NUFAC 03 Institute Lectures

Shelter island, NY
May 27-June 3, 2003

R. B. Palmer (BNL)

- Pion Capture and Phase Rotation
- Solenoid Focus &
  Transverse Ionization Cooling
- 6 Dimension Ionization Cooling Rings

Contents

1 Preface

1.1 Units

When discussing the motion of particles in magnetic fields, I will use MKS units, but this means that momentum, energy, and mass are in Joules and kilograms, rather than in the familiar 'electron Volts'. To make the conversion easy, I will introduce these quantities in the forms: $[pc/e]$, $[E1/e]$, and $[mc^2/e]$, respectively. Each of these expressions are then in units of straight Volts corresponding to the values of p, E and m expressed in electron Volts. For instance, I will write, for the bending radius in a field B:

$$\rho = \frac{[pc/e]}{B\ c}$$

meaning that the radius for a 3 GeV/c particle in 5 Tesla is

$$\rho = \frac{3\ 10^9}{5 \times 3\ 10^8} = 2m$$

This units problem is often resolved in accelerator texts by expressing parameters in terms of $(B\rho)$ where this is a measure of momentum: the momentum that would have this value of $B \times \rho$, where

$$(B\rho) = \frac{[pc/e]}{c}$$

For 3 GeV/c, $(B\rho)$ is thus 10 (Tm), and the radius of bending in a field B=5 (T) is:

$$\rho = \frac{(B\rho)}{B} = \frac{10}{5} = 2m$$

1.2 Useful Relations

$$dE = \beta_v \, dp \tag{1}$$

$$\frac{dE}{E} = \beta_v^2 \, \frac{dp}{p} \tag{2}$$

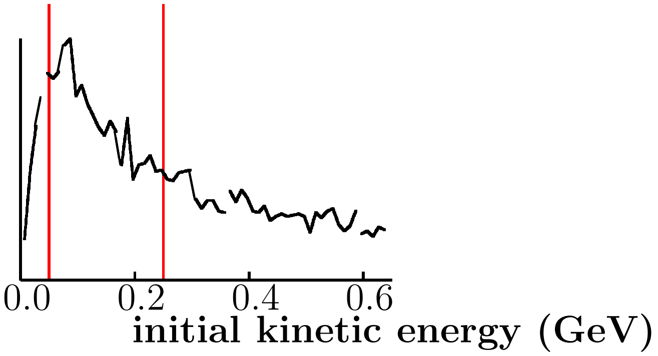
$$d\beta = \frac{dp}{\gamma^2} \tag{3}$$

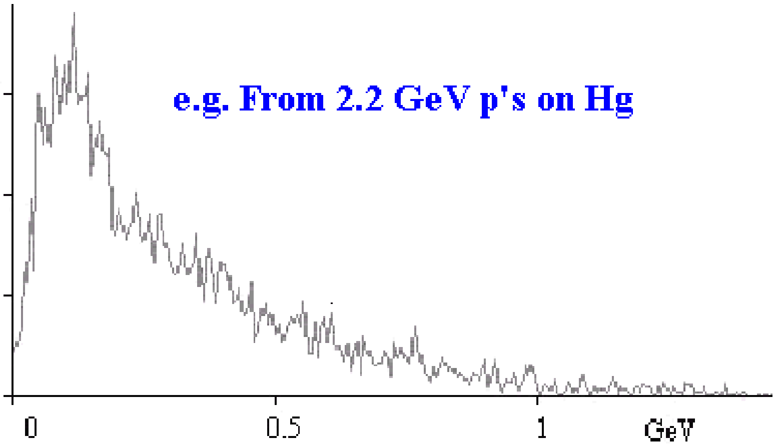
2 Pion Capture, Phase Rotation and Bunching

2.1 Production

2.1.1 Initial KE Distribution

e.g. from 24 GeV p's on Hg





- similar distributions
- Reasonable bight to accept: 50-250 MeV
- $< p > \approx$200 MeV/c
- $\sigma_{p\perp} > \approx$200 MeV/c
- rms angles ≈45 degrees !

2.2 Pion Capture

2.2.1 Magnetic Horn Capture

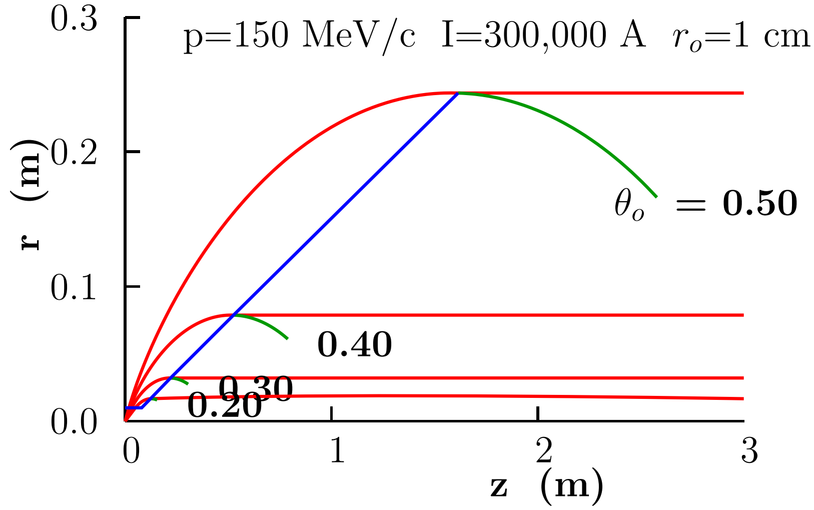
Horn theory Outside an axial conductor

$$B = \frac{\mu_o\, I}{2\,\pi}\,\frac{1}{r}$$

Bending:

$$\frac{d\theta}{ds} = \frac{B\ c}{[pc/e]}$$

Minimum radius set by inward forces. Find exit shape to focus mom=p:

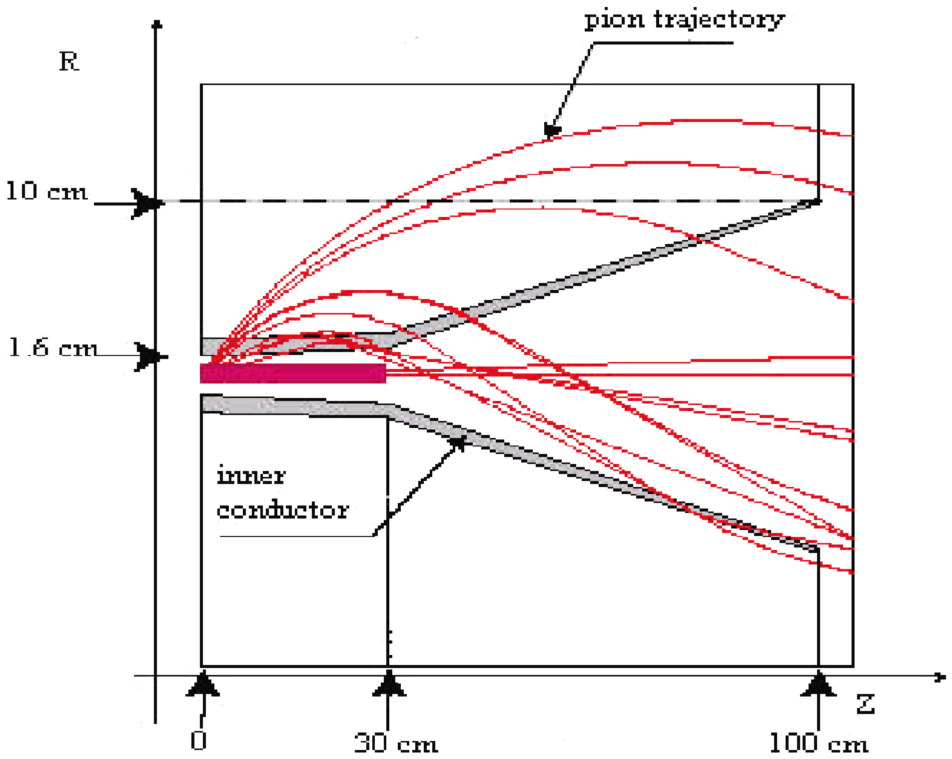


Shape is close to a conical "horn"

Note that it does not focus small angle particles

Example

CERN Design



Horns have been long used for neutrino beams, but at a repetition rate of 1/5 Hz or less

Challenge is:

- Run at a higher Rep rate (≈15 Hz)
- Withstand Radiation Damage
- Allow cooling of Target, or use of mercury inside r_o

2.3 Solenoid Capture

In the transverse plane:

$$r \;=\; \frac{[pc/e]_\perp}{c\;B}$$

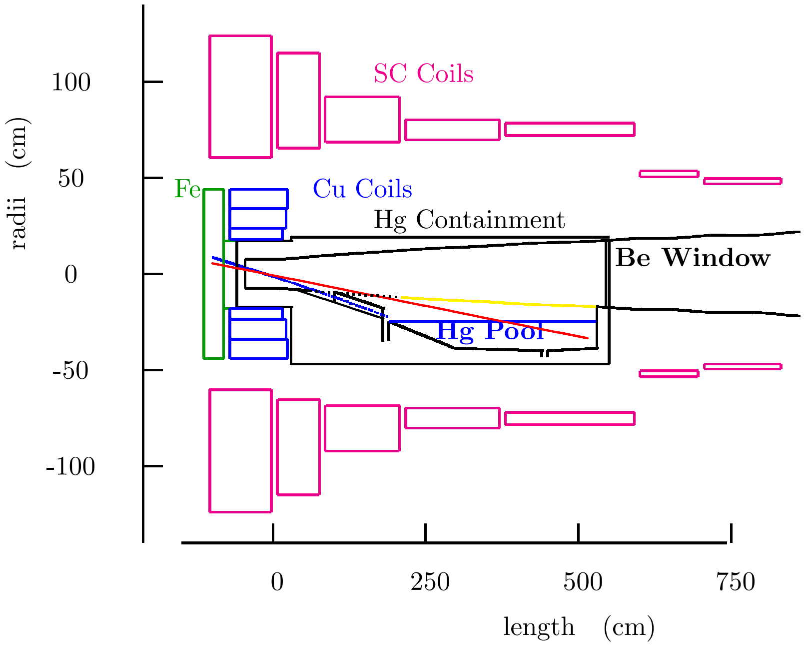
For particles generated in a thin target on the axis, inside a solenoid of inside radius R, the maximum transverse momenta captured will be:

$$[pc/e]_\perp(max) \;=\; \frac{c\;B_z\;R}{2} \tag{4}$$

e.g. For a 20 T solenoid of 8 cm radius, (These are the dimensions of an existing resistive solenoid at FSU)

$$p_\perp(max) \;=\; 240 MeV/c$$

Contains å80% of π's below 250 MeV



2.4 Adiabatic Matching

The match between a target capture Solenoid and a decay channel solenoid can be made, with negligible loss, by gently tapering the magnetic field[1] .

The condition for "gentleness" is that $d\beta/\beta$, is small in a distance equal to the current β:

$$\frac{d\beta}{\beta} \ll \frac{dz}{\beta}$$

or

$$\frac{d\beta}{dz} = \epsilon \ll 1$$

Since $\beta \propto 1/B_{solenoid}$:

$$\frac{d(1/B)}{dz} = \epsilon \ll 1$$

which gives:

$$B(z) = \frac{B_o}{1 + k\ z} \tag{5}$$

where

$$k = \epsilon \frac{B_o\ c}{2\ [pc/e]} \tag{6}$$

Note that the B drops initially very fast, corresponding to the short β's at the high initial field, but falls much slower at the lower later fields where the β's are long.

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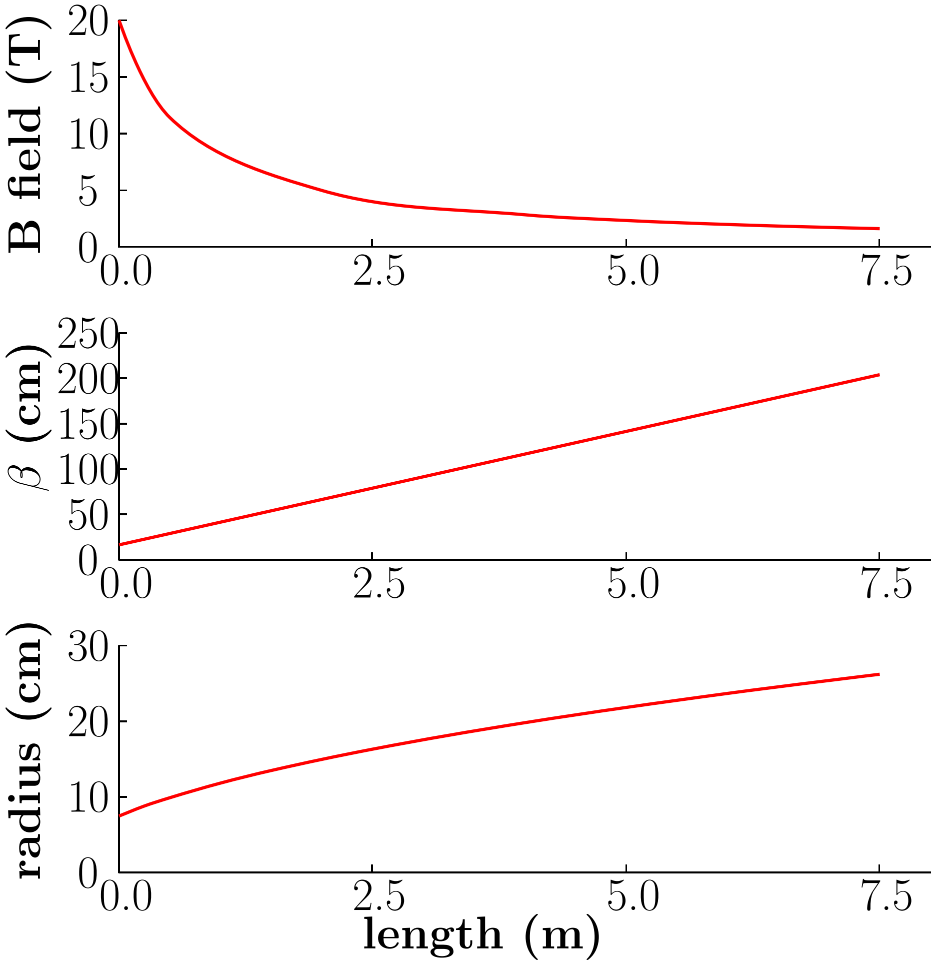
[1] R. Chehab, J. Math. Phys. 5 (1978) 9.

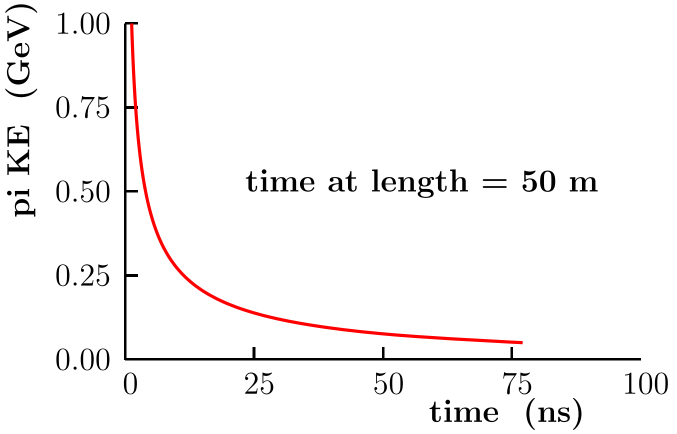
For a taper from 20T to 1.25T at momenta less than 1 GeV and $\epsilon = .5$, the taper length should be approximately 6 m.



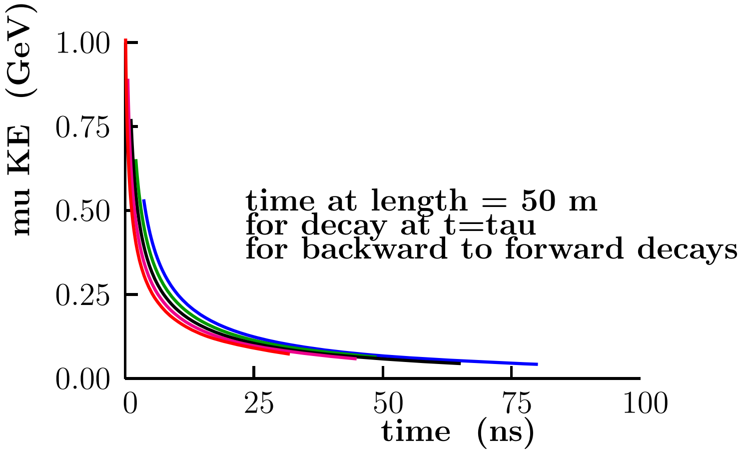
2.5 Time Jitter from Pion Decay

Note that in this section I will the HEP convention of $c = 1$ so that p and m are in the same eV units.

If the p bunch had zero length, and there was no decay, then after a drift the momentum vs. time distribution has zero width and phase rotation is ideal.



But since the pions decay to muons ($\pi \rightarrow \mu + \nu$) there is a spread from the random pion decay angle and decay position:



Decay in Center of Mass

$p_\mu \approx 30$ MeV/c $\approx m_\pi - m_\mu$

Isotropic, so
$\dfrac{dn}{dp_{\mu z}}$ is flat from -30 to 30 MeV/c

$$E_\mu \;=\; \sqrt{p_\mu^2 + m_\mu^2} \;\approx m_\mu$$

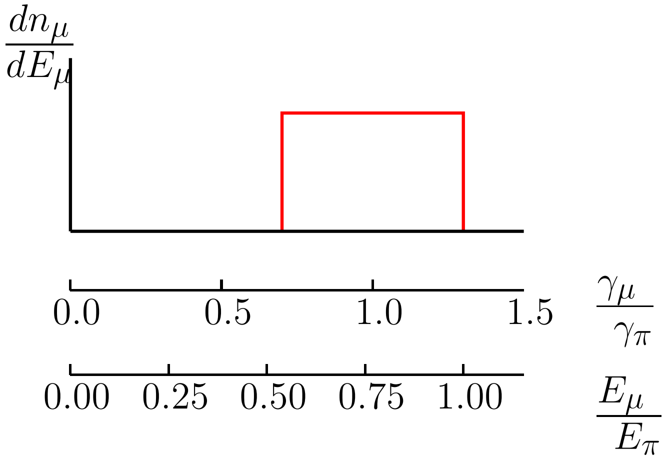
Lorentz Boost to velocity of initial π

$$\gamma_\pi \;=\; \frac{\mathrm{KE} + m_\pi}{m_\pi}$$

$$\beta_\pi \;=\; \sqrt{1 - \frac{1}{\gamma_\pi^2}}$$

$$E_\mu(\mathrm{final}) \;=\; \gamma_\pi \quad E_\mu(\mathrm{c\ of\ m}) \;+\; \beta_\pi\gamma_\pi \quad p_z(\mathrm{c\ of\ m})$$

$$\gamma_\mu(\mathrm{final}) \;\approx\; \gamma_\pi \quad \pm \quad \beta_\pi\gamma_\pi \quad \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$



$$< \gamma_\mu > = \ \gamma_\pi \ = \ \gamma$$

$$\Delta\gamma_\mu \ = \ \pm \ \beta\gamma \quad \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

$$\Delta\beta_\mu \ \approx \ \frac{d\beta}{d\gamma} \ \Delta\gamma_\mu \ = \ \frac{1}{\gamma^3\beta} \ \beta\gamma \quad \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

$$\Delta\beta_\mu \ = \ \frac{1}{\gamma^2} \ \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

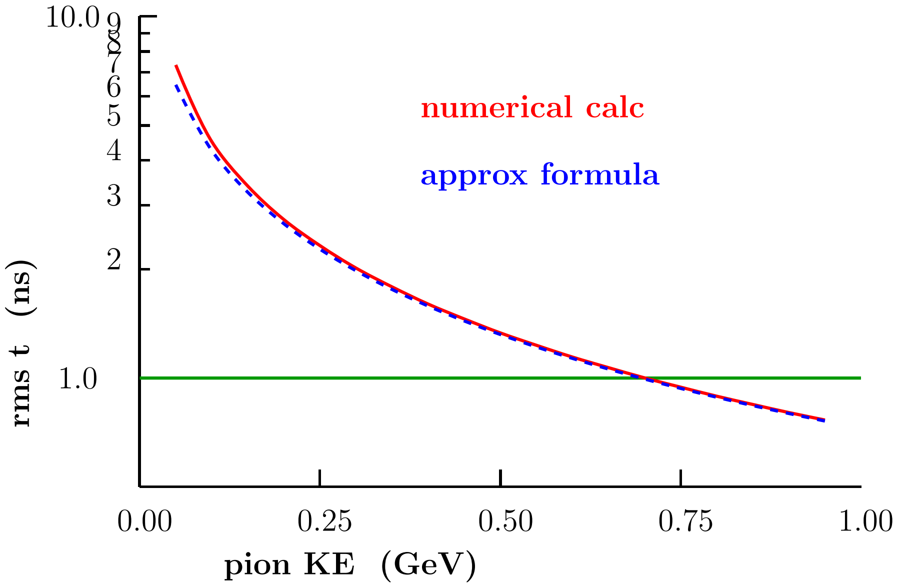
If decay occured at distance $\ell \ = \ \beta c\gamma\tau$ then Δt between forward and backward cases:

$$\Delta t \approx \frac{d}{d\beta} \left(\frac{L}{\beta c}\right) \Delta\beta \ = \ \frac{1}{\beta^2 c} \beta c\gamma\tau \ \Delta\beta$$

$$\Delta t \approx \ \frac{\tau}{\beta\gamma} \ \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

The rms spread of a uniform distribution = $\sqrt{1/3}$ × max, and the rms of the exponential is =$\sqrt{2} \ \times \ \tau$

$$\sigma_t \approx \ \sqrt{\frac{2}{3}} \ \frac{\tau}{\beta\gamma} \quad \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$



Conclusion on jitter from decay

If we capture muons from 50 to 250 MeV, the average $KE_\pi \approx$ 125 MeV, where $\sigma_t \approx$ 4 ns. If we want the broadening from the proton σ_t to be $< 25\%$ then σ_t(beam)<3 (nsec).

If we capture higher momentum muons, then we need a shorter p bunch length.

