Reconstructing Interaction Vertices in x, y, and z using the DØ Luminosity System

John Hauptman,[2] Sung Hwan Ahn,[3] Michael Begel,[4] Seh Wook Lee,[3] Chyi-Chang Miao,[1] Jae Won Park,[3] Richard Partridge,[1] Joseph Tuggle[7]

[1] *Brown University, Providence, Rhode Island*

[2] *Iowa State University, Ames, Iowa*

[3] *Korea University, Seoul, Korea*

[4] *University of Rochester, Rochester, New York*

May 27, 2003

Abstract

We describe the reconstruction of both the z-positions of single and multiple interaction vertices in individual events, and the (x, y) mean beam position during one Luminosity Block Number (LBN) for the Run 2 counters and electronics.

1 Introduction

The luminosity system is essentially an experiment [1] unto itself, comprising an independent, continuously running data acquisition stream, 48 scintillation counters that read out pulse height and time, real-time reconstuction and analysis with Field Programmable Gate Arrays (FPGAs), monitoring, and communications with both the Tevatron and all of the DØ subsystems. In addition to the basic luminosity measurements [2, 3], the luminosity system will also provide measurements of the z-positions of interaction vertices and the mean (x, y) location of the interaction diamond during one LBN (about one minute in duration). We describe in this note the methods developed for these measurments of (x, y) and z.

2 Geometry

The luminosity counters are located outside the Silicon Micro Tracker (SMT) on the front faces of the liquid-argon cryostat. A schematic layout is shown in Fig. 1, where the c-travel time is about 4.67 ns from $z = 0$ to the luminosity counters. There are 24 scintillation

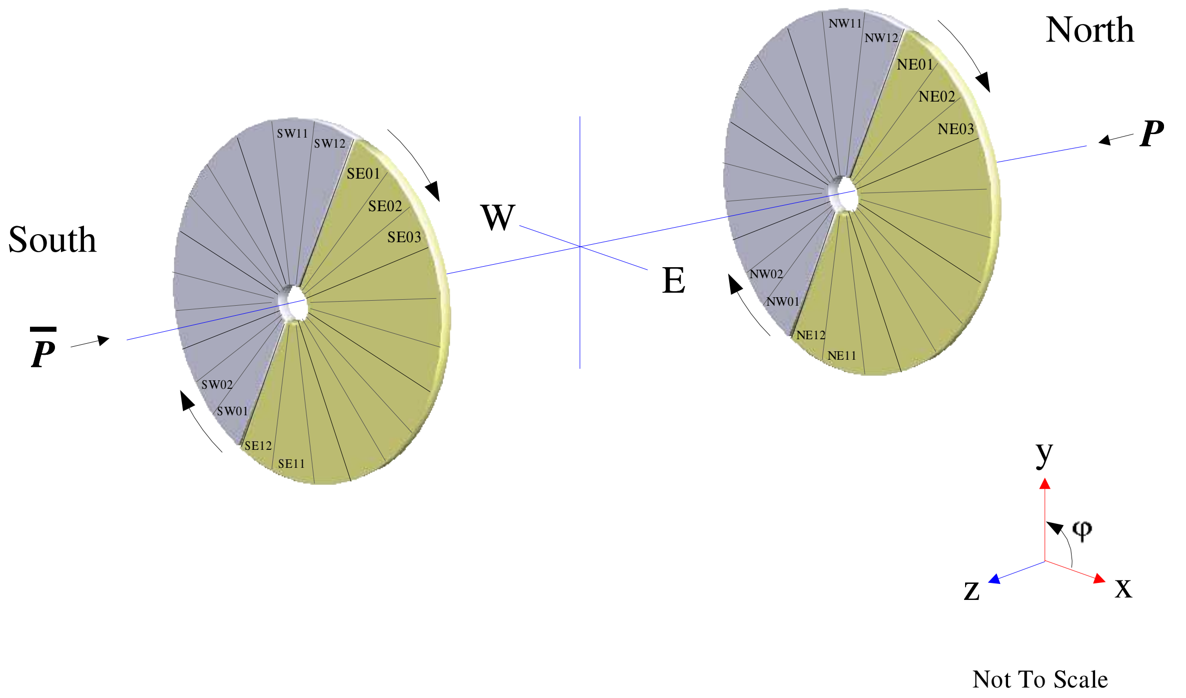

Figure 1: Geometry of the Luminosity counters, with labels as defined in control room GUIs.

counters on each end, with inner radius 3.2 cm and outer radius 19.7 cm. Each scintillator is viewed by a fast PMT and read out through a constant-fraction discriminator and a Time-to-Digital Converter (TDC). The time resolution of the counters is approximately

$$\sigma_t \approx 0.30 \quad \text{ns},$$

or about 9 cm in z. The luminous interaction region is about ± 30 cm.

3 z Reconstruction

3.1 Least Squares Fit

The strategy we use is, first, to do a least squares fit to all time-hits in the counters to a single vertex z position, z_V, and an interaction time, t_V, for the interaction. Let s index the counters on the South side (at positive z), and n index the counters on the North side (at negative z). The expected time, t_s, for a hit in the South counters at $z = +140$ cm is

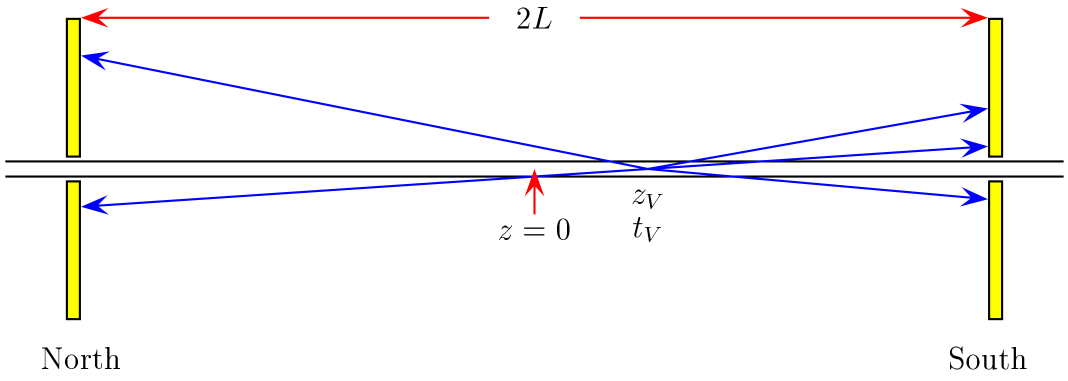

Figure 2: Single-vertex event. The interaction is at z_V and happened at time t_V.

