Capture & Cooling

NUFAC School Lectures

R.B. Palmer

Rutherford 6/24-29/2002

Contents

1 Preface

1.1 Units

I will use MKS units, except that momentum (p), kinetic Energy (E), and mass (m) will be measured in Volts. These variables will always be given in parentheses. To change to true MKS units, they may be replaced by $(p\ c/e)$, (E/e), and $(m\ c^2/e)$, respectively.

e.g.

$$\rho \;=\; \frac{(p)}{B\ c}$$

rather than

$$\rho \;=\; \frac{(p\ c/e)}{B\ c} \;=\; \frac{p}{B\ e}$$

1.2 Useful Relations

$$dE = \beta_v \, dp \tag{1}$$

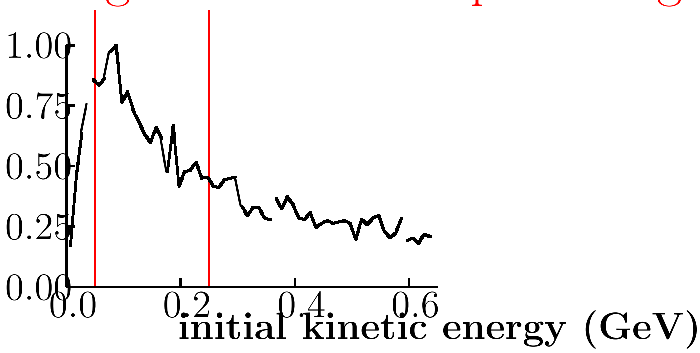
$$\frac{dE}{E} = \beta_v^2 \, \frac{dp}{p} \tag{2}$$

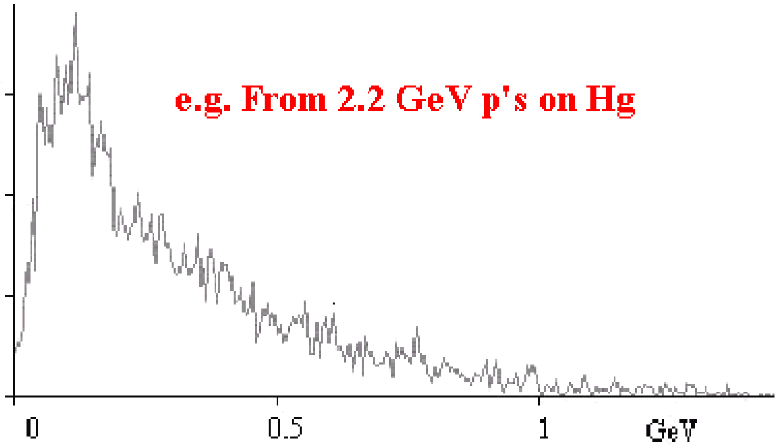
$$d\beta = \frac{dp}{\gamma^2} \tag{3}$$

2 Pion Capture

2.0.1 Initial KE Distribution

e.g. from 24 GeV p's on Hg





- similar distributions
- Reasonable ≈50-250 MeV
- $\sigma_{p\perp}$≈150 MeV/c

2.1 Magnetic Horn Capture

2.1.1 Horn theory

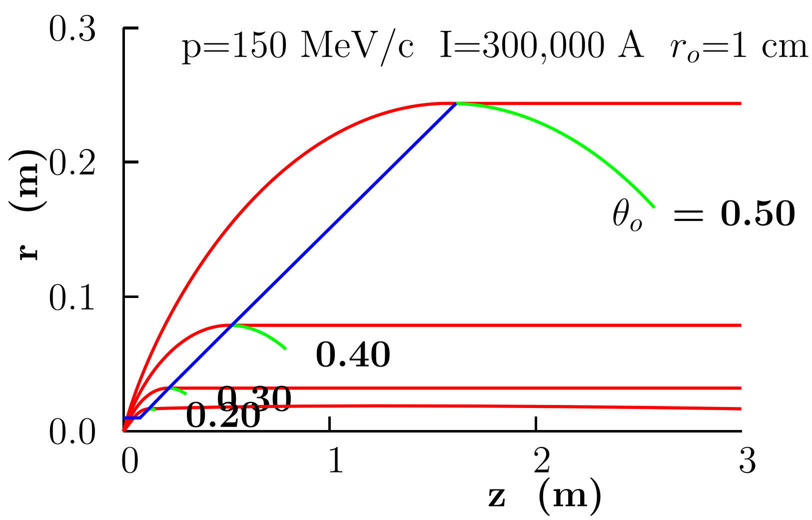
Outside an axial conductor

$$B = \frac{\mu_o I}{2\pi} \frac{1}{r}$$

Bending:

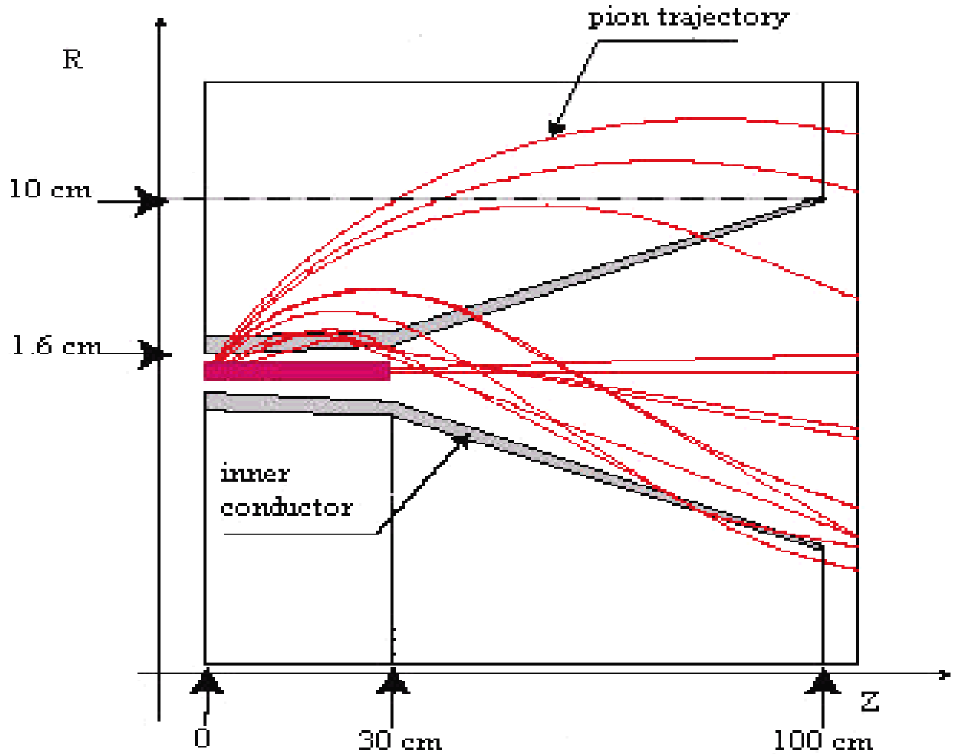
$$\frac{d\theta}{ds} = \frac{B\,c}{(p)}$$

Minimum radius set by inward forces. Find exit shape to focus mom=p:



2.1.2 Example

CERN Design



2.2 Solenoid Capture

In the transverse plane:

$$r = \frac{(p_\perp)}{c\ B}$$

For particles generated in a thin target on the axis, inside a solenoid of inside radius R, the maximum transverse momenta captured will be:

$$(p_\perp(max)) = \frac{c\ B_z\ R}{2} \qquad (4)$$

e.g. For a 20 T solenoid of 8 cm radius, (These are the dimensions of an existing resistive solenoid at FSU)

$$p_\perp(max) = 240 MeV/c$$

Contains å80% of π's below 250 MeV

2.3 Adiabatic Matching

The match between a target capture Solenoid and a decay channel solenoid can be made, with negligible loss, by gently tapering the magnetic field[1] .

The condition for "gentleness" is that $d\beta/\beta$, is small in a distance equal to the current β:

$$\frac{d\beta}{\beta} \ll \frac{dz}{\beta}$$

or

$$\frac{d\beta}{dz} = \epsilon \ll 1$$

Since $\beta \propto 1/B_{solenoid}$:

$$\frac{d(1/B)}{dz} = \epsilon \ll 1$$

which gives:

$$B(z) = \frac{B_o}{1 + k\ z} \tag{5}$$

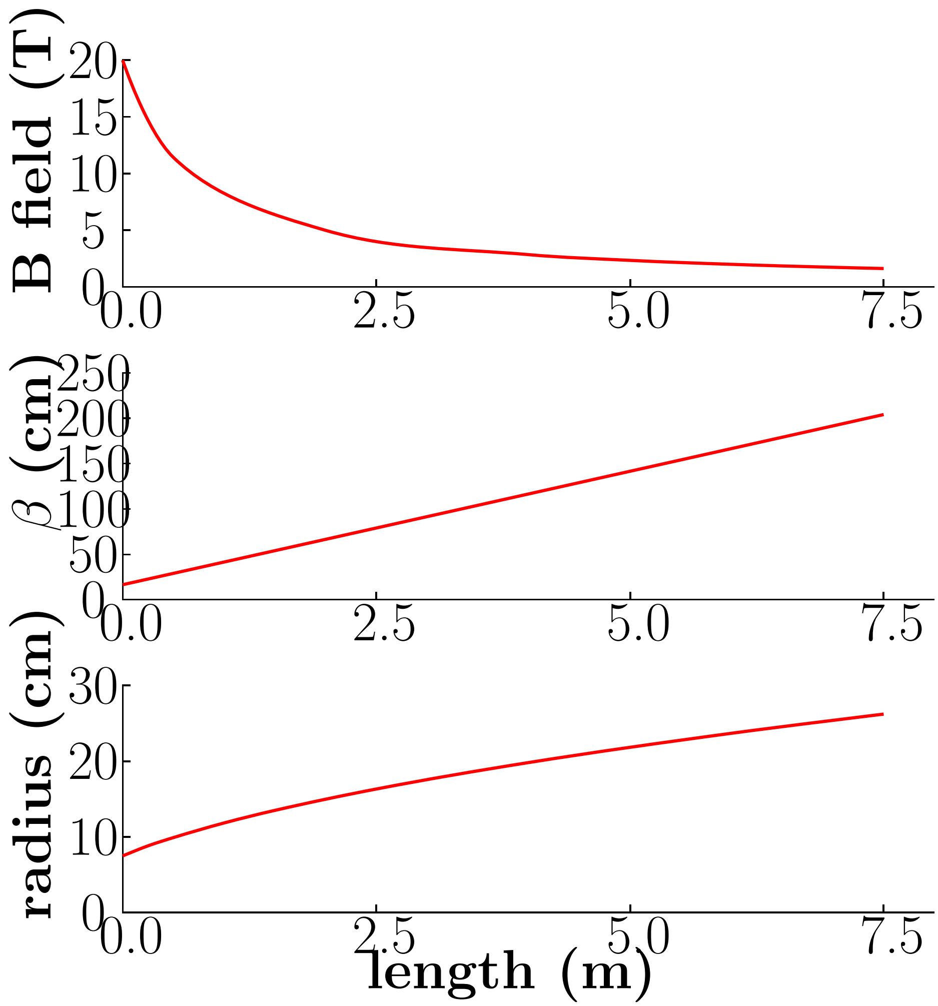
[1] R. Chehab, J. Math. Phys. 5 (1978) 9.

where

$$k \;=\; \epsilon \, \frac{B_o \, c}{2 \; (pc/e)} \tag{6}$$

Note that the B drops initially very fast, corresponding to the short β's at the high initial field, but falls much slower at the lower later fields where the β's are long.

For a taper from 20T to 1.25T at momenta less than 1 GeV and $\epsilon = .5$, the taper length should be approximately 6 m.



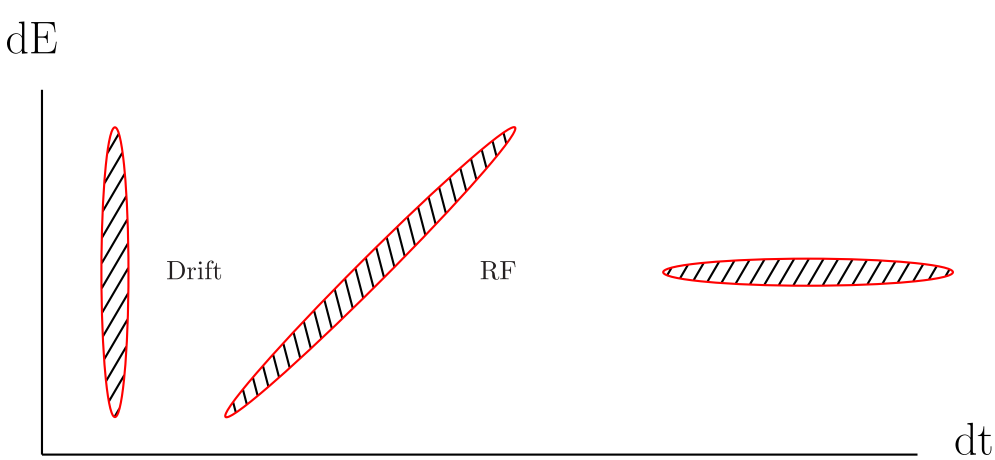
2.4 Phase Rotation

2.4.1 Introduction

- Initial pions have rms dp/p ≈ 100%
- rms Acceptance of cooling ≈ 8%

Phase Rotate

- Increase dt
- Decrease dE



2.4.2 Phase Space Conservation

For initial $\Delta E = 200$ MeV (full width) $\times\ \delta t = 4$ nsec (rms) (time is set by fluctuations in decay)

If final δE/E 8% (rms) at 200 MeV (δE=16 MeV (rms)):

$$\Delta t(final) \;=\; \frac{200(full) \times 4(rms)}{16(rms)} \;=\; 50nsec(full)$$

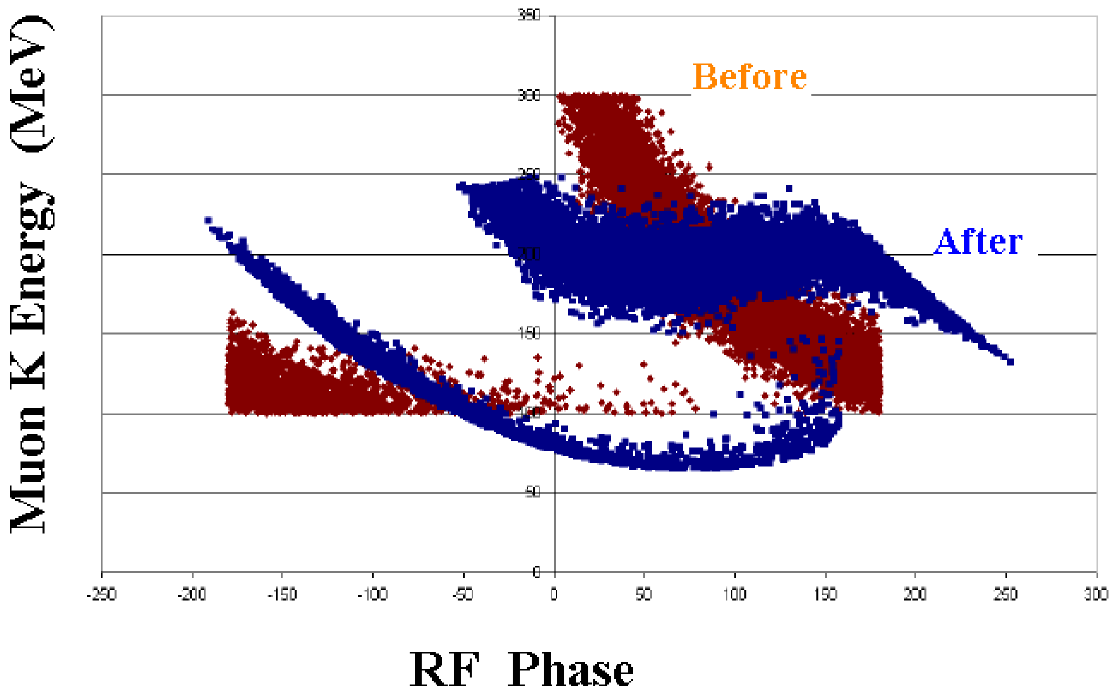
To capture and accelerate this we need frequency $\ll$ 1/50(nsec), i.e. $\ll$ 20 MHz

- KEK: 5 MHz which would allow only low gradients.
- CERN: 44 or 88 MHz
- PJK: 30 MHz but got dp/p $\approx$15 %

2.4.3 Examples without re-bunching

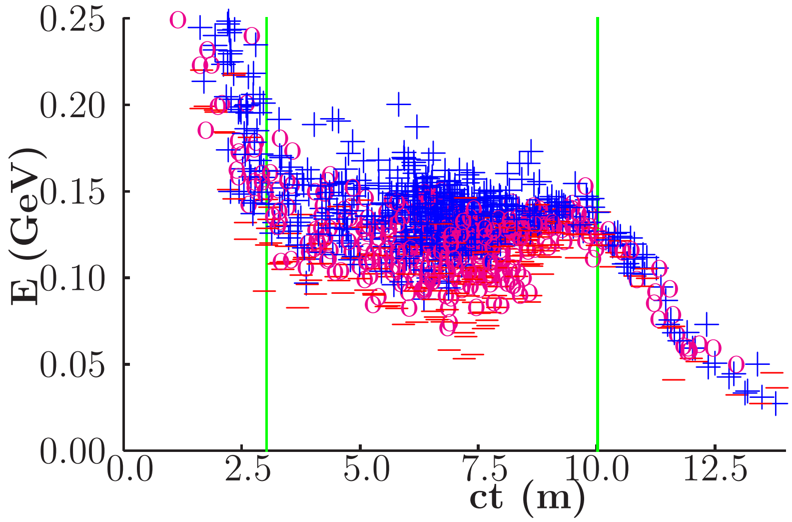
e.g. CERN

- 30 m decay channel
- 30 m 2 MV/m 44 MHz RF
- Captures ≈120-300 MeV
- Gives ≈4 m long bunch
- and ≈± 5%



e.g. PJK

	Len m	freq MHz	Grad MV/m
Drift	6		
RF	12	40	6
RF	24	30	5
RF	5	45	6



- ≈6 m long bunch
- ≈12 % dE/E

2.4.4 Examples with Re-Bunching

Alternative allowing higher frequencies:

Re-bunching increases dE/E by $\approx 4\times$
So require dE/E $\approx$2% before re bunching
And $\Delta t \approx$50 nsec $\times 4 \approx$ 200 nsec

