0.645 0	+0.237 -0.654 0	+0.219 -0.660 0
	2 +0.232 -0.623 +0.164 3 +0.155 -0.561 +0.299	+0.211 -0.632 +0.166	+0.195 -0.637 +0.168
	4 +0.042 -0.471 +0.380 5 -0.086 -0.369 +0.387	+0.031 -0.478 +0.387	+0.026 -0.482 +0.391
	6 -0.202 -0.276 +0.309	-0.202-0.278 +0.319	-0.192 -0.280 +0.324
	8 -0.289 -0.191 -0.125	-0.288 -0.190 -0.115	-0.273 -0.190 -0.109
	9 -0.230 -0.144 -0.457	-0.227-0.145-0.447	-0.210 -0.147 -0.441
	11 +0.106 -0.752 -0.309	+0.076-0.725-0.355	+0.059 -0.699 -0.404
$\frac{\pi}{(r+t)}$ wr ²	13 +0.324 -1.271 -0.230	+0.304 -1.256 -0.266	+0.300 -1.245 -0.309
	14 10.302 1.355 0	<u> +0.345 -1.346 _0</u>	<u> +0.346 -1.340 </u>
Pressure distribution along			MTS
vertical C of conduit-		wr ³ wr ² wr ²	wr3 wr2 wr2
	2 +0.237 -0.632 +0.166	+0.215 -0.638 +0.168	+0.209 -0.655 +0.172
	3 +0.159 -0.570 +0.304 4 +0.045 -0.478 +0.387	+0.141 -0.575 +0.307 +0.033 -0.482 +0.391	+0.139 -0.590 +0.315 +0.035 -0.494 +0.403
	5 -0.086 -0.374 +0.395 6 -0.205 -0.278 +0.318	-0.090 -0.377 +0.401	-0.083 -0.385 +0.416
	7 -0.284 -0.215 +0.154	-0.279 -0.215 +0.160	-0.268 -0.215 +0.177
	9 -0.229 -0.135 -0.492	-0.221 -0.138 -0.484	-0.212 -0.137 -0.467
	10 -0.151 -0.416 -0.298 11 -0.017 -0.667 -0.545	-0.163 -0.402 -0.305	-0.175 -0.380 -0.305
$\frac{\pi}{(r+t)} wr^2$	12 +0.149 -0.925 -0.699	+0.139 - 0.920 -0.697	+0.126 -0.904 -0.689
	14 +0.344 -1.346 0	+0.335 -1.340 0	+0.321 -1.323 0
volume of water in units consistent with those of the radius r.			
T x x			
μ	SINGLE B	ARREL CONDU	ит
*******	BEGGS DEFORME	TER STRESS A	NALYSIS
	COEFFICIENTS FOR MO	MENT, THRUST, AI	ND SHEAR
+ Sign convention TRIANG	ULAR INTERNAL RADIAL LO	AD - TRIANGULAR FO	UNDATION REACTION
	SHAPES	A, B, AND C	
SEP. 28, 1964			X-PEL-1038