(Fig. 2):

$$t_s = \frac{L - z_V}{c} + t_V,$$

and the expected time in the North counters at $z = -140$ cm is

$$t_n = \frac{L + z_V}{c} + t_V.$$

The χ^2 is a function of the unknowns z_V and t_V,

$$\chi^2(z_V, t_V) = \sum_s \left[\frac{t_s - [(L - z_V)/c + t_V]}{\sigma_s}\right]^2 + \sum_n \left[\frac{t_n - [(L + z_V)/c + t_V]}{\sigma_n}\right]^2$$

The least squares solution is the minimum χ^2 in the parameters z_V and t_V given by

$$0 = \frac{1}{2}\frac{\partial \chi^2}{\partial z_V}$$

and

$$0 = \frac{1}{2}\frac{\partial \chi^2}{\partial t_V}.$$

The resulting equations are easily written in terms of a weighted deviation from the $z = 0$ time (ξ) and the weight (w) for the South counters

$$\xi_S = \sum_s \frac{t_s - L/c}{\sigma_s^2} \qquad w_S = \sum_s \frac{1}{\sigma_s^2}$$

and for the North counters,

$$\xi_N = \sum_n \frac{t_n - L/c}{\sigma_n^2} \qquad w_N = \sum_n \frac{1}{\sigma_n^2},$$

yielding

$$[\xi_S + \xi_N] = z_V[-\frac{w_S}{c} + \frac{w_N}{c}] + t_V[w_S + w_N],$$

and

$$[\xi_s - \xi_N] = z_V[-\frac{w_s}{c} - \frac{w_N}{c}] + t_V[-w_N + w_S].$$

The least squares solutions for z_V and t_V are

$$z_V = \frac{c}{2}[\frac{\xi_N}{w_N} - \frac{\xi_S}{w_S}],$$

and

$$t_V = \frac{1}{2}[\frac{\xi_N}{w_N} + \frac{\xi_S}{w_S}].$$

In the simple case of one hit in each counter, and with unit weights ($\sigma = 1$), ξ and w become

$$\xi_S \to t_S - L/c \qquad w_S \to 1$$

and

$$\xi_N \to t_N - L/c \qquad w_N \to 1,$$

so that z_V and t_V reduce to their obvious values

$$z_V = \frac{c}{2}(t_N - t_S) \quad \text{and} \quad t_V = \frac{1}{2}(t_N + t_S).$$

3.2 Pythia event sample

We generate single minimum bias (MB) events with Pythia [4] to test the code. For the following plots in this note, we generate 'low p_T' QCD interactions, smear vertex position by $\sigma_{\text{vertex}} = 25$ cm as a Gaussian, track each Pythia charged particle out to the luminosity counters, and for each counter, record the pulse height and the time. The time is smeared by $\sigma_t = 0.269$ ns to simulate the PMT time jitter. The TDC is simulated by taking the earliest time hit as the TDC value.

Halo p and $\bar{p}$ are simulated as a rate comparable to the beam rates during March 2002 and are generated in time with their respective p and $\bar{p}$ bunches on both North and South counter arrays. Noise has not been included.

3.3 1-Vertex Fits

The $\Delta z \equiv (z_V - z_{\text{Pythia}})$ distribution for single vertex fits to single-vertex MB events is shown in Fig. 3 (upper left plot), giving an rms width of 3.1 cm in z. The confidence level (CL) distribution of these fits is shown in the upper right plot of Fig. 3. It is nearly flat, as expected.

The two lower plots ('CL 2-vertex fit' and 'CL 3-vertex fit') are the CL distributions for tries at two and three vertex fits, attempted when the 1-vertex fit falls below a confidence level of 1%. These multi-vertex fits are described below. In Fig. 3, there are $75 \cdot 10^3$ single minimum bias vertices attempted. Therefore, for a perfectly flat CL distribution, 750 fits will have CL below 1%; the resulting CL distribution for the 2-vertex fit is shown in the lower left figure. In turn, 2-vertex fit failures are attempted as 3-vertex fits, and a neglible fraction ($\sim 4 \cdot 10^{-3}$) appear in this category.

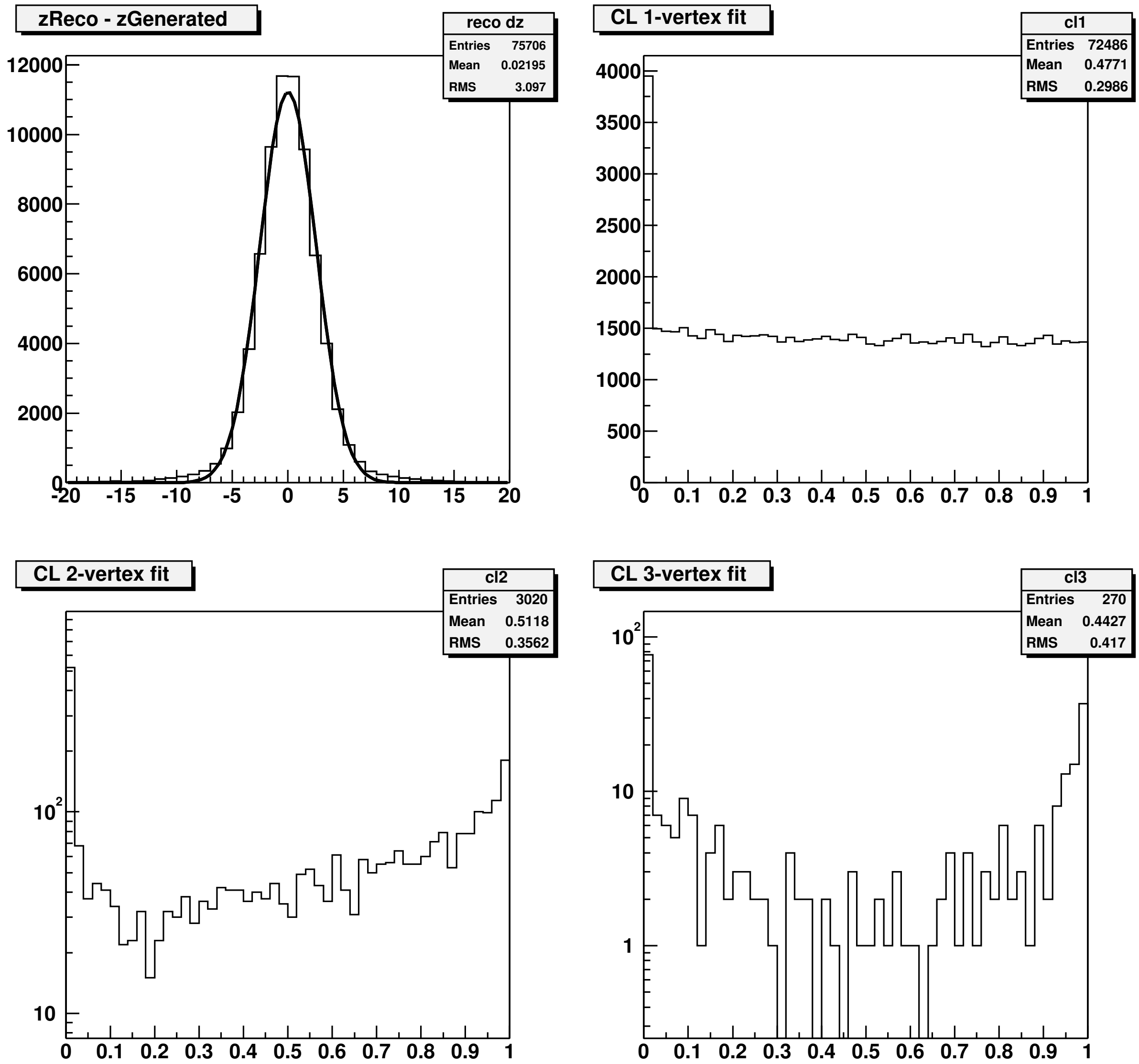

Figure 3: The Δz distribution for one-vertex minimum bias events (a), and confidence level distributions of fits to 1-vertex assumptions (b), to 2-vertex assumptions for CL(1-vertex) < 1% (c), and to 3-vertex assumptions for events in (c) with CL(2-vertex) < 10% (d).