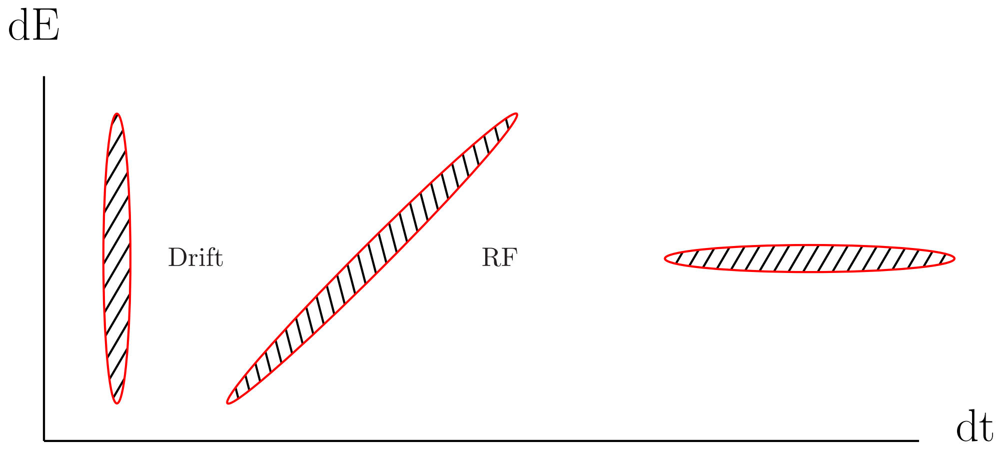
2.6 Phase Rotation

2.6.1 Introduction

- Initial pions have rms dp/p ≈ 100%
- rms Acceptance of cooling ≈ 8%

Phase Rotate

- Increase dt
- Decrease dE



2.6.2 Phase Space Conservation

For initial

$$\Delta \mathrm{E} = 200 \text{ MeV (full width)} \quad \times \quad \delta \mathrm{t} = 4 \text{ nsec (rms)}$$

(time is set by fluctuations in decay. See section 2.5)

and final

$$\delta E = 16 \text{ MeV (rms)}$$

δE/E 8% (rms) at 200 MeV

Then

$$\Delta t(final) = \frac{200(full) \times 4(rms)}{16(rms)} = 50nsec(full)$$

To capture and accelerate this, without re-bunching, we need frequency $\ll$ 1/50(nsec), i.e. $\ll$ 20 MHz

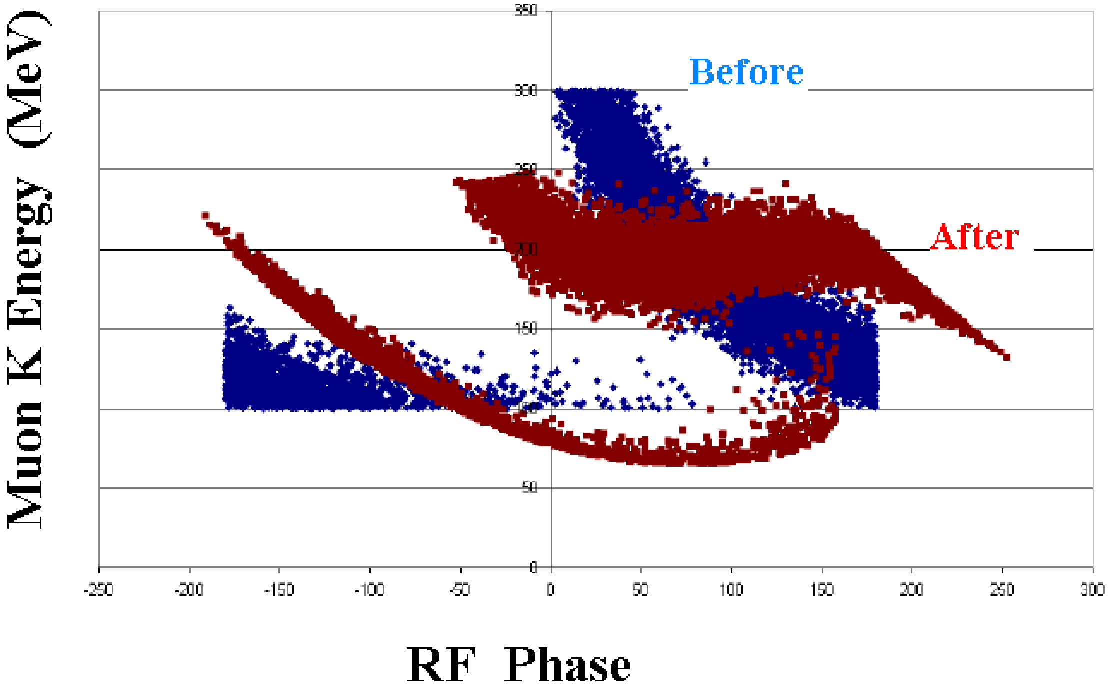
- KEK: 5-25 MHz which meets the requirement, but which allows only low gradients.
- PJK: 30 MHz but gave final ≈15 % (rather than above 8%) dp/p
- CERN: 44 or 88 MHz neither can meet this specification

Solution for ν factory (but not collider) :
rebunch into multiple higher frequency RF cycles

2.6.3 Phase Rotation without re-bunching

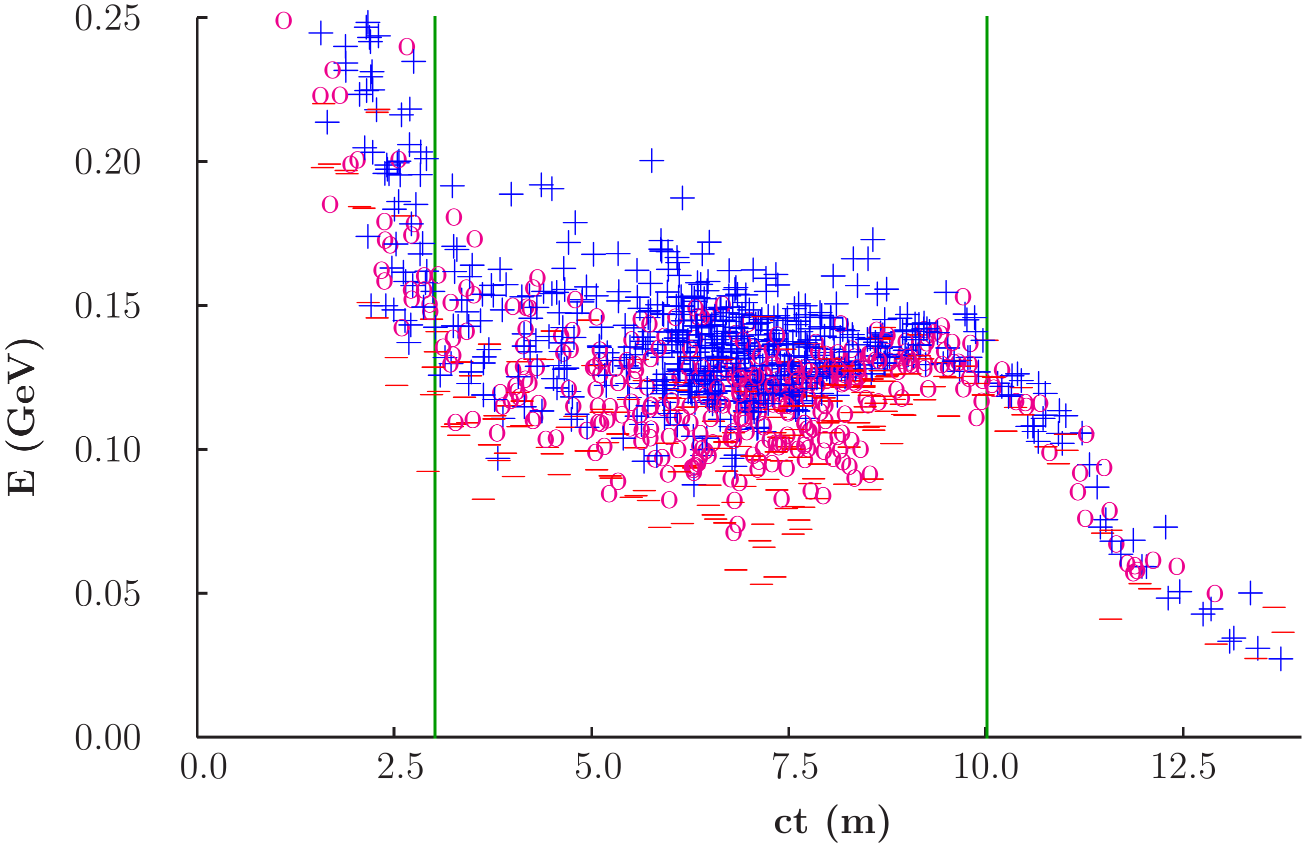
e.g. CERN

- 30 m decay channel
- 30 m 2 MV/m 44 MHz RF
- Captures ≈120-300 MeV
- Gives ≈4 m long bunch
- and ≈± 5%



e.g. PJK

	Len m	freq MHz	Grad MV/m
Drift	6		
RF	12	40	6
RF	24	30	5
RF	5	45	6



- ≈6 m long bunch
- ≈12 % dE/E

2.6.4 Phase Rotation with Re-Bunching

Alternative allowing higher frequencies:

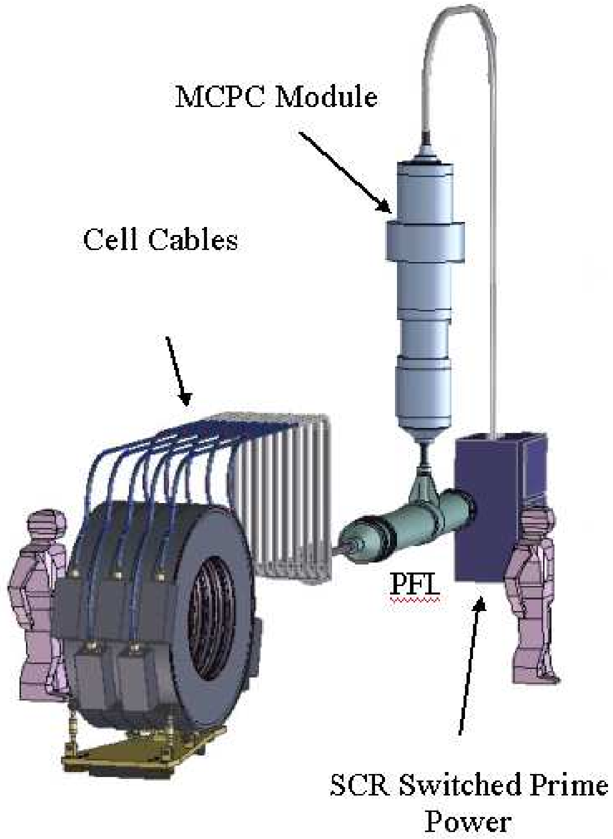
Re-bunching increases dE/E by ≈4 ×
So require dE/E ≈2% before re bunching
And Δt ≈50 nsec ×4 ≈ 200 nsec

US Study 1 had ≈150 nsec
US Study 2 had ≈300 nsec

Too long for conventional rf,

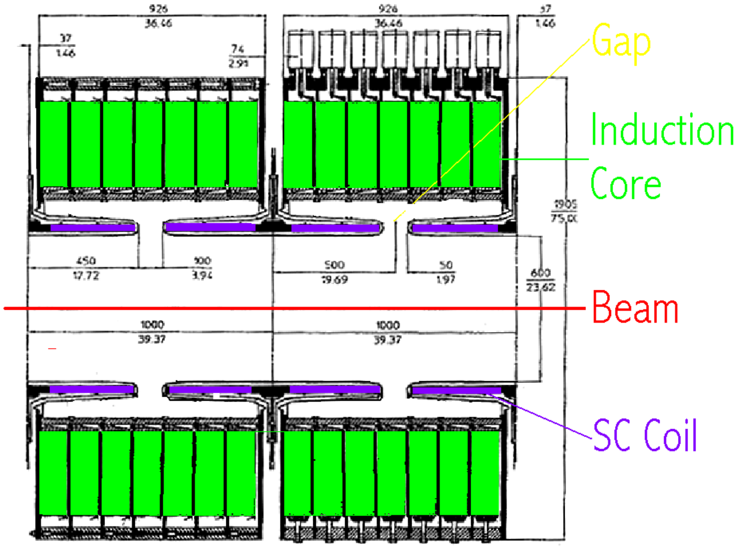
Use Induction Linacs

- pulses 50-500 nsec
- Grad's ≈1 MV/m



2m Section

95 cm radius similar to ATA or DARHT but Superconducting inside coil



e.g. From US Study 1

- Energy spread non uniform
  "Distorted"
- dp/p rms ≈ 6%
- → 18% after bunching
- particles lost

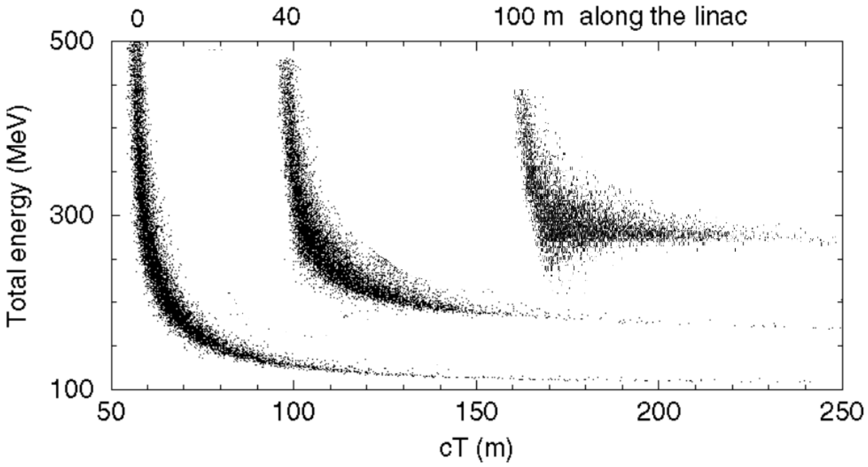
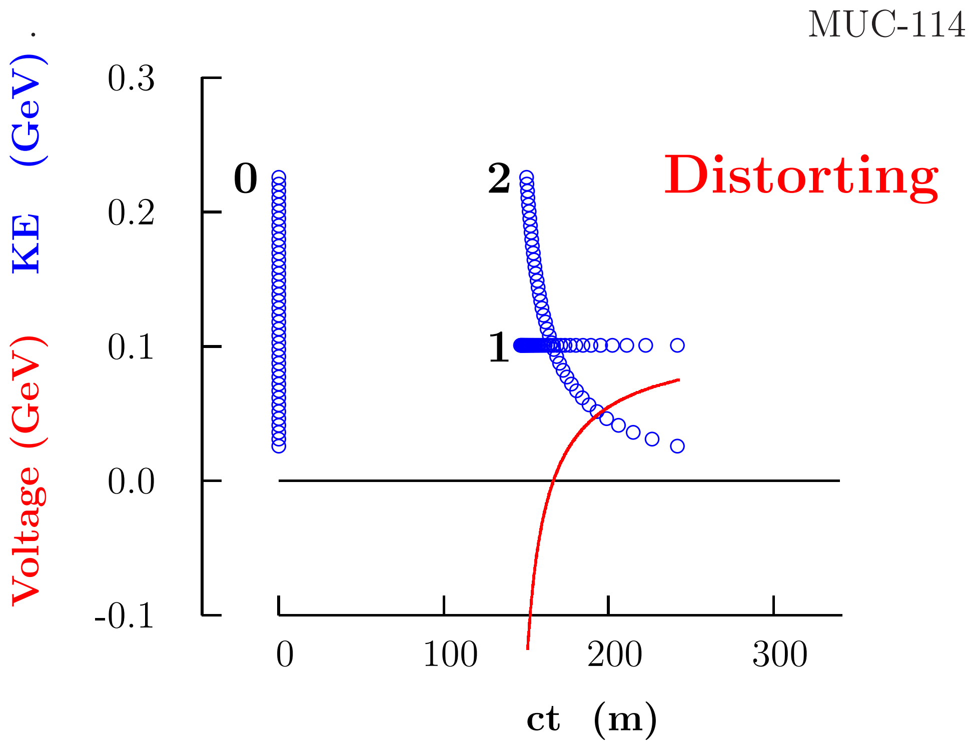
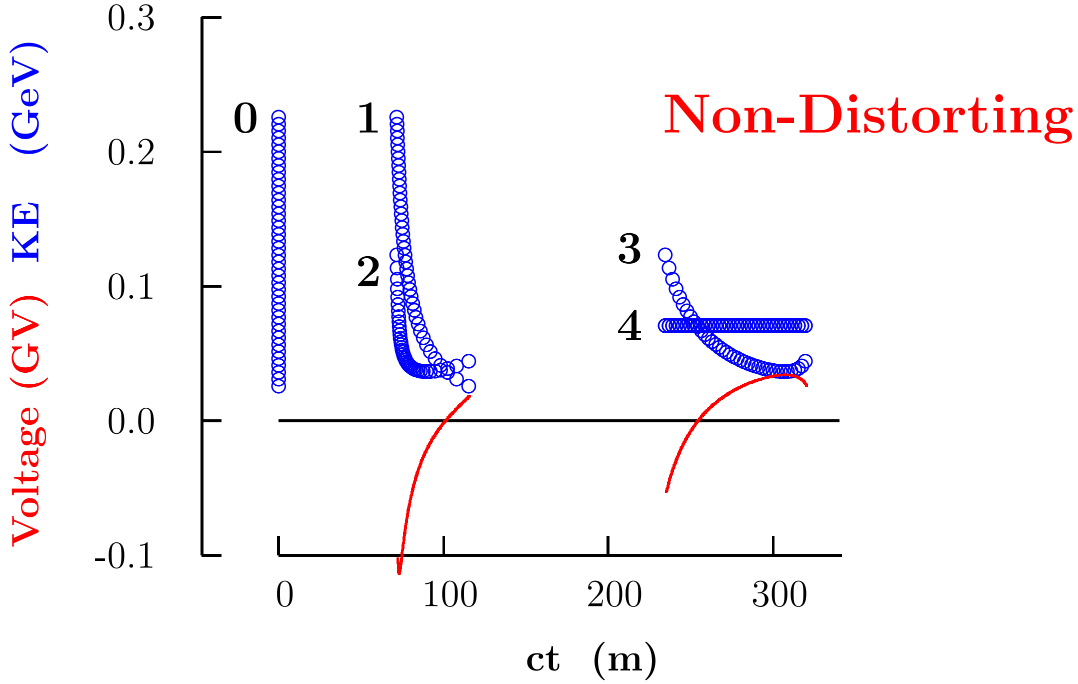


Figure 6: Beam distributions in E-cT phase space along the induction linac. Distributions from L = 0, 20, 60, and 100 m are shown.

Note: above phase space after rotation has funny shape: large dp at low E, and small dp at high E. i.e. The phase space is distorted. This can be fixed.

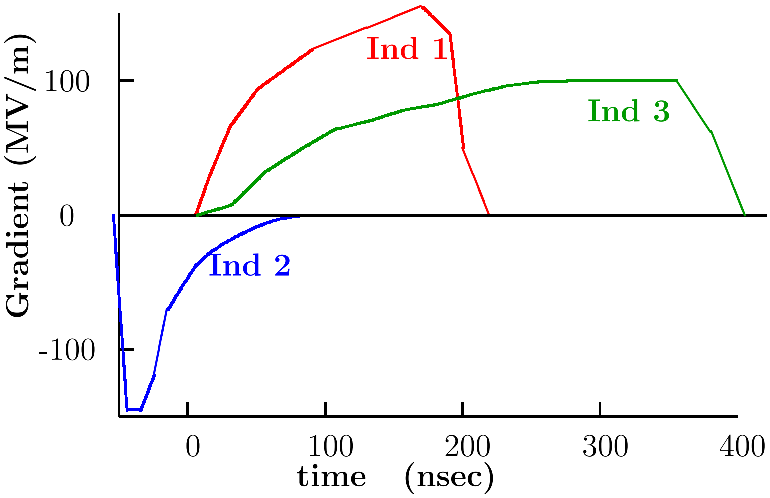
2.6.5 Non-Distorting Phase Rotation

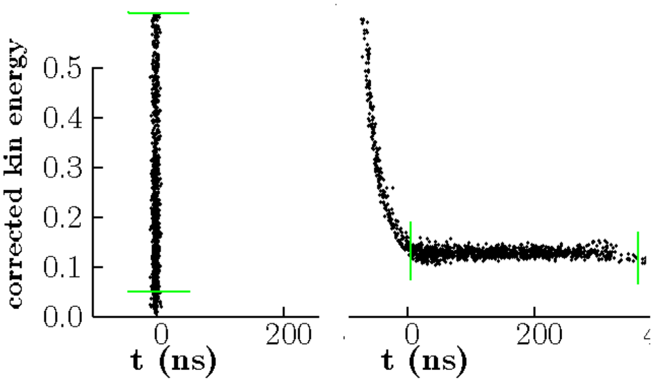




Study 2 with 3 Linacs

1. 30 m Drift
2. 100 m induction Linac to modify E vs t
3. Second drift (≈ 100 m)
4. 2 × 80 m induction Linacs to reduce dE/E





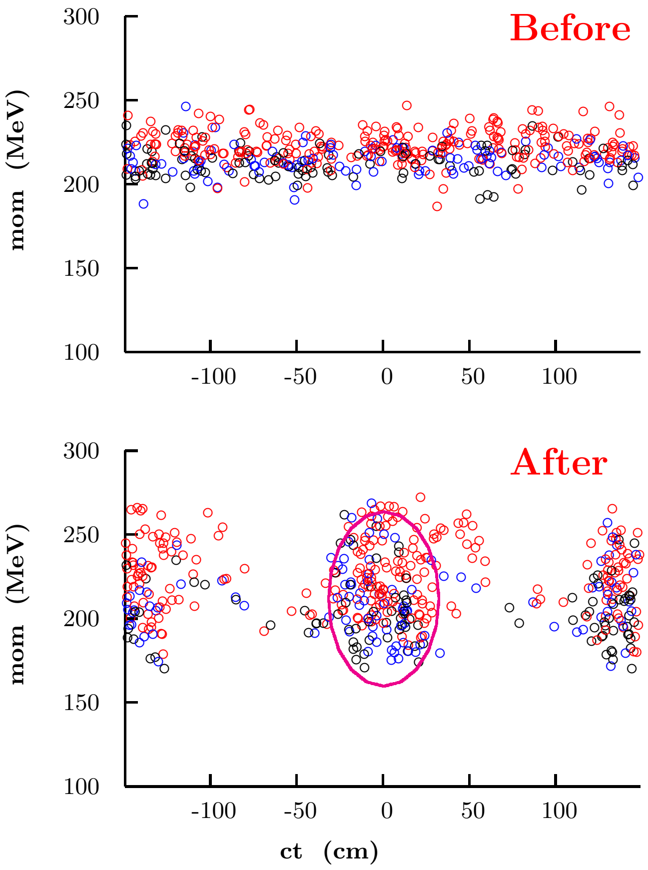
- Energy spread more uniform
- dp/p rms ≈ 3%
- OK for bunching
- But Expensive

2.7 RF Buncher

e.g. from Study-2: Three stages:

stage		len	400 MHz	200 MHz
		m	MV	MV
1	RF	2.75	-2.38	9.55
	Drift	22		
2	RF	5.5	-4.46	17.9
	Drift	8.25		
3	RF	8.25		35.8
	Drift	5.5		

Similar to Study 1



2.8 Bunched Phase Rotation

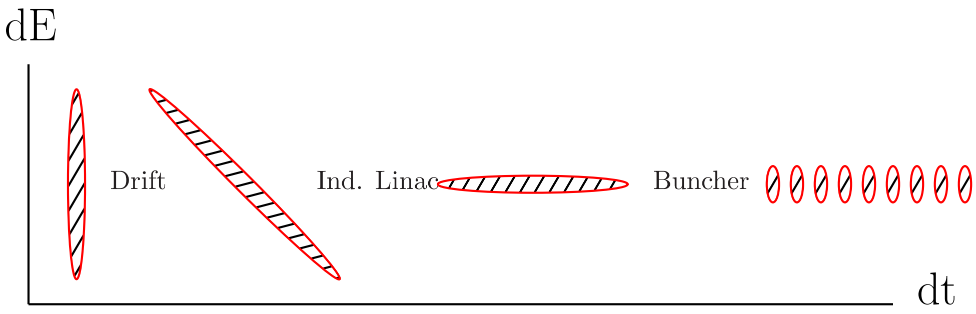
2.8.1 Introduction

1. Drift
2. Bunch
3. Rotate with high freq. rf

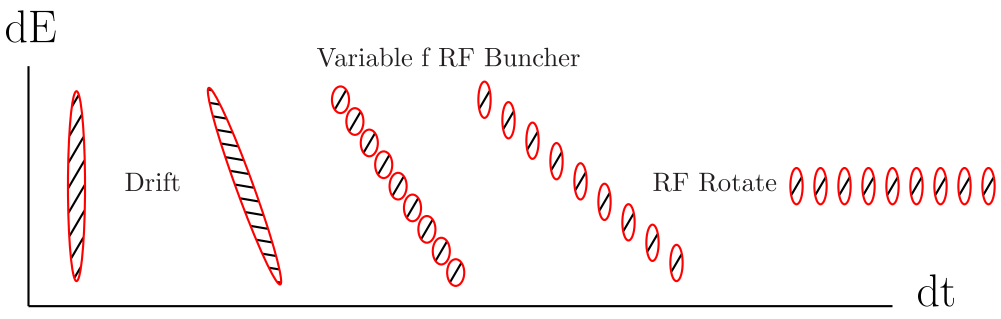
vs. Conventional

1. Drift
2. Rotate with induction linac
3. Bunch

Study 2 with Induction Linacs



Bunched Beam Rotation with 200 MHz RF (Neuffer)



2.8.2 Simulation

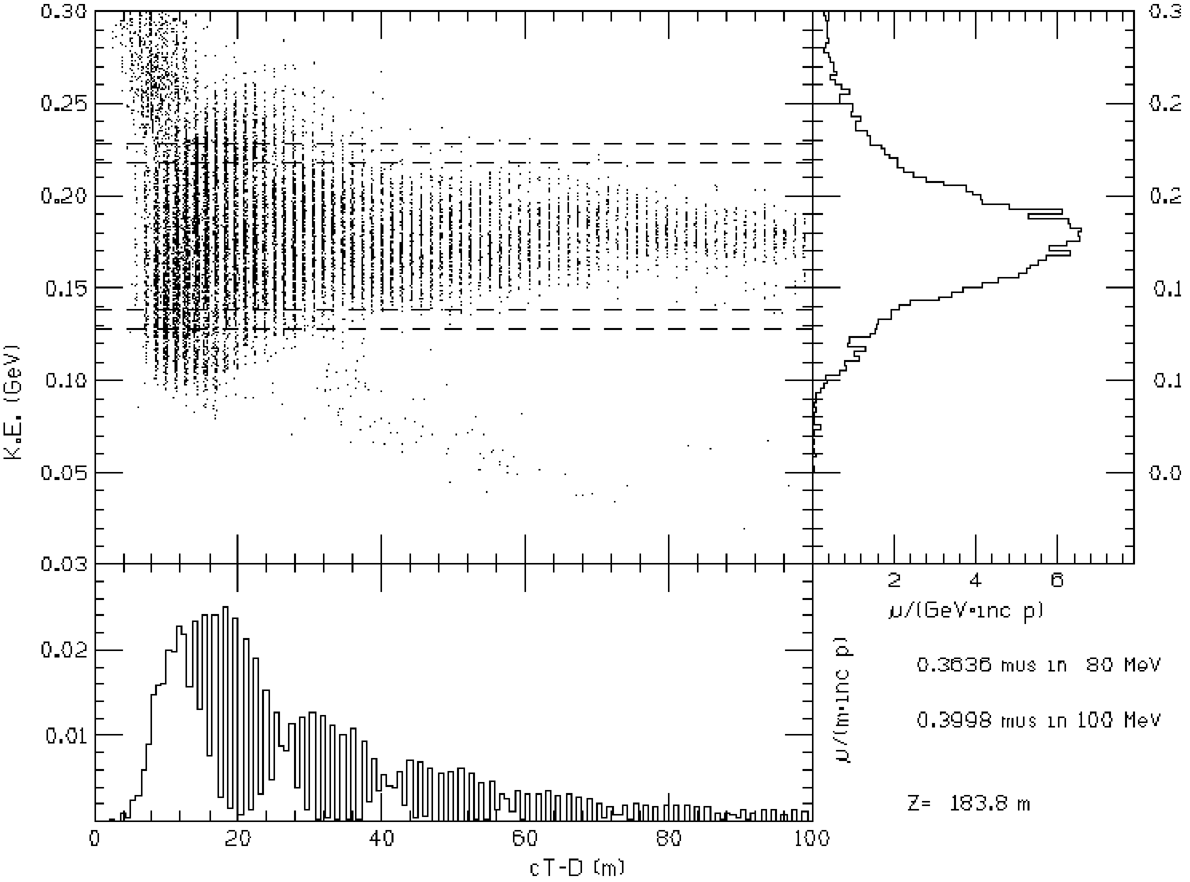
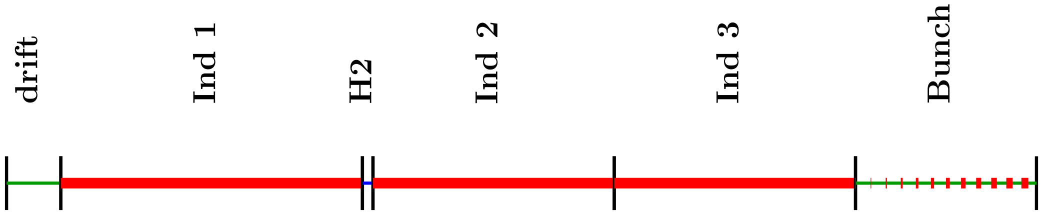


Figure 7: Muon distribution in (E, t)-space along with marginal distributions for 38 vernier (d=0.16) cavities followed by 23 (matched) fixed frequency cavities *generated with* ICOOL *program.* N_b=20 in buncher part. Plots and numbers quoted are based on 188 000 incident protons.