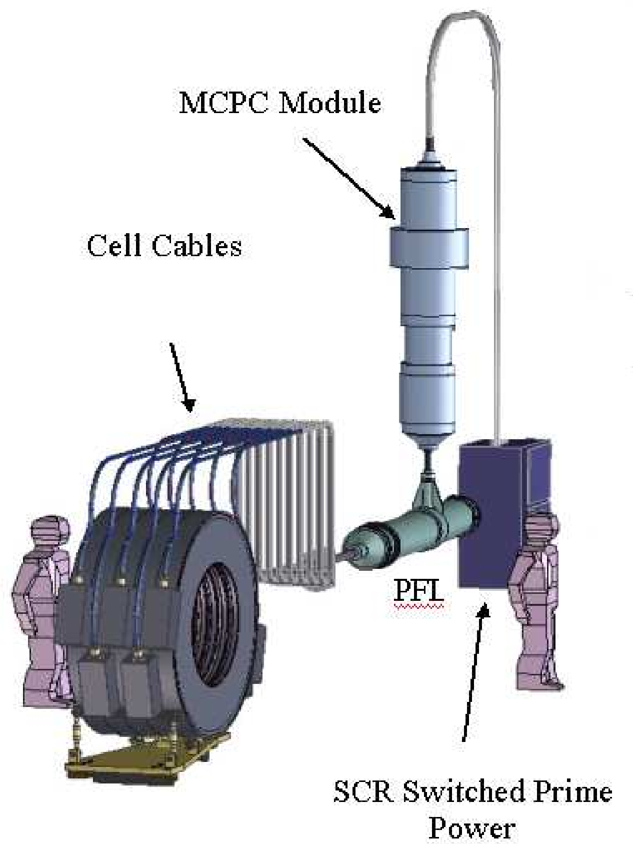
US Study 1 had $\approx$150 nsec
US Study 2 had $\approx$300 nsec

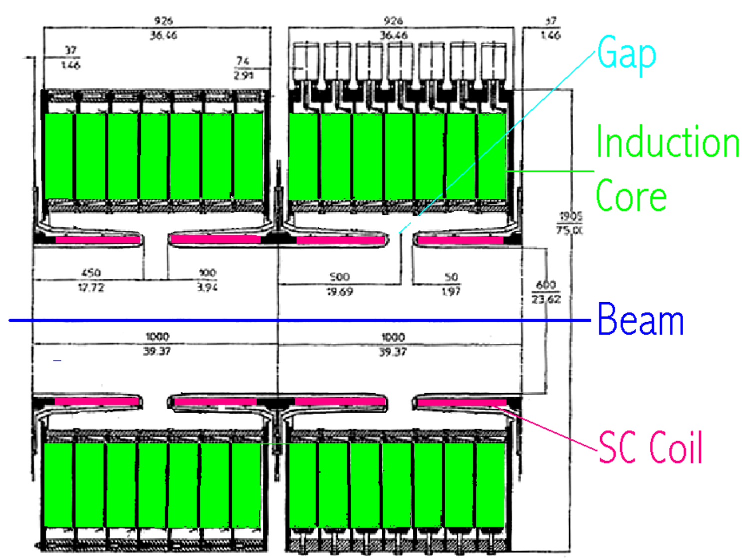
Too long for conventional rf,

Use Induction Linacs

- pulses 50-500 nsec
- Grad's $\approx$1 MV/m

2.5





2m Section

95 cm radius similar to ATA or DARHT but Superconducting inside coil

2.5.1 Example of Single Linac PR

US Study 1

- Energy spread non uniform
  "Distorted"
- dp/p rms ≈ 6%
- → 18% after bunching
- particles lost

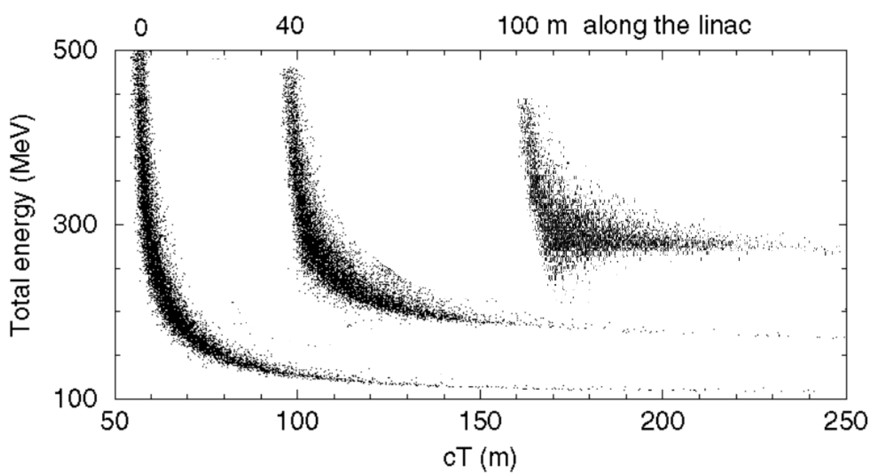
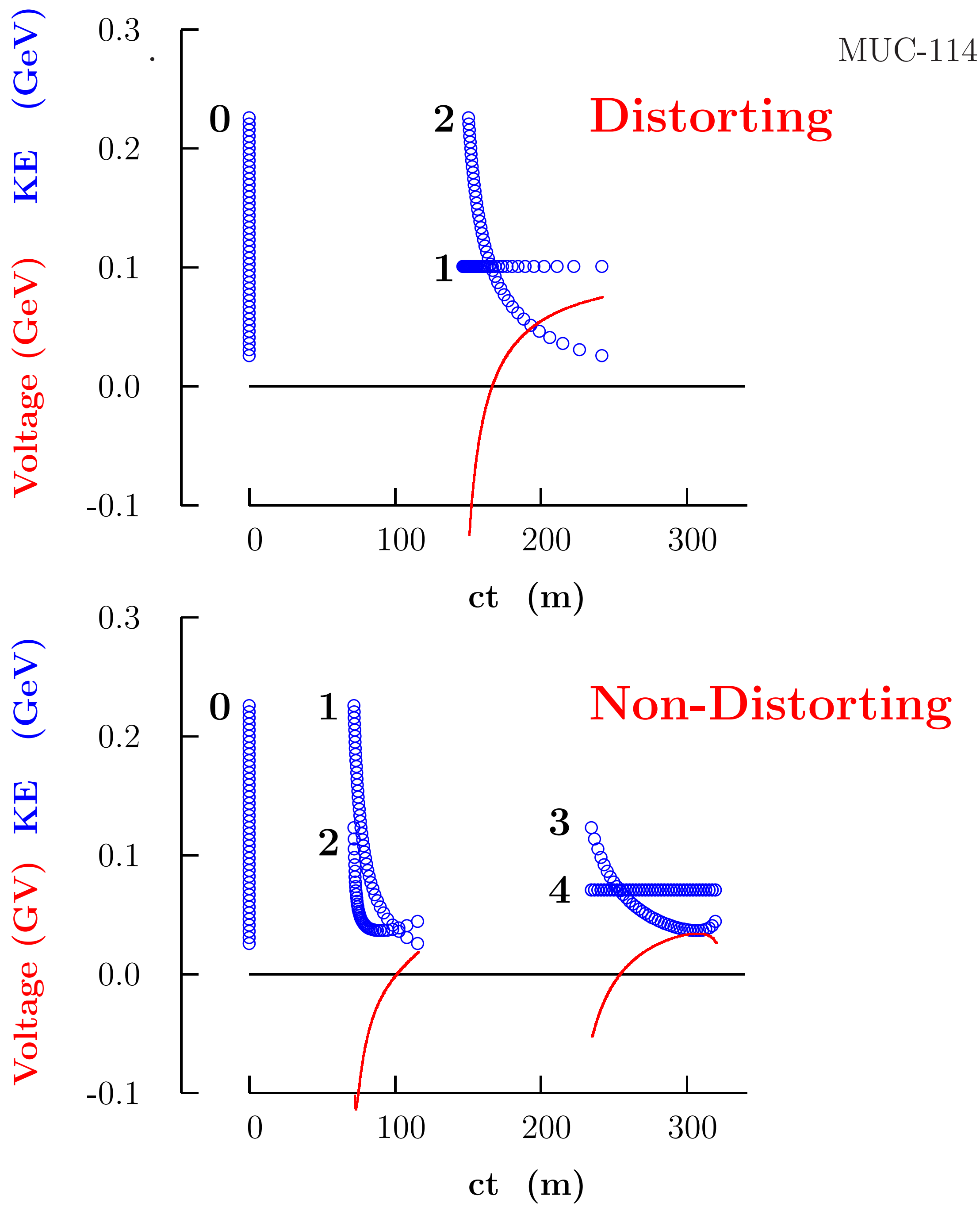


Figure 6: Beam distributions in E-cT phase space along the induction linac. Distributions from L = 0, 20, 60, and 100 m are shown.

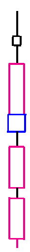
2.5.2 Non-Distorting Phase Rotation



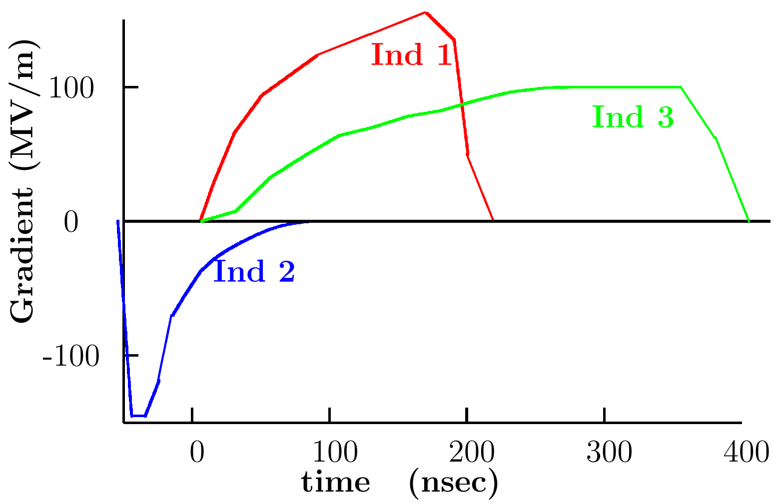
2.5.3 Example of Non-Distorting

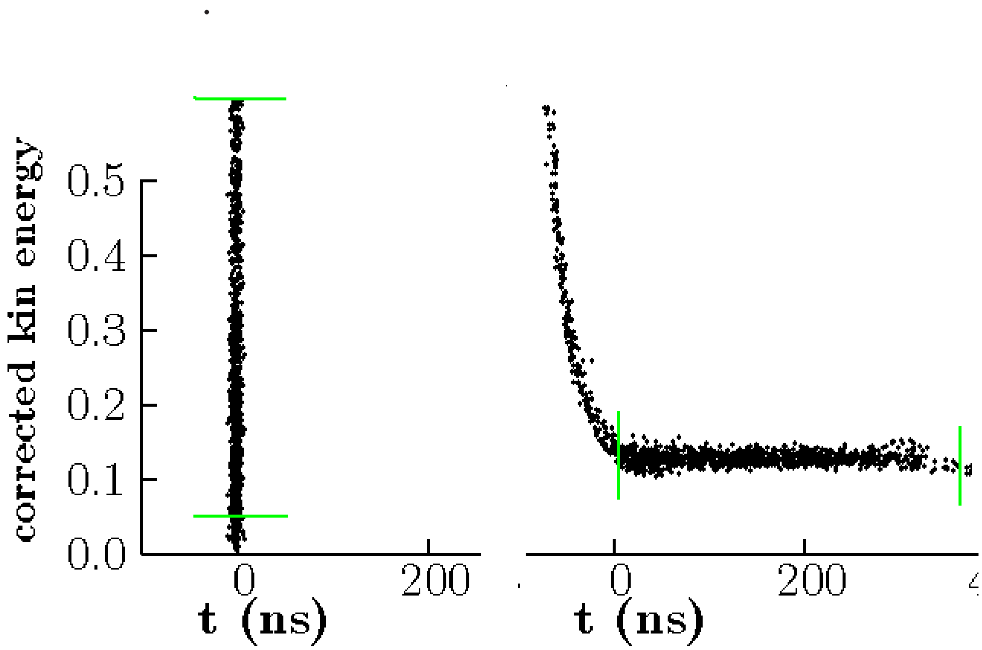
Study 2 2-3 Linacs

1. 30 m Drift
2. Induction Linac to modify E vs t
3. Second drift ($\approx$ 100 m)
4. 2nd Induction Linac to reduce dE/E



Hg Target	**(.45 m)**
Induction #1	**(100 m)**
Mini Cooling	**(3.5 m H_2)**
Induction #2	**(80 m)**
Induction #3	**(80 m)**





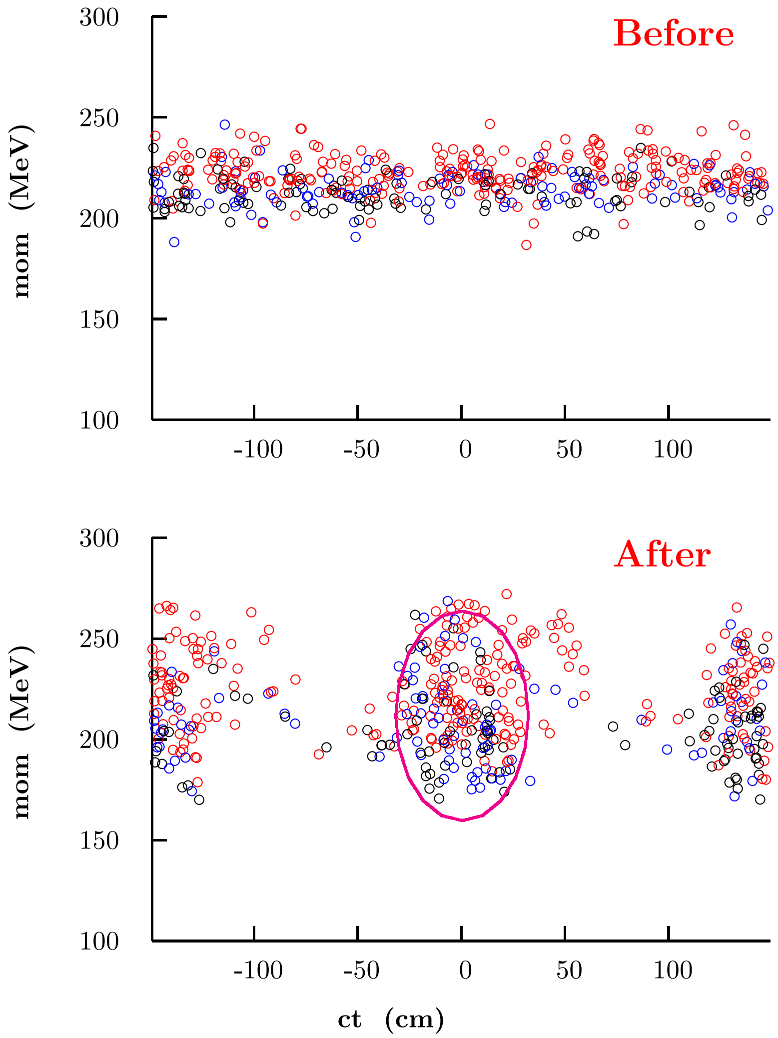
- Energy spread more uniform
- dp/p rms ≈ 3%
- OK for bunching

2.6 RF Buncher

Three stages:

stage		len	400 MHz	200 MHz
		m	MV	MV
1	RF	2.75	-2.38	9.55
	Drift	22		
2	RF	5.5	-4.46	17.9
	Drift	8.25		
3	RF	8.25		35.8
	Drift	5.5		

Similar to Study 1


Before
mom (MeV)
300
250
200
150
100
-100
-50
0
50
100
After
mom (MeV)
300
250
200
150
100
-100
-50
0
50
100
ct (cm)