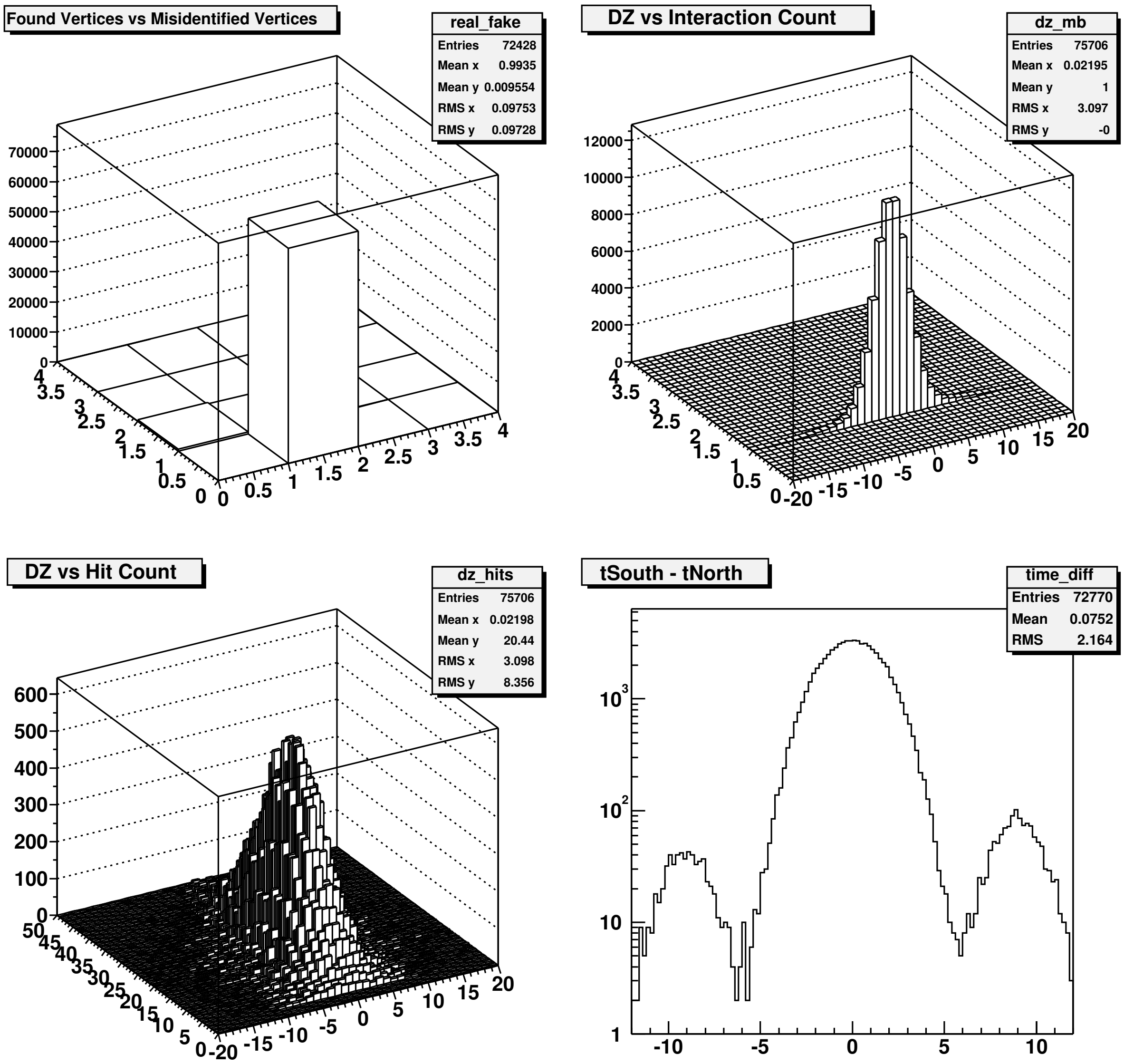

Figure 4: (a) Number of "found" vertices *vs.* number of "fake" vertices for 1-MB events, (b) the distribution in $\Delta z \equiv (z_V - z_{\mathrm{Pythia}})$, (c) Δz *vs.* the number of counter hits, and (d) the qVt distribution as seen at rack M11.

3.3.1 Fitted uncertainties in z

The z position is just the difference of the weighted times (scaled by $c/2$). Because the weighted times have an uncertainty that is mostly statistical, we expect that gross spatial resolution in z will scale like the time resolution, σ_t, divided by the square root of the number of tracks participating in the vertex:

$$\sigma_z \sim c \cdot \frac{\sigma_t}{\sqrt{N}} \oplus \text{systematic terms}$$

where N is the number of time hits used in the fit. For one hit, $\sigma = \sigma_t$, and so an event with many hits on one side N, but a single hit on the other side, will be dominated by the time measurement on the single-hit side,

$$\sigma_z \approx c \cdot \sqrt{\sigma_t^2/N + \sigma_t} \to c \cdot \sigma_t.$$

Formally, the second derivative matrix of χ^2 is the inverse matrix of the fitted uncertainties (or, the 'error matrix', sometimes called the Hessian matrix).

$$\frac{1}{2}\begin{pmatrix} \partial^2\chi^2/\partial z_V^2 & \partial^2\chi^2/\partial z_V \partial t_V \\ \partial^2\chi^2/\partial t_V \partial z_V & \partial^2\chi^2/\partial t_V^2 \end{pmatrix} = \begin{pmatrix} \sigma_z^2 & \sigma_{tz}^2 \\ \sigma_{zt}^2 & \sigma_t^2 \end{pmatrix}^{-1}$$

For z_V,

$$\frac{1}{2}\frac{\partial^2\chi^2}{\partial z_V^2} = w_S/c^2 + w_N/c^2,$$

so that the z-resolution in a single vertex fit is

$$\sigma_z = c/\sqrt{w_S + w_N} \approx c/\sqrt{2\bar{w}} \sim c \cdot \sigma_t/\sqrt{N} \sim 5 \text{ cm}$$

for $N \sim 4$ hits on each side. σ_{z_V} is correlated with σ_{t_V}, but we ignore this correlation and also ignore t_V itself, although it is fitted correctly in simulated events.