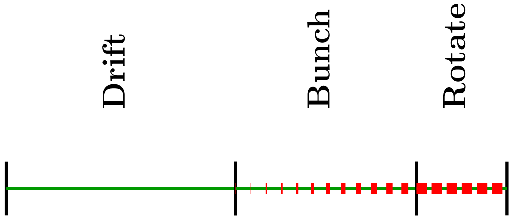
2.8.3 Compare with conventional

1. Inevitably Distorting
2. But shorter, giving less decay
3. Similar efficiency for one sign
4. But both signs rotated
5. Much less cost than induction

- **Study 2**



- **e.g. Bunch Beam Rotation**



3 Transverse Cooling

3.1 Recap Beam Definitions

3.1.1 Emittance

$$\text{normalized emittance} \ = \ \frac{\text{Phase Space Area}}{\pi \text{ m c}}$$

The phase space can be transverse:p_x vs x, p_y vs y, or longitudinal Δp_z vs z, where Δp_z and z are with respect to the moving bunch center.

If x and p_x are both Gaussian and uncorrelated, then the area is that of an upright ellipse, and:

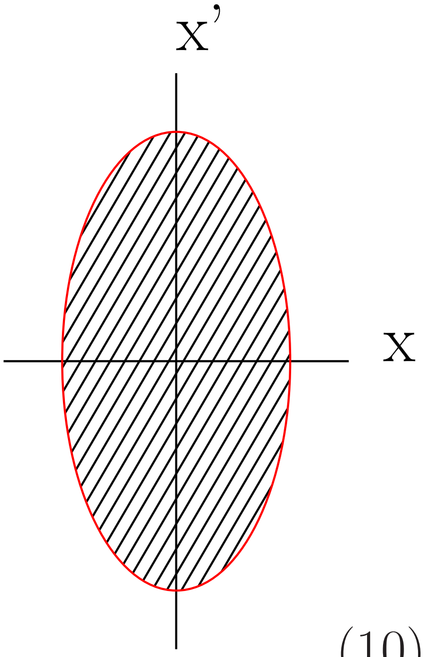
$$\epsilon_\perp = \frac{\pi \ \sigma_{p_\perp}\sigma_x}{\pi \ mc} \ = \ (\gamma\beta_v)\sigma_\theta\sigma_x \qquad\qquad (\pi \ m \ rad) \quad (7)$$

$$\epsilon_\parallel = \frac{\pi \ \sigma_{p_\parallel}\sigma_z}{\pi \ mc} \ = \ (\gamma\beta_v)\frac{\sigma_p}{p} \ \sigma_z \qquad\qquad (\pi \ m \ rad) \quad (8)$$

$$\epsilon_6 = \epsilon_\perp^2 \ \ \epsilon_\parallel \qquad\qquad (\pi \ m)^3 \qquad (9)$$

Note that, by convention, the π is not included in the calculated values, but added to the dimension

3.1.2 Beta$_{Courant-Schneider}$ of Beam



Again upright ellipse in x' vs x,

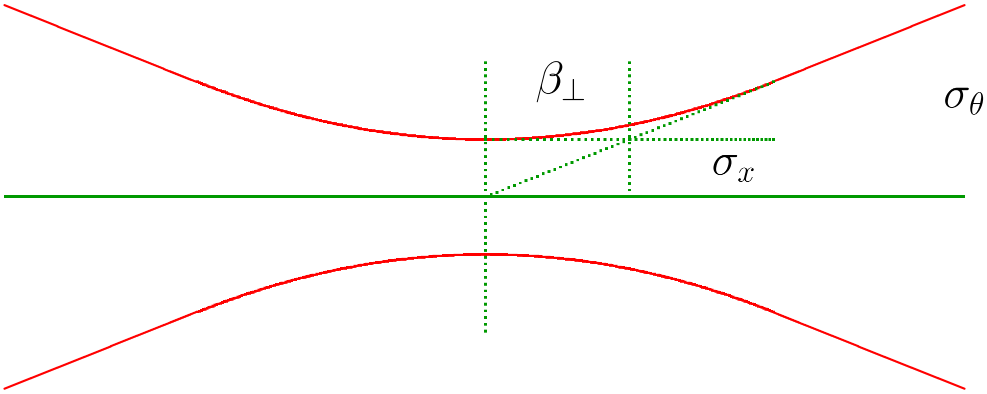
$$\beta_\perp = \frac{\sigma_x}{\sigma_\theta} \qquad (10)$$

Then, using emittance definition:

$$\sigma_x = \sqrt{\epsilon_\perp \, \beta_\perp \, \frac{1}{\beta_v \gamma}} \qquad (11)$$

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp} \, \frac{1}{\beta_v \gamma}} \qquad (12)$$

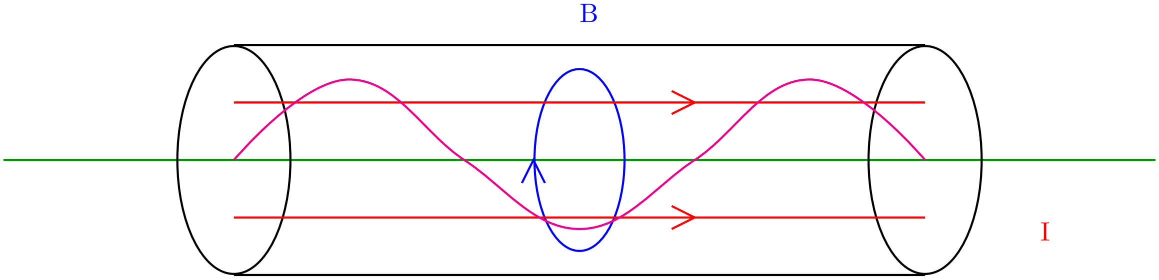
3.1.3 Beta$_{Courant-Schneider}$ at focus



β is like a depth of focus

3.1.4 Beta$_{Courant-Schneider}$ of a Lattice

$\beta_\perp$ above was defined by the beam, but a lattice can have a β_o that "matches" a beam



e,g. if continuous inward focusing force, as in a current carrying lithium cylinder (lithium lens), then

$$\frac{d^2u}{dz^2} = -k\ u$$

$$y = A\ \sin\left(\frac{z}{\beta_o}\right)$$

$$y' = \frac{A}{\beta_o}\ \cos\left(\frac{z}{\beta_o}\right)$$

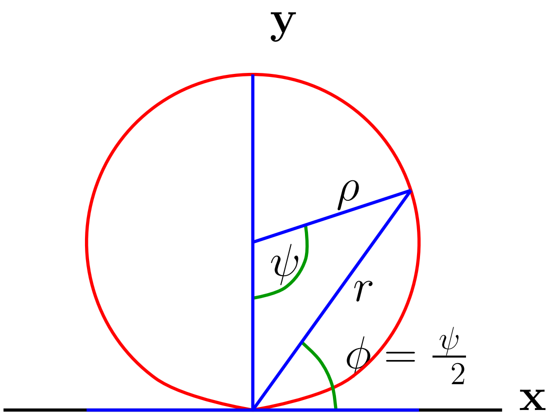
where $\beta_o = 1/k$ If $\beta_o = \beta_{\text{beam}}$ then all particles move arround the ellipse, and the shape, and thus β_{beam} remains constant. i.e. the beam is matched to this lattice. If $\beta_o \neq \beta_{\text{beam}}$, then β_{beam} oscillates about β_o: often refered to as a "beta beat".

3.2 Introduction to Solenoid Focussing

3.2.1 Motion in Long Solenoid

Note: $[pc/e]$ is the momentum in units of Volts

$$\rho \;=\; \frac{[pc/e]_\perp}{c\,B_z}$$

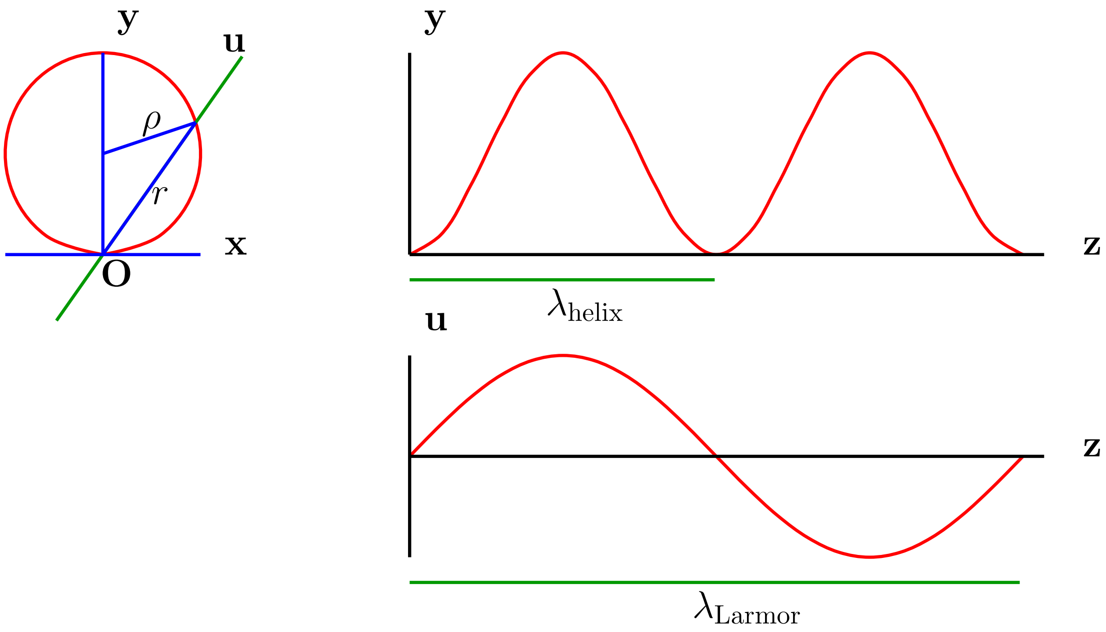


$$x \;=\; \rho\sin(\psi)$$

$$y \;=\; \rho\,(1-\cos(\psi))$$

3.2.2 Larmor Plane

If The center of the solenoid magnet is at **O**, then consider a plane that contains this axis and the particle:



$$\begin{aligned} r &= \left|2\rho\,(\sin(\phi))\right| \\ u &= 2\rho\,(\sin(\phi)) \\ \lambda_{\text{Helix}} &= 2\pi\,\frac{[pc/e]_z}{c\;B_z} \\ \lambda_{\text{Larmor}} &= 2\pi\,\frac{[pc/e]_z}{2\;c\;B_z} \end{aligned}$$

In this case, the lattice parameter β_o is defined in the Larmor frame, so

$$\beta_o = \frac{[pc/e]_z}{2\;c\;B_z} \tag{13}$$

3.2.3 Aparent Focusing "Force"

In this constant B case, the observed sinusoidal motion in the u plane is 'as though' there was a restoring force

$$\frac{d^2u}{dz^2} = -k\ u$$

where

$$k = \left(\frac{c\ B_z}{2\ [pc/e]_z}\right)^2$$

This is true, even for varying fields

Note: the focusing "Force" $\propto\ B_z^2$ so it works the same for either sign, and $\propto\ 1/p_z^2$. Wheras in a quadrupole the force $\propto\ 1/p$

So solenoids are not good for high p, but beat quads at low p.

Angular Momentum

The momentum perpendicular to the Larmor plane:

$$p_{\perp,\mathrm{Larmor}} = p_{\perp\mathrm{lab}}\ \sin(\phi)$$

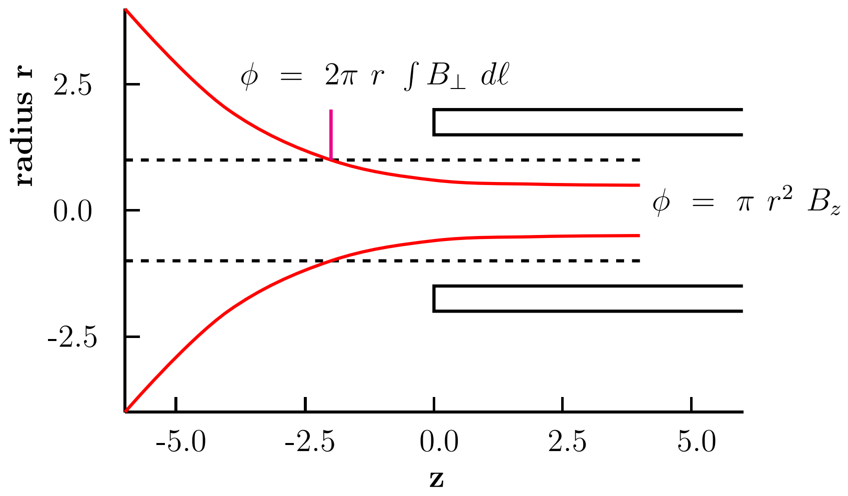
The angular momentum $\mathcal{M}$ about the Larmor axis **O**:

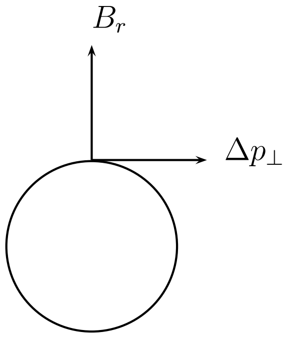
$$\mathcal{M} = r\ p_\perp\ \sin(\phi)$$

$$[\mathcal{M}c^2/e] = \frac{r^2\ B_z\ c}{2}$$

3.2.4 Entering a solenoid

If no angular momentum outside





$$\Delta[pc/e]_\perp \;=\; \int B_r\ dz \;=\; \frac{B_z\ r\ c}{2} \tag{14}$$

$$[\mathcal{M}c/e] \;=\; p_\perp\ r \;=\; \frac{r^2\ B_z\ c}{2}$$

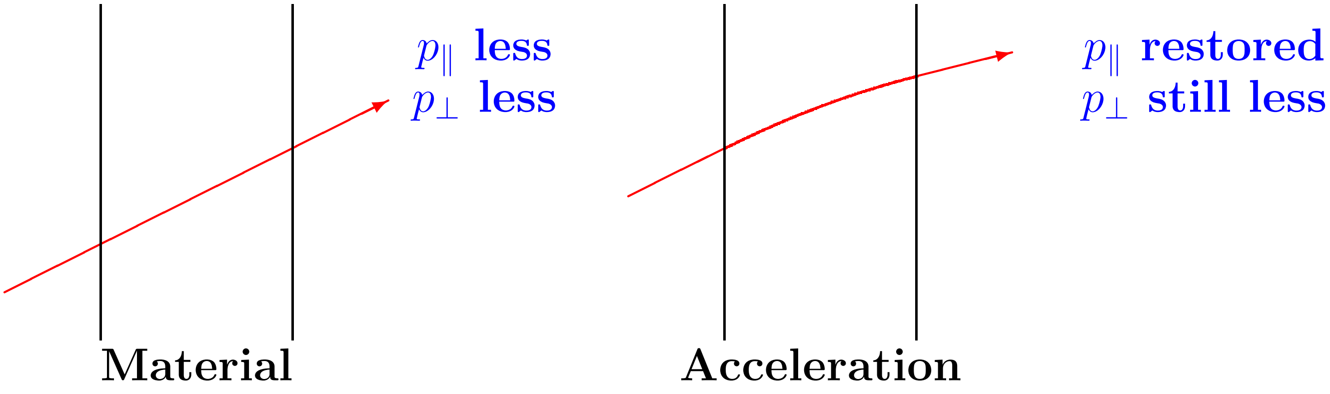
Same as that for motion in the Larmor plane. i.e. if no angular momentum outside then motion remains in Larmor plane independent of field shape.

But if angular momentum outside M_o (known as Canonical Angular Momentum) is not zero, then motion is in a $u\ v$ frame, but still as given by an aparent centering force.

and inside B_z:

$$[\mathcal{M}c/e] \;=\; [\mathcal{M}c/e]_o \;+\; \frac{r^2\ B_z\ c}{2}$$

3.3 Transverse Cooling



3.3.1 Cooling rate vs. Energy

$$\text{(eq 7)} \quad \epsilon_{x,y} \ = \ \gamma\beta_v \, \sigma_\theta \, \sigma_{x,y}$$

If there is no Coulomb scattering, or other sources of emittance heating, then σ_θ and $\sigma_{x,y}$ are unchanged by energy loss, but p and thus $\beta\gamma$ are reduced. So the fractional cooling $d\epsilon\ /\epsilon$ is:

$$\frac{d\epsilon}{\epsilon} = \frac{dp}{p} = \frac{dE}{E}\,\frac{1}{\beta_v^2} \tag{15}$$

which, for a given energy change, strongly favors cooling at low energy.

But if total acceleration were not important, e.g. if the cooling is done in a ring, then there is another criterion: The cooling per fractional loss of particles by decay:

$$Q = \frac{d\epsilon/\epsilon}{dn/n} = \frac{dp/p}{d\ell/c\beta_v\gamma\tau}$$

$$= \frac{dE/E \ \ 1/\beta_v^2}{d\ell/(c\gamma\beta_v\tau)}$$

$$= (c\tau/m_\mu) \ \ \frac{dE}{d\ell} \ \frac{1}{\beta_v}$$

Which only mildly favours low energy

3.3.2 Heating Terms

$$\epsilon_{x,y} \;=\; \gamma\beta_v\, \sigma_\theta\, \sigma_{x,y}$$

Between scatters the drift conserves emittance (Liouiville).

When there is scattering, $\sigma_{x,y}$ is conserved, but σ_θ is increased.

$$\Delta(\epsilon_{x,y})^2 \;=\; \gamma^2\beta_v^2\, \sigma_{x,y}^2\Delta(\sigma_\theta^2)$$

$$2\epsilon\; \Delta\epsilon \;=\; \gamma^2\beta_v^2 \left(\frac{\epsilon\beta_\perp}{\gamma\beta_v}\right)\; \Delta(\sigma_\theta^2)$$

$$\Delta\epsilon \;=\; \frac{\beta_\perp\gamma\beta_v}{2}\; \Delta(\sigma_\theta^2)$$

e.g. from Particle data booklet

$$\Delta(\sigma_\theta^2) \approx \left(\frac{14.1\ 10^6}{[pc/e]\beta_v}\right)^2\; \frac{\Delta s}{L_R}$$

$$\Delta\epsilon \;=\; \frac{\beta_\perp}{\gamma\beta_v^3}\,\Delta E\;\; \left(\left(\frac{14.1\ 10^6}{2[mc^2/e]_\mu}\right)^2\; \frac{1}{L_R dE/ds}\right)$$

Defining

$$C(mat,E) \;=\; \frac{1}{2}\left(\frac{14.1\ 10^6}{[mc^2/e]_\mu)}\right)^2 \frac{1}{L_R\ d\gamma/ds} \tag{16}$$

then

$$\frac{\Delta\epsilon}{\epsilon} \;=\; dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat,E) \tag{17}$$

Equating this with equation 15

$$dE\ \frac{1}{\beta_v^2\ E} \;=\; dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ C(mat,E)$$

gives the equilibrium emittance ϵ_o:

$$\epsilon_{x,y}(min) \;=\; \frac{\beta_\perp}{\beta_v}\ C(mat,E) \tag{18}$$

At energies such as to give minimum ionization loss, the constant C_o for various materials are approximately:

material	T ^{o}K	density kg/m^3	dE/dx MeV/m	L_R m	C_o 10^{-4}
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

Clearly Liquid Hydrogen is far the best material, but has cryogenic and safety complications, and requires windows made of Aluminum or other material which will significantly degrade the performance.

3.3.3 Rate of Cooling

$$\frac{d\epsilon}{\epsilon} = \left(1 - \frac{\epsilon_{\min}}{\epsilon}\right) \frac{dp}{p} \tag{19}$$

3.3.4 Beam Divergence Angles

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp \beta_v \gamma}}$$

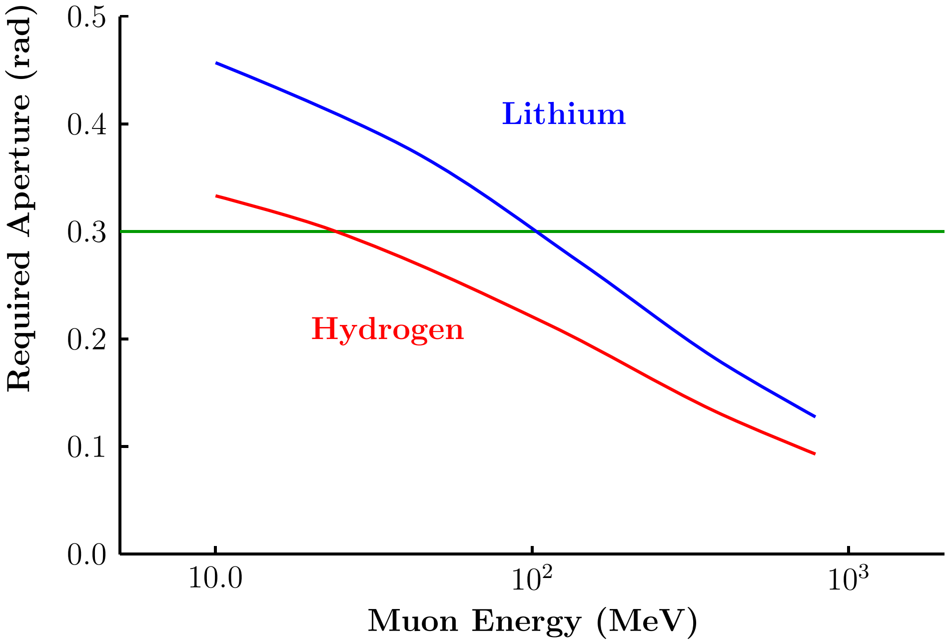
so, from equation 18, for a beam in equilibrium

$$\sigma_\theta = \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}}$$

and for 50 % of maximum cooling rate and an aperture at 3 σ, the angular aperture $\mathcal{A}$ of the system must be

$$\mathcal{A} = 3\sqrt{2} \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \tag{20}$$

Apertures for hydrogen and lithium are plotted vs. energy below. These are **very large angles**, and if we limit apertures to less than 0.3, then this requirement sets lower energy limits of about 100 MeV (≈ 170 MeV/c) for Lithium, and about 25 MeV (≈ 75 MeV/c) for hydrogen.