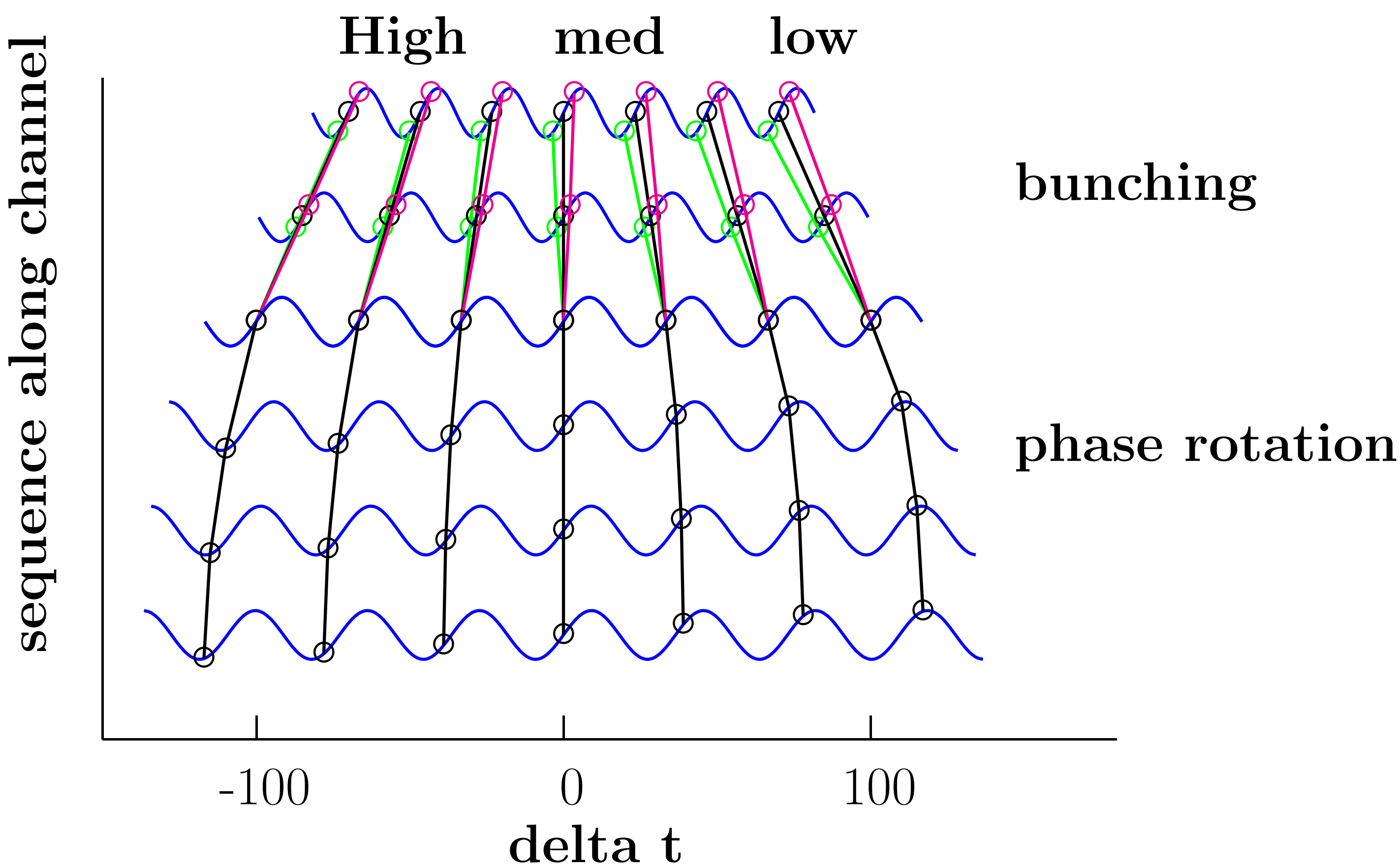
Bunched Phase Rotation

1. Drift
2. Bunch
3. Rotate with high freq. rf

vs. Conventional

1. Drift
2. Rotate with induction linac
3. Bunch

Bunched Phase Rotation



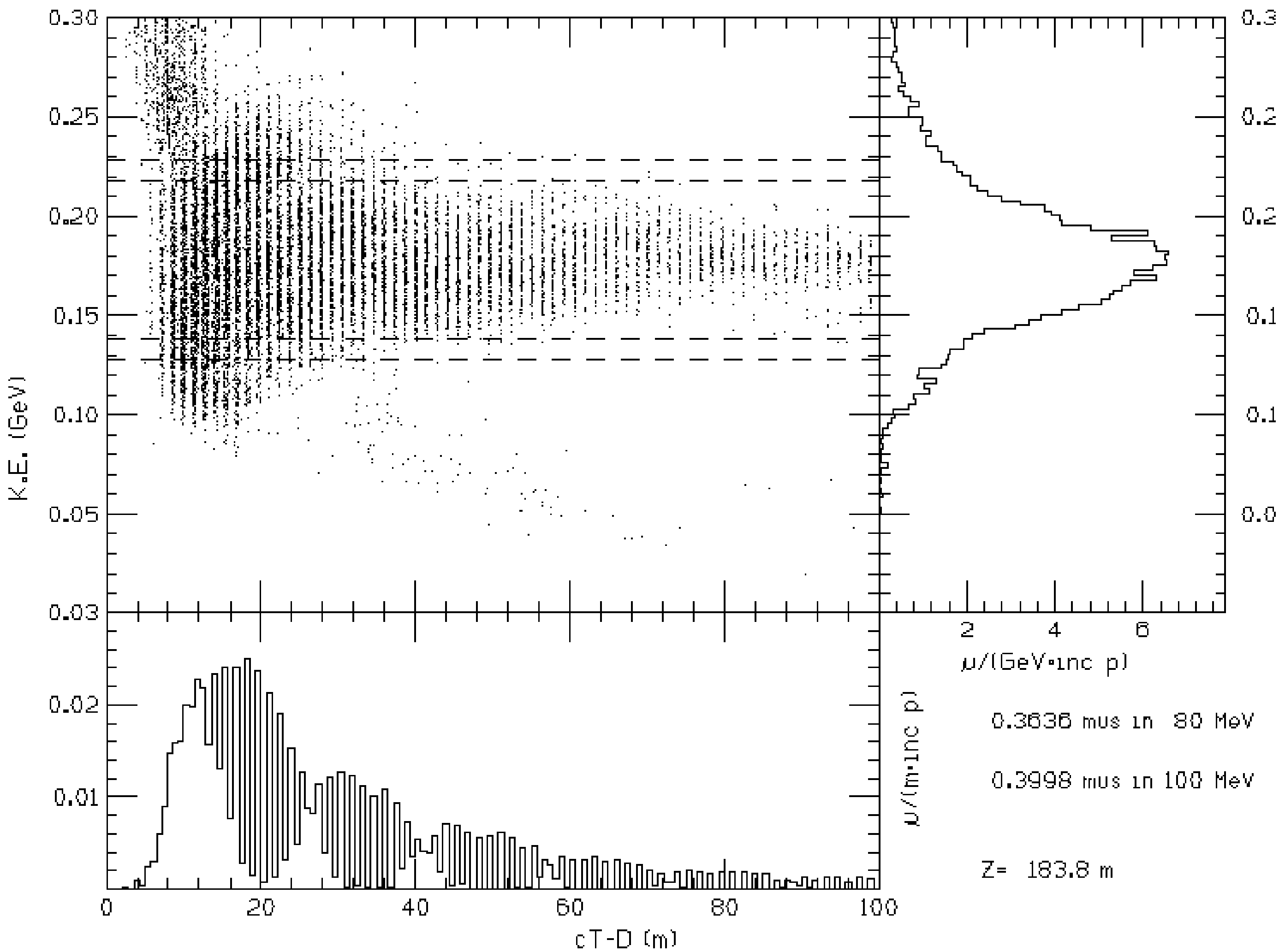


Figure 7: Muon distribution in (E,t)-space along with marginal distributions for 38 vernier (d=0.16) cavities followed by 23 (matched) fixed frequency cavities *generated with* ICOOL *program.* N_b=20 in buncher part. Plots and numbers quoted are based on 188 000 incident protons.

Compare with conventional

1. Inevitably Distorting
2. Probably less efficient for one sign
3. But both signs rotated
4. Much less cost than induction

3 Transverse Cooling

3.1 Recap Beam Definitions

3.1.1 Emittance

$$\text{normalized emittance} \;=\; \frac{\text{Phase Space Area}}{\text{m c}}$$

If x and p_x both Gaussian and uncorrelated, then area is that of an upright ellipse

$$\epsilon_\perp = \frac{\sigma_{p_\perp}\sigma_x}{mc} = \sigma_\theta \sigma_x \ (\gamma\beta_v) \qquad\qquad (\pi\, m\, rad)$$

$$\epsilon_\parallel = \frac{\sigma_{p_\parallel}\sigma_z}{mc} = \frac{\sigma_p}{p}\sigma_z \ (\gamma\beta_v) \qquad\qquad (\pi\, m\, rad)$$

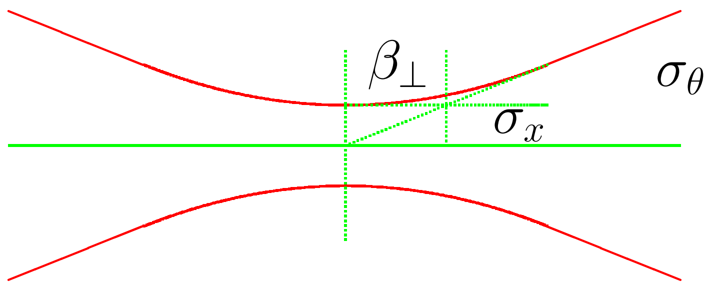
$$\epsilon_6 = \epsilon_\perp^2 \ \ \epsilon_\parallel \qquad\qquad (\pi\ m)^3$$

Note that, by convention, the π is not included in the calculated values, but added to the dimension

3.1.2 $\mathbf{Beta}_{Courant-Schneider}$

Again upright ellipse, e.g. at Focus:

$$\beta_\perp = \frac{\sigma_x}{\sigma_\theta}$$



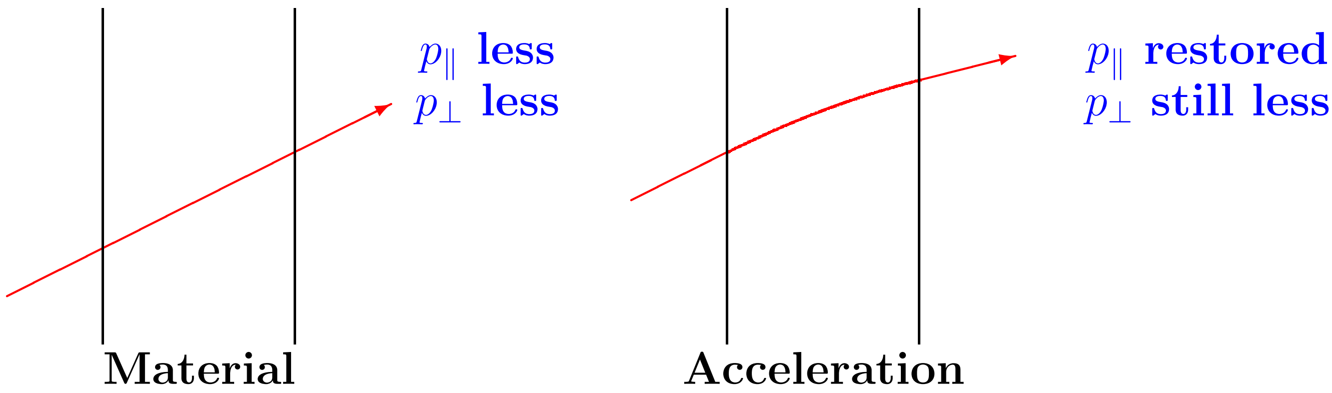
Then, using emittance definition:

$$\sigma_x = \sqrt{\epsilon_\perp \, \beta_\perp \, \frac{1}{\beta_v \gamma}}$$

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp} \, \frac{1}{\beta_v \gamma}}$$

$\beta_\perp$ is defined by the beam, but a lattice can have a β_o that "matches" a beam with that $\beta_\perp$

3.2 Transverse Cooling



3.2.1 Cooling rate vs. Energy

$$\epsilon_{x,y} \;=\; \gamma\beta_v \; \sigma_\theta \; \sigma_{x,y} \tag{7}$$

If there is no Coulomb scattering, or other sources of emittance heating, then σ_θ and $\sigma_{x,y}$ are unchanged by energy loss, but p and thus $\beta\gamma$ is reduced. So the fractional cooling $d\epsilon\ /\epsilon$ is:

$$\frac{d\epsilon}{\epsilon} \;=\; \frac{dp}{p} \;=\; \frac{dE}{E}\,\frac{1}{\beta_v^2} \tag{8}$$

which, for a given energy change, strongly favors cooling at low energy.

But if total acceleration were not important, e.g. if the cooling is done in a ring, then there is another criterion: The cooling per fractional loss of particles by decay:

$$Q \;=\; \frac{d\epsilon/\epsilon}{dn/n} \;=\; \frac{dp/p}{d\ell/c\beta_v\gamma\tau}$$

$$=\; \frac{dE/E \;\; 1/\beta_v^2}{d\ell/(c\gamma\beta_v\tau)}$$

$$=\; (c\tau/m_\mu) \;\; \frac{dE}{d\ell} \;\; \frac{1}{\beta_v}$$

Which only mildly favours low energy

3.2.2 Heating Terms

$$\epsilon_{x,y} = \gamma\beta_v \, \sigma_\theta \, \sigma_{x,y} \tag{9}$$

Between scatters the drift conserves emittance (Liouiville).

When there is scattering, $\sigma_{x,y}$ is conserved, but σ_θ is increased.

$$\begin{aligned}
\Delta(\epsilon_{x,y})^2 &= \gamma^2\beta_v^2 \, \sigma_{x,y}^2 \Delta(\sigma_\theta^2) \\
2\epsilon \, \Delta\epsilon &= \gamma^2\beta_v^2 \left(\frac{\epsilon\beta_\perp}{\gamma\beta_v}\right) \Delta(\sigma_\theta^2) \\
\Delta\epsilon &= \frac{\beta_\perp\gamma\beta_v}{2} \, \Delta(\sigma_\theta^2)
\end{aligned}$$

e.g. from Particle data booklet

$$\Delta(\sigma_\theta^2) \approx \left(\frac{14.1 \; 10^6}{(p)\beta_v}\right)^2 \frac{\Delta s}{L_R}$$

$$\Delta\epsilon = \frac{\beta_\perp}{\gamma\beta_v^3} \, dE \left(\left(\frac{14.1 \; 10^6}{2(m_\mu)}\right)^2 \frac{1}{L_R \Delta E/ds}\right)$$

Defining

$$C(mat,E) \;=\; \frac{1}{2}\;\left(\frac{14.1\;10^6}{(m_\mu)}\right)^2\;\frac{1}{L_R\;d\gamma/ds} \tag{10}$$

then

$$\frac{\Delta\epsilon}{\epsilon} \;=\; dE\;\;\frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\;\;C(mat,E) \tag{11}$$

Equating this with equation 8

$$dE\;\frac{1}{\beta_v^2\;E} \;=\; dE\;\;\frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\;\;C(mat,E)$$

gives the equilibrium emittance ϵ_o:

$$\epsilon_{x,y}(min) \;\;=\;\; \frac{\beta_\perp}{\beta_v}\;\;C(mat,E) \tag{12}$$

Rate of Cooling

$$\frac{d\epsilon}{\epsilon} \;=\; \left(1\;-\;\frac{\epsilon}{\epsilon(min.)}\right)\;\frac{dp}{p} \tag{13}$$

At energies such as to give minimum ionization loss, the constant C_o for various materials are approximately:

material	T ^{o}K	density kg/m^3	dE/dx MeV/m	L_R m	C_o 10^{-4}
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

Clearly Liquid Hydrogen is far the best material, but has cryogenic and safety complications, and requires windows made of Aluminum or other material which will significantly degrade the performance.

3.2.3 Beam Divergence Angles

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp\ \beta_v \gamma}}$$

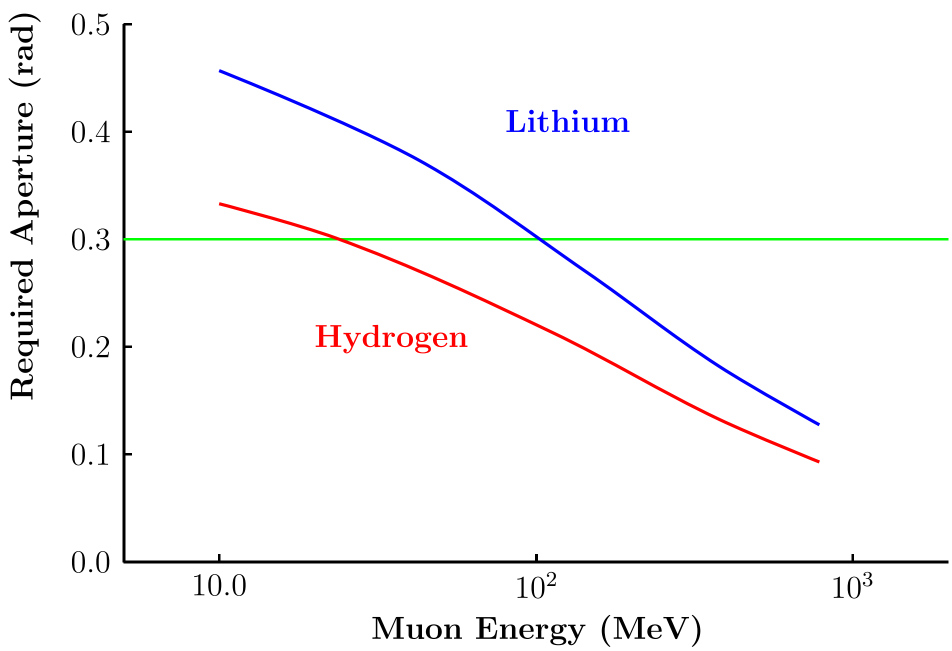
so, from equation 12, for a beam in equilibrium

$$\sigma_\theta = \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}}$$

and for 50 % of maximum cooling and an aperture at 3 σ, the aperture $\mathcal{A}$ of the system must be

$$\mathcal{A} = 3\sqrt{2}\sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}} \qquad (14)$$

Apertures for hydrogen and lithium are plotted vs. energy below. These are very large angles, and if we limit apertures to less than 0.3, then this requirement sets lower energy limits of about 100 MeV ($\approx$ 170 MeV/c) for Lithium, and about 25 MeV ($\approx$ 75 MeV/c) for hydrogen.



3.3 Focusing Systems

3.3.1 Solenoid

∗∗∗

In a solenoid with axial field B_{sol}

$$\beta_\perp \;=\; \frac{2\ (p)}{c\ B_{sol}}$$

so

$$\epsilon_{x,y}(min) \;=\; C(mat, E)\ \frac{2\ \gamma\ (m_\mu)}{B_{sol}\ c} \tag{15}$$

For $E = 100\ MeV$ $(p \approx 170\ MeV/c)$, $B = 20\ T$, then $\beta \approx 5.7\ cm.$ and

$\epsilon_{x,y} \approx 266(\pi mm\ mrad)$.

3.3.2 Current Carrying Rod

In a rod carrying a uniform axial current, the azimuthal magnetic field B varies linearly with the radius r.

A muon traveling down it:

$$\frac{d^2r}{dr^2} = \frac{B\ c}{(p)} = \frac{r\ c}{(p)}\frac{dB}{dr}$$

so orbits oscillate with

$$\beta_\perp^2 = \frac{\gamma\beta_v}{dB/dr}\frac{(m_\mu)}{c} \tag{16}$$

If we set the rod radius a to be f_{ap} times the rms beam size $\sigma_{x,y}$,

$$\sigma_{x,y} = \sqrt{\frac{\epsilon_{x,y}\ \beta_\perp}{\beta_v\gamma}}$$

and if the field at the surface is B_{max}, then

$$\beta_\perp^2 = \frac{\gamma\beta_v(m_\mu)\ f_{ap}}{B_{max}\ c}\sqrt{\frac{\epsilon_{x,y}\ \beta}{\gamma\ \beta_v}}$$

from which we get:

$$\beta_\perp \;=\; \left(\frac{f_{ap}\ (m_\mu)}{B_{max}\ c}\right)^{2/3}\ (\gamma\beta_v\ \epsilon_{x,y})^{1/3}$$

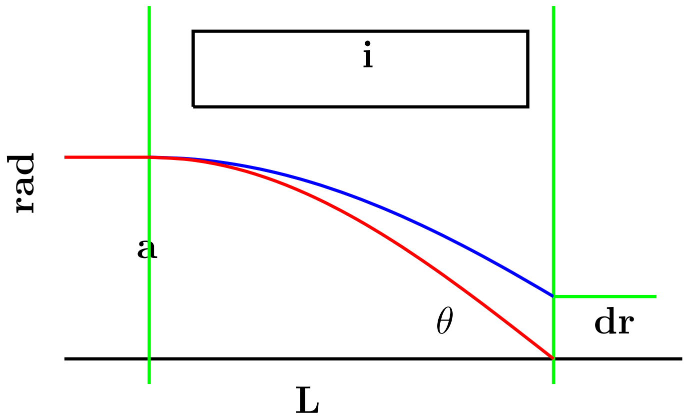
puting this in equation 12

$$\epsilon_{x,y}(min) \;=\; (C(mat,E))^{1.5}\left(\frac{f_{ap}\ (m_\mu)}{B_{max}\ c\ \beta_v}\right)\ \sqrt{\gamma} \tag{17}$$

e.g. B_{max}=10 T, f_{ap}=3, E=100 MeV, then $\beta_\perp \;=\; 1.23$ cm, and

$\epsilon(min)$=100 (π mm mrad)

3.3.3 At a Focus



The minimum beta obtainable at a focus is set by chromatic abberations, i.e. momentum dependent effects. Assuming no external correction:

$$\beta(min) \ = \ \frac{dr}{\theta} \ = \ \frac{a}{\theta}\, dp/p \ = \ L\ dp/p$$

For a solenoid with axial filed B, and momentum p

$$L \ = \ \frac{\pi}{2}\, \beta_o \ = \ \frac{\pi\ (p)}{c\ B}$$

so

$$\beta(min) \;=\; \left(\frac{\pi\ (p)}{c\ B}\right)\ \frac{dp}{p}$$

$$\epsilon(min) \;=\; C_{H_2}\ \left(\frac{\pi(E)}{c\ B}\right)\ \frac{dp}{p}$$

e.g. p=.17 MeV, B=5 T, dp/p=5%, $\beta(min)$ = 1.8 cm, and

$\epsilon(min) = 82\ \pi$ mm mrad

But as p falls, the possible coil thickness also falls. Below some mom we may have to fix the current density i:

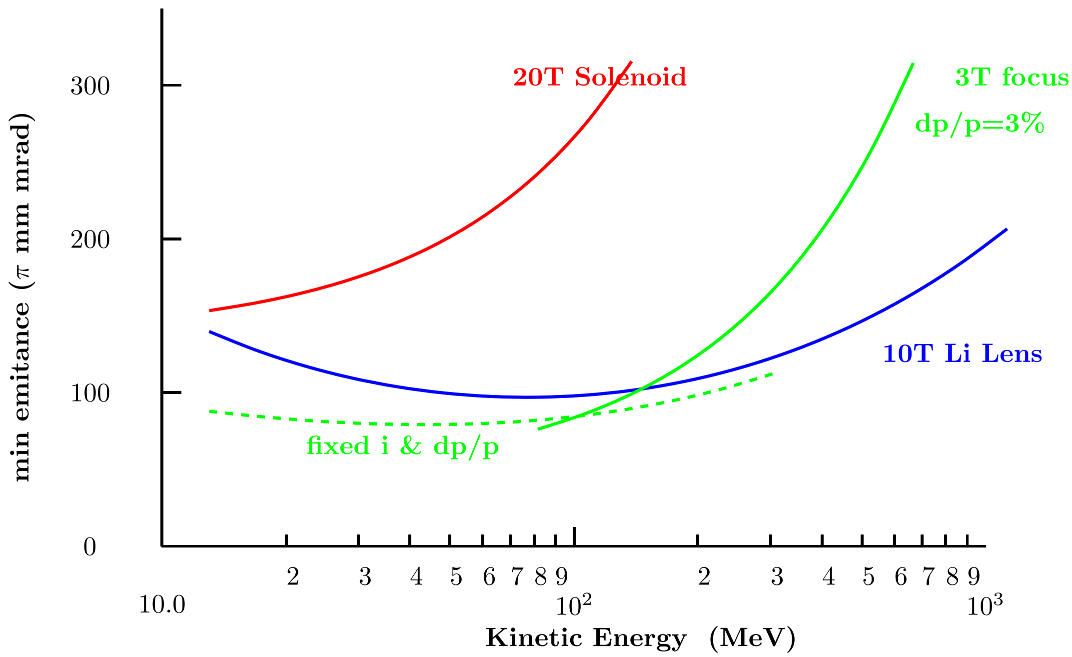
$$B \;\propto\; \frac{p}{B}$$

and so

$$\beta(min) \;\propto\; \sqrt{\frac{\gamma}{\beta}}$$

3.3.4 Compare Focusing

Assuming that the current limits the focus beta below 100 MeV, then we can compare the methods as a function of the beam kinetic energy.