3.3.2 Efficiency and Fake rates

The single vertex fit is easy. The distribution of the number of "found" vertices *vs.* the number of "fake" vertices for single-vertex MB events is shown in Table 1. A vertex is labelled "found" if its z position lies within $\sigma_z = 9$ cm of a true vertex.

We show in Fig. 4(a) the distribution of the number of "found" vertices against the number for "fake" vertices in a LEGO plot (the same numbers as in Table 1).

3.4 Two-vertex events; or, 'divide and conquer'

If the fit is poor, that is, if the confidence level of the fit is less than 1%, we assume there is more than one interaction vertex. This poorly fitted z_V will represent an approximate average of the two (or more) vertices, and so we artificially generate two z_V positions: $z_1 = z_V + 10$ cm and $z_2 = z_V - 10$ cm (Fig. 5). The interaction time, t_V, of

# Fake ↓				
3				
2				
1	582		110	
0		71,736		
# Fake ↑ # Found →	0	1	2	3

Table 1: Number "found" vertices *vs.* number of "fake" vertices for a sample of 75,000 generated one-vertex minimum bias (1-MB) events (low-p_T QCD). The finding efficiency was about 95.6%. In this sample of events, the trigger efficiency was 96.6%.

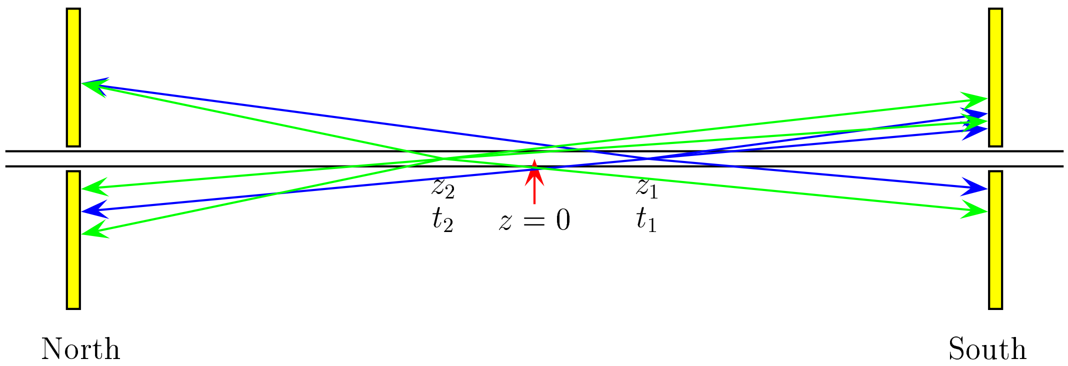

Figure 5: Two-vertex MB event. The vertices are at (z_1, t_1) and (z_2, t_2).

# Fake ↓				
3				
2	6101			
1		11,846		254
0			16,778	
# Fake ↑ # Found →	0	1	2	3

Table 2: Number of "found" vertices *vs.* number of "fake" vertices for a sample of 35,000 generated two-vertex minimum bias (2-MB) events (low-p_T QCD). The trigger efficiency was 99.96%.

the 1-vertex fit is also presumably a mean time of the two (or more) interactions. We try two pre-fits to estimate first approximation values for the 4-parameter two-vertex fit. We generate two possible interactions times, $t_1 = t_V + 0.25$ ns and $t_2 = t_V - 0.25$ ns, and then associate (z_1, t_1) and (z_2, t_2) to calculate the total two-vertex χ^2 (without doing a fit). Then we switch the times, *i.e.*, associate (z_1, t_2) and (z_2, t_1) and again calculate χ^2. The lower χ^2 association is taken as the first approximation for the 2-vertex fit.

The main problem with two vertices and many hits is how to assign the hits to the vertices. We choose an extremely simple algorithm: for each hit, we calculate the contribution to χ^2 at each vertex and assign that hit to the vertex contributing the lesser χ^2.

The fit is repeated for these two vertices. The code is identical to the single-vertex fit solution above since the hits are already assigned to their separate vertices, and the total χ^2 for the fit is the sum of the χ^2 from the separate vertices,

$$\chi^2 = \chi_1^2 + \chi_2^2.$$

The distribution of "found" vertices *vs.* "fake" vertices is shown in Table 2. A vertex is labelled "found" if its z position lies within 9 cm of a true vertex.

3.4.1 Efficiency and Fake rate

The efficiency and fake rates for two-vertex events are not expected to be good, although the overall probability to find both vertices correctly and with no additional fakes is nearly 50%.