Required Aperture (rad)
0.5
0.4
0.3
0.2
0.1
0.0
Lithium
Hydrogen
10.0
10^2
10^3
Muon Energy (MeV)

3.4 Focusing Systems

3.4.1 Solenoid

In a solenoid with axial field B_{sol} (from eq 13)

$$\beta_\perp = \frac{2\,[pc/e]}{c\,B_{sol}}$$

so

$$\epsilon_{x,y}(min) = C(mat, E)\ \frac{2\,\gamma\,[mc^2/e]_\mu}{B_{sol}\ c} \tag{21}$$

For $E = 100\ MeV$ $(p \approx 170\ MeV/c)$, $B = 20\ T$, then $\beta \approx 5.7\ cm.$ and

$\epsilon_{x,y} \approx 266(\pi mm\ mrad).$

3.4.2 Current Carrying Rod

In a rod carrying a uniform axial current, the azimuthal magnetic field B varies linearly with the radius r. A muon traveling down it is focused:

$$\frac{d^2r}{dr^2} = \frac{B\ c}{[pc/e]} = \frac{r\ c}{[pc/e]}\ \frac{dB}{dr}$$

so orbits oscillate with

$$\beta_\perp^2\ =\ \frac{\gamma\beta_v}{dB/dr}\ \frac{[mc^2/e]_\mu}{c} \tag{22}$$

If we set the rod radius a to be f_{ap} times the rms beam size $\sigma_{x,y}$ (from eq.11),

$$\sigma_{x,y} = \sqrt{\frac{\epsilon_{x,y}\ \beta_\perp}{\beta_v \gamma}}$$

and if the field at the surface is B_{max}, then

$$\beta_\perp^2\ =\ \frac{\gamma\beta_v[mc^2/e]_\mu\ f_{ap}}{B_{max}\ c}\ \sqrt{\frac{\epsilon_{x,y}\ \beta}{\gamma\ \beta_v}}$$

from which we get:

$$\beta_\perp\ =\ \left(\frac{f_{ap}\ [mc^2/e]_\mu}{B_{max}\ c}\right)^{2/3}\ \ (\gamma\beta_v\ \epsilon_{x,y})^{1/3}$$

puting this in equation 18

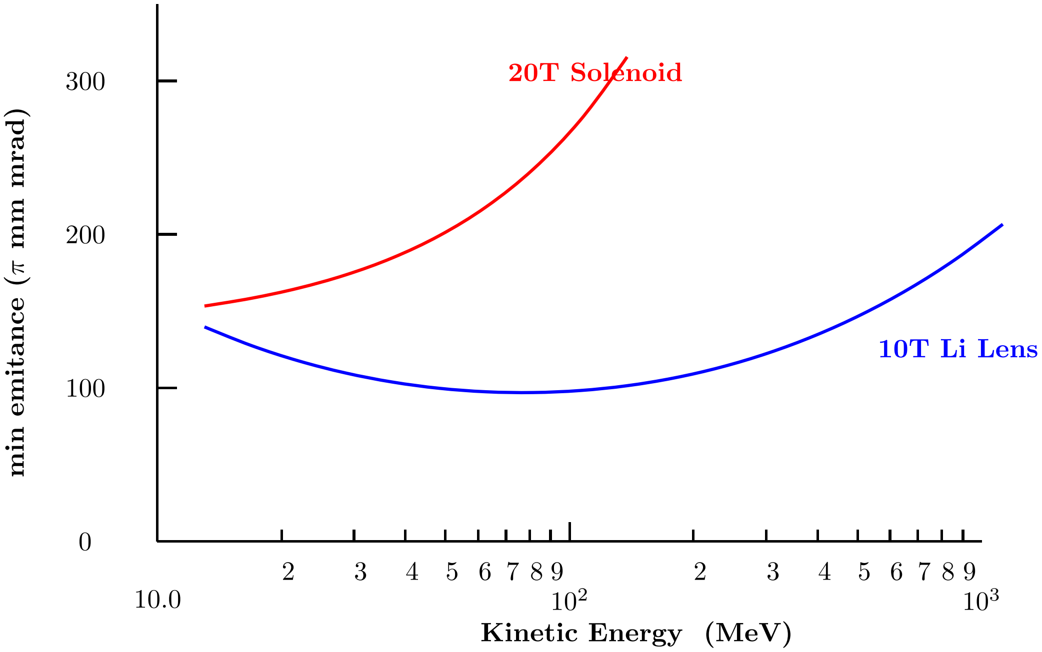
$$\epsilon_{x,y}(min)\ =\ (C(mat,E))^{1.5}\left(\frac{f_{ap}\ [mc^2/e]_\mu}{B_{max}\ c\ \beta_v}\right)\ \sqrt{\gamma} \tag{23}$$

e.g. B_{max}=10 T, f_{ap}=3, E=100 MeV, then $\beta_\perp$ = 1.23 cm, and $\epsilon(min)$=100 (π mm mrad)

The choice of a maximum surface field of 10 T is set by breaking of the containing pipe in current solid Li designs. With liquid Li a higher field may be possible.

3.4.3 Compare Focusing

Comparing the methods as a function of the beam kinetic energy.



We see that, for the parameters selected, The lithium rod achieves a lower emittance than the solnoid despite its higher C value. Neither method allows transverse cooling below about 80 (π mm mrad)

3.5 Angular Momentum Problem

or: Why we reverse the solenoid directions

In the absence of external fields and energy loss in materials, the angular momentum of a particle is conserved.

But a particle entering a solenoidal field will cross radial field components and its angular ($r\ p_\phi$) momentum will change (eq.14).

$$\Delta([pc/e]_\phi) \ = \ \Delta\left(\frac{c\ B_z\ r}{2}\right)$$

If, in the absence of the field, the particle had "canonical" angular momentum $(p_\phi\ r)_{\rm can}$, then in the field it will have angular momentum:

$$[pc/e]_\phi\ r \ = \ (p_\phi\ r)_{\rm can} \ + \ \left(\frac{c\ B_z\ r}{2}\right)\ r$$

so

$$[pc/e]_\phi\ r)_{\rm can} \ = \ [pc/e]_\phi\ r \ - \ \left(\frac{c\ B_z\ r}{2}\right)\ r \qquad (24)$$

If the initial average canonical angular momentum is zero, then in B_z:

$$< [pc/e]_\phi\ r > = \left(\frac{c\ B_z\ r}{2}\right)\ r$$

Material introduced to cool the beam, will reduce all momenta, both longitudinal and transverse, random and average.

Re-acceleration will not change the angular momenta, so the average angular momentum will continuously fall.

Consider the case of almost complete transverse cooling: all transverse momenta are reduced to near zero leaving the beam streaming parallel to the axis.

$$[pc/e]_\phi \, r \;\approx\; 0$$

and there is now a finite average canonical momentum (from eq.24):

$$< [pc/e]_\phi \, r >_{\rm can} = -\left(\frac{c \, B_z \, r}{2}\right) \, r$$

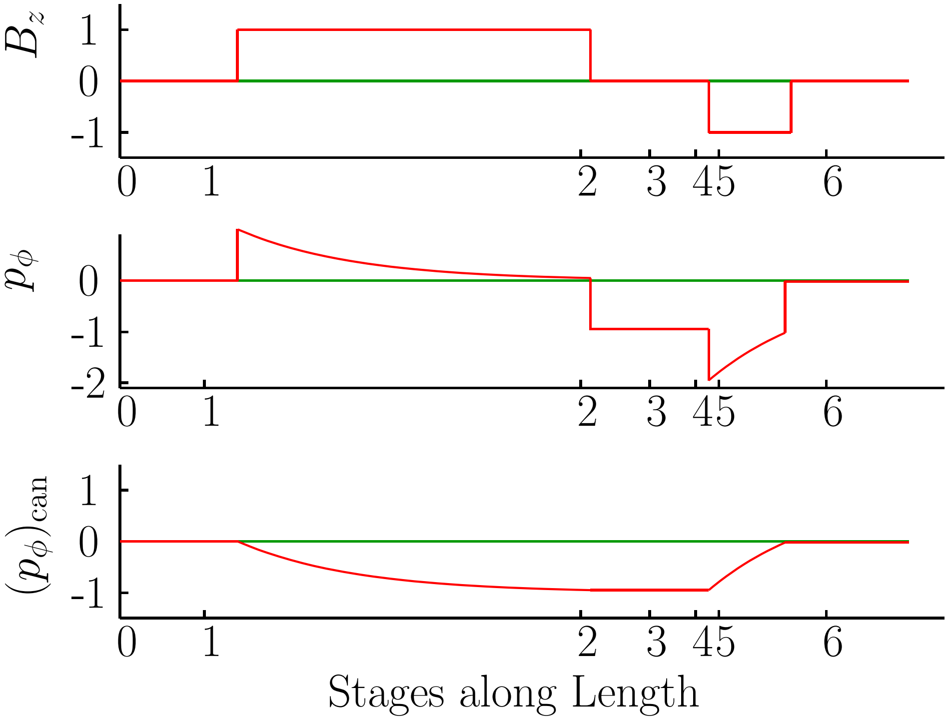
When the beam exits the solenoid, then this canonical angular momentum becomes a real angular momentum and represents an effective emittance, and severely limits the possible cooling.

$$< [pc/e]_\phi \, r >_{\rm end} = \; -\left(\frac{c \, B_z \, r}{2}\right) \, r$$

The only reasonable solution is to reverse the field, either once, a few, or many times.

3.5.1 Single Field Reversal Method

The minimum required number of field "flips" is one.



After exiting the first solenoid, we have real coherent angular momentum:

$$([pc/e]_\phi \; r)_3 \;=\; -\left(\frac{c\;B_{z1}\;r}{2}\right)\; r$$

The beam now enters a solenoid with opposite field $B_{z2} = -B_{z1}$.

The canonical angular momentum remains the same, but the real angular momentum is doubled.

$$([pc/e]_\phi \; r)_4 \;=\; -2\left(\frac{c\;B_{z1}\;r}{2}\right)\; r$$

We now introduce enough material to halve the transverse field components. Then

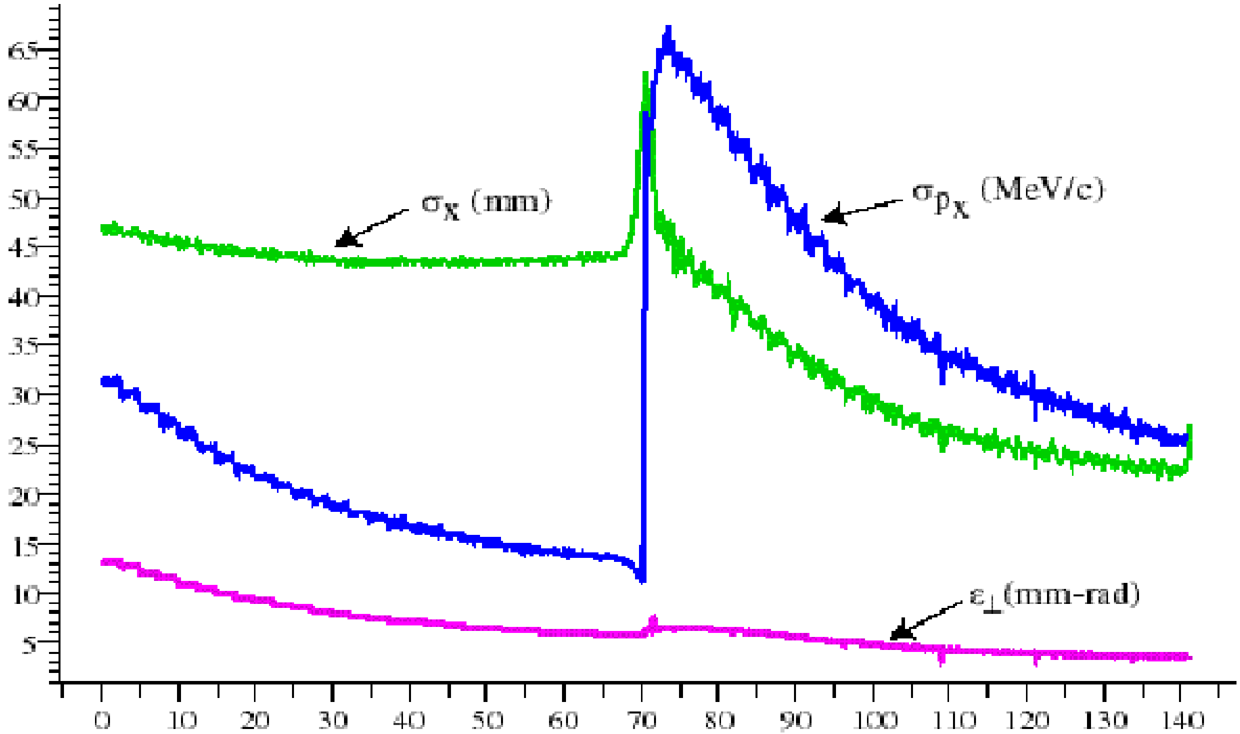
$$([pc/e]_\phi\ r)_5 \;=\; -\left(\frac{c\ B_{z1}\ r}{2}\right)\ r$$

This is inside the field $B_{z2} \;=\; -B_{z1}$. The canonical momentum, and thus the angular momentum on exiting, is now:

$$([pc/e]_\phi\ r)_6 \;=\; -\left(\frac{c\ B_{z1}\ r}{2}\right)\ r \;-\; -\left(\frac{c\ B_{z1}\ r}{2}\right)\ r \;=\; 0$$

3.5.2 Example of "Single Flip"

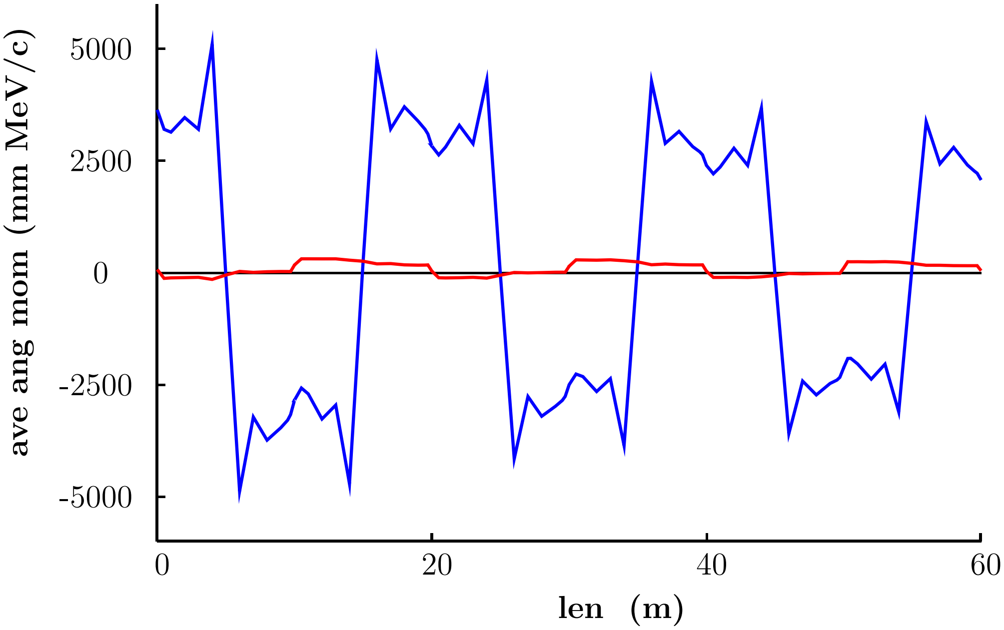
From "single flip alternative" in US Study 2



3.5.3 Alternating Solenoid Method

If we reverse the field frequently enough, no significant canonical angular momentum is developed.

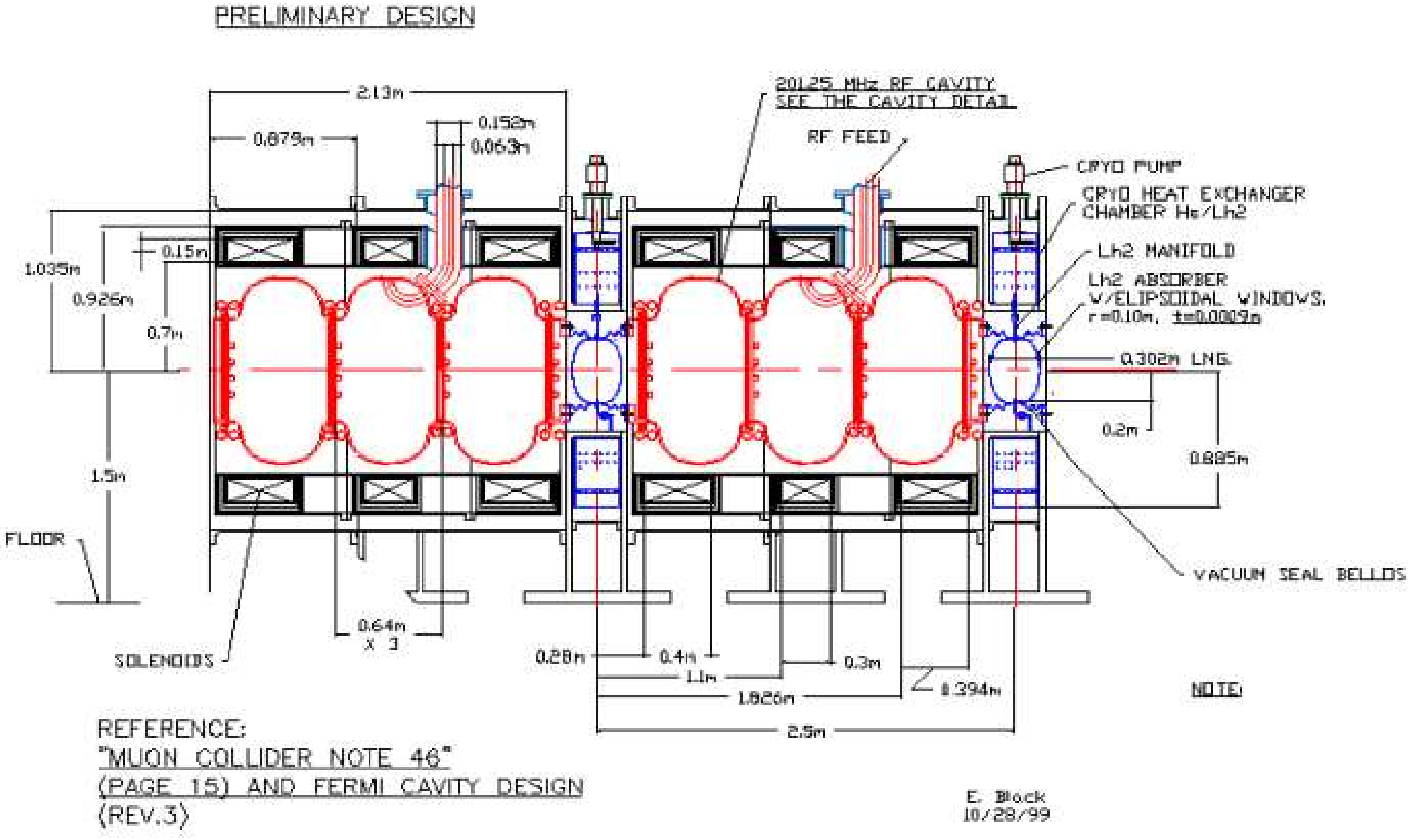
The Figure below shows the angular momenta and canonical angular momenta in a simulation of an ”alternating solenoid” cooling lattice. It is seen that while the coherent angular momenta are large, the canonical angular momentum (in red) remains very small.



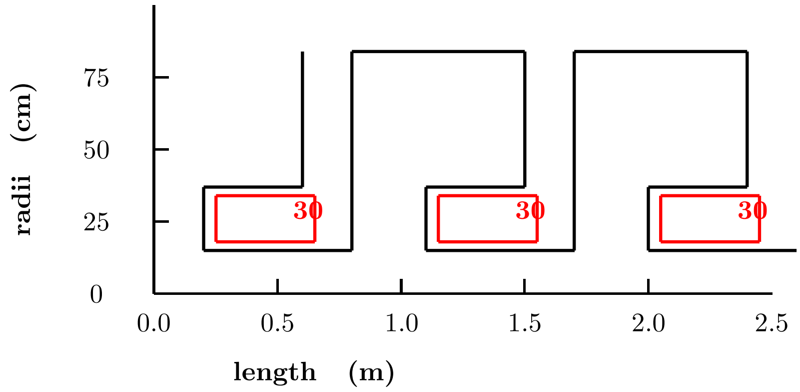
3.6 Focussing Lattice Designs

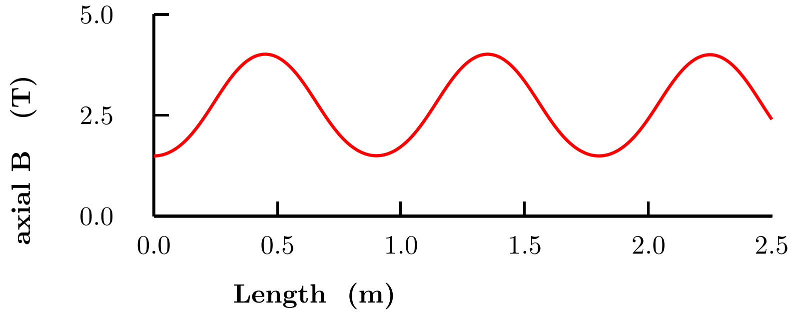
3.6.1 Solenoids with few "flips"

- Coils Outside RF: e.g. FNAL 1 flip



- Coils interleeved: e.g. CERN



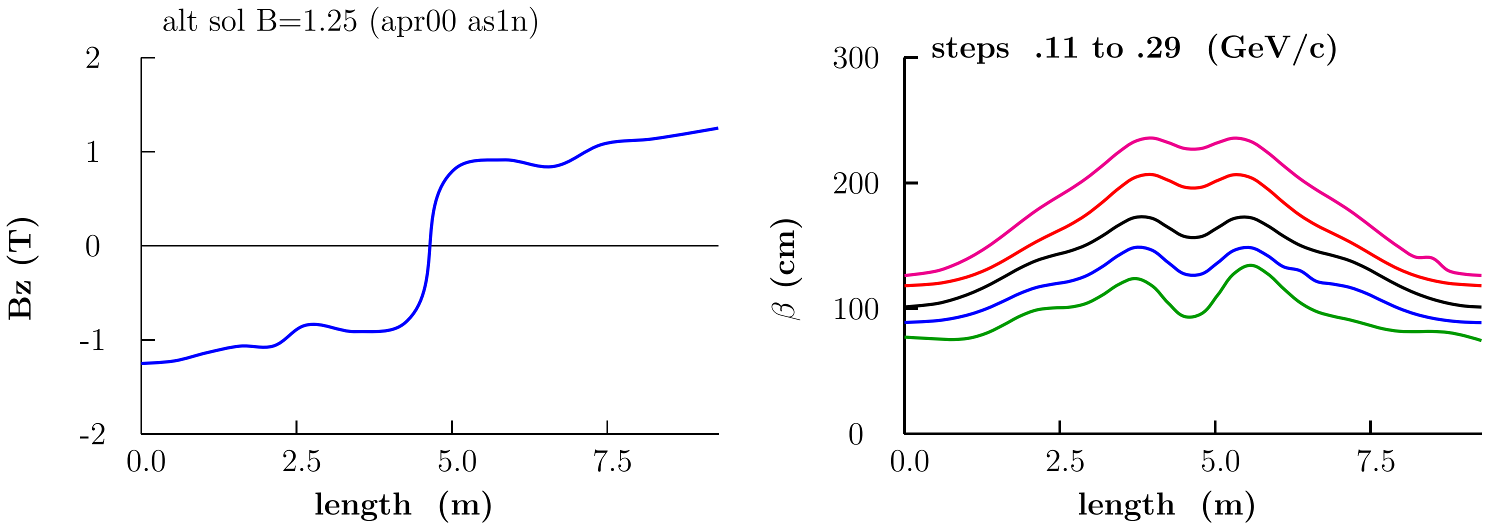


In this design, the field must be flipped, as in a uniform field case. But here the Field is far from uniform and must be treated as a lattice and will have ”stop bands”.

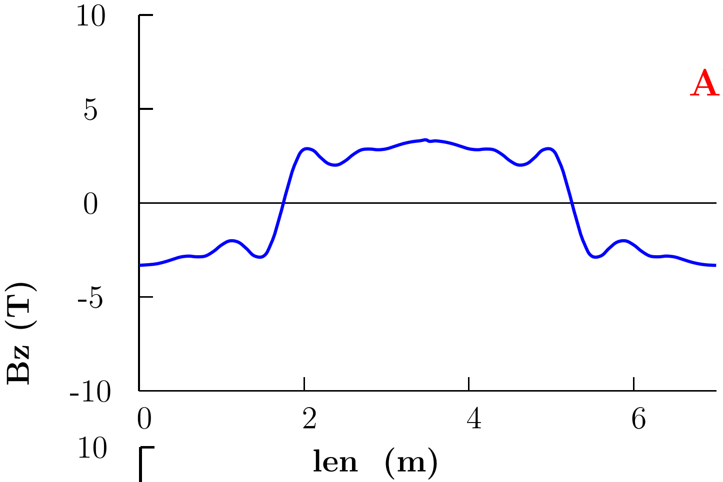
"Flips"

One must design the flips to match the betas from one side to the other.

For a computer designed matched flip between uniform solenoidal fields: the following figure shows B_z vs. z and the $\beta_\perp$'s vs. z for different momenta.