We see that, for the parameters selected, no method allows transverse cooling below about 80 (π mm mrad)

3.4 Simulation

- Calculations assume Gaussian scatter and straggling, and small angles, and thus approximate.
- Accurate results require simulation
- Several "local" codes
  Two Documented codes: GEANT & ICOOL
  Both have:
  - Choices of scattering and straggling formulations
  - Standing Wave RF fields
  - allow use of both
    1. Maxwellian, or
    2. "hard edged" magnetic fields
  - flexible Geometries
  - Good tracking

The differences in handling bends discussed in section 4.3

3.5 Angular Momentum Problem

or: Why we reverse the Solenoids

In the absence of external fields and energy loss in materials, the angular momentum of a particle is conserved.

But a particle entering a solenoidal field will cross radial field components and its angular ($r\ p_\phi$) momentum will change.

$$\Delta(p_\phi) \ = \ \Delta\left(\frac{c\ B_z\ r}{2}\right)$$

If in the absence of the field the particle had "canonical" angular momentum $(p_\phi\ r)_{\rm can}$, then in the field it will have angular momentum:

$$p_\phi\ r \ = \ (p_\phi\ r)_{\rm can} \ + \ \left(\frac{c\ B_z\ r}{2}\right)\ r$$

so

$$(p_\phi\ r)_{\rm can} \ = \ p_\phi\ r \ - \ \left(\frac{c\ B_z\ r}{2}\right)\ r$$

If the initial canonical angular momentum is zero, then in B_z:

$$p_\phi \, r \; = \; \left(\frac{c \; B_z \; r}{2}\right) \; r$$

Material will reduce all momenta, both longitudinal and transverse.

Re-acceleration will not change the angular momenta.

The angular momentum will continuously fall.

Consider the case of almost complete transverse cooling: all transverse momenta are reduced to near zero leaving the beam streaming parallel to the axis.

$$p_\phi \, r \; \approx \; 0$$

and

$$(p_\phi \, r)_{\rm can} \; = \; p_\phi \, r - \left(\frac{c \; B_z \; r}{2}\right) \; r \; = \; -\left(\frac{c \; B_z \; r}{2}\right) \; r$$

When the beam exits the solenoid, then this canonical angular momentum becomes a real angular momentum and represents an effective emittance, and severely limits the possible cooling.

$$p_\phi\, r(end) \ = \ -\left(\frac{c\ B_z\ r}{2}\right)\ r$$

The only reasonable solution is to reverse the field, either once, a few, or many times.

3.5.1 Single Field Reversal Method

The minimum required number of field "flips" is one.

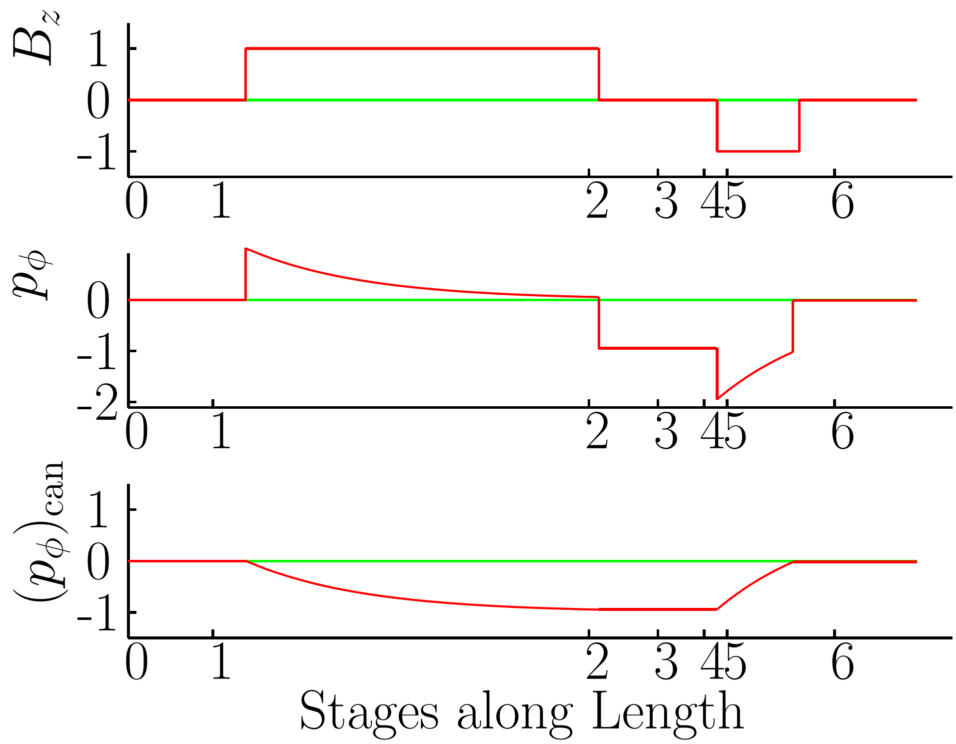


Figure: Axial Field, Angular Momentum, and Canonical Angular Momentum, in an Ideal, Single Field Reversal, Solenoid Cooling System.

After exiting the first solenoid, we have real coherent angular momentum:

$$(p_\phi \, r)_3 \; = \; - \left(\frac{c \; B_{z1} \; r}{2} \right) \; r$$

The beam now enters a solenoid with opposite field $B_{z2} = -B_{z1}$.

The canonical angular momentum remains the same, but the real angular momentum is doubled.

$$(p_\phi \, r)_4 \; = \; -2 \left(\frac{c \; B_{z1} \; r}{2} \right) \; r$$

We now introduce enough material to halve the transverse field components. Then

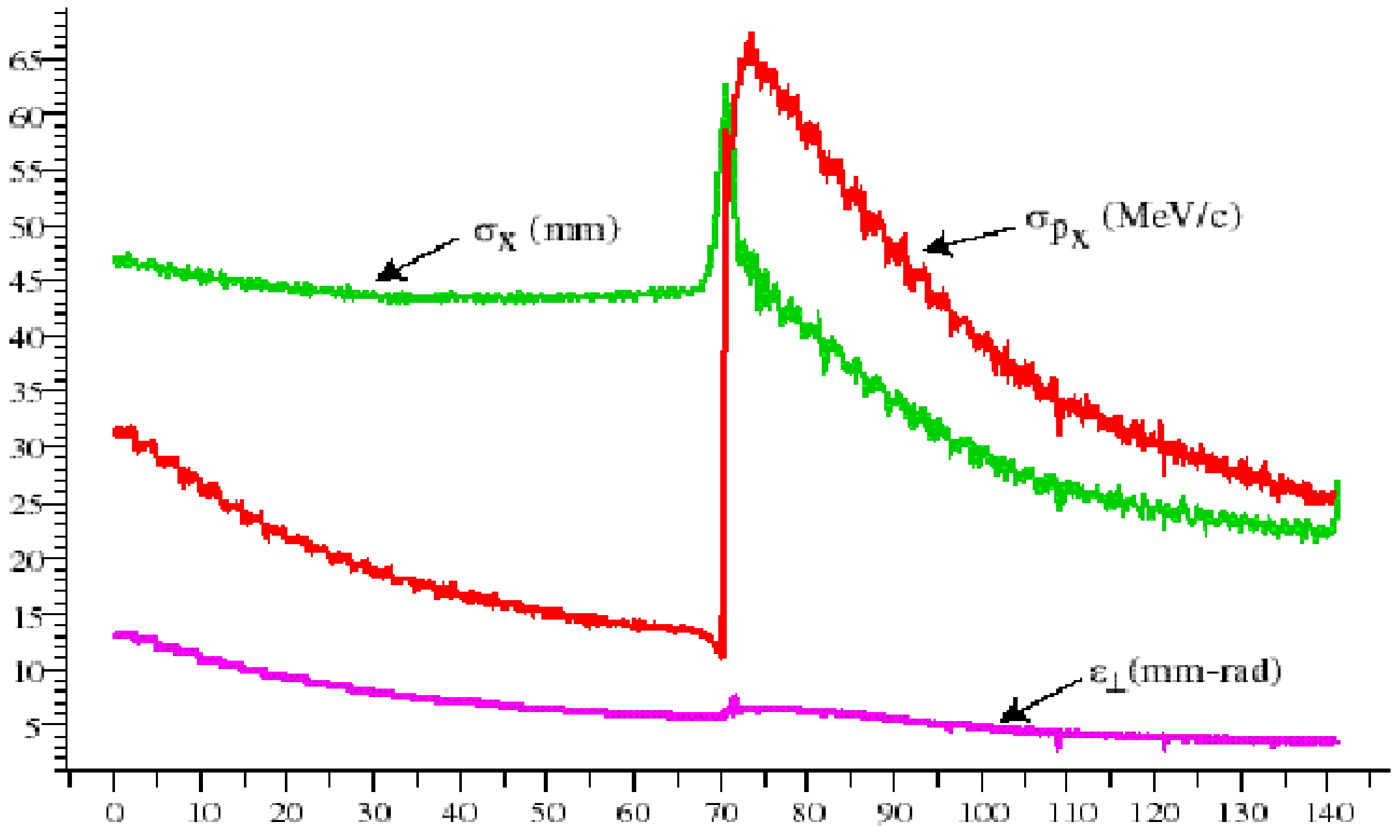
$$(p_\phi \, r)_5 \; = \; - \left(\frac{c \; B_{z1} \; r}{2} \right) \; r$$

This is inside the field $B_{z2} \; = \; -B_{z1}$. The canonical momentum, and thus the angular momentum on exiting, is now:

$$(p_\phi \, r)_6 \; = \; - \left(\frac{c \; B_{z1} \; r}{2} \right) \; r - - \left(\frac{c \; B_{z1} \; r}{2} \right) \; r \; = \; 0$$

3.5.2 Example of "Single Flip"

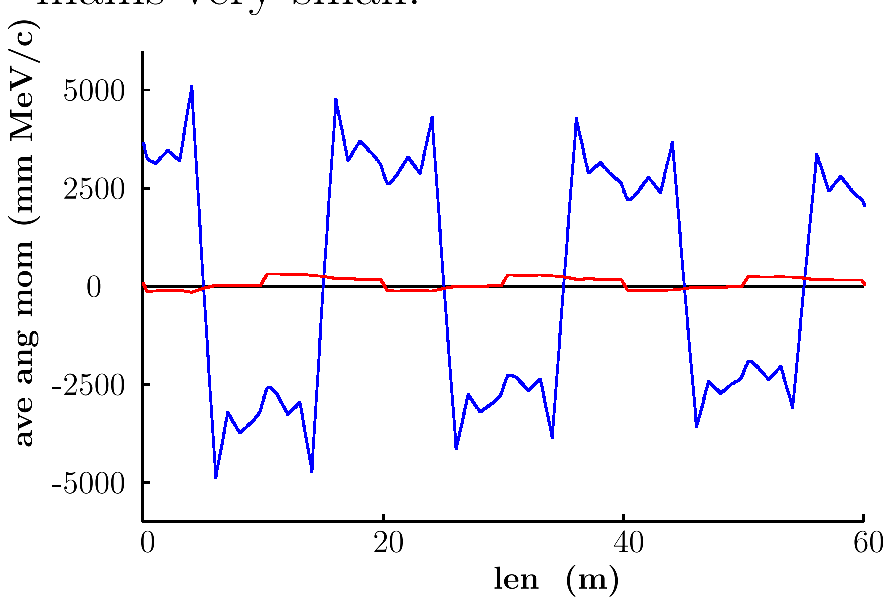
From "single flip alternative" in US Study 2



3.5.3 Alternating Solenoid Method

If we reverse the field frequently enough, no significant canonical angular momentum is developed.

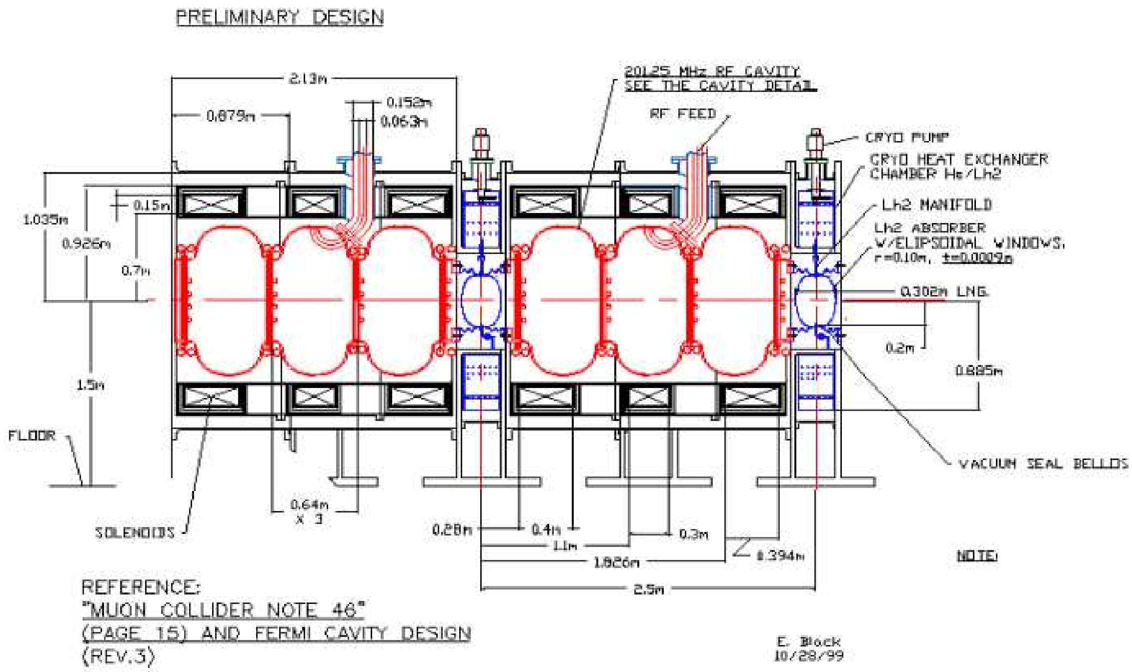
The Figure below shows the angular momenta and canonical angular momenta in a simulation of an "alternating solenoid" cooling lattice. It is seen that while the coherent angular momenta are large, the canonical angular momentum (in red) remains very small.



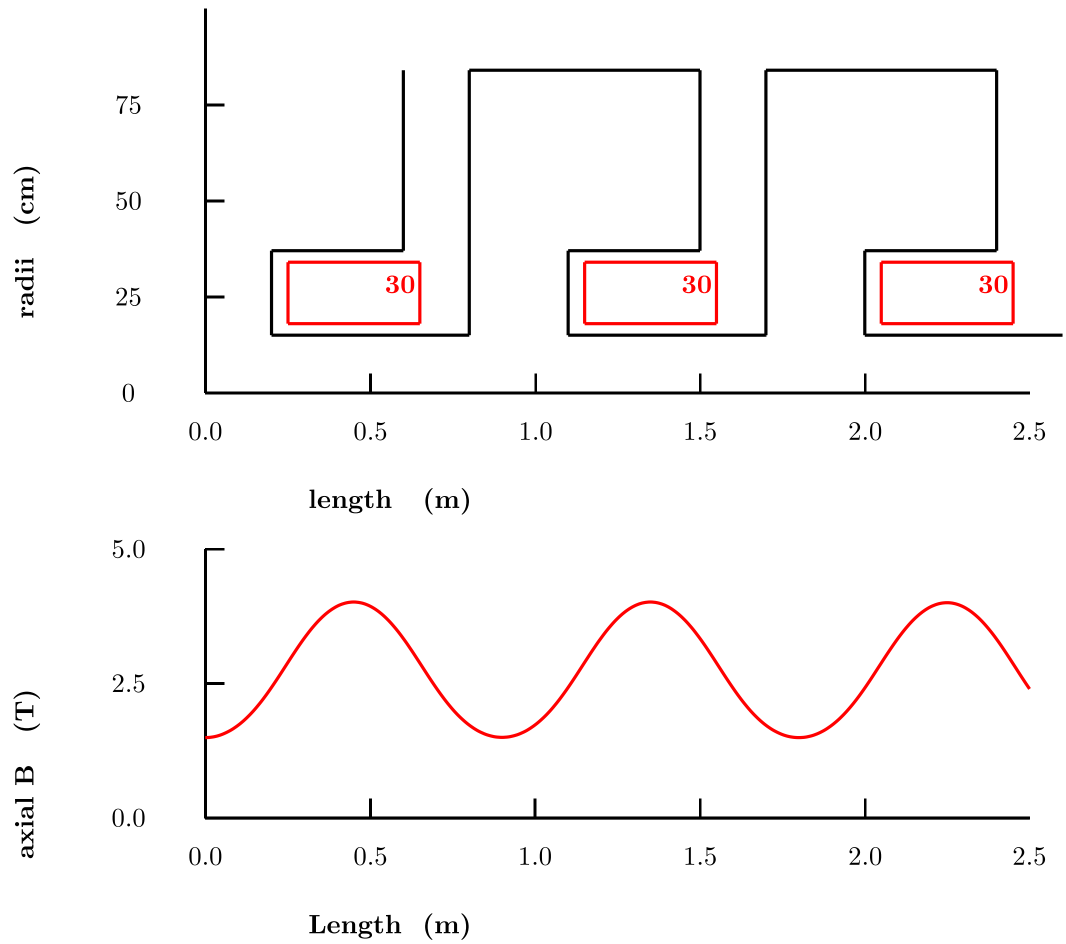
3.6 Focussing Lattice Design

3.6.1 Solenoids with few "flips"

- Coils Outside RF: e.g. FNAL 1 flip



- Coils interleeved: e.g. CERN

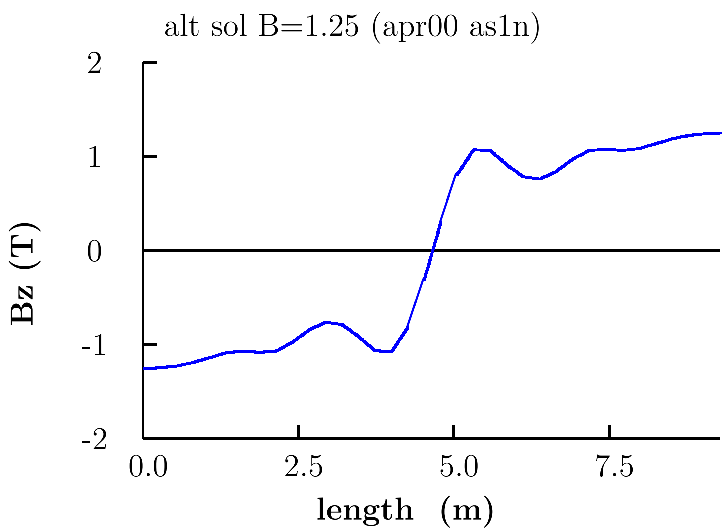


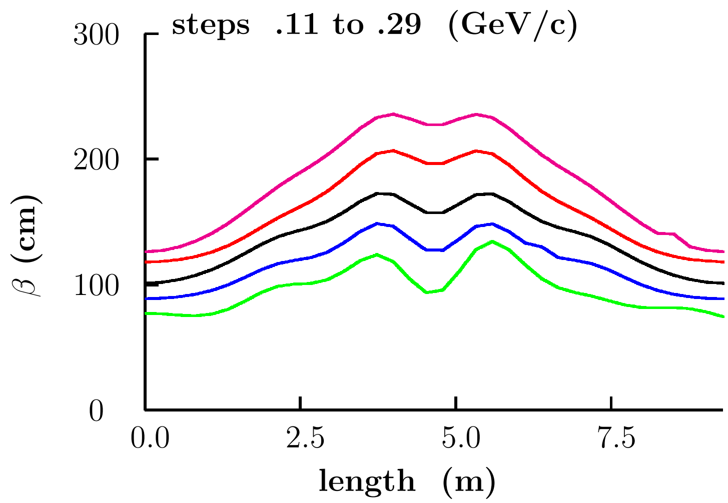
Note: Field is far from uniform and must be treated as a lattice.