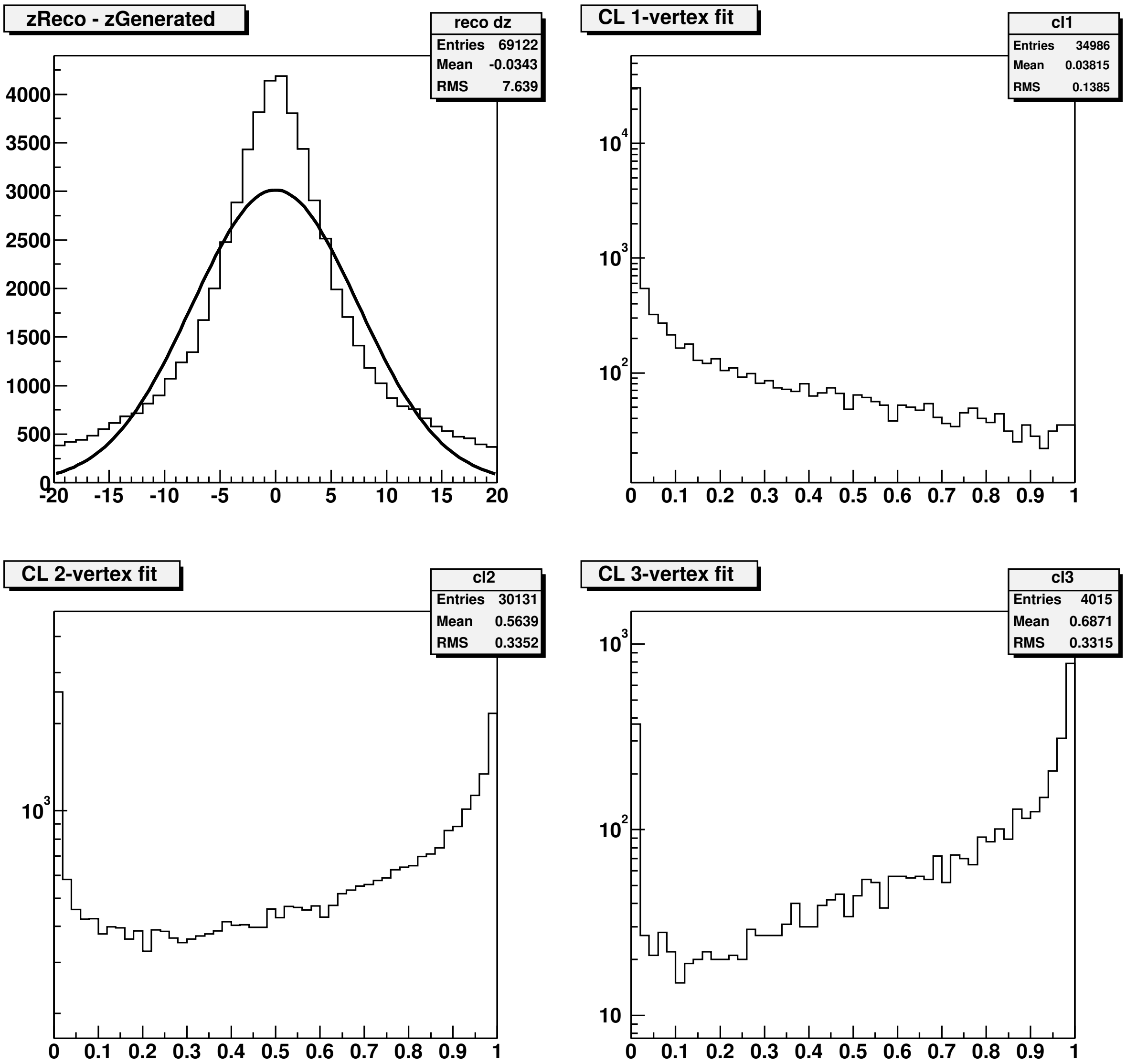

Figure 6: The $\Delta z \equiv (z_V - z_{\rm Pythia})$ distribution for a sample of two minimum bias events (upper left). The CL distribution for the 1-vertex fit is poor, as expected, and the CL distribution for the 2-vertex fit is normal (lower left). A small fraction of the two-MB events fail the 2-vertex fit and a 3-vertex fit is attempted (lower right).

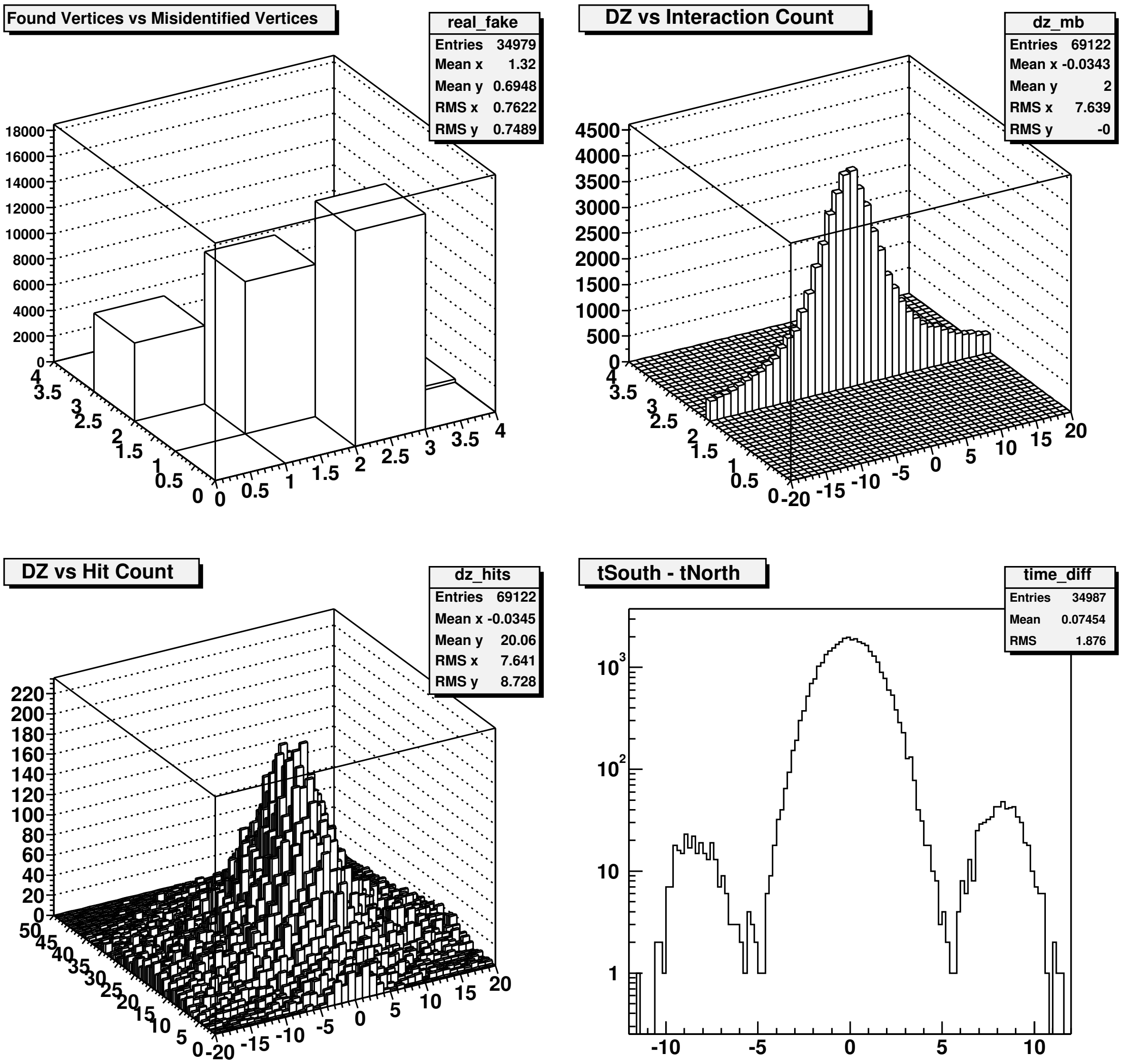

Figure 7: The fit statistics for 35,000 2-MB events. Plots are the same as in Fig. 4.

# Fake ↓				
3	2203			
2		8874		
1			11,565	
0				2358
# Fake ↑ # Found →	0	1	2	3

Table 3: Number of "found" vertices *vs,* number of "fake" vertices for a sample of 25,000 generated three-vertex minimum bias events (low-p_T QCD). The trigger efficiency was 100%.

3.5 Three-vertex events

'Divide and conquer' worked once, so we try it again. Of the two fitted vertices, one has the larger χ^2, and this one we split into two. We then re-assign all counter hits to one of the three hypothesized vertices and re-fit. This works fairly well. The z resolution is non-Gaussian, as shown in Fig. 8, and the CL distribution for these 3-vertex events is skewed to the high-side. This skew is expected since each counter hit is assigned to that vertex which contributes the lower χ^2. The distribution of "found" *vs.* number of "fake" vertices is shown in Table 3.