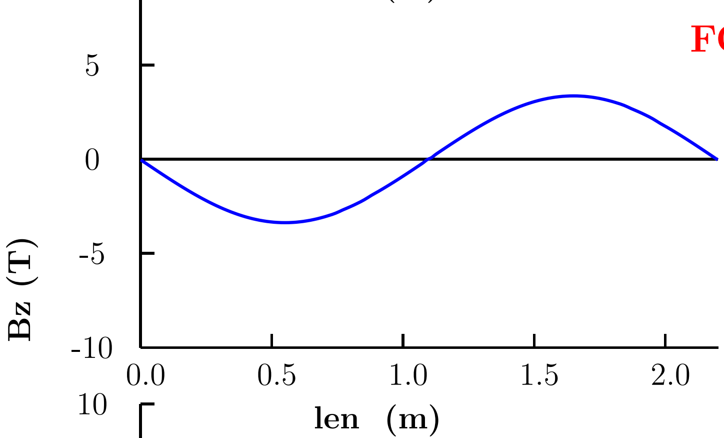


3.6.2 Lattices with many "flips"



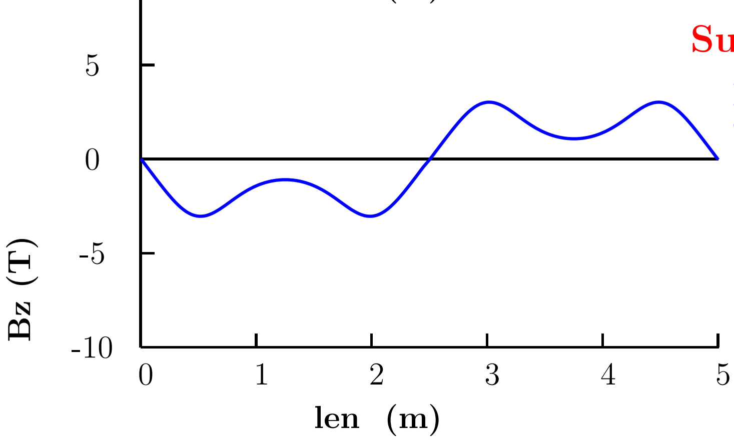
Alternating Solenoid

$B_z(\text{max}) = 3.4$ (T)
$dB_z/dz(\text{max}) = 15$ (T/m)



FOFO

$B_z(\text{max}) = 3.4$ (T)
$dB_z/dz(\text{max}) = 9.4$ (T/m)



Super FOFO

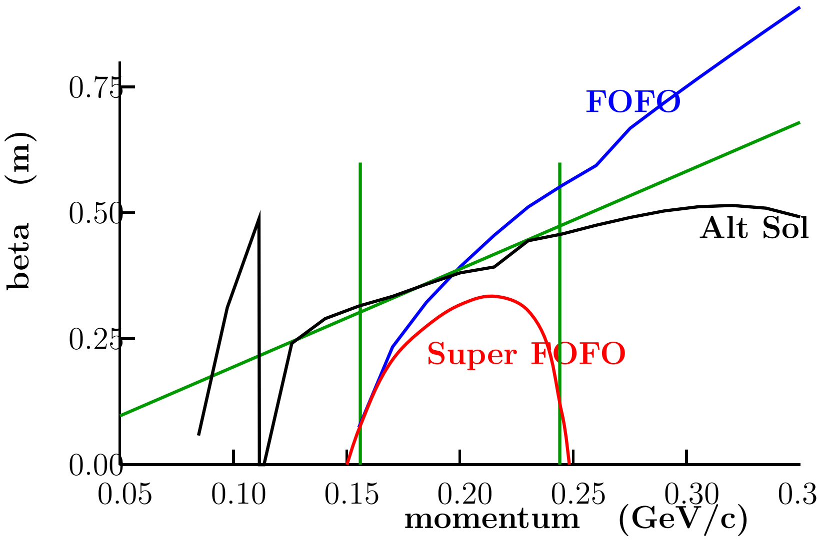
$B_z(\text{max}) = 2.6$ (T)
$dB_z/dz(\text{max}) = 7$ (T/m)

Determination of lattice betas

- Track single near paraxial particle through many cells
- plot θ_x vs x after each cell
- fit ellipse: $\beta_{x,y}$ = A((x) / A(θ_x)

beta vs. Momentum

Note "stop bands" where particles are not transmitted

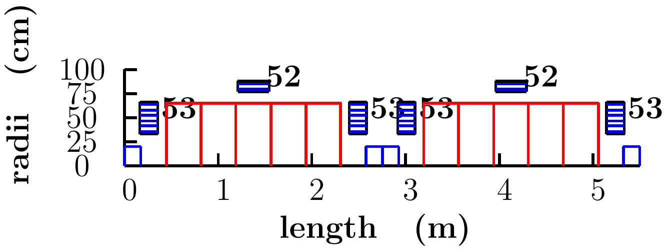


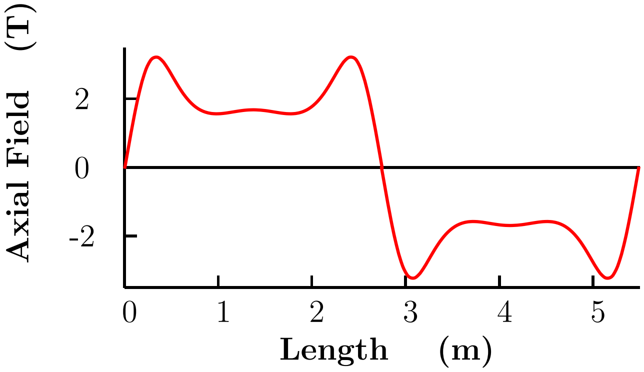
- Alternating Solenoid has largest p acceptance
- FOFO shows $\beta \propto dp/p$
- SFOFO more complicated, and better

3.6.3 Example of Multi-flip lattice

US Study 2 Super FOFO

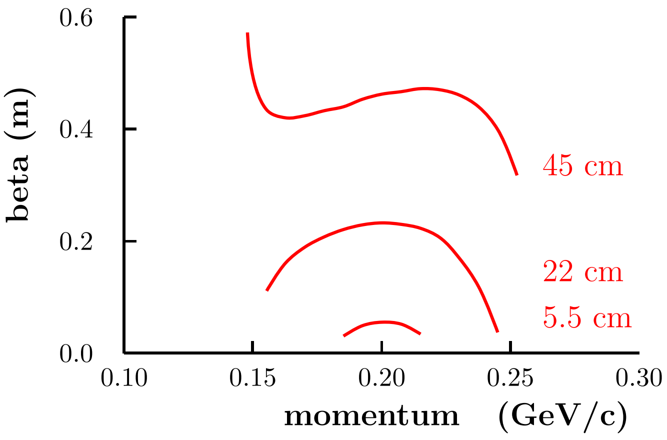
Smaller Stored E than continuous solenoid over RF ($\approx 1/5$)





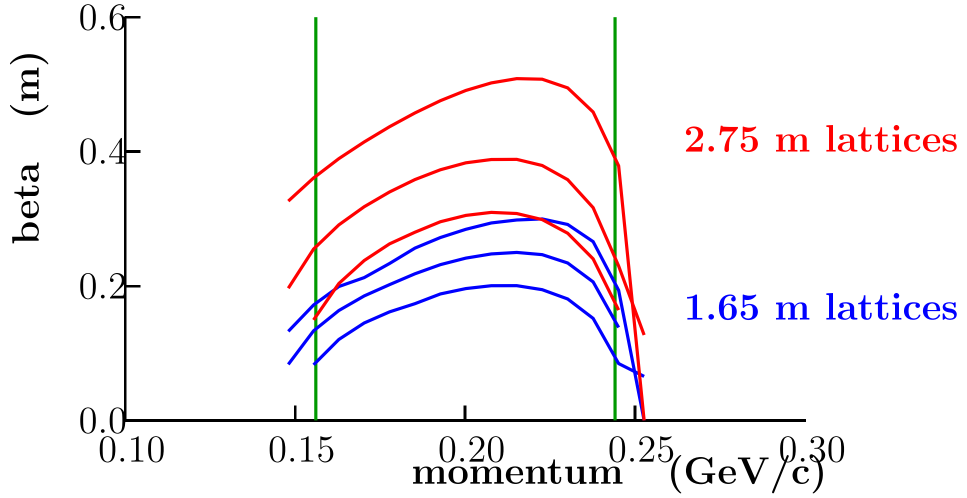
Adjusting Currents adjusts $\beta_\perp$'s

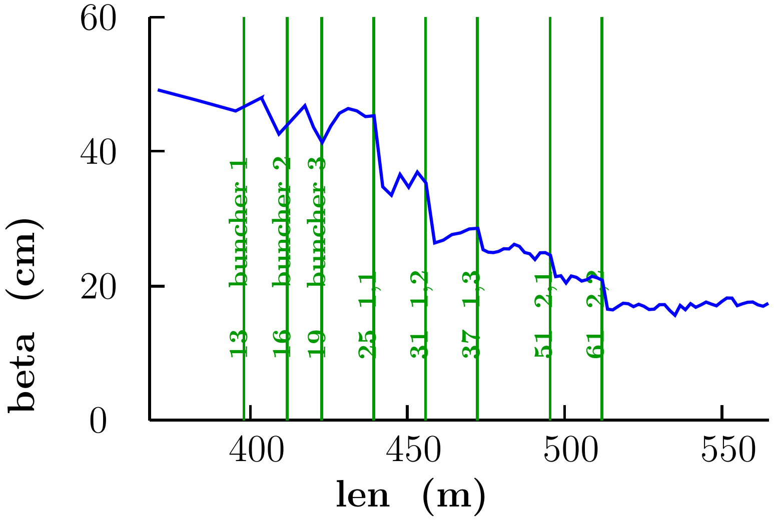
But mom acceptance falls with $\beta_\perp$



3.6.4 Tapering the Cooling Lattice

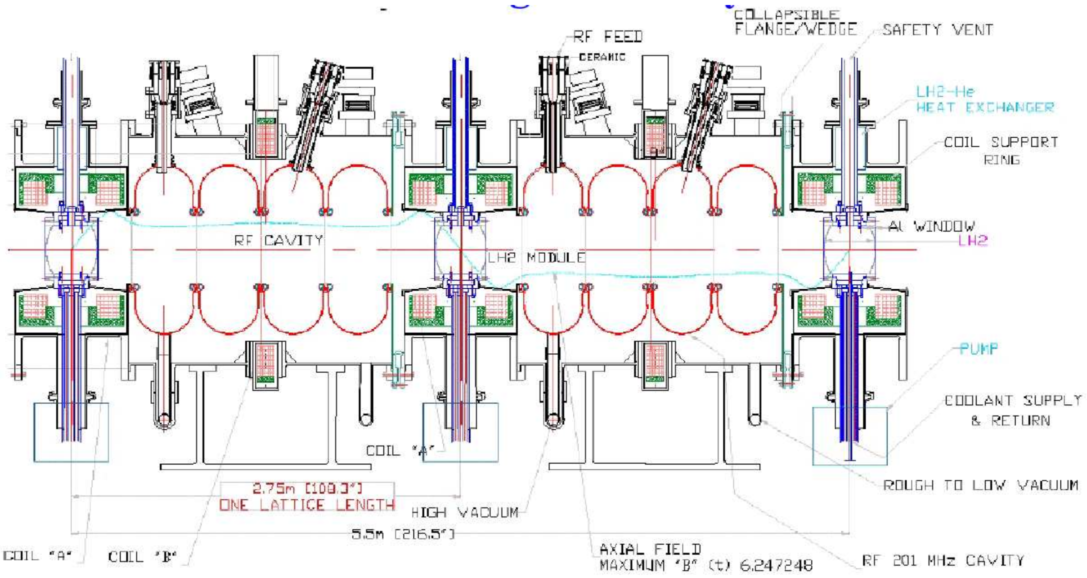
- as emittance falls, lower betas
- maintain constant angular beam size
- maximizes cooling rate
- Adjust current, then lattice



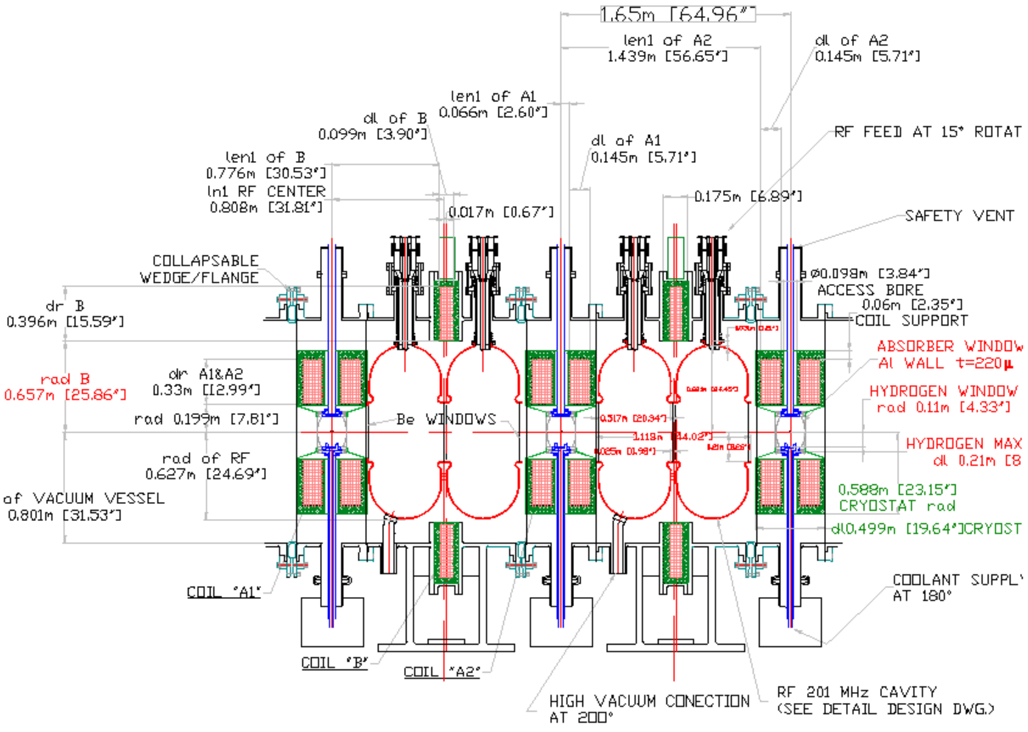


3.6.5 Hardware

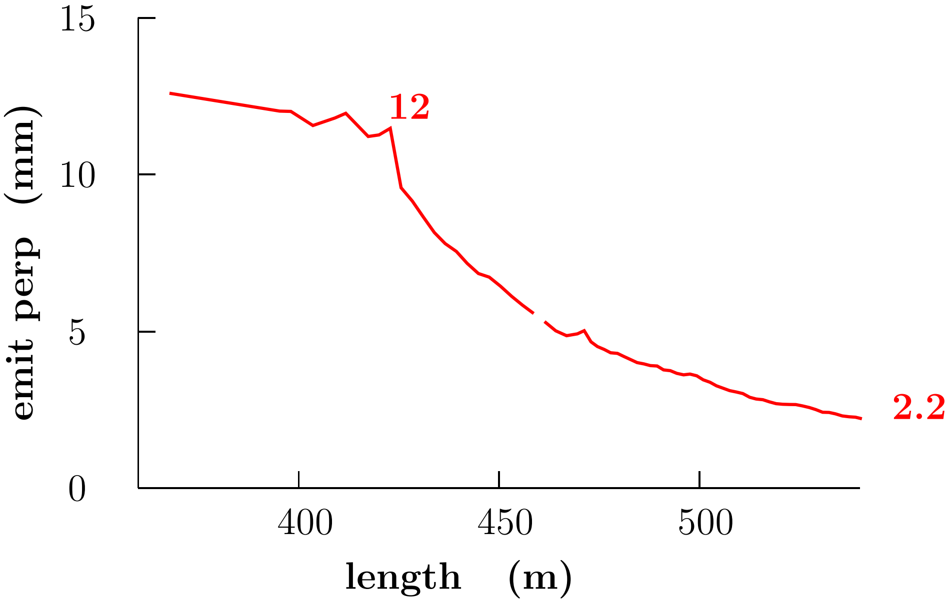
At Start of Cooling



At end of Cooling



3.6.6 Study 2 Performance



With RF and Hydrogen Windows, $C_o \approx 45\ 10^{-4}$
$\beta_\perp$(end)=.18 m, β_v(end)=0.85, So

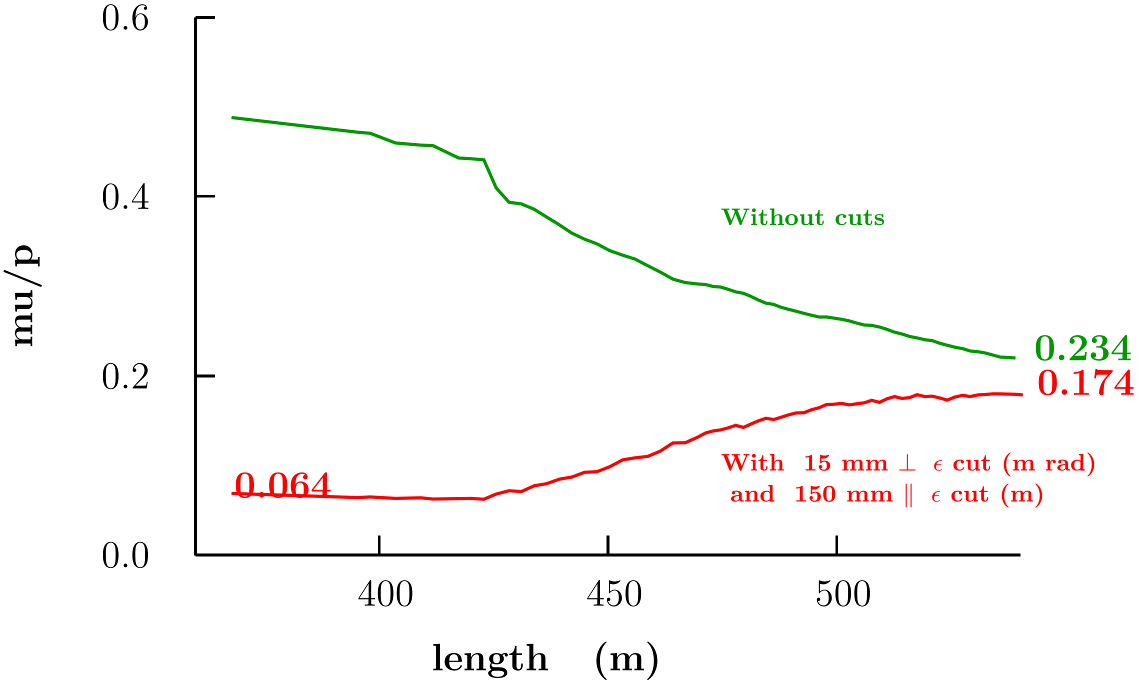
$$\epsilon_\perp(\min) = \frac{45\ 10^{-4}\ 0.18}{0.85} = 0.95 \ \ (\pi mm\ mrad)$$

$$\frac{\epsilon_\perp}{\epsilon_\perp(\min)} \approx 2.3$$

so from eq. 19

$$\frac{d\epsilon}{\epsilon}(\mathrm{end}) = \left(1 - \frac{\epsilon}{\epsilon(\min)}\right) \frac{dp}{p} \approx 0.57 \frac{dp}{p}$$

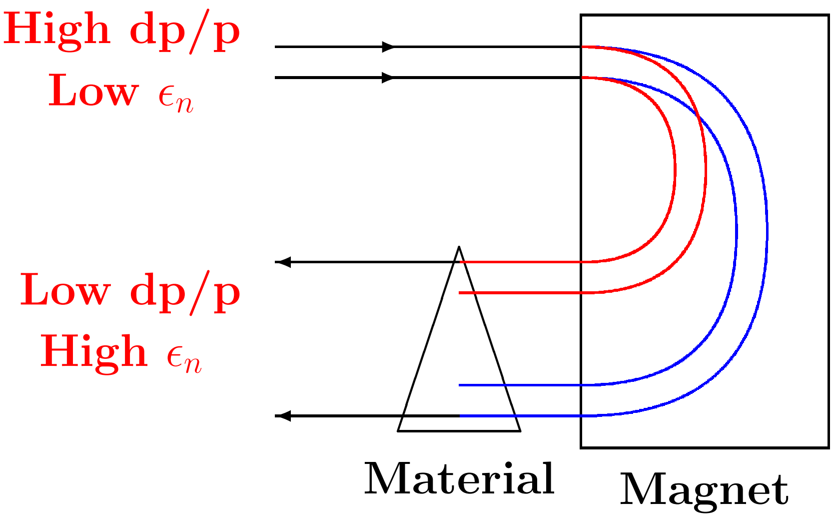
Muons accepted by Acceleration



- Gain Factor = 3
- No Further gain from length
- Loss from growth of long emit.
- Avoided if longitudinal cooling

4 Longitudinal Cooling

4.1 Introduction



- dp/p reduced
- But σ_y increased

- Long Emittance reduced
- Trans Emittance Increased

- "Emittance Exchange"

4.2 Partition Functions

Following the convention for synchrotron cooling we define partition functions:

$$J_{x,y,z} \;=\; \frac{\frac{\Delta\;(\epsilon_{x,y,z})}{\epsilon_{x,y,z}}}{\frac{\Delta p}{p}} \tag{25}$$

$$J_6 = J_x + J_y + J_z \tag{26}$$

where the $\Delta\epsilon$'s are those induced directly by the energy loss mechanism (ionization energy loss in this case). Δp and p refer to the loss of momentum induced by this energy loss.

In the synchrotron case, in the absence of gradients fields, $J_x = J_y = 1$, and $J_z = 2$.

In the ionization case, as we shall show, $J_x = J_y = 1$, but J_z is negative or small.

4.2.1 Transverse

From last lecture:

$$\frac{\Delta\sigma_{p\perp}}{\sigma_{p\perp}} \;=\; \frac{\Delta p}{p}$$

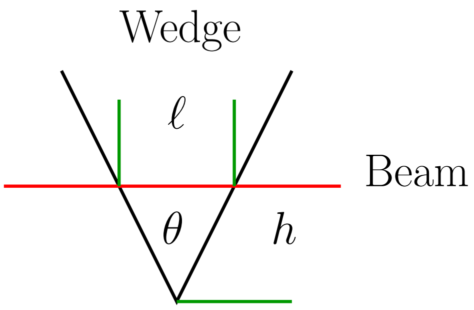
and $\sigma_{x,y}$ does not change, so

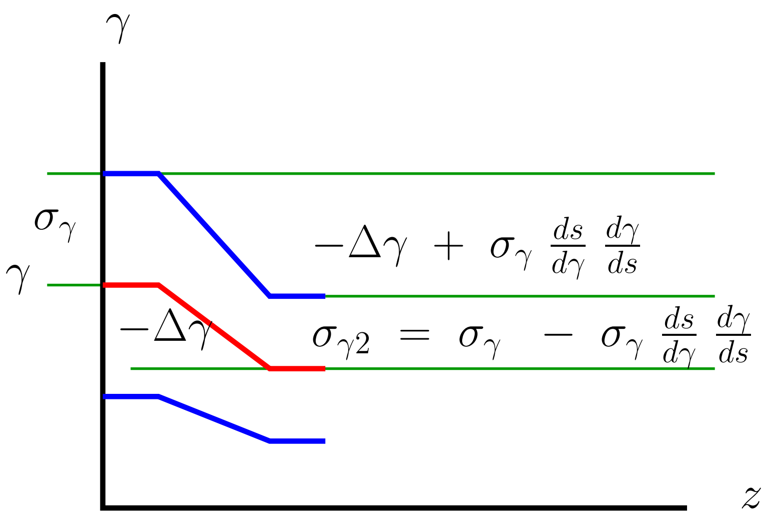
$$\frac{\Delta\epsilon_{x,y}}{\epsilon_{x,y}} \;=\; \frac{\Delta p}{p} \tag{27}$$

and thus

$$J_x \;=\; J_y \;=\; 1 \tag{28}$$

4.2.2 Longitudinal





The emittance in the longitudinal direction ϵ_z is (eq.8):

$$\epsilon_z \ = \ \gamma\beta_v \, \frac{\sigma_p}{p} \, \sigma_z \ = \ c \, \sigma_\gamma \, \sigma_t$$

where σ_t is the rms bunch length in time, and c is the velocity of light. Drifting between interactions will not change emittance (Louville), and an interaction will not change σ_t, so emittance change is only induced by the energy change in the interactions:

For a wedge with center thickness ℓ and height from center h ($2h \ \tan(\theta/2) = \ell$), in dispersion D ($D = \frac{dy}{dp/p}$) (see fig. above):

$$\frac{\Delta\epsilon_z}{\epsilon_z} \ = \ \frac{\Delta\sigma_\gamma}{\sigma_\gamma} \ = \ \frac{d\ell}{d\gamma}\left(\frac{d\gamma}{ds}\right) \ = \ \left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2 \ \gamma} \ \left(\frac{d\gamma}{ds}\right)$$

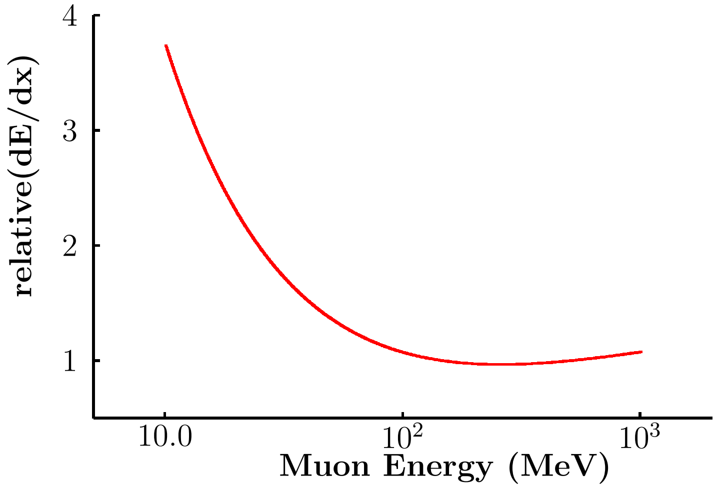
and

$$\frac{\Delta p}{p} \ = \ \frac{\Delta\gamma}{\beta_v^2\gamma} \ = \ \frac{\ell}{\beta_v^2\gamma} \ \left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

$$J_z \ = \ \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} \ = \ \frac{\left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2 \ \gamma} \ \left(\frac{d\gamma}{ds}\right)}{\frac{\ell}{\beta_v^2\gamma} \ \left(\frac{d\gamma}{ds}\right)} \ = \ \frac{D}{h} \qquad (29)$$

Energy Loss



A typical relative energy loss as a function of energy is shown above (this example is for Lithium). It is given approximately by:

$$\frac{d\gamma}{ds} \; = \; B \, \frac{1}{\beta_v^2} \, (\frac{1}{2} \, ln(A \; \beta_v^4 \gamma^4 \; - \; \beta_v^2) \tag{30}$$

where

$$A \; = \; \frac{(2m_e c^2/e)^2}{I^2} \tag{31}$$

$$B \approx \frac{0.0307}{(m_\mu c^2/e)} \, \frac{Z}{A} \tag{32}$$

where Z and A are for the nucleus of the material, and I is the ionization potential for that material.