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		$t = \frac{r}{2}$	$t = \frac{r}{3}$	$t = \frac{r}{6}$
		POINT M T S		$\frac{M}{vr^2} \frac{T}{vr} \frac{S}{vr}$
	2 ² / ₄	1 +0.551 +0.108 0	+0.465 +0.096 0	+0.386 +0.086 0
		2 +0.476 +0.172 +0.553	+0.394+0.158+0.51	9 +0.321 +0.146 +0.482
		4 -0.010 +0.546 +1.213	-0.054 +0.510 +1.10	8 -0.087 +0.472 +0.995
		5 -0.311 +1.449 +0.401	-0.270 +1.285 +0.37	0 -0.228 +1.121 +0.335
٥		7 -0.446 +1.503 +0.064	-0.391 +1.335 +0.06	1 -0.334 +1.169 +0.056
PE		8 -0.440 +1.500 -0.108	-0.386 +1.333 -0.09	6 -0.330 +1.167 -0.086
SHA	-+- 7	10 -0.280 +1.437 -0.444	-0.245 +1.274 -0.40	4 -0.207 +1.112 -0.365
			-0.111 +1.218 -0.55	1 -0.091 +1.059 -0.497
	10 114 13 14	13 +0.277 +0.041 -0.653	+0.235 +0.048 -0.61	6 +0.198 +0.054 -0.577
	╒┿┽╅╪┿┽╡┥╡┊┊╡┽┽┼┼┼┼┼┼	14 +0.402 -0.071 -0.328	+0.353 -0.060 -0.31	0 +0.309 -0.052 -0.291
		15 1+0.445[-0.108] 0	[+0.333 [-0.036] 0	1+0.347 -0.086 0
	ī			
	źv	POINT M T S		
	ATTITUTE 1	1 +0.513 +0.166 0	+0.448 +0.144 0	+0.387 +0.121 0
		3 +0.212 +0.707 +0.891	+0.184 +0.625 +0.79	4 +0.158 +0.542 +0.698
		4 -0.044 +1.087 +0.852	-0.038 +0.964 +0.76	0 -0.032 +0.839 +0.669
ل يا	HI STE	5 -0.265 + 1.359 +0.592 6 -0.395 + 1.490 +0.227	-0.227 +1.206 +0.53	0 -0.191+1.052+0.468
Ц Ц		7 -0.404 + 1.500 -0.166	-0.347 +1.333 -0.14	4 -0.292 +1.167 -0.121
SHA SHA	┝┥╴┩	8 -0.349 + 1.500 -0.166	-0.299 +1.333 -0.14	4 -0.252 +1.167 -0.121
0/		10 -0.238 + 1.500 -0.166	-0.203 +1.333 -0.14	4 -0.172 +1.167 -0.121
		11 +0.054 -0.166 -1.000	-0.012 -0.144 -1.00	0 -0.078 -0.121 -1.000
		12 +0.331 -0.166 -0.667	+0.265 -0.144 -0.66	3 +0.366 -0.121 -0.333
		14 +0.554 -0.166 0	+0.488 -0.144 0	+0.422-0.121 0
	î			
		POINT M T S	M T S	M T S
	20	V P V V V	+0.464 +0.105 0	VF ² VF VF
		2 +0.439 +0.295 +0.621	+0.385 +0.257 +0.55	3 +0.327 +0.220 +0.485
		3 +0.220 +0.670 +0.912	+0.193 +0.591 +0.81	4 +0.161 +0.513 +0.714
4.	HAT XA	5 -0.276 +1.338 +0.629	-0.234 +1.186 +0.56	4 -0.201 +1.036 +0.498
u	H 111	6 -0.419 +1.479 +0.268	-0.356 +1.314 +0.24	4 -0.302 +1.148 +0.218
APE		8 -0.376 +1.443 -0.331	-0.320 +1.282 -0.28	8 -0.273 +1.121 -0.247
HS		9 -0.259 +1.311 -0.510	-0.218 +1.162 -0.44	7 -0.184 +1.014 -0.385
	File Internet	10 -0.103 +1.114 -0.639	-0.080 +0.984 -0.55	9 -0.064 +0.853 -0.480
	13 13	12 +0.257 +0.075 -0.704	+0.222 +0.081 -0.65	4 +0.181 +0.085 -0.605
	ſ ╄╊┞╄┨<u></u>╋╋╋╋╋╋╋╋╋╋╋╋╋╋╋╋╋╋╋	13 +0.411 -0.073 -0.366	+0.364 -0.057 -0.34	0 +0.312 -0.043 -0.314
	57 - E			
	1/(SINGLE	BARREL CON	DUIT
	r'al Dis .	BEGGS DEFOR	METER STRES	SS ANALYSIS
	*	COEFFICIENTS FOR	MOMENT. THRUS	ST. AND SHEAR
		TRIANGULAR VERTICAL	LOAD - UNICOPH	OUNDATION PEACTION
	+ Sign convention	CHA	PES D F AND F	CONDATION REACTION
	-	511A		
	e 1044			Y - DEL - 104 4
JEP. Z	0, 1004			A "FEL" 1044