3.6 Multiple Interaction Flag

It seems reasonable that an effective and efficient multiple interaction flag would select on the χ^2 (or CL) of the single vertex fit.

4 (x, y) Reconstruction

The 24 luminosity counters on each end are relatively fine-grained in azimuth: 15^o per wedge. They define a small inner circular boundary of radius 3.2 cm. The particle hit distribution in radius is shown in Fig. 9. If the beam is off center, then the reaction products from underlying events, diffractive events, etc., which are symmetric about the interaction diamond, will non-uniformly populate the luminosity counter wedges. It will be like two circles, one off center from the other, and the resulting hit distribution in azimuthal angle ϕ will be modulated by a cosine function, as shown in Fig. 10 for two LBNs generated with low-p_T QCD Pythia

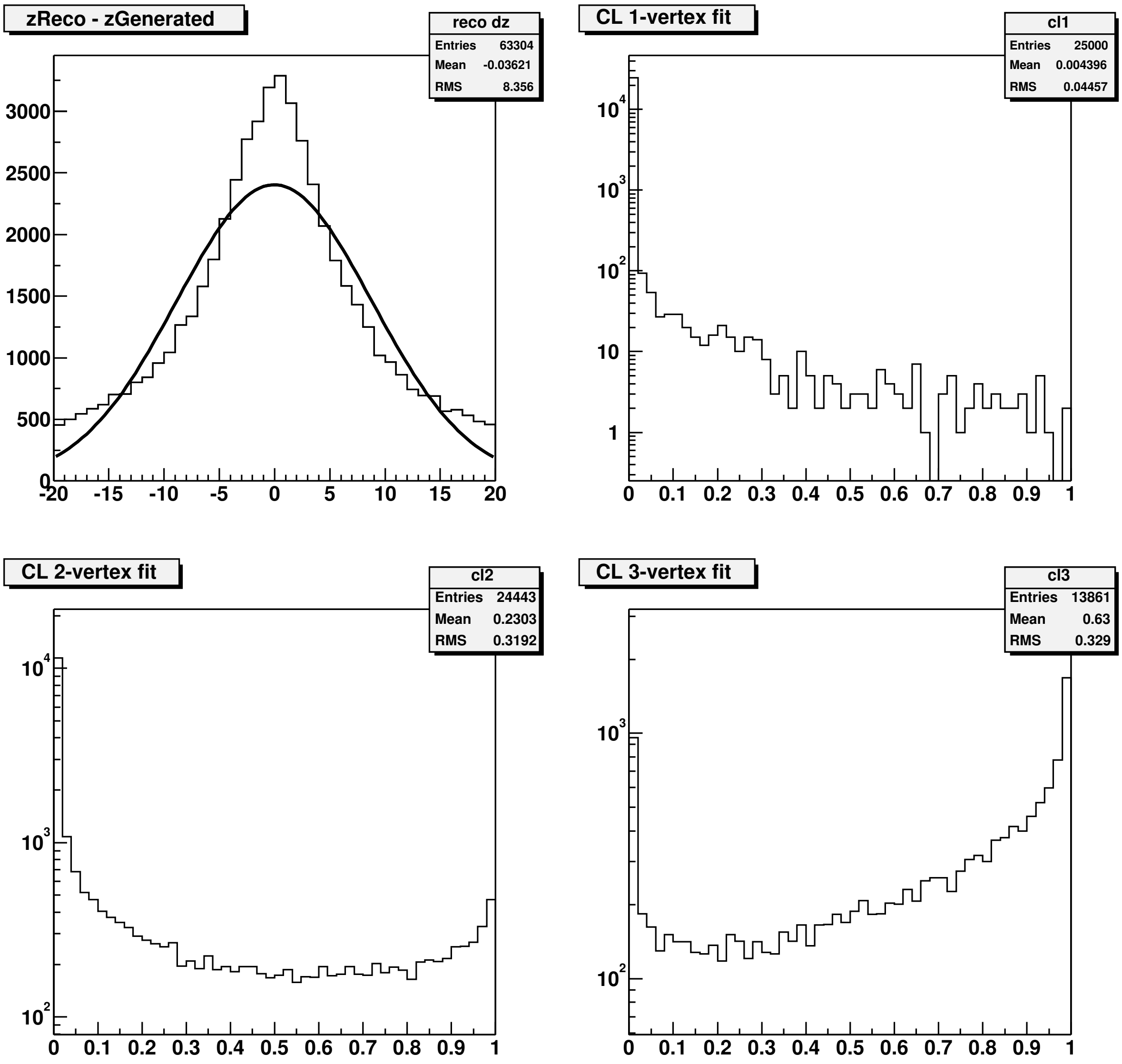

Figure 8: $\Delta z \equiv (z_V - z_{\mathrm{Pythia}})$ distribution for Pythia-generated three-vertex minimum bias events. Clearly, the CL distributions for 1-vertex and 2-vertex fits are expected to be poor. The 3-vertex fit CL distribution is biased to high CL because the algorithm assigns hits to vertices based on lower chi-squared.

events.

We note at the outset that this (x, y) position is a relative position of the counters to the beam center; we do not make an absolute measurement of position: only the SMT can do that in the DØ coordinate system. In fact, the luminosity counters are not surveyed into position even to 1 mm, and certainly not to 30 μm. Furthermore, the inner edges of each counter are unknown to within 1 mm, as well. These facts do not matter for this relative measurement of (x, y), which we can consider as a beam monitoring device.

The fit to an (x, y) beam position is very simple.[1] We histogram the hit rates in each wedge, then fit this distribution to the function

$$\frac{dN}{d\phi_k} = A + B\cos(\phi_k - \phi_0),$$

where k labels the k^{th} counter. The fitted parameter ϕ_0 is the maximum in the ϕ_k distribution, and therefore the azimuthal direction in which the beam is offset. The parameter A merely measures the rate and B is the amplitude of the cosine modulation. The ratio B/A directly measures the offset of the beam from the counter center. The radial deviation of the beam, R_{beam}, we estimate by generating many LBNs and fitting them. We find that

$$R_{\mathrm{beam}} \approx \rho[\frac{B}{A}],$$

where $\rho \approx 4.05$ cm. The beam (x, y) position is calculated as

$$x_{\mathrm{beam}} = R_{\mathrm{beam}} \cdot \cos\phi_0$$

and

$$y_{\mathrm{beam}} = R_{\mathrm{beam}} \cdot \sin\phi_0.$$

The fitted uncertainties in the parameters, σ_A, σ_B and σ_ϕ, translate into uncertainties on the final (x, y) as

$$\sigma_x = \frac{\rho}{A}\sqrt{[B\sin\phi_0]^2\sigma_\phi^2 + [[\cos\phi_0]^2\sigma_B^2 + [x_{\mathrm{beam}}^2[\frac{\sigma_A}{A}]^2}$$

and

$$\sigma_y = \frac{\rho}{A}\sqrt{[B\cos\phi_0]^2\sigma_\phi^2 + [[\sin\phi_0]^2\sigma_B^2 + [B\sin\phi_0]^2[\frac{\sigma_A}{A}]^2}.$$

The resulting fitted beam positions and their fitted uncertainties are shown in Fig. 11 for 10^5 QCD inelastic collisions per LBN. We would like to have a plot like this one in a GUI some day. The spread of the x and y positions about the Pythia beam position is about 82 μm, compared to the fitted uncertainty on each vertex of about 90 μm for single-vertex minimum bias events.