Differentiating the above:

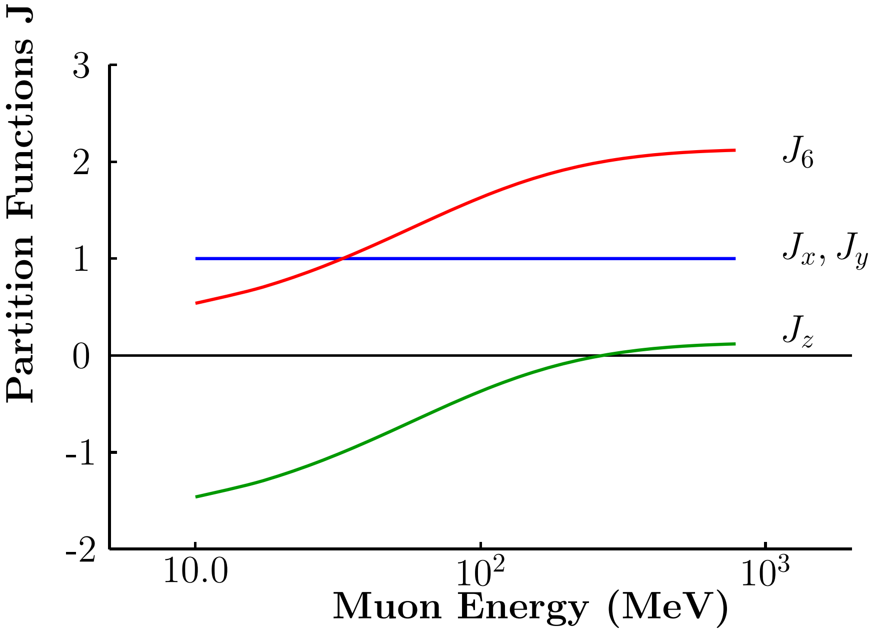
$$\frac{\delta(d\gamma/ds)}{\delta\gamma} \; = \; \frac{B}{\beta_v} \left(\frac{2}{\beta_v \gamma} \; - \; \frac{1}{(\beta_v \gamma)^3} \, ln(A \; \beta_v^4 \gamma^4) \; + \; \frac{2}{(\beta_v \gamma)^3} \right)$$

Substituting this into equation 29:

$$J_z \; \approx \; - \frac{\left(\frac{2}{\beta_v \gamma} \; - \; \frac{1}{(\beta_v \gamma)^3} \, ln(A \; \beta_v^4 \gamma^4) \; + \; \frac{2}{(\beta_v \gamma)^3} \right)}{\left(\frac{1}{2} \, ln(A \; \beta_v^4 \gamma^4 \; - \; \beta_v^2 \right)} \, \beta_v^3 \gamma \tag{33}$$

4.2.3 6D Partition Function J_6

J_z, $J_{x,y}$ and $J_6 \;=\; J_x + J_y + J_z$ are plotted below



It is seen that despite the heating implicit in the negative values of J_z at low energies, the six dimensional cooling J_6 remains positive. In fact the relative cooling for a given acceleration ΔE :

$$\frac{\Delta\epsilon_6/\epsilon}{\Delta E} \;=\; \frac{J_6}{E\ \beta_v^2}$$

rises without limit as the energy falls. This suggests that, for economy of acceleration, cooling should be done at a very low energy. In practice there are many difficulties in doing this, but it remains desirable to use the lowest practical energy: typically around 250 MeV/c, where:

$$J_x \;+\; J_y \;+\; j_z \;=\; J_6 \;\approx\; 2.0 \tag{34}$$

4.2.4 Longitudinal Heating Terms

and from Perkins text book, converted to MKS:

$$\Delta(\sigma_\gamma^2) \;=\; 2\sigma_\gamma\;\Delta\sigma_\gamma \approx 0.06\,\frac{Z}{A}\;\left(\frac{m_e}{m_\mu}\right)^2\;\gamma^2\;\left(1-\frac{\beta_v^2}{2}\right)\;\rho\;\Delta s$$

Since $\epsilon_z \;=\; \sigma_\gamma\;\sigma_t\;c$, and t and thus σ_t is conserved in an interaction

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

and using eq. 2:

$$\Delta s \;=\; \frac{\Delta p\;\frac{ds}{dp/p}\;\frac{1}{p} \;=\; \Delta p}{p}\;\frac{\beta_v^2\;\gamma}{d\gamma/ds}$$

so

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{0.06}{2\sigma_\gamma^2}\;\frac{Z}{A}\;\left(\frac{m_e}{m_\mu}\right)^2\;\gamma^2\;\left(1-\frac{\beta_v^2}{2}\right)\;\rho\;\frac{\beta_v^2\;E}{dE/ds}\;\frac{\Delta p}{p}$$

This can be compared with the cooling term

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; -\;J_z\;\frac{dp}{p}$$

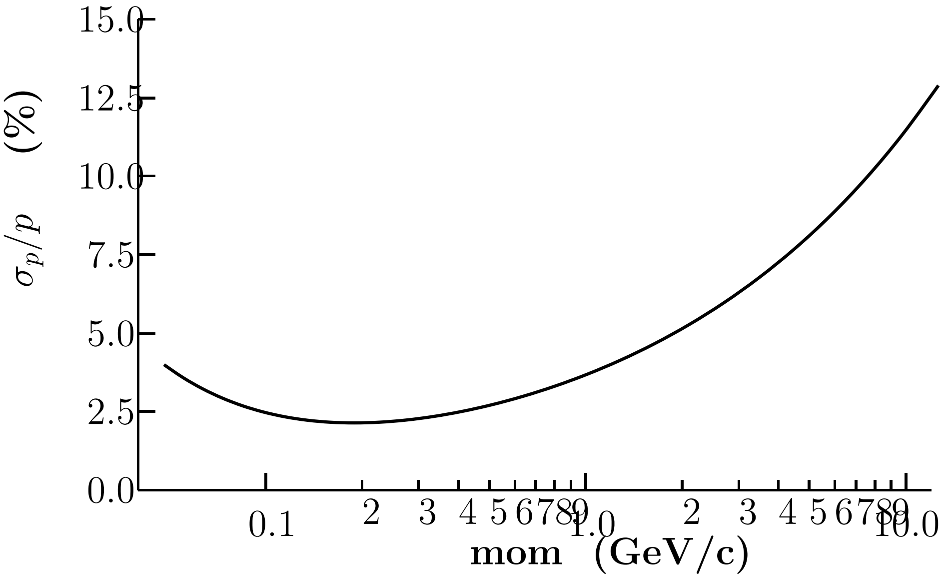
giving an equilibrium:

$$\frac{\sigma_p}{p} \;=\; \left(\left(\frac{m_e}{m_\mu}\right)\;\sqrt{\frac{0.06\;Z\;\rho}{2\;A\;(d\gamma/ds)}}\right)\;\sqrt{\frac{\gamma}{\beta_v^2}\;\left(1-\frac{\beta_v^2}{2}\right)\quad\frac{1}{J_z}} \qquad (35)$$

For Hydrogen, the value of the first parenthesis is ≈1.36 %.

Without coupling, J_z is small or negative, and the equilibrium does not exist. But with equal partition functions giving $J_z \approx 2/3$ then this expression, for hydrogen, gives: the values ploted below.

The following plot shows the dependency for hydrogen



It is seen to favor cooling at around 200 MeV/c, but has a broad minimum.

4.2.5 rf and bunch length

To obtain the Longitudinal emittance we need σ_z.

If the rf acceleration is relatively uniform along the lattice, then we can write the synchrotron wavelength[2] :

$$\lambda_s \;=\; \sqrt{\frac{2\pi\beta_v^2\lambda_{rf}\gamma\ \ [mc^2/e]_\mu)}{\mathcal{E}_{\mathrm{rf}}\alpha\ \cos(\phi)}} \tag{36}$$

where, in a linear lattice, the "momentum compaction" is:

$$\alpha \;=\; \frac{\frac{dv_z}{v_z}}{\frac{dp}{p}} \;=\; \frac{1}{\gamma^2} \tag{37}$$

and the field $\mathcal{E}_{\mathrm{rf}}$ is the rf accelerating field; ϕ is the rf phase, defined so that for $\phi = 0$ there is zero acceleration.

The bunch length, given the relative momentum spread $dp/p = \delta$, is given by[3]:

$$\sigma_z \;=\; \delta\ \beta_v \frac{\alpha\ \lambda_s}{2\pi} \;=\; \delta\ \beta_v^2\ \sqrt{\frac{\lambda_{rf}\ [mc^2/e]_\mu}{2\pi\ \gamma\ \mathcal{E}\ \ \cos(\phi)}} \tag{38}$$

This, in the following plot, is seen to be only weakly dependent on the energy, but the longitudinal emittance $\epsilon_z \;=\; \beta_v\gamma\ \sigma p/p\ \sigma_z$ rises almost linearly with momentum, strongly favouring low momenta.

It is also apparent that the emittance can be reduced if a higher frequency and higher gradient rf is used. The limit her is when the ratio of σ_z/λ becomes too large and particles do not remain in the bucket.

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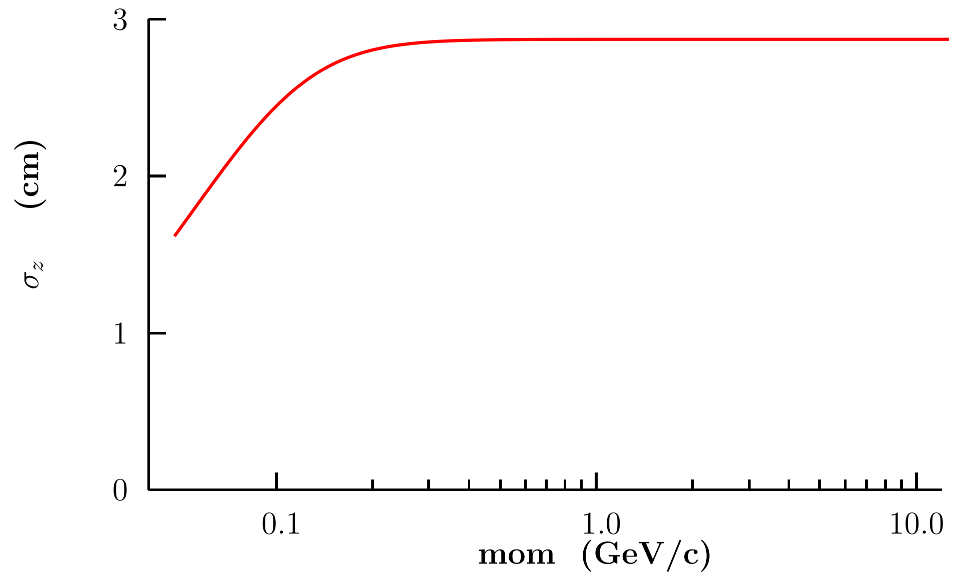
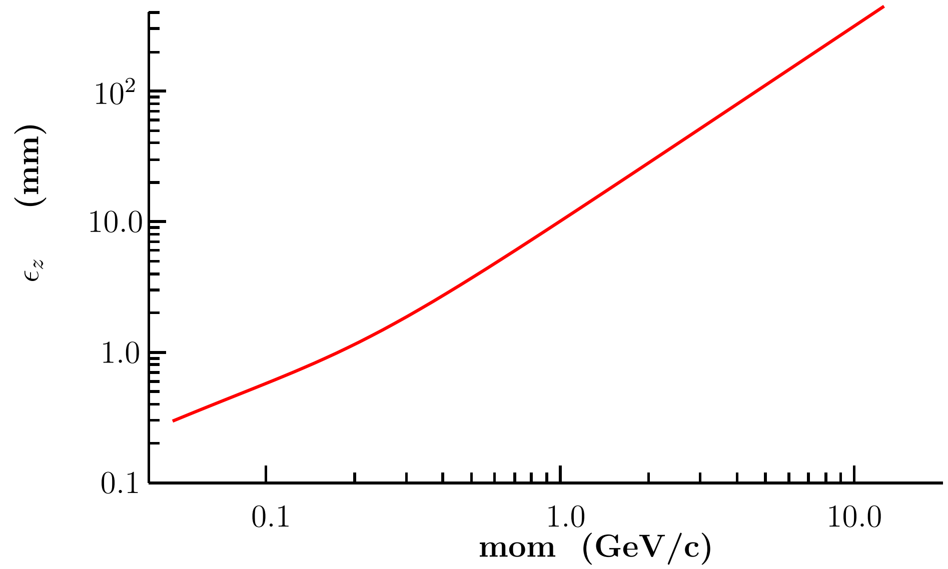
[2] e.g. s y Lee "Accelerator Physics", eq 3.27

[3] e.g. s y Lee "Accelerator Physics", eq 3.55



σ_z (cm)
3
2
1
0
0.1
1.0
10.0
mom (GeV/c)



ϵ_z (mm)
10^2
10.0
1.0
0.1
0.1
1.0
10.0
mom (GeV/c)

4.3 Emittance Exchange Studies

- Attempts at separate cooling & exch.
  - Wedges in Bent Solenoids
  - Wedges in Helical Channels[4]

  Poor performance & problems matching between them
- Attempts in rings with alternate cooling & exchange
  - Balbakov[5] with solenoid focus
    achieved Merit=90
- Attempts in rings with combined cooling & exchange
  - Garren et al[6] Quadrupole focused ring
    achieved Merit ≈15
  - Garren et al: Bend only focusing
    achieved Merit ≈100
  - Palmer et al[7]
    achieved Merit ≈140

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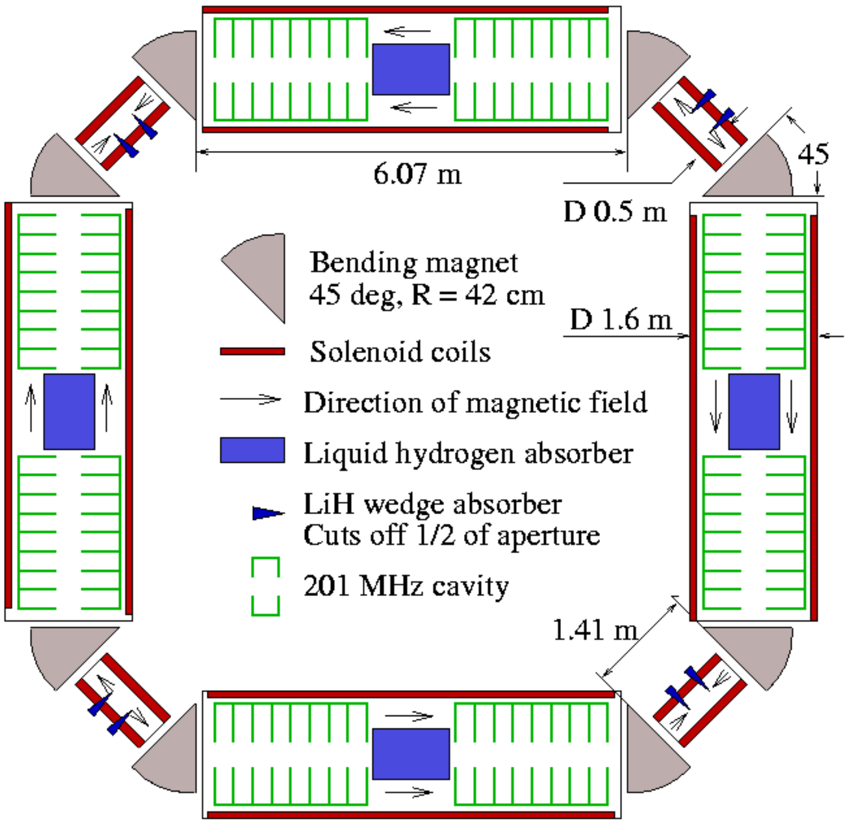
[4] MUC-146, 147, 187, & 193
[5] MUC-232 & 246
[6] Snowmass Proc.
[7] MUC-239

4.4 Balbekov 6D Cooling Ring

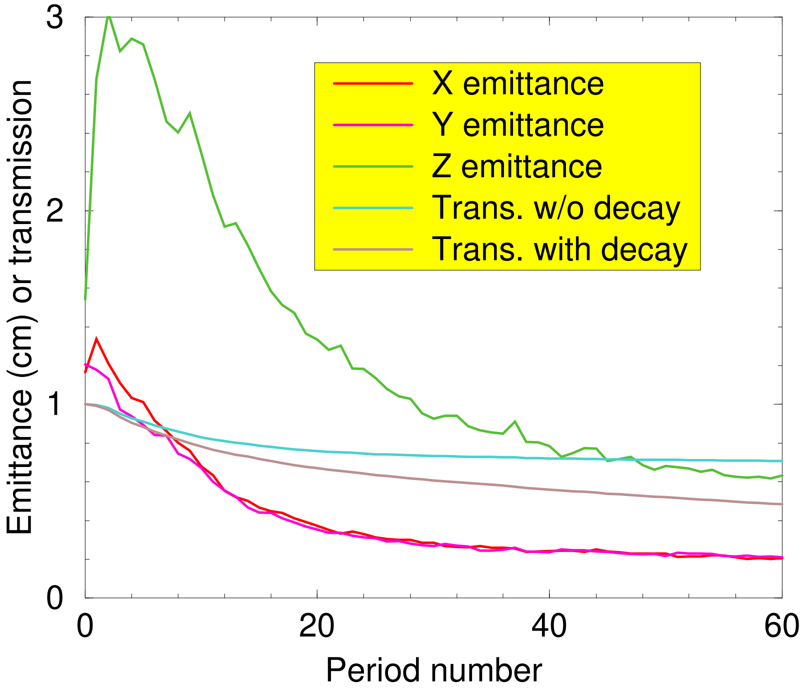
Alternate transverse cooling with H2 with emittance exchange in Li wedge



Circumference	36.963 m
Cell Length	2.27+6.97=9.25 m
Energy	250 MeV
Max B_z	5.155 T
RF Frequency	205.69 MHz
Gradient	15 MV/m

4.4.1 Performance

	Before	After
$\epsilon_\perp$ (cm)	1.2	0.21
$\epsilon_\parallel$ (cm)	1.5	0.63
ϵ_6 (cm^3)	2.2	0.028
ϵ_6/ϵ_{60}	1	79
N/N_0, no decay	1	0.71
N/N_0, inc. decay	1	0.48
Merit	1	38



- Good cooling in all dimensions
- Merit Factor 38 c.f. Study 2 Linear: Merit=15

BUT

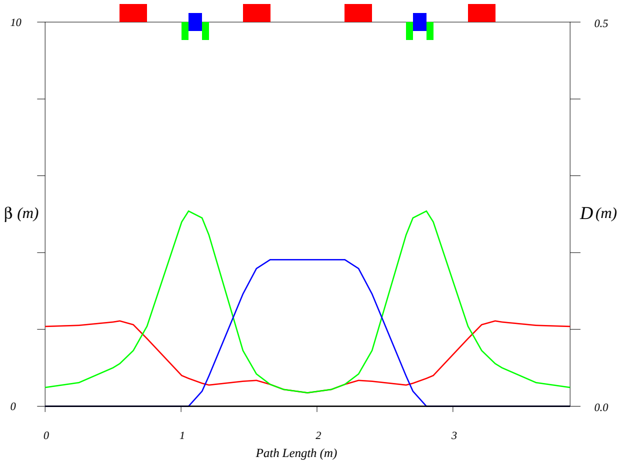
- Calculated without Maxwellian fields
- Design of bends proving hard
- Injection/extraction hard Merit → 3.9 with missing rf
- Non-linear effects with real fields not yet examined

4.5 Quadrupole Focused Rings

Garren, Kirk

- Easier to design lattice (dispersion suppression, etc.)
- Thick wedge: both cooling and longitudinal/transverse coupling

Circumference	31 m
Cell Length	3.8 m
Momentum	250 MeV/c
RF Frequency	200 MHz
RF Gradient	16 MV/m



	Before	After	ratio
ϵ_x (mm)	8.5	3.4	2.5
ϵ_y (mm)	5.2	1.2	4.2
$\epsilon_z\vert$ (mm)	14	3.8	3.7
ϵ_6 (mm^3)	0.62	0.015	39
N/N_0, inc. decay	1	0.41	.41
Merit	1	16	**16**

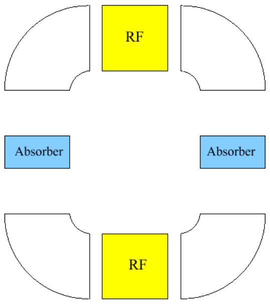
- Final trans. emittance similar to Balbakov
- Longitudinal emittance lower than Balbakov

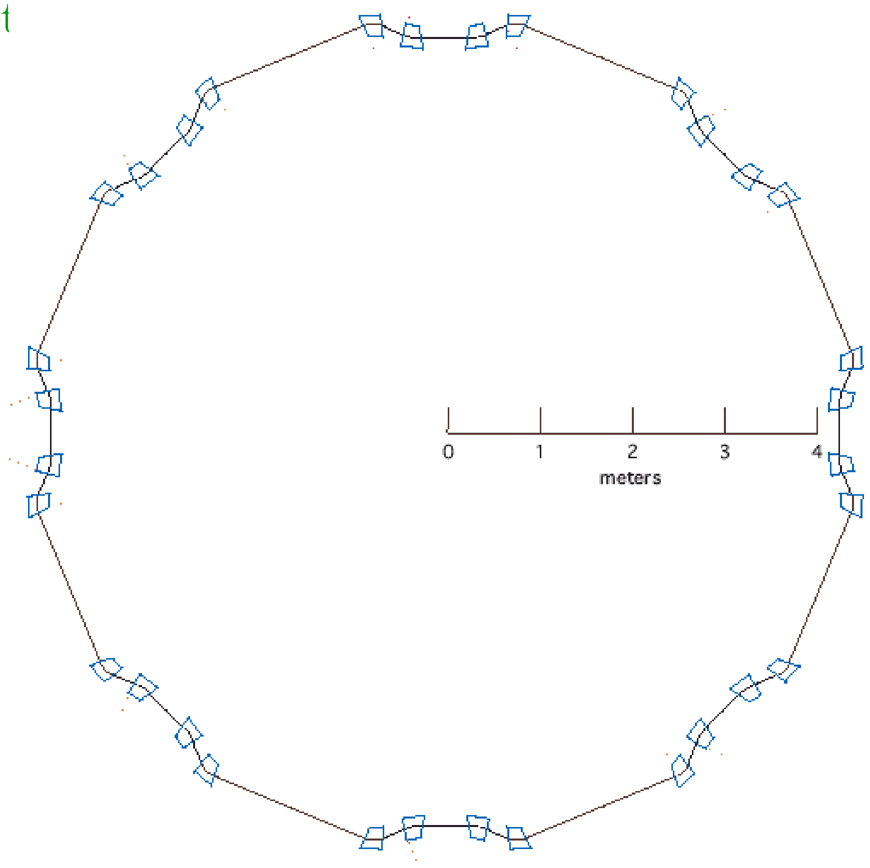
BUT

- Currently Less acceptance, and thus less Merit
- Probably due to use of Quads vs Solenoids
- Problems with real fields