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FIGURE ** 45

	$t = \frac{r}{2}$	$t = \frac{r}{3}$	(: 	
	$\begin{array}{c c} POINT & \underline{M} & \underline{T} & \underline{S} \\ \hline pr^{\sharp} & pr & pr \\ I & 0 & -I & 000 & 0 \end{array}$	<u>M</u> <u>T</u> pr ² pr	s M pr pre	T pr -1.000	s pr 0
A STATISTY'S		0 -1.000	0 0	-1.000	0
E ALLER	4 0 -1.000 0	0 -1.000	0 0	-1.000	0
	5 0 -1.000 0	0 -1.000	0 0	-1.000	0
or HELS	7 0 -1.000 0	0 -1.000	0 0	-1.000	0
	8 <u>0</u> -1.000 0 9 0 -1.000 0	0 -1.000	0 0	-1.000	0
	10 0 -1.000 0	0 -1.000	0 0	-1.000	0
		0 -1.000	0 0	-1.000	0
	13 0 -1.000 0	0 - 1.000	0 0	-1.000	0
	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c c} \hline & M \\ \hline & pr^2 \\ \hline & pr^2 \\ \hline & 0 \\ \hline & -0.196 \\ \hline & 0.500 \\ \hline & -0.071 \\ \hline & 0.000 \\ \hline & +0.304 \\ \hline & .500 \\ \hline & -0.196 \\ \hline & 0.500 \\ \hline & -0.071 \\ \hline & 0 \\ \hline & -0.304 \\ \hline & .500 \\ \hline & -0.304 \\ \hline & 0.500 \\ \hline & -0.071 \\ \hline & 0 \\ \hline & -0.196 \\ \hline \\ \hline & -0.196 \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$	T Pr -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 - -1.000 -	B pr 0 0.500 1.000 0.500 0.500 0.500 0.500 0.500 0.500 0.500 0.500
BHAPE 6	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	M T pr* pr +0.024 -1.065 +0.013 -1.055 +0.021 -1.065 +0.021 -1.065 +0.001 -1.048 +0.001 -1.044 -0.035 -1.000 -0.035 -1.000 -0.035 -1.000 +0.036 -1.000 +0.036 -1.000 +0.036 -1.000 +0.336 -1.000 +0.356 -1.000 +0.036 -1.000 +0.036 -1.000 +0.1036 -1.000 +0.036 -1.000 +0.036 -1.000 +0.102 -0.009 -0.038 -0.000 +0.209 -0.22 -0.009 -0.933 -0.009 -0.933 -0.102 -0.933	S M pr pr ⁸ 0.017 +0.019 0.034 +0.000 0.056 -0.013 0.067 -0.028 0.067 -0.029 0.433 +0.050 .433 +0.050 .454 +0.223 .550 +0.223 .550 +0.016 .250 -0.076 0 -0.076	T - 1.057 - 1.057 - 1.057 - 1.057 - 1.057 - 1.030 + - 1.030 + - 1.030 + - 1.000 - 1.000 - 1.000 - 1.000 - 1.000 - 1.000 - 1.040 + - 1.010 + - 0.941 + - 0.941 - 0.941	S pr 0 0.015 0.030 0.042 0.051 0.059 0.191 0.451 0.451 0.527 0.521 0.521 0.521 0.522 0.520 0
+ Sign convention	SINGLE BEGGS DEFORM COEFFICIENTS FOR UNIFORM SHAPES CI	BARREL C METER STR MOMENT, THR INTERNAL RADIA RCULAR, SQUAR	CONDUIT ESS AN/ RUST, AND L LOAD RE, AND G	ALYSIS SHEAT	S R
SEP. 28. 1964				X - PEL	- 1066