"Flips"

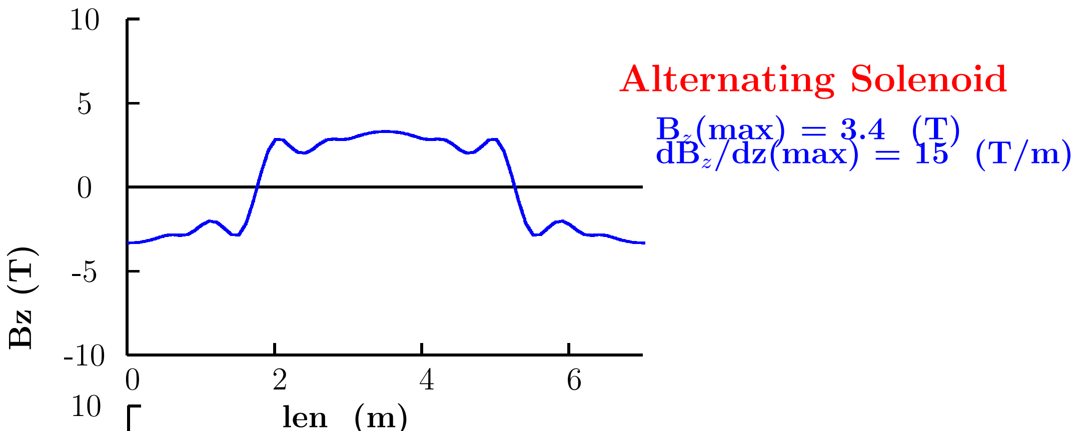
One must design the flips to match the betas from one side to the other.

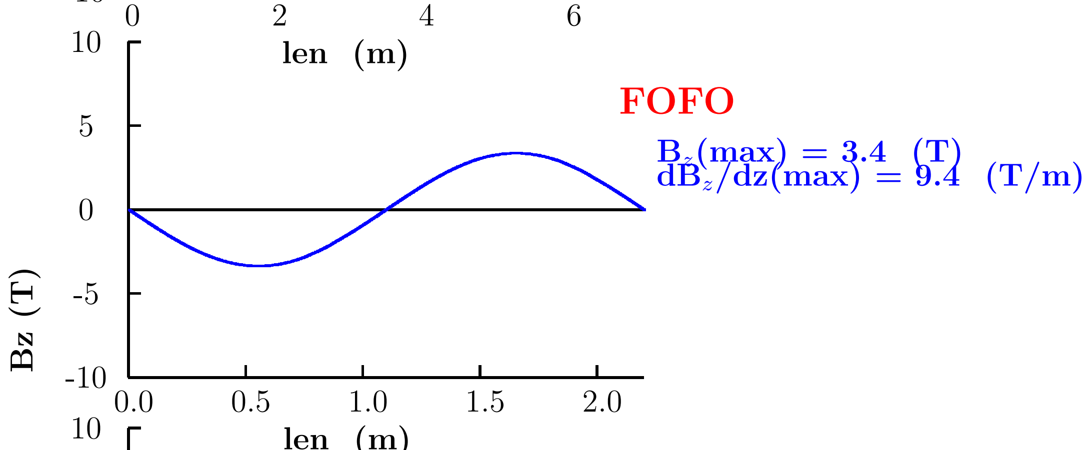
For a computer matched flip, the following figure shows B_z vs. z and the $\beta_\perp$'s vs. z for different momenta.

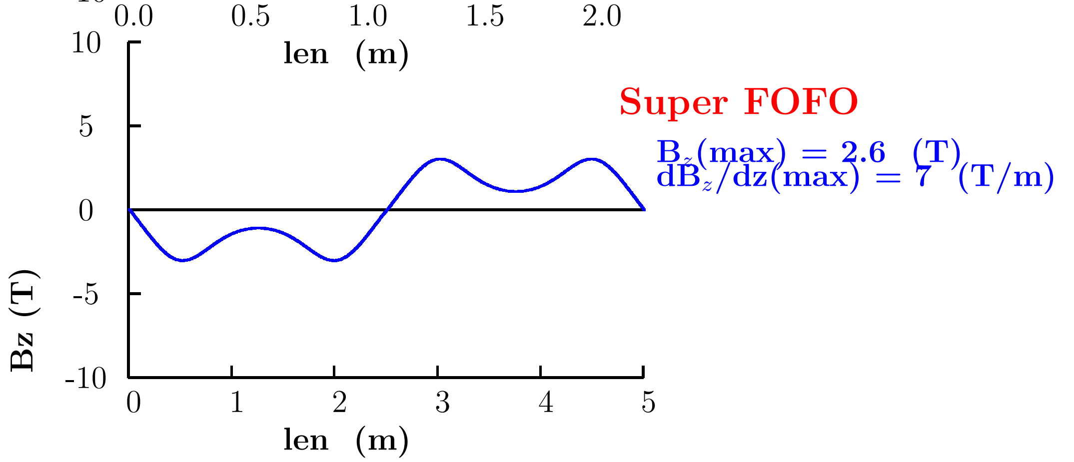




3.6.2 Lattices with many "flips"



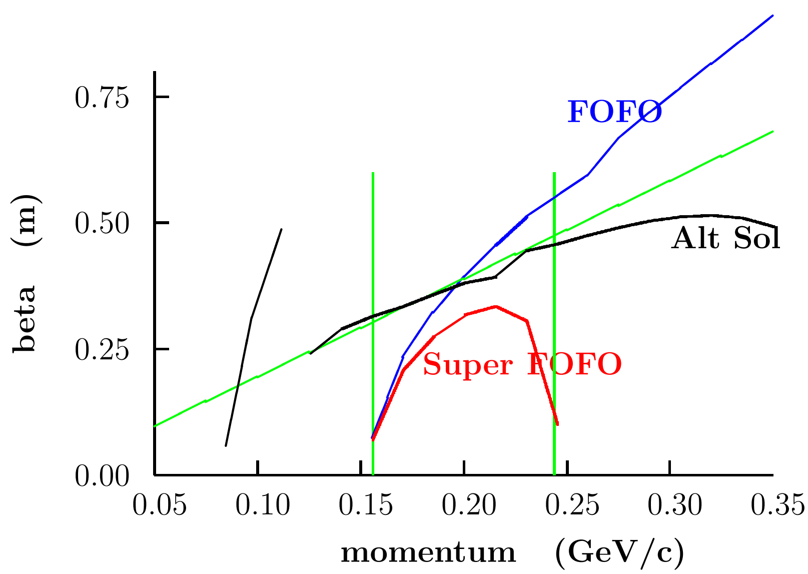




Determination of lattice betas

- Track single near paraxial particle through many cells
- plot θ_x vs x after each cell
- fit ellipse: $\beta_{x,y}$ = A((x) / A(θ_x)

beta vs. Momentum

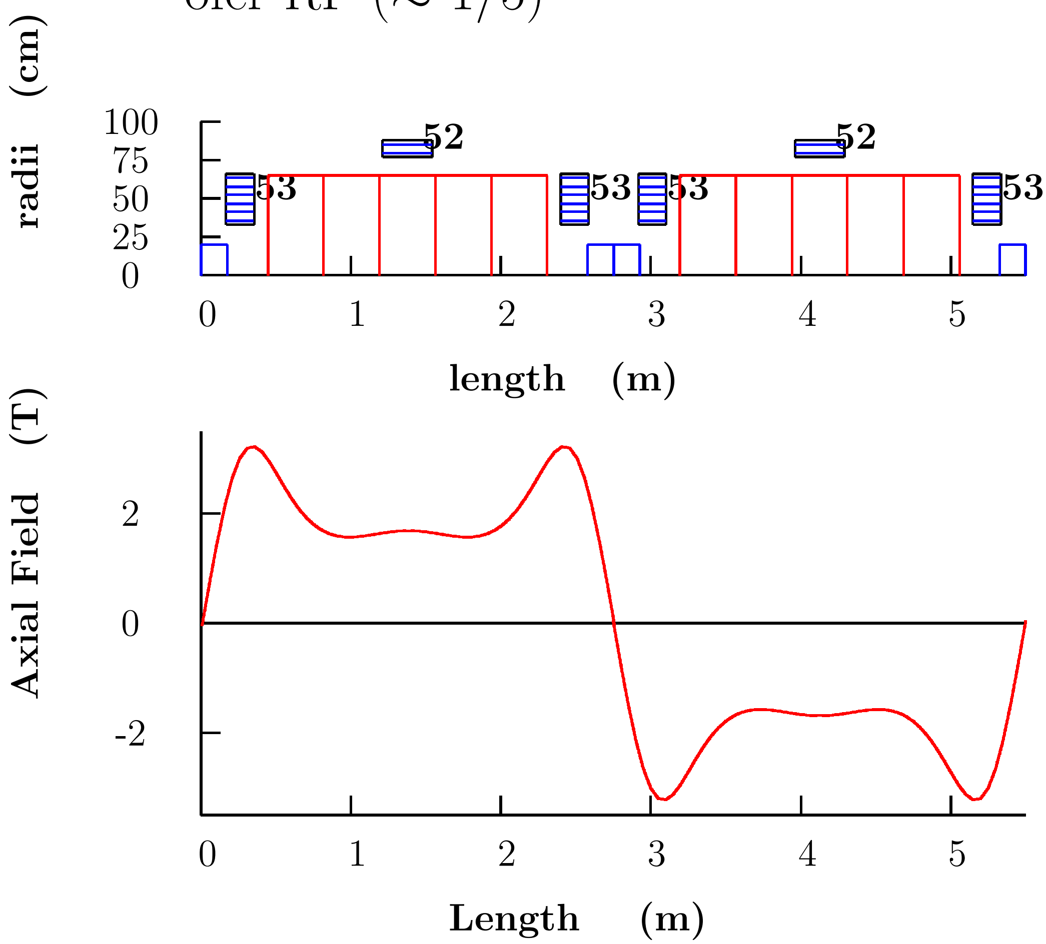


- Solenoid has largest p acceptance
- FOFO shows $\beta \ \propto \ dp/p$
- SFOFO more complicated, and better

3.6.3 Example of Multi-flip lattice

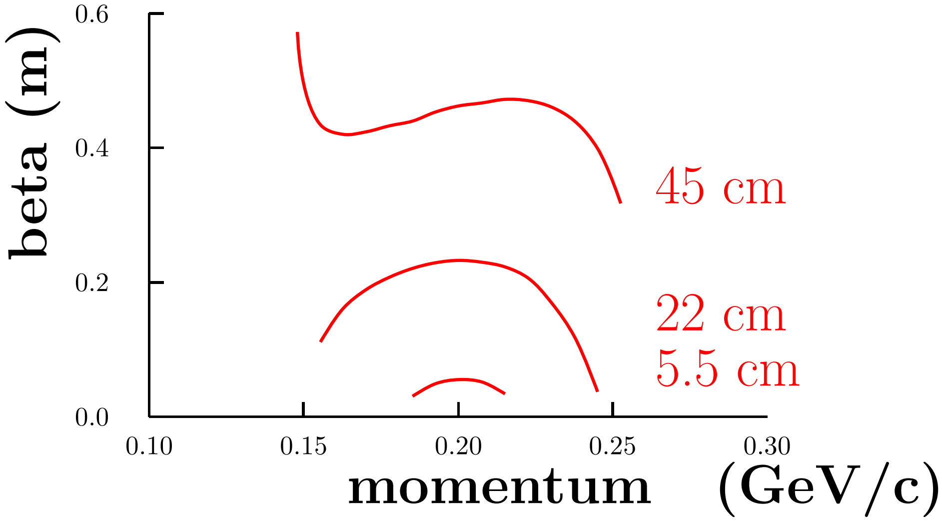
US Study 2 Super FOFO

Smaller Stored E than continuous solenoid ofer RF ($\approx 1/5$)



Adjusting Currents adjusts $\beta_\perp$'s

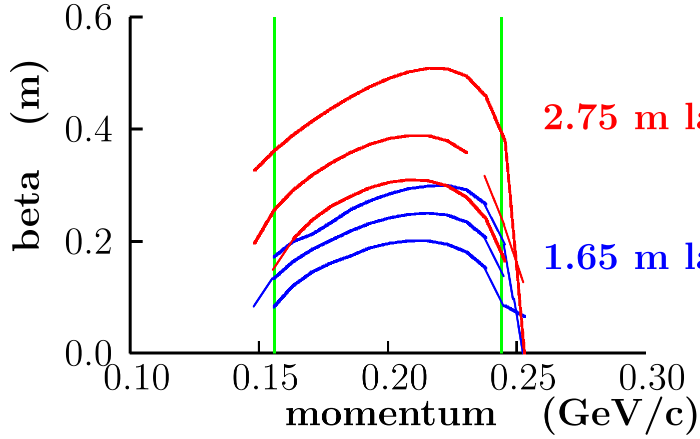
But mom acceptance falls with $\beta_\perp$

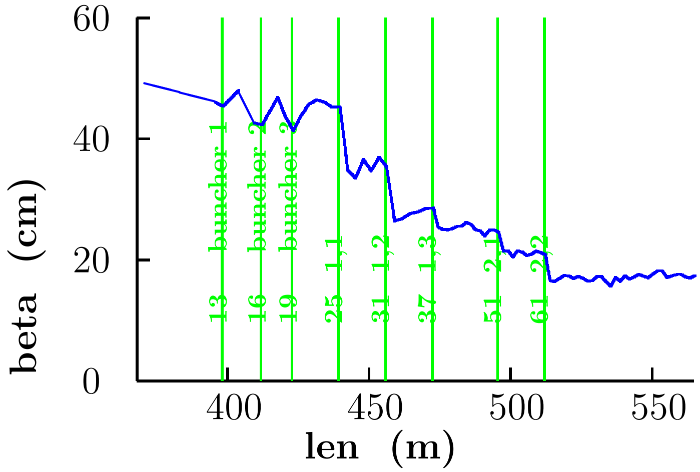


This allows:

3.6.4 Tapering the Cooling Lattice

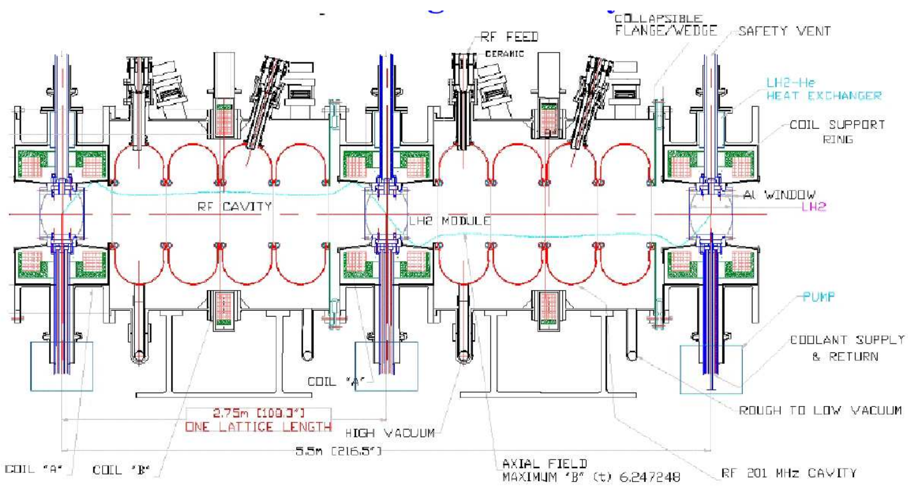
- as emittance falls, lower betas
- maintain constant angular beam size
- maximizes cooling rate
- Adjust current, then lattice