4.6 Bend (weak) Focused Rings

(Garren, Kirk, Fukui et



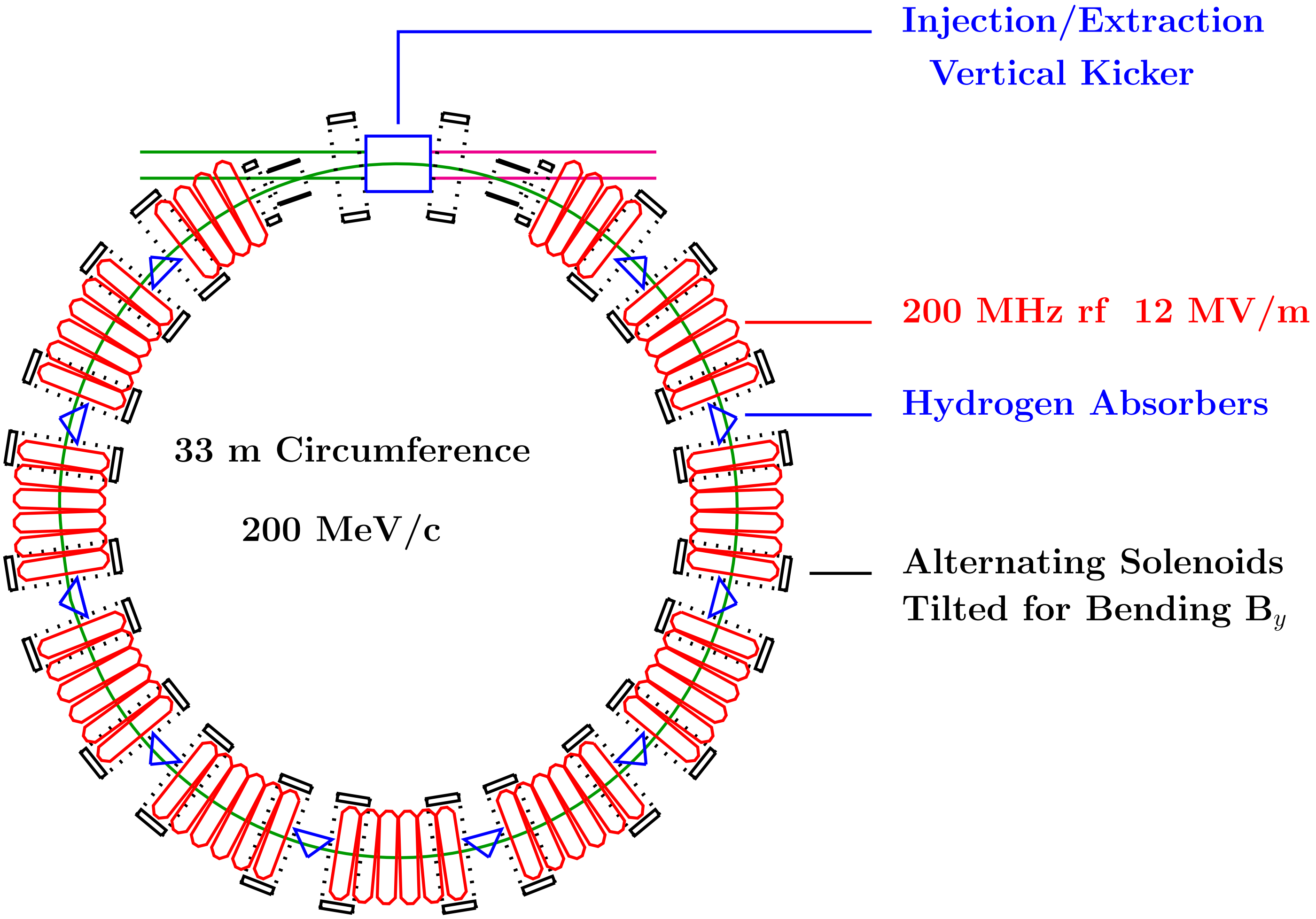


- Good focusing requires strong bending
  - Very small, or with alternate bends
  - Or reverse bends
- Good Acceptance with Ideal Fields
- Problems with Real Fields
- Now working on Quad ring with Li Lens cooling for Final Collider Cooling Ring

4.7 RFOFO Ring

4.7.1 Introduction

R.B. Palmer R. Fernow J. Gallardo[8], and Balbekov[9]



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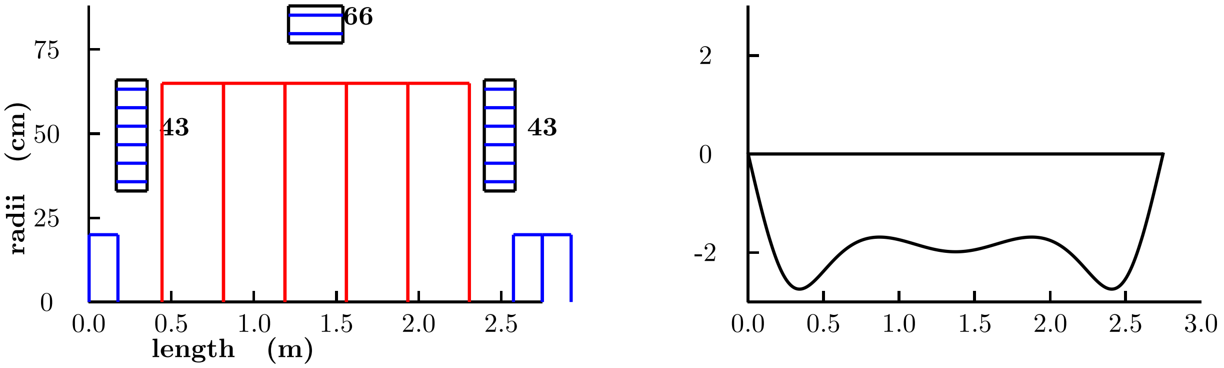
[8] Fernow and others: MUC-232, 265, 268, & 273

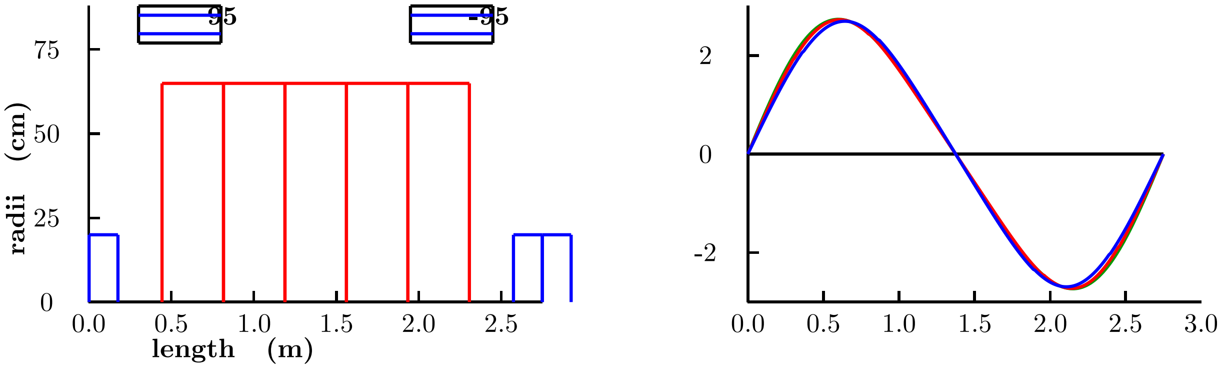
[9] V.Balbekov "Simulation of RFOFO Ring Cooler with Tilted Solenoids" MUC-CONF–0264

4.7.2 Lattice

SFOFO as in Study 2



RFOFO has Reversed Fields

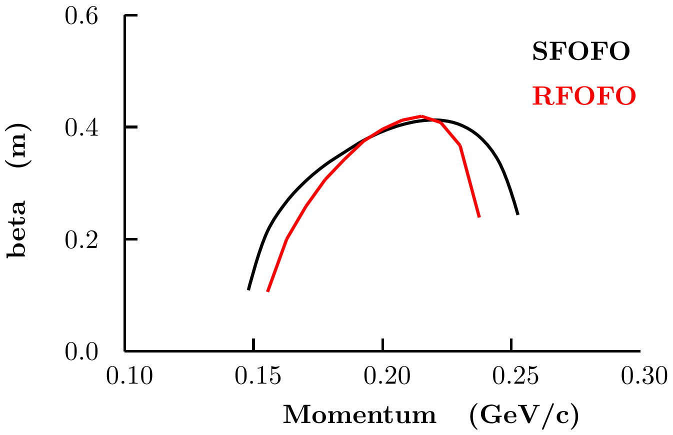


RFOFO chosen

- Less Mom acceptance

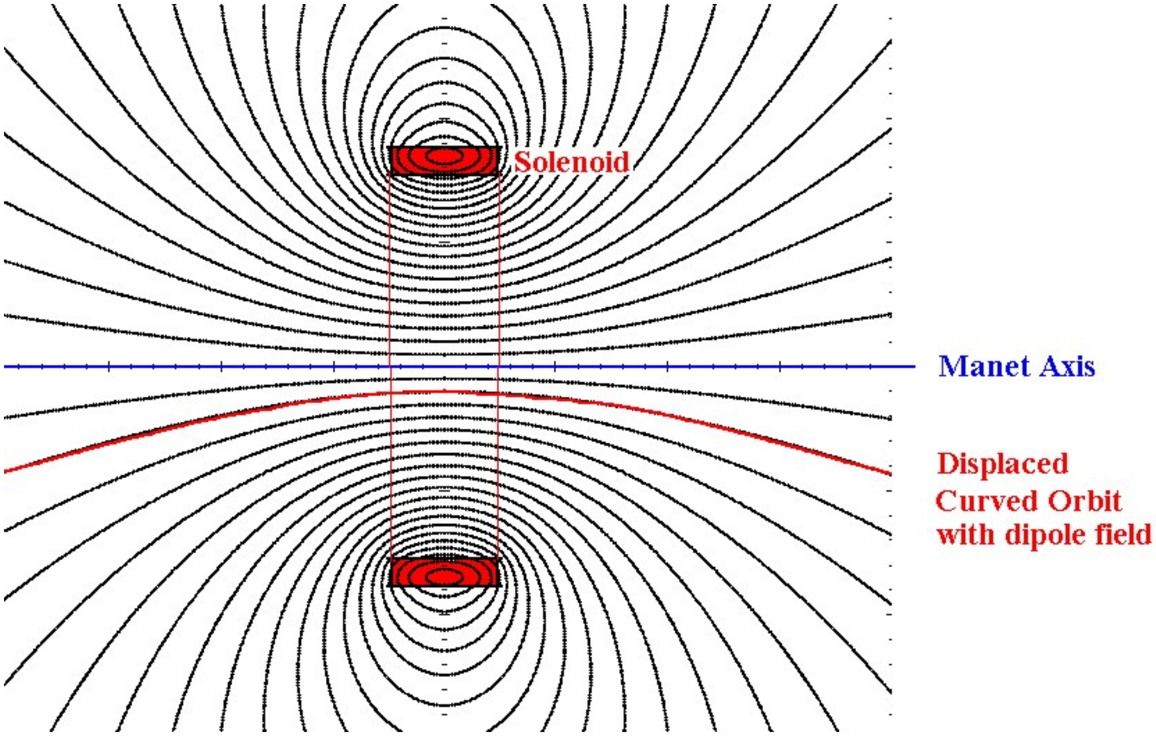
BUT

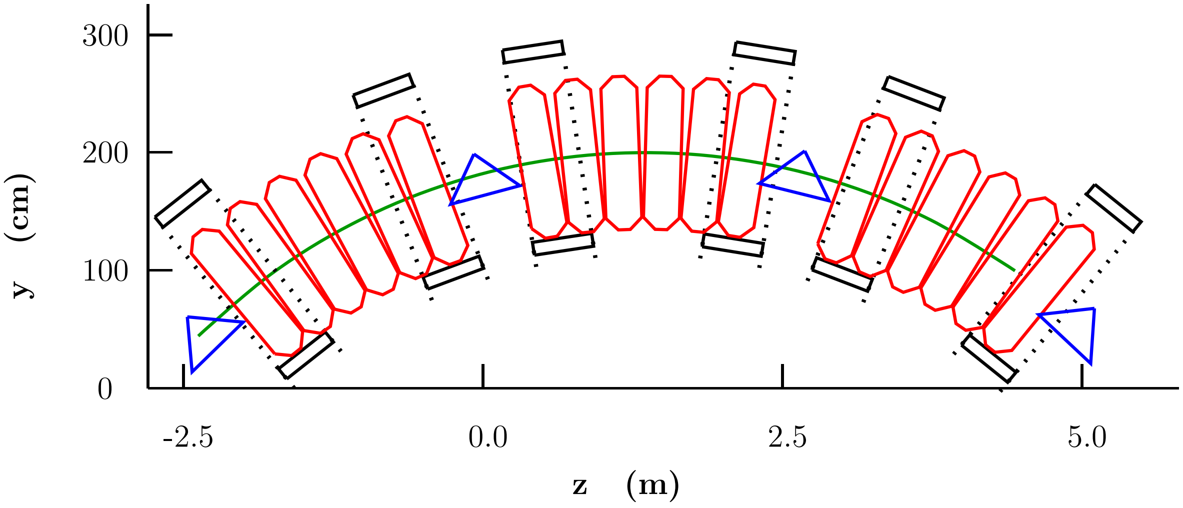
- All cells the same
- Fewer resonances



4.7.3 Coil Layout

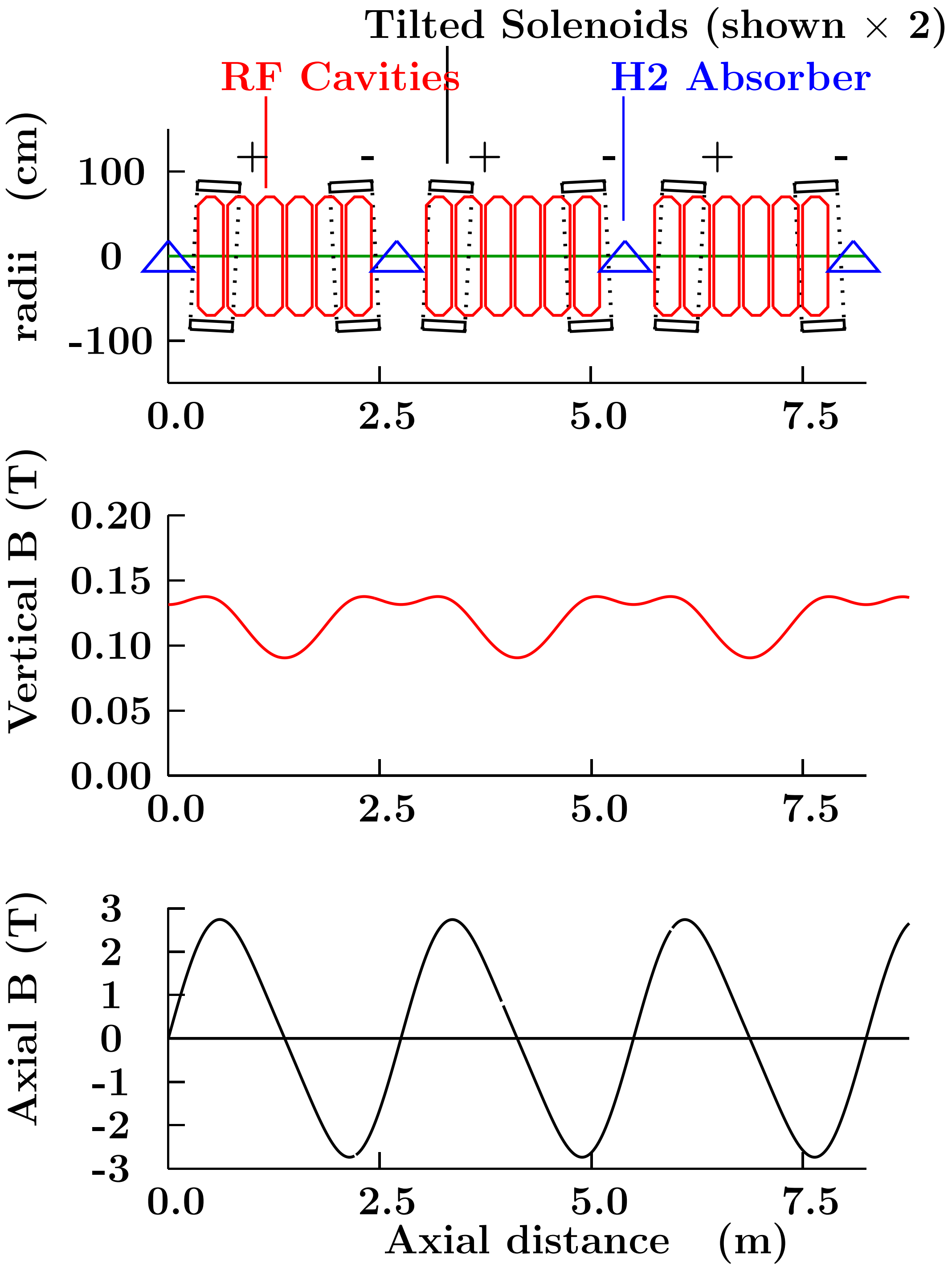
Shifted Coils so beam follows field lines



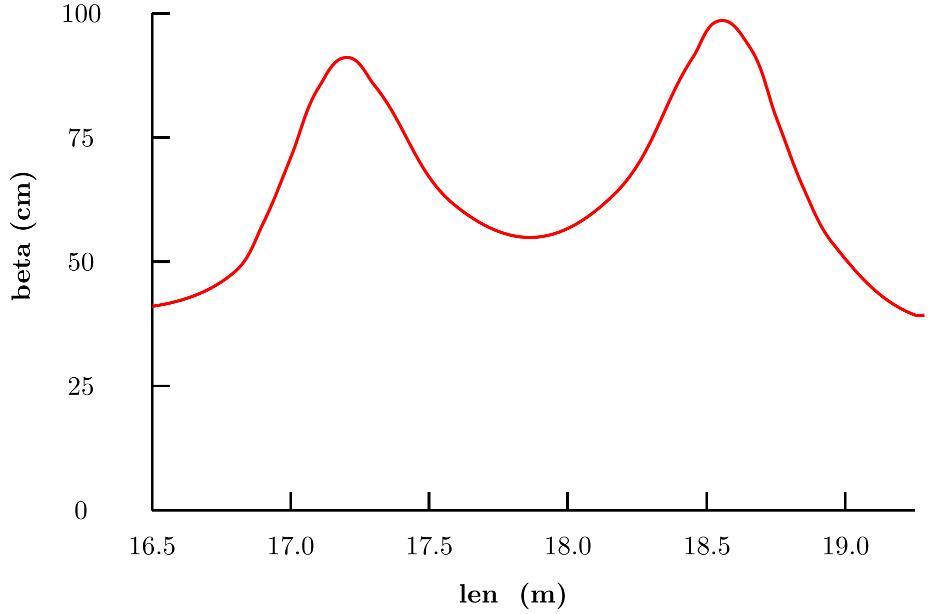


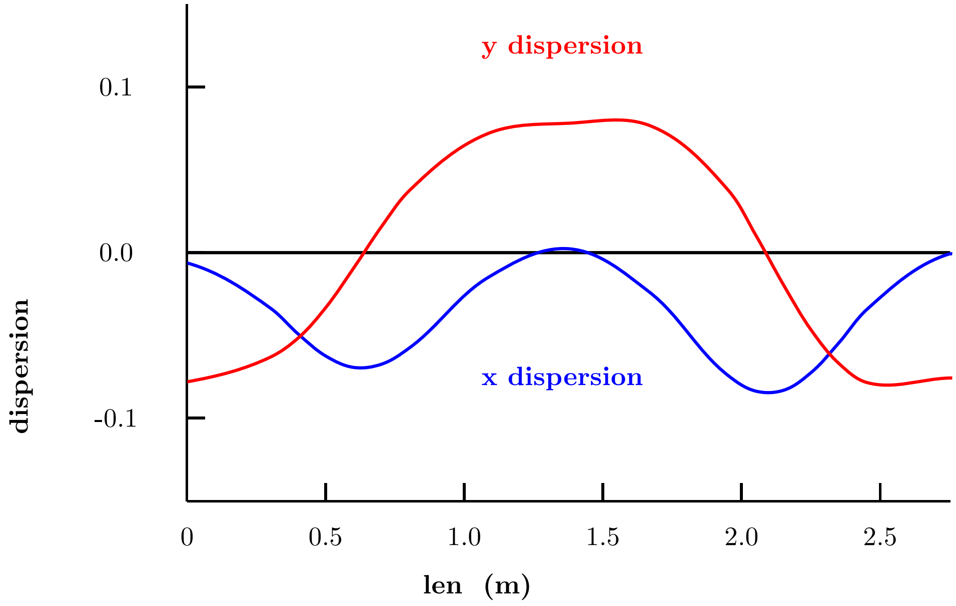
Tilt Coils to get Bend

.



4.7.4 Beta and Dispersion





Dispersion is rotating back and forth

4.7.5 Params for Simulation

Coils

gap m	start m	dl m	rad m	dr m	tilt rad	I/A A/mm^2
0.310	0.310	0.080	0.300	0.200	0.0497	86.25
0.420	0.810	0.080	0.300	0.200	0.0497	86.25
0.970	1.860	0.080	0.300	0.200	-.0497	-86.25
0.420	2.360	0.080	0.300	0.200	-.0497	-86.25

amp turns 5.52 (MA)

amp turns length 13.87326 (MA m)

cell length 2.750001 (m)

Wedge

Material		H2
Windows		none
Radius	cm	18
central thickness	cm	28.6
min thickness	cm	0
wedge angle	deg	100
wedge azimuth from vertical	deg	30

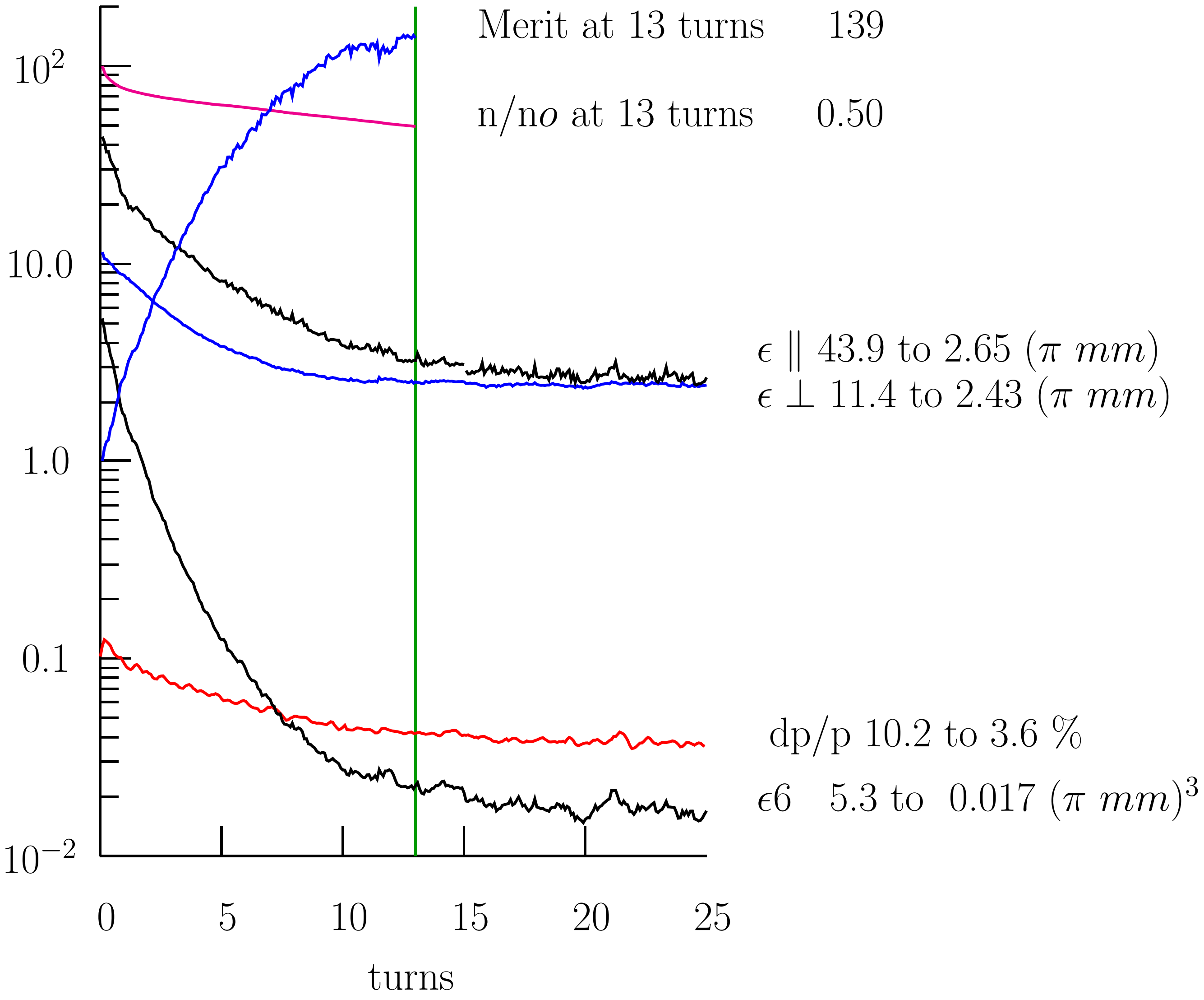
RF

Cavities		6
Lengths	cm	28
Central gaps	cm	5
Radial aperture	cm	25
Frequency	MHz	201.25
Gradient	MV/m	16
Phase rel to fixed ref	deg	25
Windows		none

4.7.6 Performance

Using Real Fields, but no windows or injection insertion

$n/n_o =$ 1543 / 4494



Merit falls after 13 turns due to decay loss

4.7.7 Compare with Linear theory

$D = 7$ cm, $\ell = 28.6$ cm, and

$$h = \frac{\ell}{2\ \tan(100^o/2)} = 12 \quad \text{cm}$$

so

$$J_z = \frac{D}{h} = 0.58$$

Since there is good mixing between x and y so $J_x = J_y$, and from equ 34, $\Sigma J_i \approx 2.0$, so

$$J_x = J_y \approx \frac{2 - 0.58}{2} = 0.71$$

i.e. The wedge angle was chosen to give nearly equal partition functions in all 3 coordinates, and gives the maximum merit factor.

