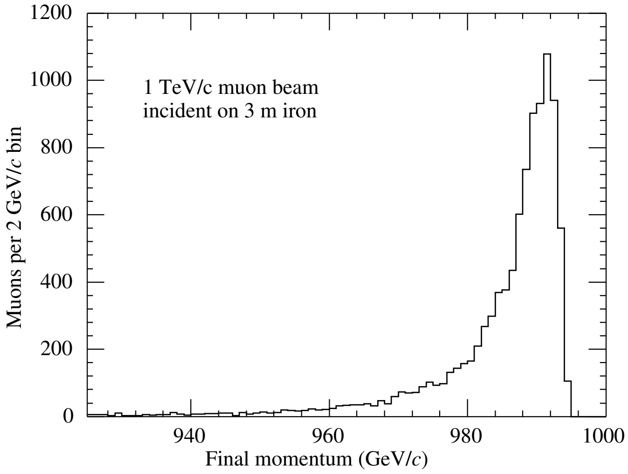

Figure 10.12: The momentum distribution of 1 TeV/c muons after traversing 3 m of iron, as obtained with Van Ginniken's TRAMU muon transport code [48].

10.10. Čerenkov and transition radiation [4,51,52]

A charged particle radiates if its velocity is greater than the local phase velocity of light (Čerenkov radiation) or if it crosses suddenly from one medium to another with different optical properties (transition radiation). Neither process is important for energy loss, but both are used in high-energy physics detectors.

Čerenkov Radiation. The half-angle θ_c of the Čerenkov cone for a particle with velocity βc in a medium with index of refraction n is

$$\begin{aligned}\theta_c &= \arccos(1/n\beta)\\ &\approx \sqrt{2(1-1/n\beta)} \quad \text{for small } \theta_c, \textit{e.g.} \text{ in gases.}\end{aligned} \tag{10.29}$$

The threshold velocity β_t is $1/n$, and $\gamma_t = 1/(1-\beta_t^2)^{1/2}$. Therefore, $\beta_t\gamma_t = 1/(2\delta+\delta^2)^{1/2}$, where $\delta = n-1$. Values of δ for various commonly used gases are given as a function of pressure and wavelength in Ref. 53. For values at atmospheric pressure, see Table 6.1. Data for other commonly used materials are given in Ref. 54.

The number of photons produced per unit path length of a particle with charge ze and per unit energy interval of the photons is

$$\begin{aligned}\frac{d^2N}{dEdx} &= \frac{\alpha z^2}{\hbar c}\sin^2\theta_c = \frac{\alpha^2 z^2}{r_e\, m_e c^2}\left(1-\frac{1}{\beta^2 n^2(E)}\right)\\ &\approx 370 \sin^2\theta_c(E)\ \text{eV}^{-1}\text{cm}^{-1} \qquad (z=1)\ ,\end{aligned} \tag{10.30}$$

or, equivalently,

$$\frac{d^2N}{dx d\lambda} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1-\frac{1}{\beta^2 n^2(\lambda)}\right)\ . \tag{10.31}$$

The index of refraction is a function of photon energy E, as is the sensitivity of the transducer used to detect the light. For practical use, Eq. (10.30) must be multiplied by the the transducer response function and integrated over the region for which $\beta\, n(E) > 1$. Further details are given in the discussion of Čerenkov detectors in the Detectors section (Sec. 12 of this *Review*).

Transition Radiation. The energy radiated when a particle with charge ze crosses the boundary between vacuum and a medium with plasma frequency ω_p is

$$I = \alpha z^2 \gamma\hbar\omega_p/3\ , \tag{10.32}$$

where

$$\begin{aligned}\hbar\omega_p &= \sqrt{4\pi N_e r_e^3}\ m_e c^2/\alpha\\ &= \sqrt{4\pi N_e a_\infty^3}\ 2\times 13.6\ \text{eV}\ .\end{aligned} \tag{10.33}$$

Here N_e is the electron density in the medium, r_e is the classical electron radius, and a_∞ is the Bohr radius. For styrene and similar materials, $\sqrt{4\pi N_e a_\infty^3} \approx 0.8$, so that $\hbar\omega_p \approx 20$ eV. The typical emission angle is $1/\gamma$.

The radiation spectrum is logarithmically divergent at low energies and decreases rapidly for $\hbar\omega/\gamma\hbar\omega_p > 1$. About half the energy is emitted in the range $0.1 \le \hbar\omega/\gamma\hbar\omega_p \le 1$. For a particle with $\gamma = 10^3$, the radiated photons are in the soft x-ray range 2 to 20 eV. The γ dependence of the emitted energy thus comes from the hardening of the spectrum rather than from an increased quantum yield. For a typical radiated photon energy of $\gamma\hbar\omega_p/4$, the quantum yield is

$$\begin{aligned}N_\gamma &\approx \frac{1}{2}\ \frac{\alpha z^2\gamma\hbar\omega_p}{3}\Big/\frac{\gamma\hbar\omega_p}{4}\\ &\approx \tfrac{2}{3}\alpha z^2 \approx 0.5\% \times z^2\ .\end{aligned} \tag{10.34}$$

More precisely, the number of photons with energy $\hbar\omega > \hbar\omega_0$ is given by [55]

$$N_\gamma(\hbar\omega > \hbar\omega_0) = \frac{\alpha z^2}{\pi}\left[\left(\ln\frac{\gamma\hbar\omega_p}{\hbar\omega_0} - 1\right)^2 + \frac{\pi^2}{12}\right]\ , \tag{10.35}$$

within corrections of order $(\hbar\omega_0/\gamma\hbar\omega_p)^2$. The number of photons above a fixed energy $\hbar\omega_0 \ll \gamma\hbar\omega_p$ thus grows as $(\ln\gamma)^2$, but the number above a fixed fraction of $\gamma\hbar\omega_p$ (as in the example above) is constant. For example, for $\hbar\omega > \gamma\hbar\omega_p/10$, $N_\gamma = 2.519\,\alpha z^2/\pi = 0.59\% \times z^2$.

The yield can be increased by using a stack of plastic foils with gaps between. However, interference can be important, and the soft x rays are readily absorbed in the foils. The first problem can be overcome by choosing thicknesses and spacings large compared to the "formation length" $D = \gamma c/\omega_p$, which in practical situations is tens of μm. Other practical problems are discussed in Sec. 12.

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