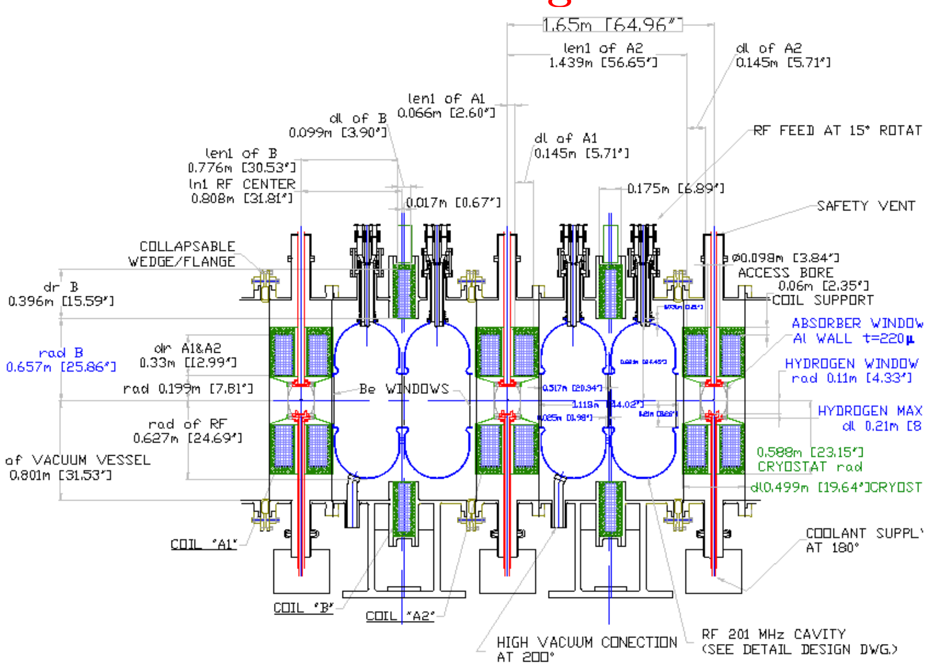


3.6.5 Hardware

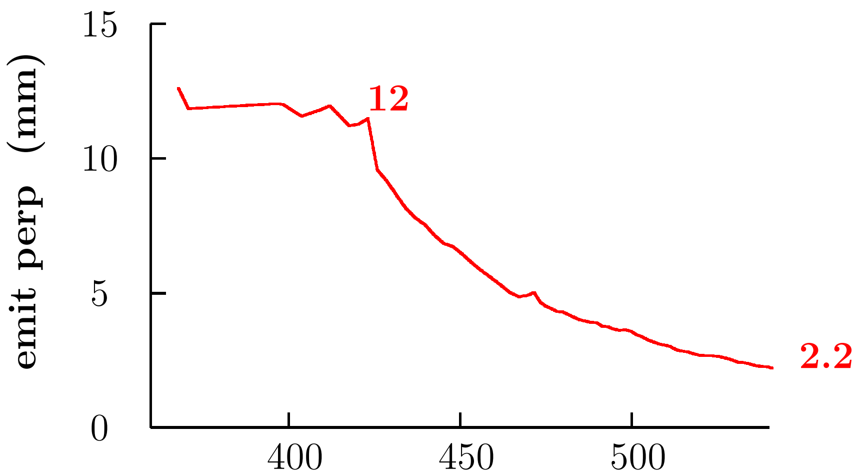
At Start of Cooling



At end of Cooling



3.6.6 Study 2 Performance



With RF and Hydrogen Windows, $C_o \approx 45\ 10^{-4}$
$\beta_\perp$(end)=.18 m, β_v(end)=0.85, So

$$\epsilon_\perp(\mathrm{min}) = \frac{45\ 10^{-4}\ 0.18}{0.85} = 0.95\ (\pi mm\ mrad)$$

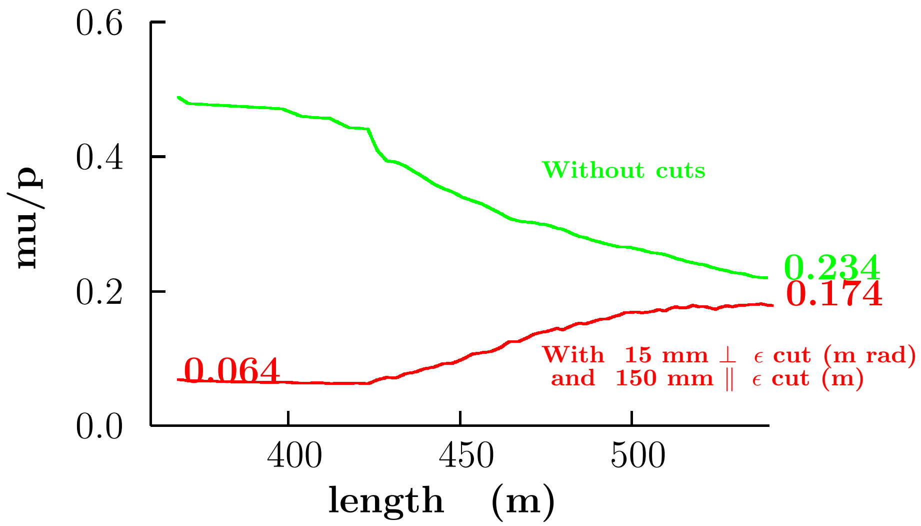
$$\frac{\epsilon_\perp}{\epsilon_\perp(\mathrm{min})} \approx 2.3$$

so from eq. 13

$$\frac{d\epsilon}{\epsilon}(\mathrm{end}) = \left(1 - \frac{\epsilon}{\epsilon(\mathrm{min})}\right) \frac{dp}{p} \approx 0.57 \frac{dp}{p}$$

- A lower emittance would req. >> length

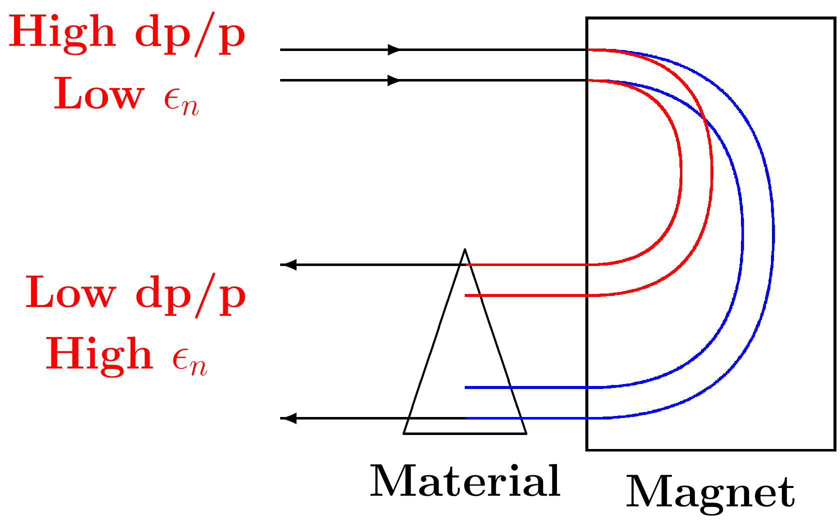
Muons accepted by Acceleration



- Gain Factor = 3
- No Further gain from length
- Loss from growth of long emit.
- Avoided if longitudinal cooling

4 Longitudinal Cooling

4.1 Introduction



- dp/p reduced
- But σ_y increased

- Long Emittance reduced
- Trans Emittance Increased

- "Emittance Exchange"

4.2 Partition Functions

Following the convention for synchrotron cooling we define partition functions:

$$J_{x,y,z} \;=\; -\,\frac{\frac{\Delta\;(\epsilon_{x,y,z})}{\epsilon_{x,y,z}}}{\frac{\Delta p}{p}} \tag{18}$$

$$J_6 = J_x + J_y + J_z \tag{19}$$

where the $\Delta\epsilon$'s are those induced directly by the energy loss mechanism (ionization energy loss in this case). Δp and p refer to the loss of momentum induced by this energy loss.

In the synchrotron case, in the absence of gradients fields, $J_x = J_y = 1$, and $J_z = 2$.

In the ionization case, as we shall show, $J_x = J_y = 1$, but J_z is negative or small.

4.2.1 Transverse

From last lecture:

$$\frac{\Delta\sigma_p}{\sigma_p} = \frac{\Delta p}{p}$$

and $\sigma_{x,y}$ does not change, so

$$\frac{\Delta\epsilon_{x,y}}{\epsilon_{x,y}} = \frac{\Delta p}{p} \tag{20}$$

and thus

$$J_x = J_y = 1 \tag{21}$$

4.2.2 Longitudinal

The emittance in the longitudinal direction ϵ_z is:

$$\epsilon_z = \gamma\beta_v \frac{\sigma_p}{p} \sigma_z = \frac{\sigma_p \ \sigma_z}{m_\mu} = \frac{c \ \sigma_E \ \sigma_t}{m_\mu}$$

where σ_t is the rms bunch length in time, and c is the velocity of light. σ_t will not change as the beam passes though material.

The relative change in the rms energy spread σ_γ will be given by

$$\frac{\Delta\sigma_\gamma}{\sigma_\gamma} = - \frac{\delta(d\gamma/ds)}{\delta\gamma} \Delta s$$

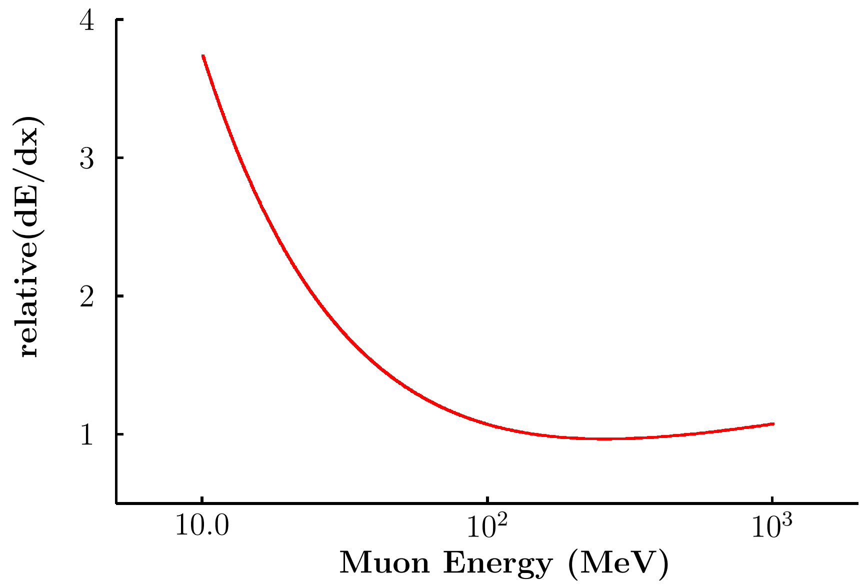
so

$$\Delta\epsilon_z = - \frac{\delta(d\gamma/ds)}{\delta\gamma} \sigma_\gamma \sigma_t c \Delta s$$

From the definition of the partition function J_z:

$$J_z = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\gamma}{\beta_v^2\gamma}} = - \frac{\frac{\delta(d\gamma/ds)}{\delta\gamma}}{d\gamma/ds} \beta_v^2\gamma \tag{22}$$

Energy Loss



A typical relative energy loss as a function of energy is shown above (this example is for Lithium). It has a minimum at about 300 MeV, a gentle rise above and a steep rise at lower energies. It is given approximately by:

$$\frac{d\gamma}{ds} = B\,\frac{1}{\beta_v^2}\,(\frac{1}{2}\,ln(A\;\beta_v^4\gamma^4\;-\;\beta_v^2) \quad (23)$$

where

$$A \;=\; \frac{(2m_e c^2/e)^2}{I^2} \qquad (24)$$

$$B \approx \frac{0.0307}{(m_\mu c^2/e)}\,\frac{Z}{A} \qquad (25)$$

where Z and A are for the nucleus of the material, and I is the ionization potential for that material.

Differentiating the above:

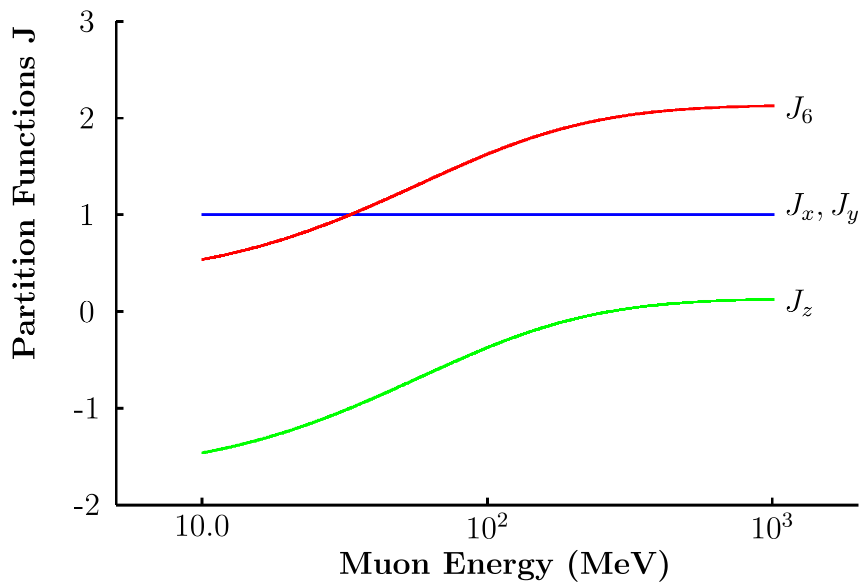
$$\frac{\delta(d\gamma/ds)}{\delta\gamma} \;=\; \frac{B}{\beta_v}\left(\frac{2}{\beta_v\gamma} \;-\; \frac{1}{(\beta_v\gamma)^3}\,ln(A\;\beta_v^4\gamma^4) \;+\; \frac{2}{(\beta_v\gamma)^3}\right)$$

Substituting this into equation 22:

$$J_z \;\approx\; -\frac{\left(\frac{2}{\beta_v\gamma} \;-\; \frac{1}{(\beta_v\gamma)^3}\;ln(A\;\beta_v^4\gamma^4) \;+\; \frac{2}{(\beta_v\gamma)^3}\right)}{\left(\frac{1}{2}\;ln(A\;\beta_v^4\gamma^4 \;-\; \beta_v^2\right)}\,\beta_v^3\gamma \qquad (26)$$

4.2.3 6D Partition Function J$_6$

J_z, $J_{x,y}$ and $J_6 \; = \; J_x + J_y + J_z$ are plotted below



It is seen that despite the heating implicit in the negative values of J_z at low energies, the six dimensional cooling J_6 remains positive. In fact the relative cooling for a given acceleration ΔE :

$$\frac{\Delta \epsilon_6/\epsilon}{\Delta E} \; = \; \frac{J_6}{E \; \beta_v^2}$$

rises without limit as the energy falls. This suggests that, for economy of acceleration, cooling should be done at a very low energy. In practice there are many difficulties in doing this, but it remains desirable to use the lowest practical energy.

4.2.4 Longitudinal Heating Terms

and from Perkins text book, converted to MKS:

$$\Delta(\sigma_\gamma^2) \approx 0.06\, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho\, \Delta s \;=\; 2\sigma_\gamma\, \Delta\sigma_\gamma$$

$$\epsilon_z \;=\; \sigma_\gamma\, \sigma_t\, c$$

Since t and thus σ_t is conserved

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

and using eq. 2:

$$\Delta s \;=\; \frac{\Delta p}{p}\, \frac{\beta_v^2\, E}{dE/ds}$$

so

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{0.06}{2\sigma_\gamma^2}\, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho\, \frac{\beta_v^2\, E}{dE/ds}\, \frac{\Delta p}{p}$$

This can be compared with the cooling term

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; -\, J_z\, \frac{dp}{p}$$

giving an equilibrium:

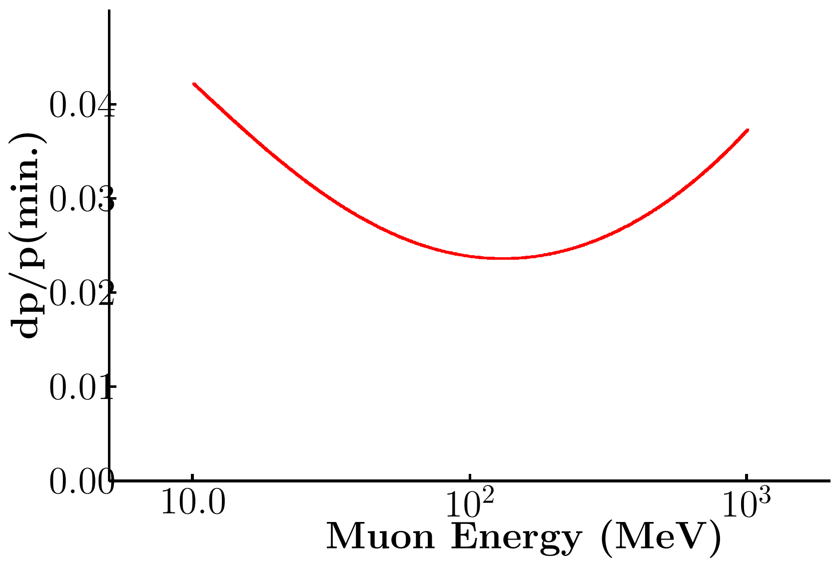
$$\frac{\sigma_p}{p} = \left(\left(\frac{m_e}{m_\mu}\right) \sqrt{\frac{0.06\ Z\ \rho}{2\ A\ (d\gamma/ds)}}\right) \sqrt{\frac{\gamma}{\beta_v^2}\ \left(1 - \frac{\beta_v^2}{2}\right)\ \frac{1}{J_z}} \tag{27}$$

For Hydrogen, the value of the first parenthesis is $\approx$1.45 %.

If there is no coupling between transverse and longitudinal emittances then J_z is small or negative, and the equilibrium does not exist or is large.

However, since J_6 is always greater than 0, we can use wedges to redistribute the J's to allow $J_z = J_6/3$.

The following plot shows the dependency for hydrogen



It is seen to favor cooling at around 300 MeV/c, but has a broad minimum.

4.2.5 rf and bunch length

To obtain the Longitudinal emittance we need σ_z.

If the rf acceleration is relatively uniform along the lattice, then we can write the synchrotron wavelength:

$$\lambda_s \;=\; \sqrt{\frac{\beta_v \gamma\ \lambda_{rf}\ (m_\mu)}{\alpha \mathcal{E}_{\mathrm{rf}}\ \cos(\phi)}} \tag{28}$$

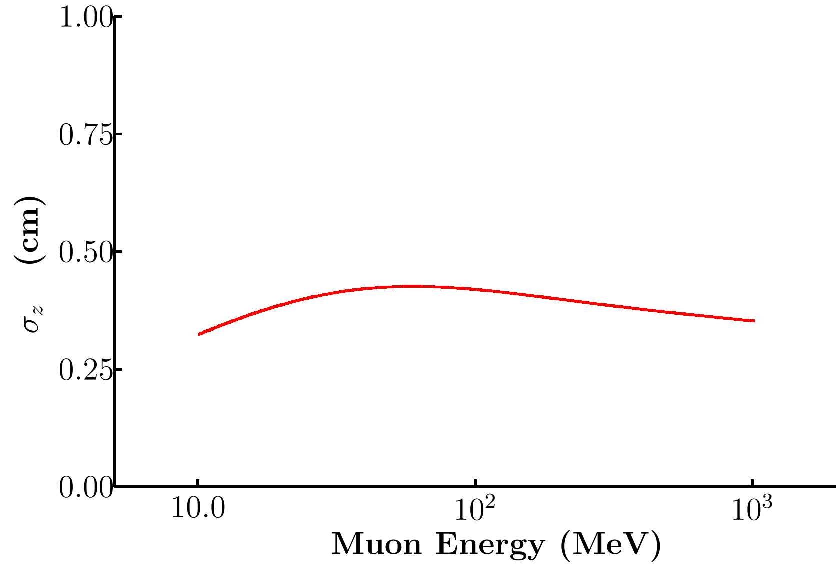
where, in a linear lattice

$$\alpha \;=\; \frac{1}{\gamma^2} \tag{29}$$

and the field $\mathcal{E}_{\mathrm{rf}}$, ie it is the rf accelerating field; ϕ is the rf phase, defined so that for $\phi = 0$ there is no acceleration.