The theoretical equilibrium emittances are now (eq.18):

$$\epsilon_{\perp}(\text{min}) = \frac{C\ \beta_{\perp}}{J\ \beta_v} = \frac{38\ 10^{-4}\ 0.4}{0.71\ 0.85} = 2.5 \ \ (\pi\ mm)$$

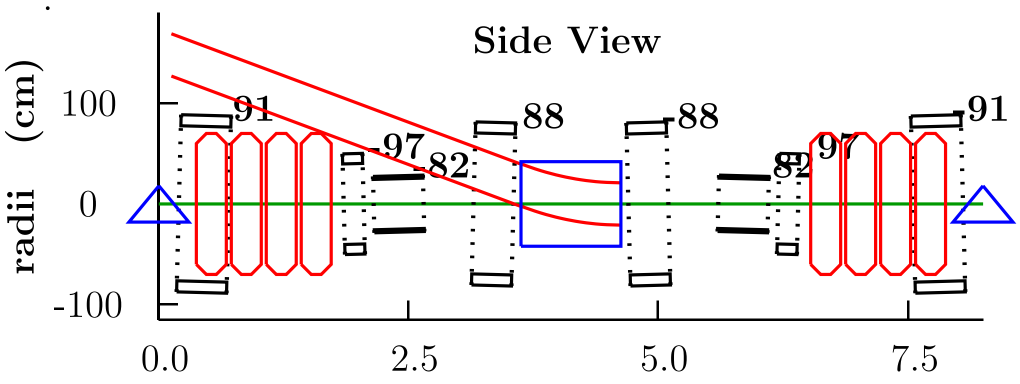
c.f. 2.43 (π mm) observed, which is very good agreement considering the approximations used.

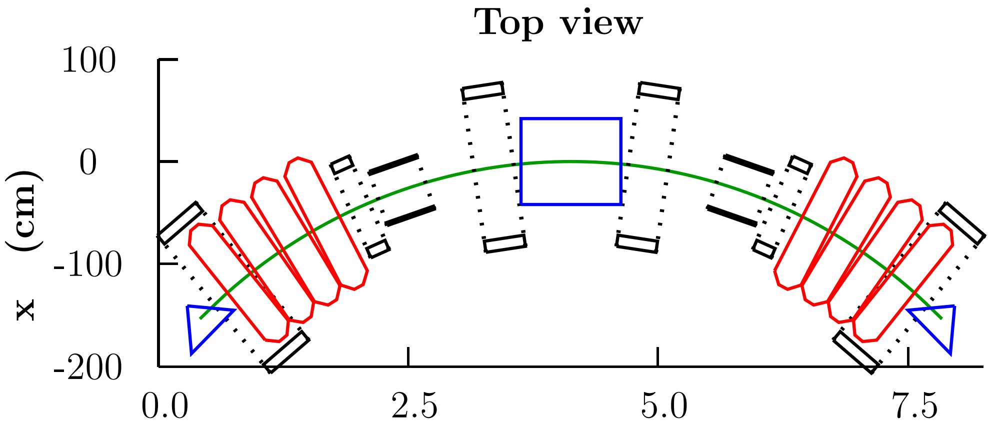
And from equation 35 we expect

$$\frac{dp}{p}(\text{min}) \ \approx 2.3\%$$

compared with 3.6% observed, which is less good agreement. This may arise from the poorer approximation of the real Landau scattering distribution by a simple gaussian.

4.7.8 Insertion for Injection/Extraction

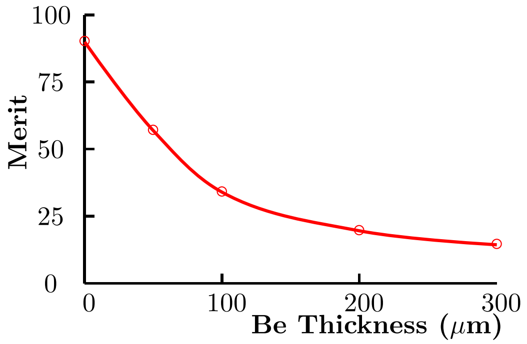

Side View


Top view

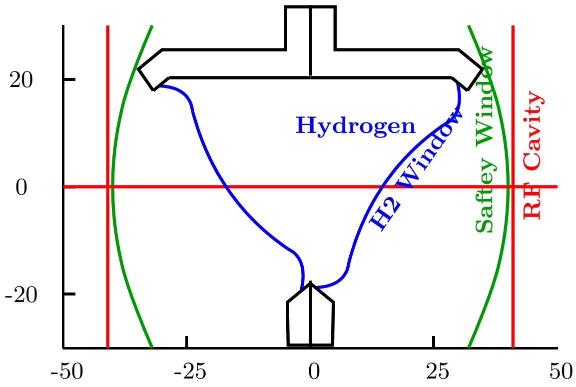
- First Simulation gave Merit = 10
  Synchrotron tune = 2.0: Integer
- Increase energy, wedge angle, and add matching.
- Merit achieved ≈ 100

4.7.9 Further Problems under study

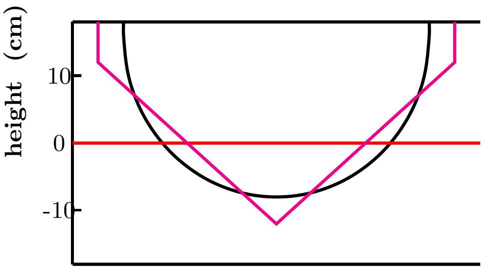
- RF windows must be very thin (≤ 50 microns)
  RF at 70 deg will help



- Design of wedge absorber



For 80 Degrees

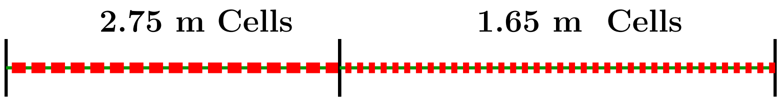


For 100 Degrees

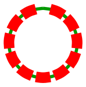
- Absorber heating is high for many passes
- The kicker (problem common to all rings)

Compare with Study 2

- **Study 2 Cooling**



- **e.g. RFOFO Cooling Ring**



- Similar transmission
- Similar Trans emittance
- Less Long Emittance

	Study 2	Now	Factor
Tot Length (m)	108	33	30 %
Acc Length (m)	54	16	30 %
Acc Grad	16 MV/m	12 MV/m	66 %

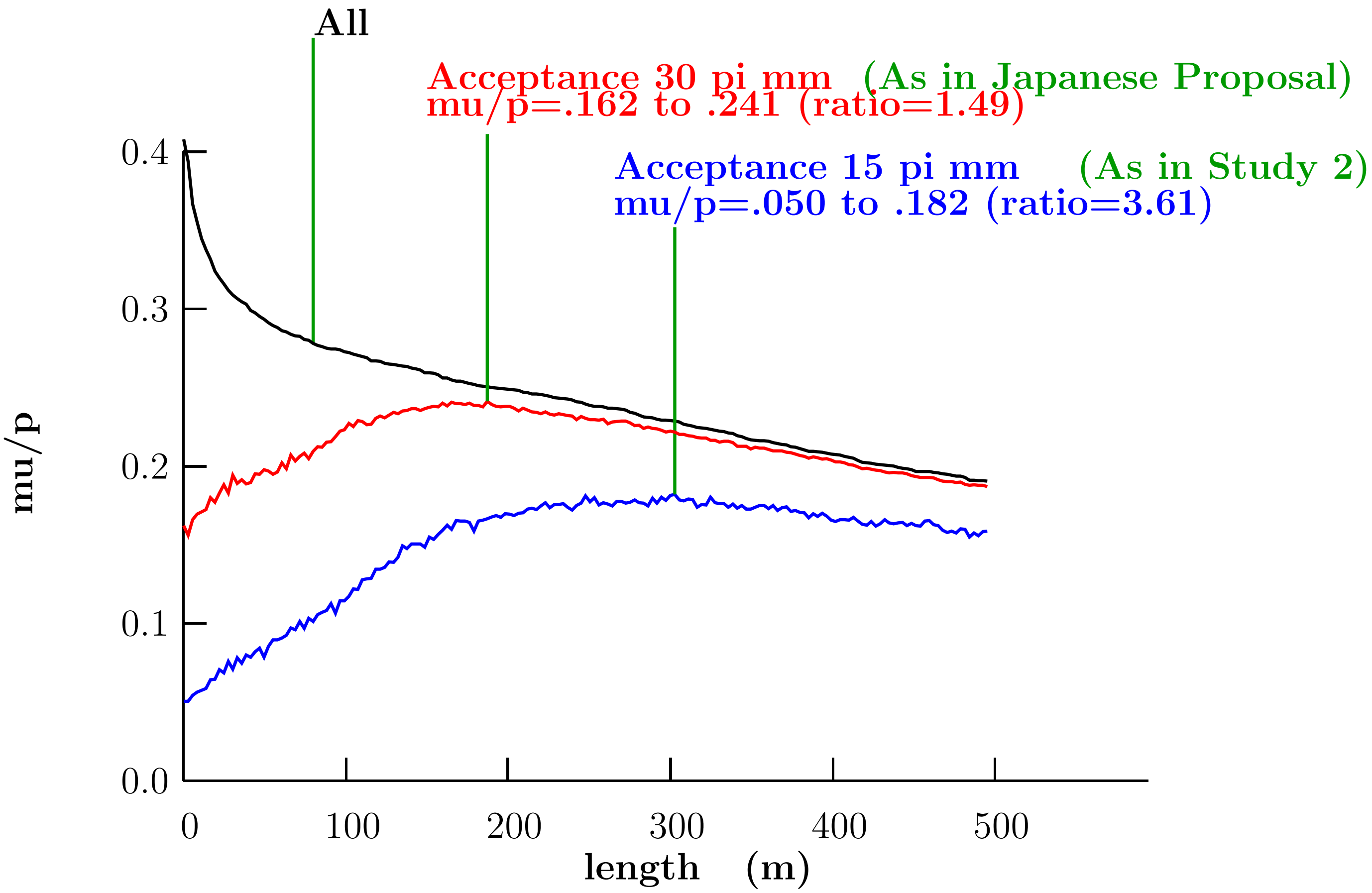
- EXPECT COST $\approx 1/3$

BUT

- Need R&D on absorber heating
- Need R&D on thin windows
- Need R&D on kicker

4.8 Mu/p with Cooling vs Accelerator Trans Acceptance

Using input from Study-2 Front-End (includes some mini-cooling)

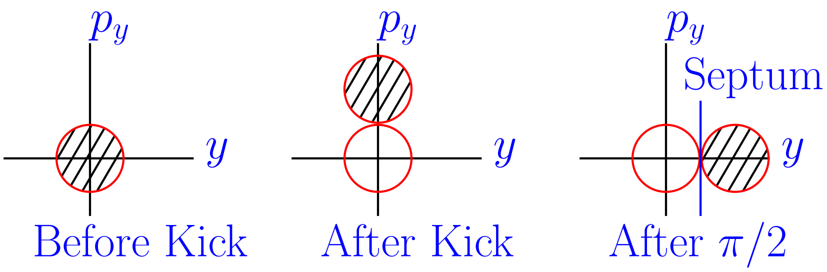


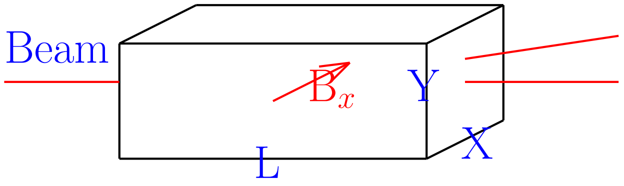
- Performance at 30 pi mm without cooling
  ≈ Performance at 15 pi mm with cooling
- Not a new idea:
  Mori at KEK has proposed no cooling for a long time
- Cost of acceptance 15→30 pi mm may be less than for cooling
- If no cooling required, less R&D required for Neutrino Factory

- But we still need (approx 3) cooling rings for a Muon Collider

4.9 Kickers

4.9.1 Minimum Required kick





$$f_\sigma = \frac{\text{Ap}}{\sigma} \qquad \mu \ = \ \text{inf} \qquad F \ = \ \frac{Y}{X}$$

$$I \ = \ F \left(\frac{4 \ f_\sigma^2 \ m_\mu}{\mu_o \ c} \right) \quad \frac{\epsilon_n}{L}$$

$$V \ = \ \left(\frac{4 \ f_\sigma^2 \ m_\mu \ R}{c} \right) \quad \frac{\epsilon_n}{\tau}$$

$$U \ = \ F \left(\frac{m_\mu^2 \ 8 \ f_\sigma^4 \ R}{\mu_o \ c^2} \right) \quad \frac{\epsilon_n^2}{L}$$

- muon ϵ_n $\gg$ other ϵ_n's
- So muon kicker Joules $\gg$ other kickers
- Nearest are $\bar{p}$ kickers

Compare with others

For $\epsilon_\perp = 10\ \pi$mm, $\beta_\perp = 1$m, & τ=50 nsec:

After correction for finite μ and leakage flux:

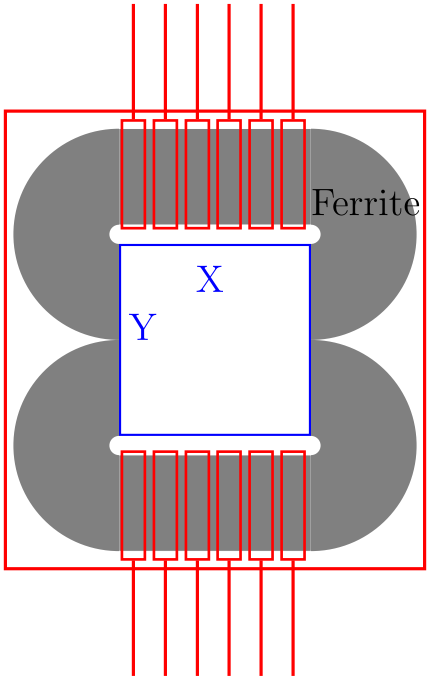
		μ Cooling	CERN $\bar{p}$	Ind Linac
$\int Bd\ell$	Tm	.30	.088	
L	m	1.0	≈5	5.0
t_{rise}	ns	**50**	**90**	**40**
B	T	**.30**	**≈0.018**	**0.6**
X	m	.42	.08	
Y	m	.63	.25	
V_{1turn}	kV	**3,970**	**800**	**5,000**
U_{magnetic}	J	**10,450**	**≈13**	**8000**

Note

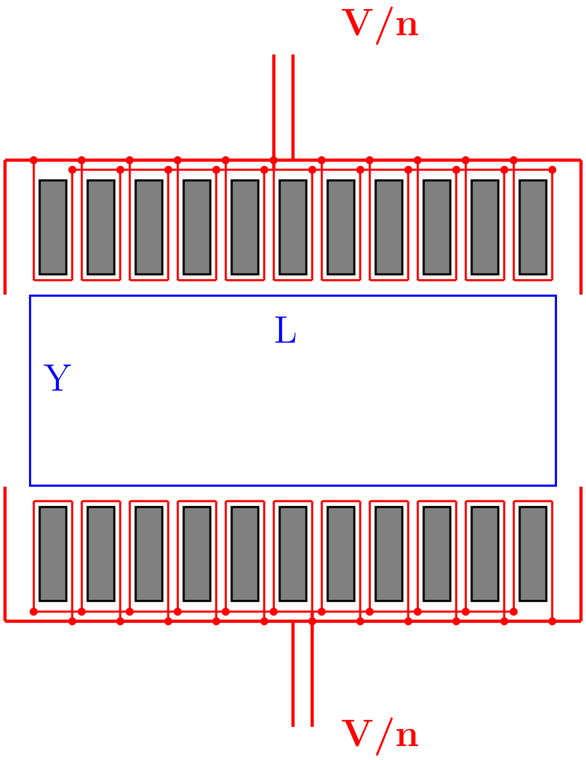
- U is 3 orders above $\bar{p}$
- Same order as Induction
- And t same order
- But V is too High

4.9.2 Induction Kicker

- Drive Flux Return
- Subdivide Flux Return Loops
  Solves Voltage Problem
- Conducting Box Removes
  Stray Field Return



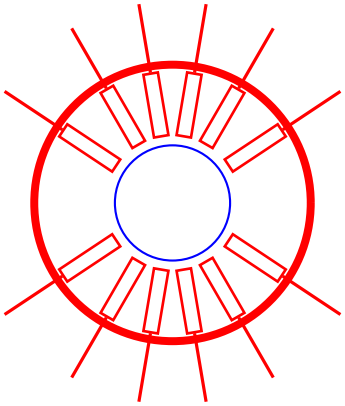
End View



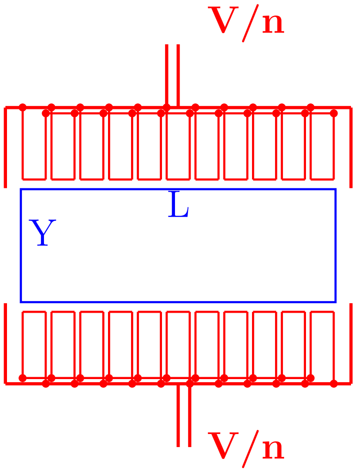
Side View

Works with no Ferrite

- V = the same
- U≈ 2.25×
- I≈ 2.25×
- No rise time limit
- Not effected by solenoid fields



End View

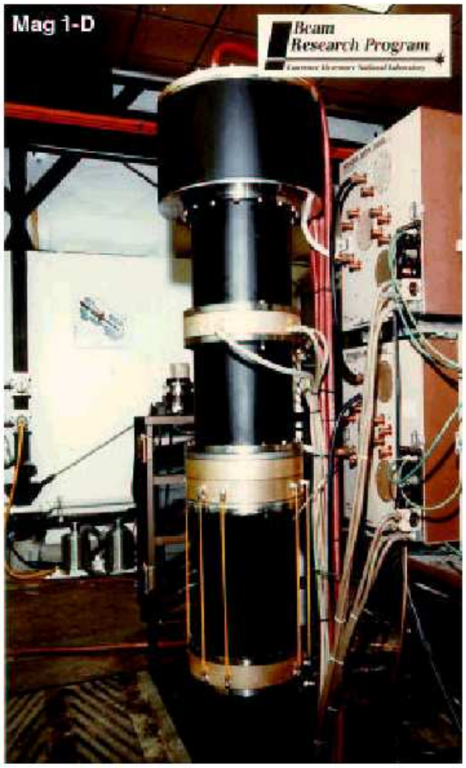


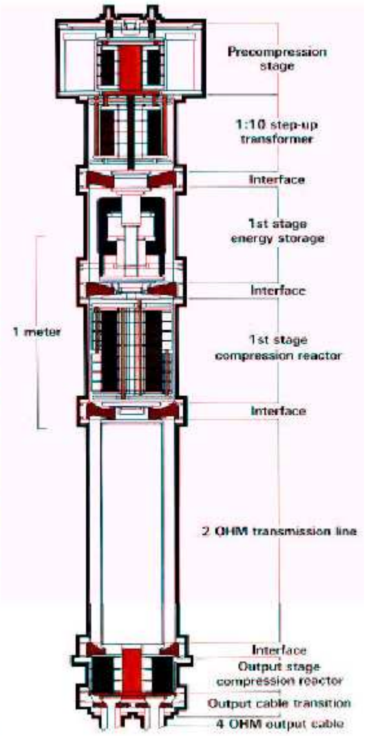
Side View

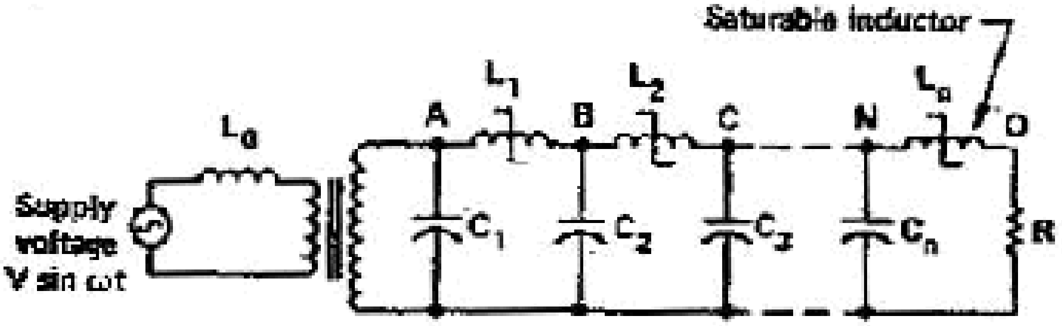
- If non Resonant: 2 Drivers for inj. & extract.
  Need 24 ×2 Magamps (≈ 20 M$)
- If Resonant: 1 Driver, 2×efficient
  Need 12 Magamps (≈ 5 M$)

4.9.3 Magnetic Amplifiers

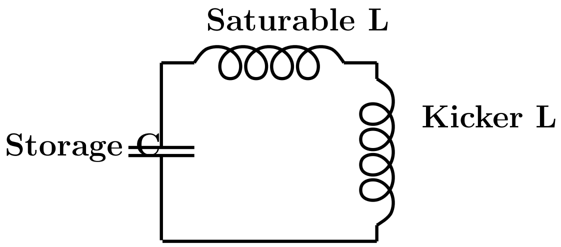
Used to drive Induction Linacs
similar to ATA or DARHT







Magamp principle



Initially Unsaturated, $L = L_1$ is large:

$$\tau_L \;=\; \sqrt{(L + L_1)C} \quad \text{is slow}$$

The current I rises slowly:

$$I \;=\; I_o \;\sin\left(\frac{t}{\tau_L}\right)$$

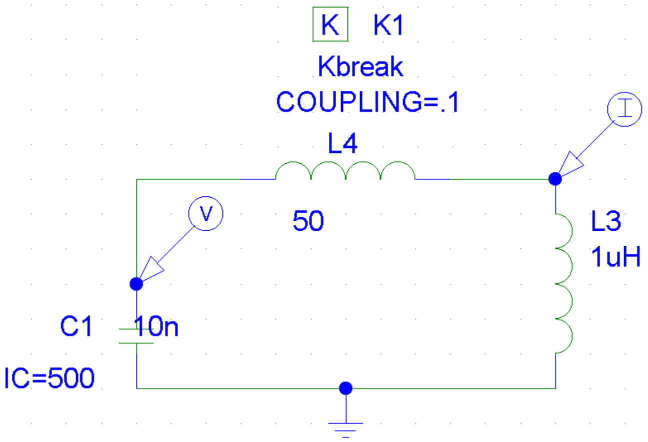
When the inductor saturates
$L = L_2$ is small:

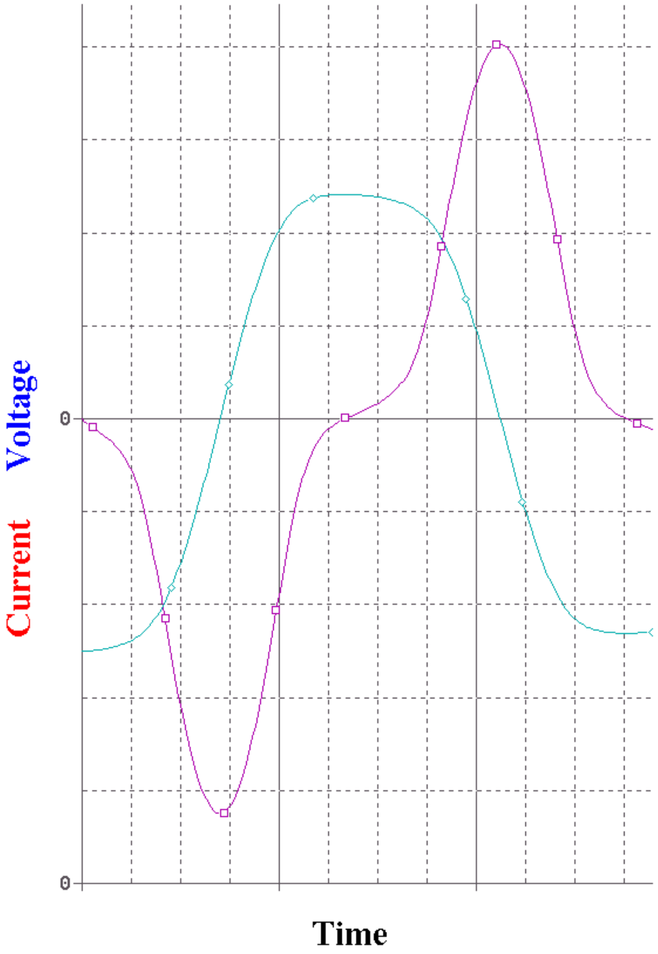
$$\tau_S \;=\; \sqrt{(L + L_2)C} \qquad \text{is fast}$$

After approx π phase
Inductor regains its high inductance
The oscillation slows before reversing.

Pspice Simulation

a) Single stage





Circuit Model (Reginato)

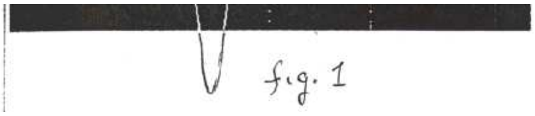


fig. 1