SHAPE A	SHAPE B	SHAPE C
POINT M T S	POINT M T S	POINT M T S
1 +0.057 +0.728 0	1 +0.019 +0.679 0	1 +0.080 +0.687 0
2 +0.049 +0.772 +0.069	2 +0.015 +0.710 +0.029	2 +0.068 +0.737 +0.099
3 +0.027 +0.892 +0.114	3 +0.007 +0.797 +0.047	3 +0.034 +0.876 +0.167
4 -0.001 +1.065 +0.117	4 -0.003 +0.924 +0.046	4 -0.010 +1.073 +0.184
5 -0.024 +1.253 +0.079	5 -0.010 +1.070 +0.030	
5 -0.032 +1.423 +0.014 7 -0.025 +1.549 -0.048	7 -0.012 +1.338 +0.002	7 -0.085 +1.625 -0.007
8 -0.048 +1.689 +0.224	8 -0.035 +1.489 +0.192	8 -0.102 +1.785 +0.157
9 -0.154 +1.913 +0.561	9 -0.106 +1.770 +0.421	9 -0.162 +2.094 +0.363
10 -0.356 +2.266 +0.954	10 -0.216 +2.256 +0.660	10 -0.148 +2.099 -0.576
11 -0.323 +2.281 -1.172	11 -0.194 +2.452 -0.696	
		13 +0.062 +2.030 -0.057
14 +0.098 +1.994 0	14 +0.067 +2.043 0	14 +0.070 +2.035 0
SHAPE E	SHAPE F	SHAPE G
POINT wrs wrs	POINT wr3 wr2 wr2	POINT wrs wrs wrs
1 +0.061 +0.907 0	1 +0.103 +0.775 0	I +0.083+0.778 0
2 +0.049+0.964+0.099	2 +0.086 +0.834 +0.121	2 +0.071+0.832+0.10
3 +0.018+1.123+0.156		
5 -0.038+1.570+0.056	5 -0.049+1.469+0.150	5 -0.037+1.410+0.11
6 -0.028+1.753-0.082	6 -0.069 + 1.674 + 0.033	6 -0.049+1.599+0.01
7 +0.019+1.854-0.226	7 -0.057 + 1.806 -0.094	7 -0.033 +1.725 -0.09
8 +0.023+1.988+0.218	8 -0.047 + 1.919 + 0.038	8 -0.048 +1.825 +0.22
9 -0.139+2.121+0.774		9 -0.151+1.925+0.61
11 -0.464+1.816-1.933	11 -0.174 + 2.071 -0.898	11 -0.400+2.535+0.66
12 +0.073+1.816-1.288	12 +0.051+2.004-0.623	12 -0.364+2.458-1.05
13 +0.395+1.816-0.644	13 +0.190 +1.962 -0.319	13 -0.163+2.160-1.04
14 +0.503+1.816 0	14 +0.237 +1.947 0	
		16 +0.286+1.944 0
CIRCULAR	SQUARE	Top reaction is assumed to b of uniform intensity, v.
1 +0.106 +0.707 0	1 +0.315 +0.753 0	SHAPE T
2 +0.090 +0.764 +0.125	2 +0.159 +0.753 +0.623	A +0.493
3 +0.047 +0.923 +0.211	3 -0.308 +0.753 +1.246	B +0.312
4 -0.009+1.149+0.232		C +0.558
6 -0.094 +1 603 +0.086	6 +0.139+1.854-0.072	E +0.734
7 -0.100 + 1.748 -0.026	7 +0.009 +2.054 +0.636	G +0.644
8 -0.080 + 1.842 -0.109	8 -0.539 +2.255 +1.594	Circular+0.664
9 -0.042+1.912-0.152	9 -0.510 +1.969 -1.933	Square +0.845
10 +0.003+1.961-0.155		For loading diagram, + sign
		convention, and the assumption
13 +0.081+2.016 0		see Figures 18, 33, and 4
		Note: Shape D does not f
BEGGS	SINGLE BARREL CONDU DEFORMETER STRESS A	NALYSIS
COEFFICIE	NTS FOR MOMENT, THRUST, AN	D SHEAR
TRI	ANGULAN EXIENNAL HIDNOSTATIC L Including dead load	VAV
	CONDUITS ASSUMED TO FLOAT All shapes	
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FI	G	υ	R	E	50