The bunch length, given the relative momentum spread $dp/p = \delta$, is given by:

$$\sigma_z \;=\; \delta\, \frac{\beta_v\ \alpha\ \lambda_s}{2\pi} \;\propto\; \frac{\beta_v^{3/2}}{\gamma} \sqrt{\frac{\lambda_{rf}}{\mathcal{E}_{\mathrm{rf}}}} \tag{30}$$



It is seen to be only weakly dependent on the energy.

4.3 Simulation

Several "local" codes, but

2 Documented codes (GEANT & ICOOL)

Both have:

- Choices of scattering and straggling formulations
- Standing Wave RF fields
- allow use of both
  1. Maxwellian, or
  2. "hard edged" magnetic fields
- Flexible Geometries
- Good tracking

4.3.1 GEANT

- CERN code
- **Works in Cartesian Coord's**
- Uses field maps in 3D
- Requires tweaking to get reference orbit
- Good graphics
- 3 versions:

  1. GEANT 3 is in Fortran single precision (not suitable)
  2. GEANT DP has been modified and has been much used
  3. GEANT 4 is new, C++, and good, but lacks some ease of use

4.3.2 ICOOL

- BNL (Rick Fernow) Fortran code
- **Works in Transport" Coords**
- Uses field maps in 2D, OR
- Field multipoles about a reference orbit
- No tweaking needed
- But does not specify exact coil locations needed
- Poor graphics
- Some Optimization Capability with "OPTICOOL"

4.4 Emittance Exchange Studies

- Attempts at separate cooling & exch.
  - Wedges in Bent Solenoids
  - Wedges in Helical Channels[2]

  Poor performance & problems matching between them

- Attempts in rings with alternate cooling & exchange
  - Balbakov[3] with solenoid focus
    achieved Merit=38 *

- Attempts in rings with combined cooling & exchange
  - Garren et al[4] Quadrupole focused ring
    achieved Merit ≈5
  - Palmer et al[5]
    achieved Merit ≈160*

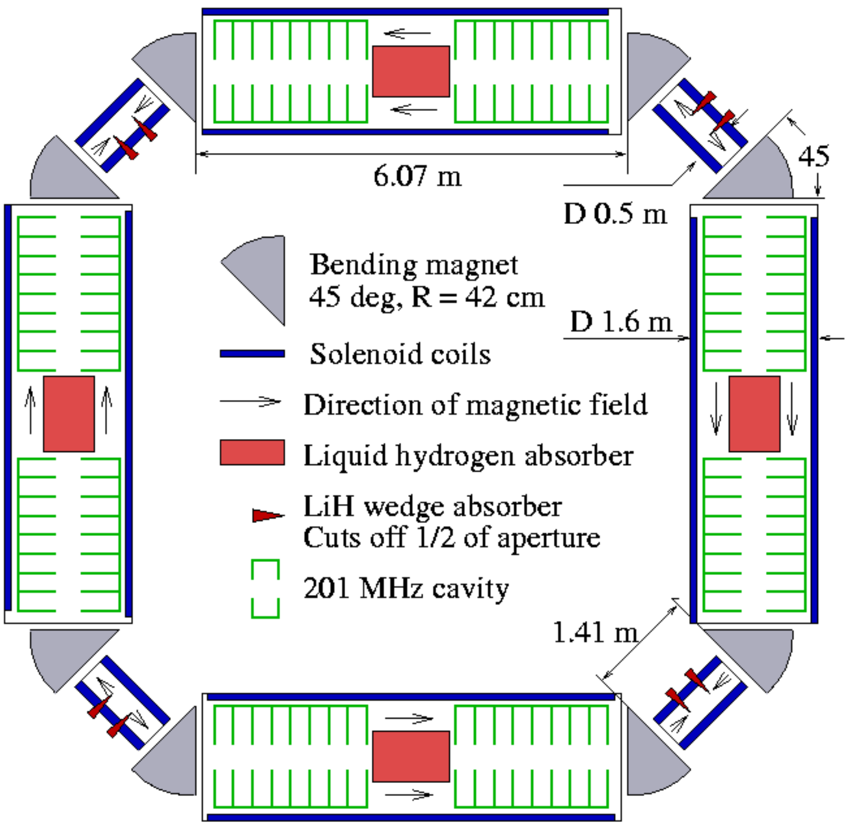
---

[2] MUC-146, 147, 187, & 193
[3] MUC-232 & 246
[4] Snowmass Proc.
[5] MUC-239

4.5 Example 1

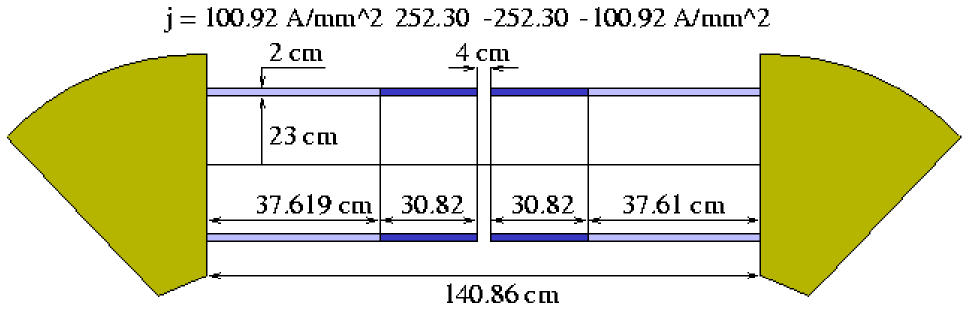
Balbekov 6D Cooling Ring

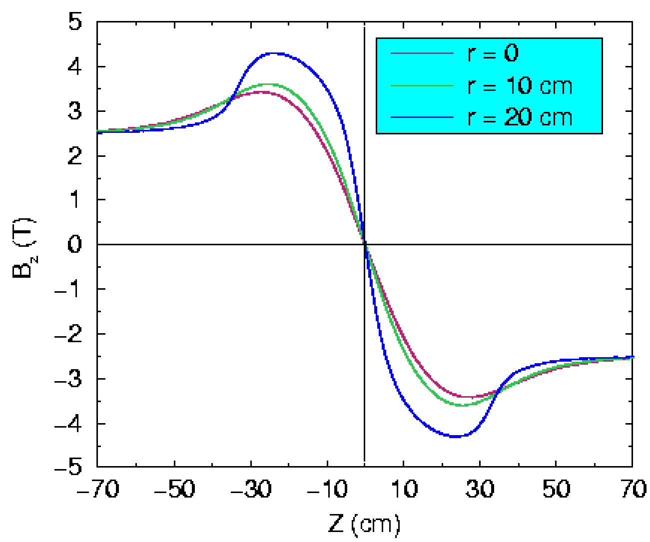
Alternate transverse cooling with H2 with emittance exchange in Li wedge



1.	Circumference	32.555 m
2.	Equilibrium energy (total)	
	minimal	232 MeV
	middle	250 MeV
	maximal	268 MeV
3.	Number of bending magnets	8
4.	Bending angle	45^o
5.	Bending radius	42 cm
6.	Bending field	1.80 T
7.	Normalized field gradient	0.5
8.	Length of short SS	1.409 m
9.	Length of long SS	6.070 m
10.	Maximal axial field of solenoid	4.32 T
11.	Revolution frequency	8.378 MHz
12.	RF harmonic number	24
13.	RF frequency	201 MHz
14.	Accelerating gradient	15 MeV/m
15.	Main absorber	LH,120 cm
16.	Wedge absorber	LiH,
	$dE/dx = \pm dE/dy = 0.3$ MeV/cm	

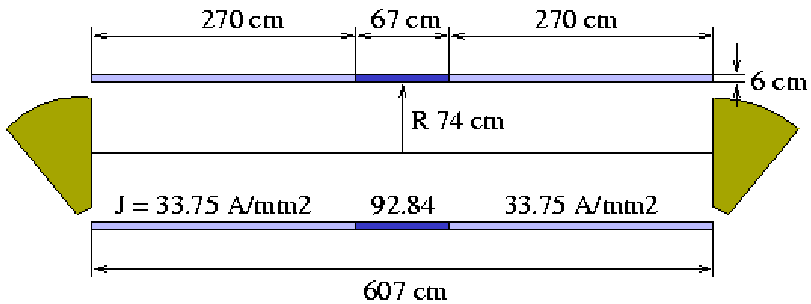
Short Straight

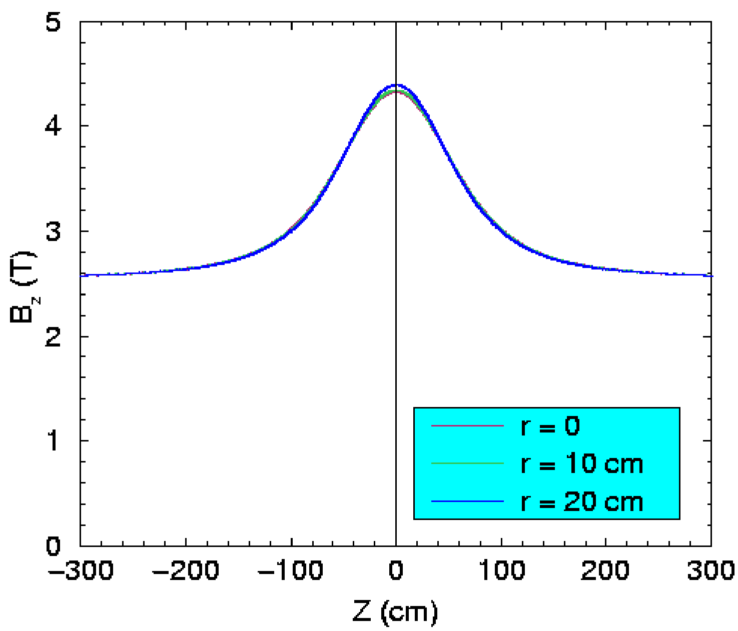




- Field flip in center
- Max Dispersion in center
- LiH Wedge in center

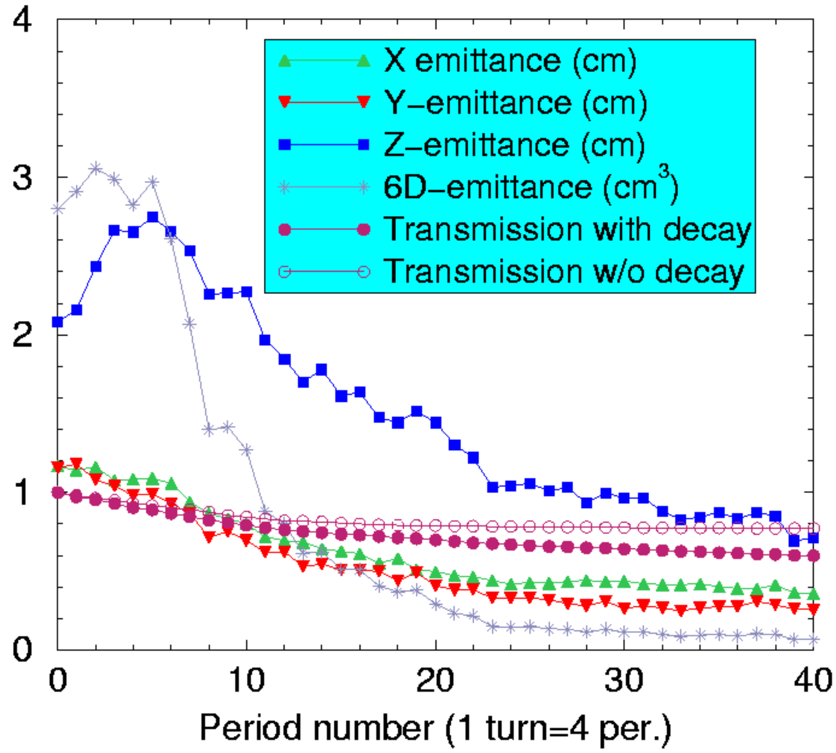
Long Straight





- No flip & No Dispersion
- Higher field & lower β at center
- Hydrogen Absorber at center
- RF on either side

4.5.1 Performance



Number of turns	0	5	10	15
X-emittance (cm)	1.2	0.37	0.24	0.21
Y-emittance (cm)	1.2	0.35	0.24	0.21
Z-emittance (cm)	1.5	1.3	0.79	0.63
6D-emittance (cm^3)	2.2	0.18	0.045	0.028
Transmission without decay	1	0.76	0.72	0.71
Transmission with decay	1	0.67	0.56	0.48
Merit factor ($Tr \times \varepsilon_{6,ini}/\varepsilon_{6,fin}$)	1	8.2	27	38

4.5.2 Conclusion for Balbakov

- Good cooling in all dimensions
- Merit Factor 38
  c.f. Study 2 Linear: Merit=15

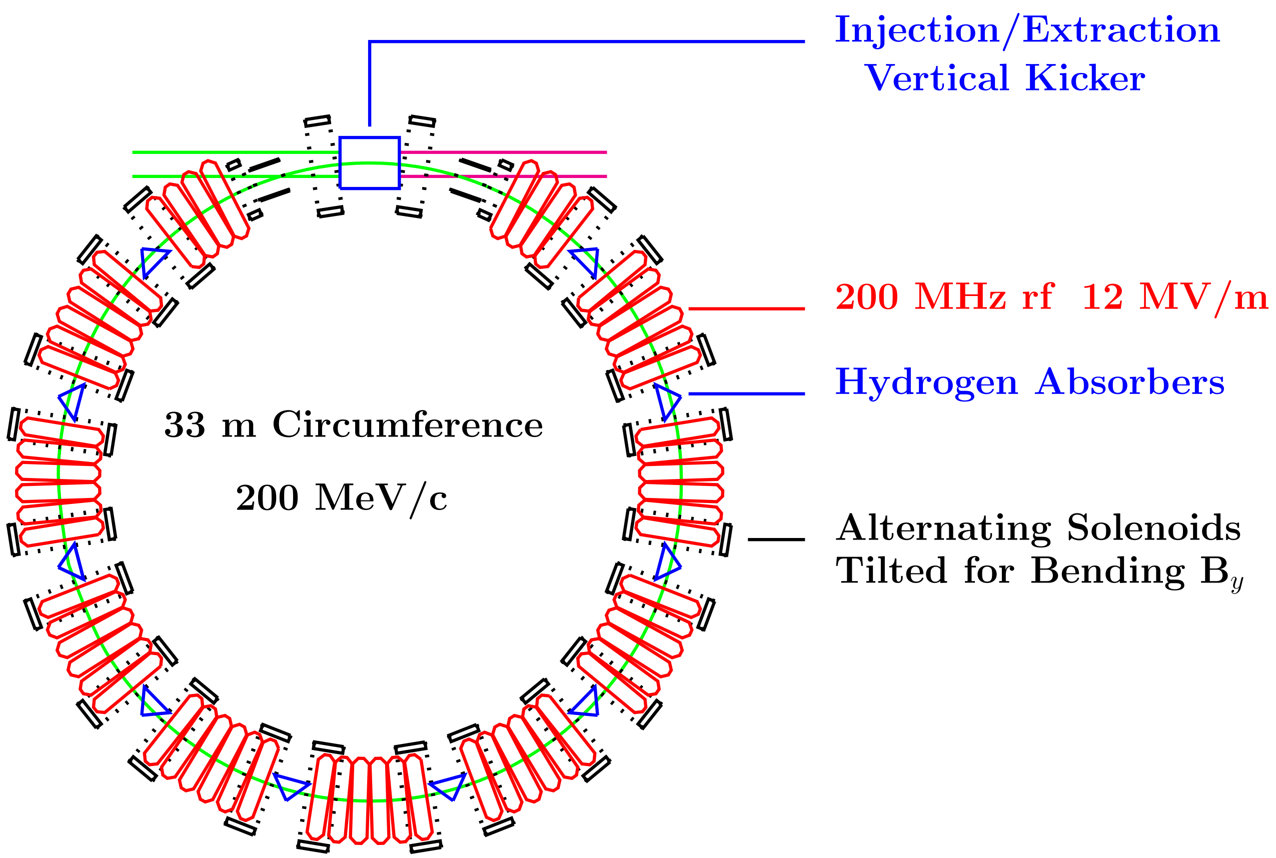
BUT

- Calculated without Maxwellian fields
- Design of bends proving hard
- Injection and extraction very hard
  Merit $\rightarrow$ 3 with missing rf
- Upward spiral an alternative

4.6 Example 2

RFOFO Ring[6]

4.6.1 Introduction

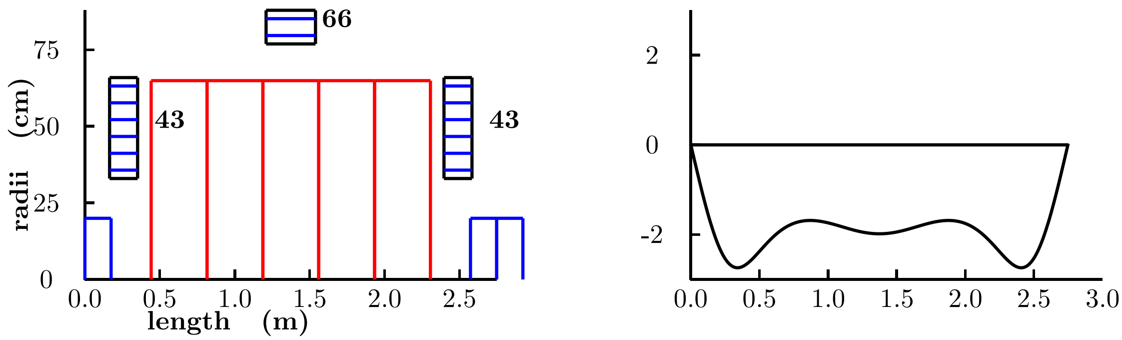


[6] MUC-232

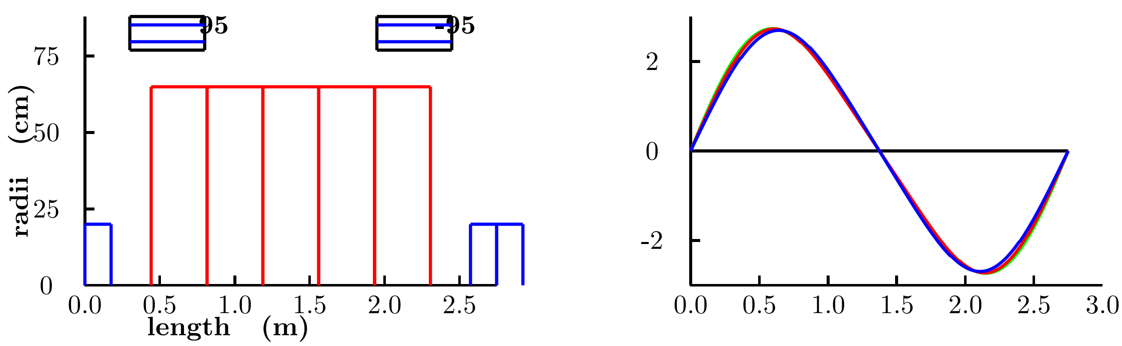
4.6.2 Lattice

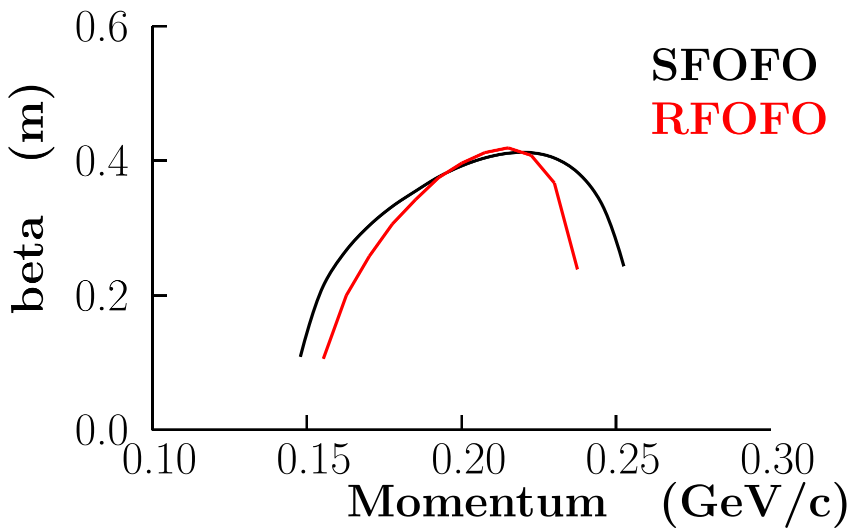
- Make all cells ≈same
  avoid matching problems