[1] In a subsequent note we will discuss calibration of the counters in pulse height, time, and space. The rates in each wedge will depend on both the radial position of the inner counter edge with respect to the beam and also the counter threshold and noise.

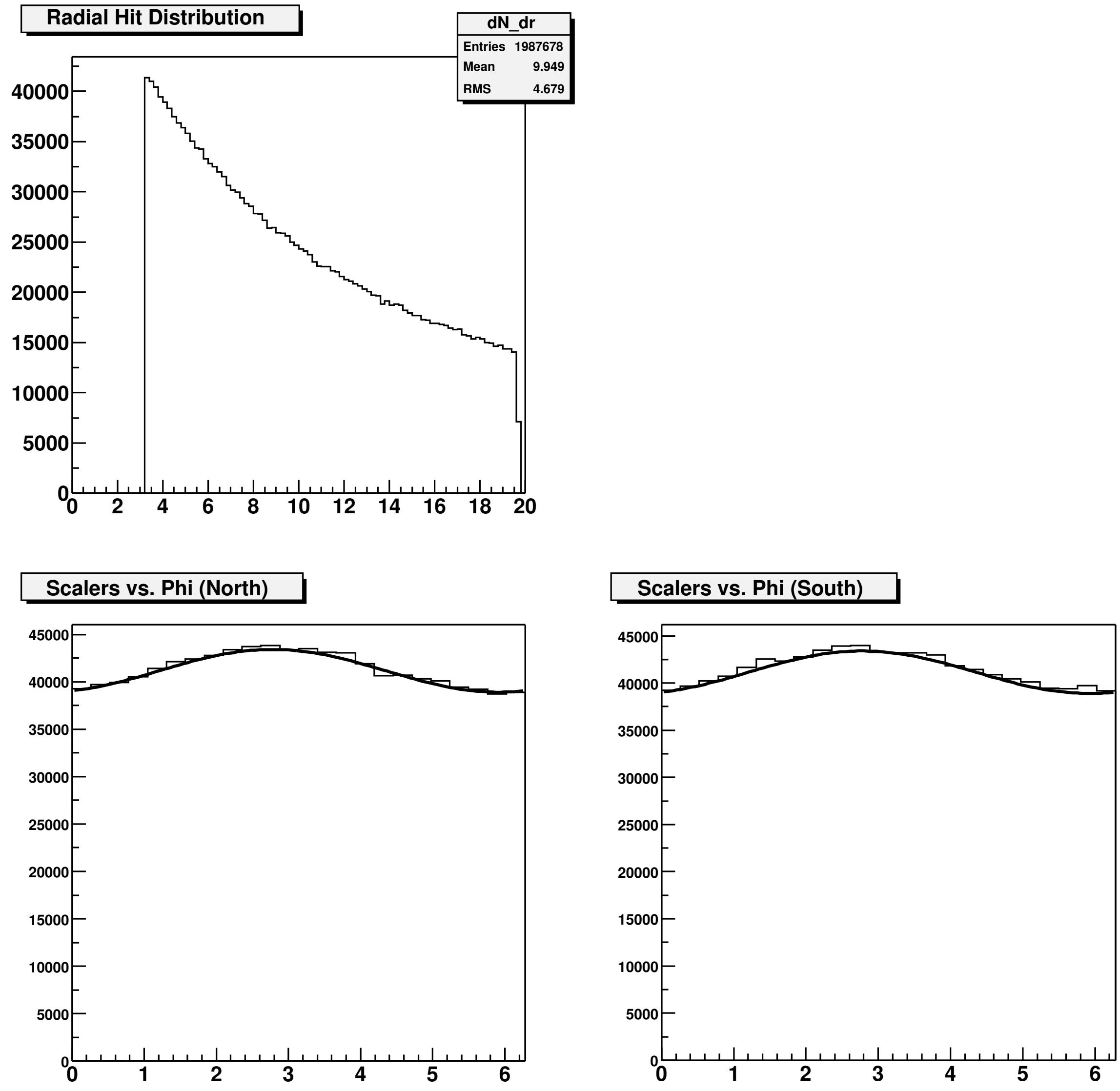

Figure 9: The radial distribution of tracks that hit the luminosity wedge counters. The inner edge of these wedges has the highest rate. The lower two figures show the asimuthal distribution of hits for minimum bias events interacting in a vertex diamond whose mean is displaced 0.2 cm in both x and y.