This study has been made using the Beggs Deformeter apparatus 456 (figure 51). The basis of the method is a direct application of Maxwell's Theorem of Reciprocal Deflections, which states that for any two points on a structure, the ratio of the displacement at the first point to the load causing it, applied at the second point, is equal to the ratio of the displacement at the second point to the load causing it, applied at the second point. Displacements are measured in the load directions.

In the general application of this method of stress analysis, an elastic scale model of the structure under consideration is deformed at a cut in the model by use of a special set of gage blocks and plugs. Three sets of plugs are used to apply a rotational, a normal, and a shearing displacement at the gage block. Microscopes equipped with filar evepieces are used to measure the model deflections at points corresponding to the load points of the actual structure. Deflections are measured in the direction of the prototype loads. No loads are applied to the model. Deflections of the model are read at prototype load points for displacements applied at the gage block. The difference in microscope readings is a measure of the model deflection induced by the change at the gage block from the first position of the plugs to the second position of the plugs.

From Maxwell's Theorem the following equations may be written for the redundant reactions at the cut section:

For a concentrated load	For a distributed load		
$M_1 = P \frac{e_M}{d_M} n$	$M_1 = \frac{n}{d_M} \int p e_M dl$		

⁴ Beggs, G. E., "An Accurate Solution of Statically Indeterminate Structures by Paper Models and Special Gages," *Proceedings* ACI, vol. XVIII, 1922, pp. 58-78. ⁵ McCullough, C. B., and Thayer, E. S., *Elastic Arch Bridges*, John Wiley and Sons, New York, 1931, pp. 282-300.

$$S_{1} = P \frac{e_{S}}{d_{S}} \qquad S_{1} = \frac{1}{d_{S}} \int pe_{S} \, dl$$
$$T_{1} = P \frac{e_{T}}{d_{T}} \qquad T_{1} = \frac{1}{d_{T}} \int pe_{T} \, dl$$

where

- d_M is the angular rotation applied at the cut by the moment plugs
- d_s is the displacement applied at the cut by the shear plugs
- d_T is the displacement applied at the cut by the thrust plugs
- e_M is the measured deflection at a load point, in the direction of the load, due to d_M
- e_s is the measured deflection at a load point, in the direction of the load, due to d_s
- e_T is the measured deflection at a load point, in the direction of the load, due to d_T l is the load length
- M_1 is the redundant moment reaction at the cut
- n is the scale factor (prototype to model)
- P is a load acting at a point on the prototype
- *p* is the load intensity on the prototype at the deflection point
- S_1 is the redundant shear reaction at the cut
- T_1 is the redundant thrust reaction at the cut.

The only unknowns in these equations are M_1 , T_1 , and S_1 .

In the actual operation of the Beggs Deformeter the arithmetic is simplified by the use of calibration factors based on the plug dimensions and the eyepiece scales. An influence line through points obtained by multiplying the deflection ordinates by the proper calibration factor gives directly the magnitude of the moment, thrust, or shear at the gage block position for a unit traveling load.

It should be pointed out that the Beggs Deformeter method automatically takes into account the strain energy in a structure due to moment, thrust, and shear as well as haunch effects and other shape changes.

^{300.} ⁶ Phillips, H. B., and Allen, I. E., "The Beggs Deformeter Theory and Technique," Bureau of Reclamation, Denver, Colo., July 1965.

FIGURE 51. Beggs Deformeter apparatus and shape B conduit model.

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