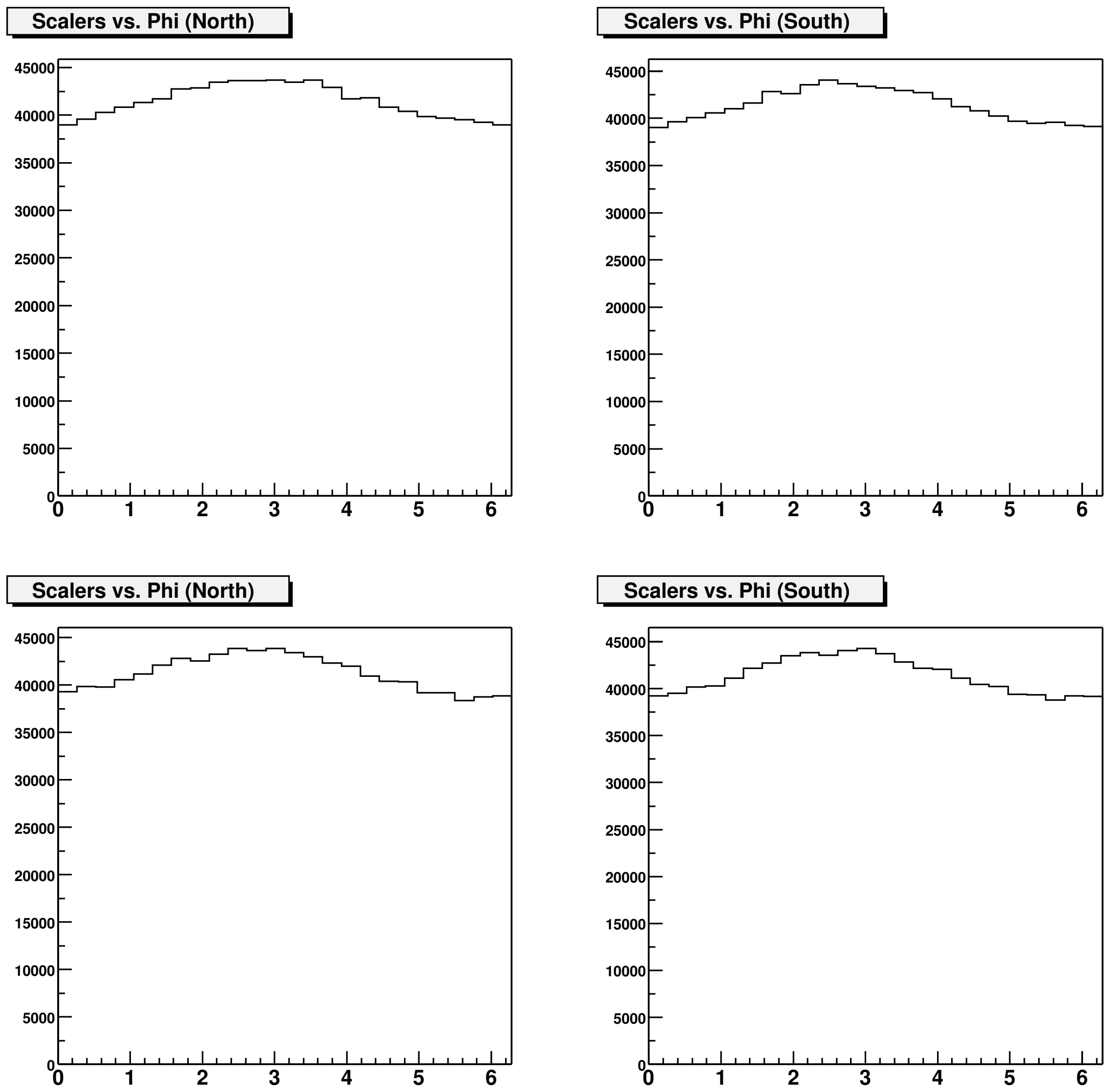

Figure 10: The $dN/d\phi_k$ distribution for two separate LNBs with about $2 \cdot 10^6$ hits per LBN. Each LBN has two estimates of (x, y), one from the North counters and one from the South counters.

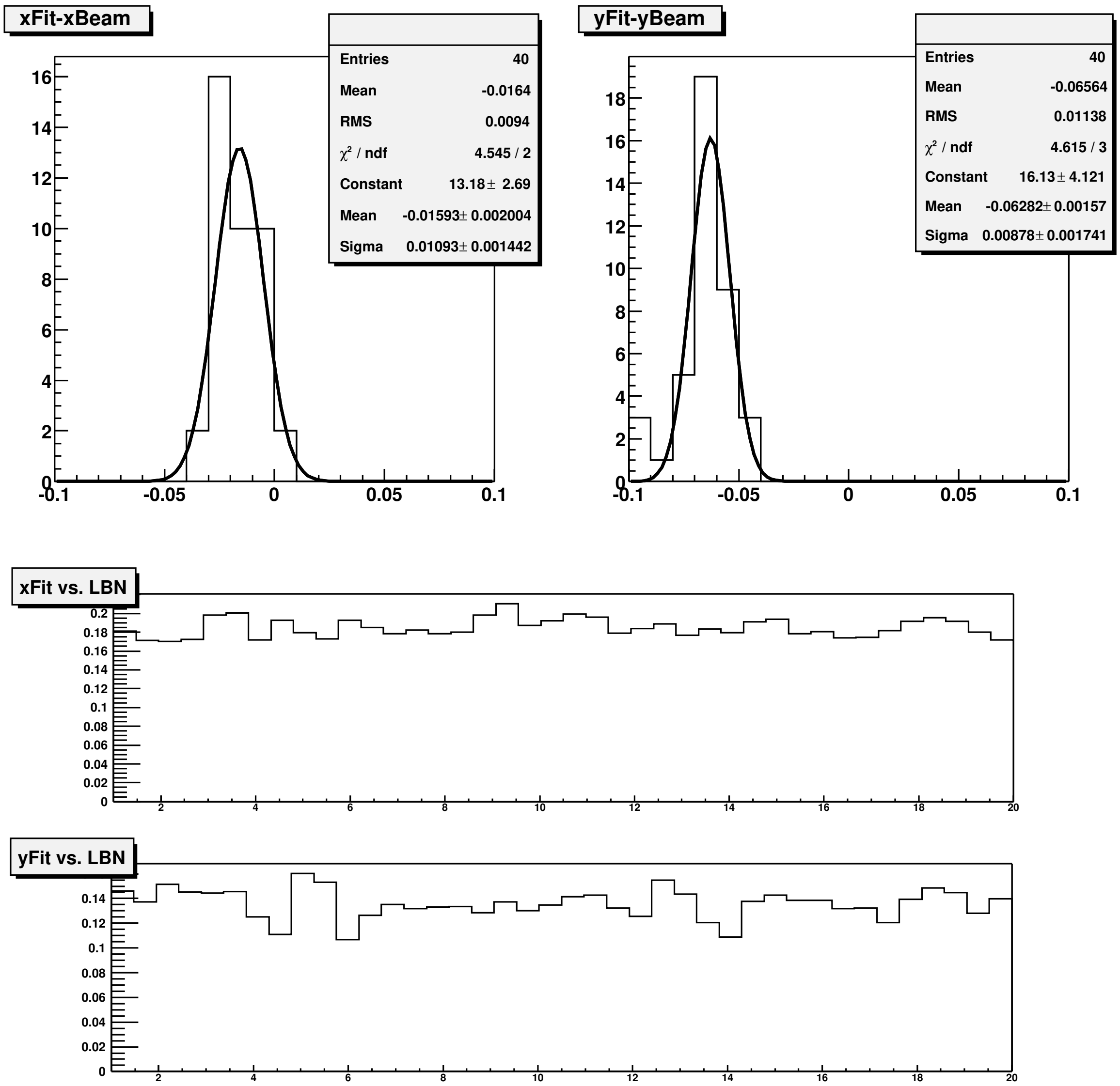

Figure 11: The fitted (x, y) mean beam position compared to the Pythia beam position (upper two figures); the x and y fitted beam position and their fits as a function of LBN for 100,000 QCD inelastic collisions per LBN. The Pythia beam was centered at $(x, y) = (0.200, 0.200)$ cm. There is obviously a bias towards fitting too low in y. The most likely source of this is the ρ parameter, used after the cosine function has been determined.

References

[1] M. Begel *et al.*, *Luminosity and Performance of DØ during the First Year of Run 2*, DØ Note 3973.

[2] M. Begel *et al.*, *DØ Luminosity in Run 2: Delivered*, DØ Note 3970.

[3] M. Begel *et al.*, *DØ Luminosity in Run 2: Triggered*, DØ Note 3971.

[4] T. Sjöstrand, Computer Physics Commun. 82 (1994) 74.