SFOFO as in Study 2



RFOFO has Reversed Fields





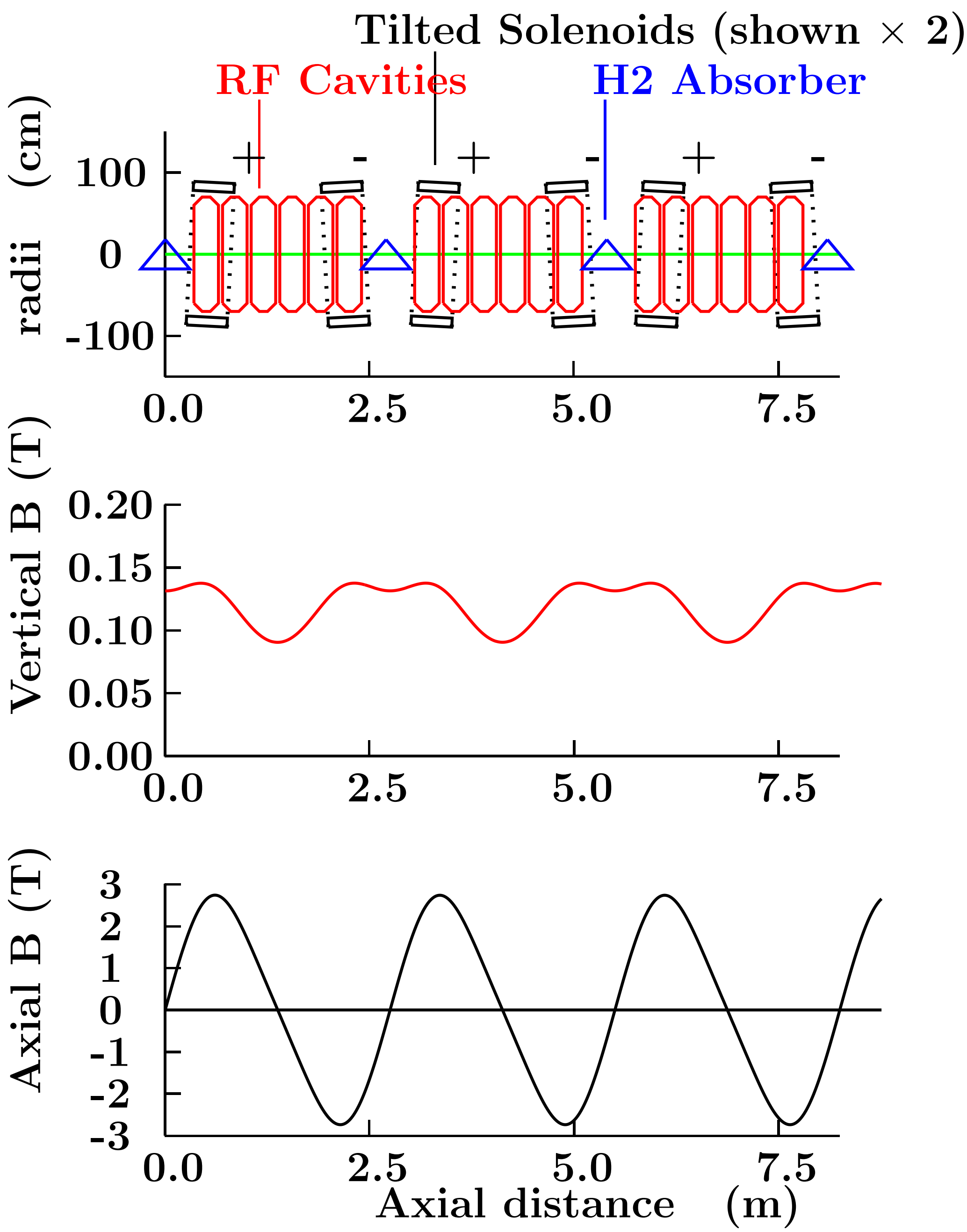
- RFOFO Mom acceptance worse

BUT

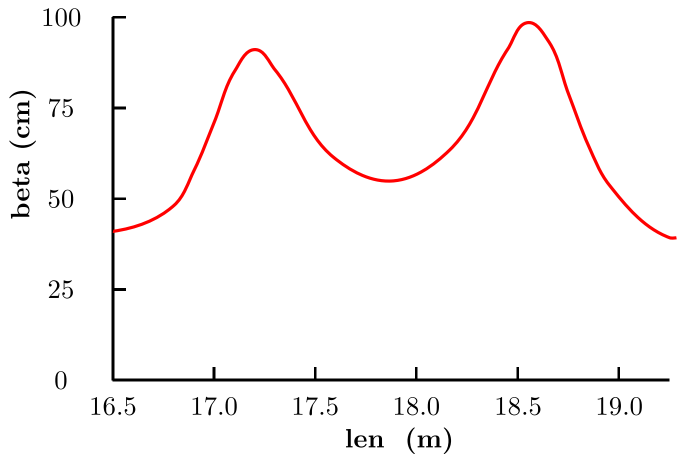
- All cells the same
- Fewer resonances
- Choose RFOFO

4.6.3 Tilt Coils to get Bend

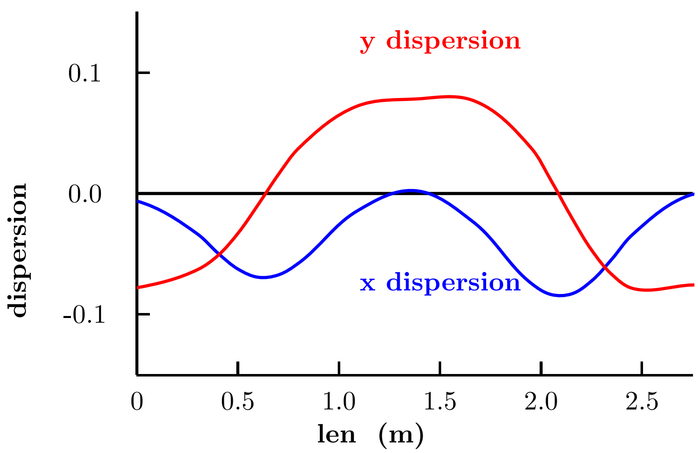
.



Beta and Dispersion .



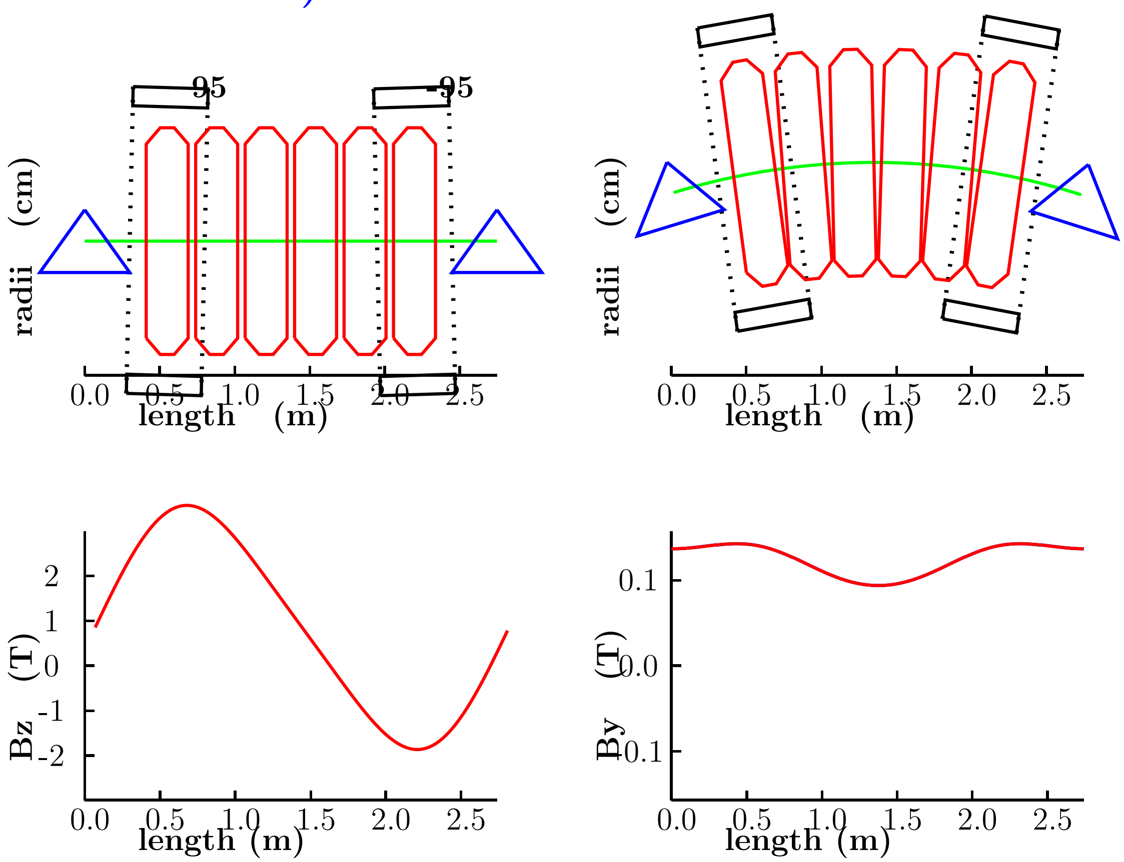
beta is ≈ straight case



Dispersion is rotating back and forth

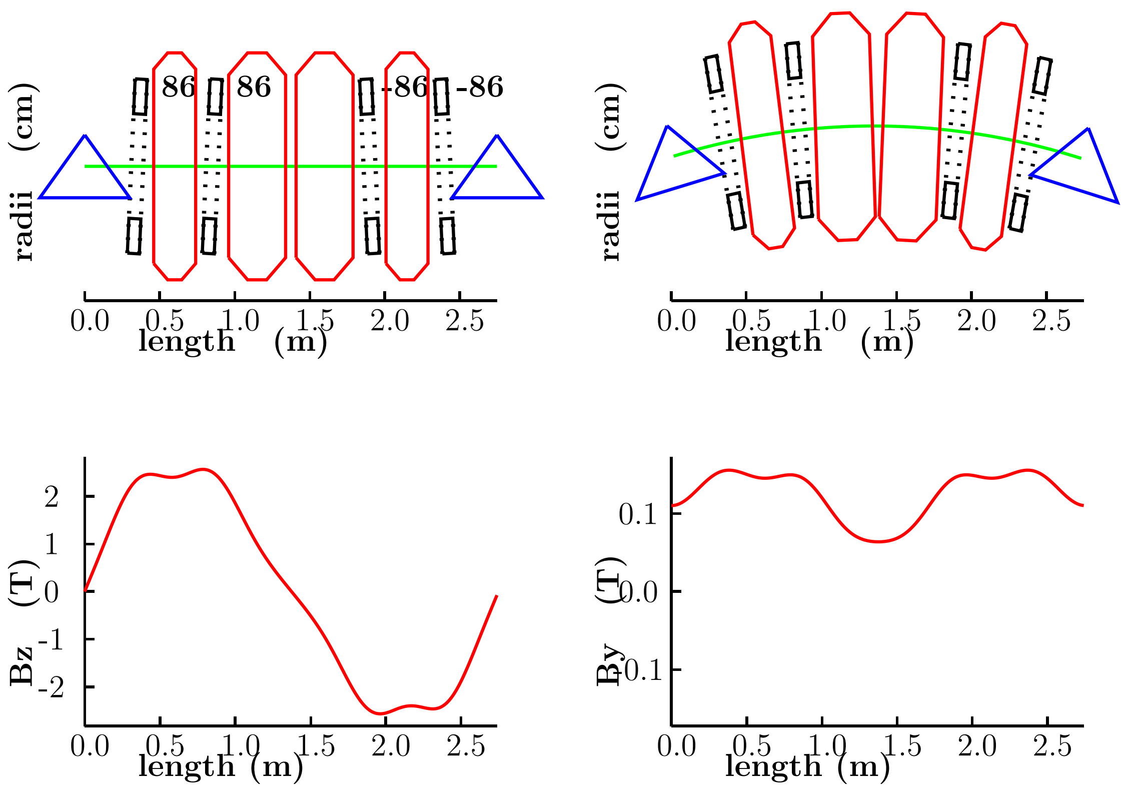
4.6.4 Cell Layouts

a) Coils outside RF



- Wedges shown 0 and 90 deg.
  true angle 30 deg
- Amp-turn-length = 54 MAm/cell
- RF Grad = 12 MV/m

b) Coils between Cavities



- Amp-turn-length = 14 MAm/cell
- RF Grad = 16 MV/m

- Performance the same
- Choice not yet made

4.6.5 Params for Simulation

Coils

gap	start	dl	rad	dr	tilt	I/A
m	m	m	m	m	rad	A/mm^2
0.310	0.310	0.080	0.300	0.200	0.0497	86.25
0.420	0.810	0.080	0.300	0.200	0.0497	86.25
0.970	1.860	0.080	0.300	0.200	-.0497	-86.25
0.420	2.360	0.080	0.300	0.200	-.0497	-86.25

amp turns 5.52 (MA)
amp turns length 13.87326 (MA m)
cell length 2.750001 (m)

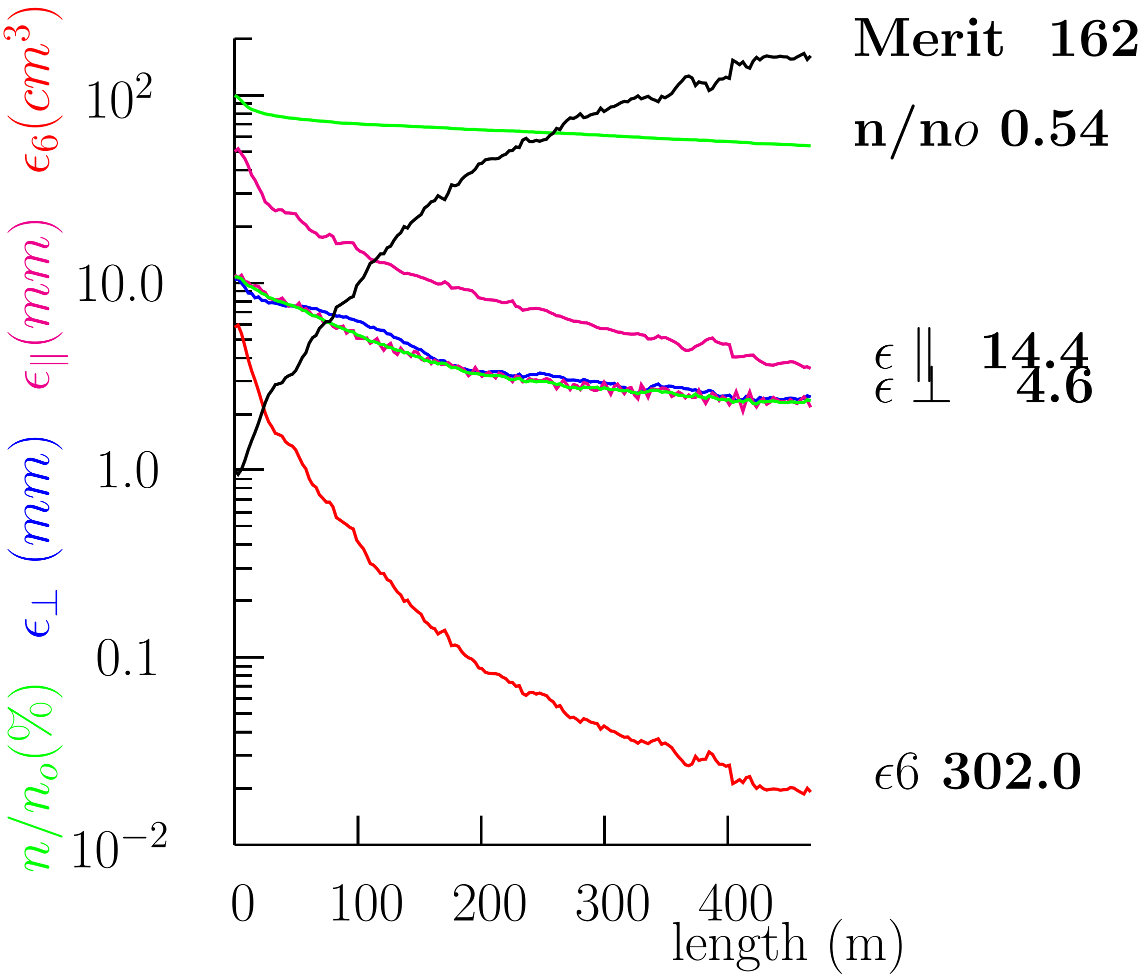
Wedge

Material		H2
Windows		none
Radius	cm	18
central thickness	cm	28.6
min thickness	cm	0
wedge angle	deg	100
wedge azimuth from vertical	deg	30

RF

Cavities		6
Lengths	cm	28
Central gaps	cm	5
Radial aperture	cm	25
Frequency	MHz	201.25
Gradient	MV/m	16
Phase rel to fixed ref	deg	25
Windows		none

4.6.6 Performance



	len	trans	$\epsilon_\perp$	dp/p	$\epsilon_\parallel$	ϵ_6	max Q	merit
	m	%	π mm	%	π mm	π^3cm^3		
final	468	54	2.3	4.0	3.5	0.019	24	**162**
initial			10.7	11.2	50.1	5.787		
ratio			4.6	2.8	14.4	302.0		

If $J_\perp = 1$ then:

$$\epsilon_\perp(\min) \; = \; \frac{38\; 10^{-4}\; 0.4}{0.85} \; = \; 1.8 \; (\pi mm\; mrad)$$

So here

$$J_\perp \approx \frac{1.8}{2.3} \; = \; 0.78$$

$$J_\parallel \approx 2 - 2\; 0.78 = 0.43$$

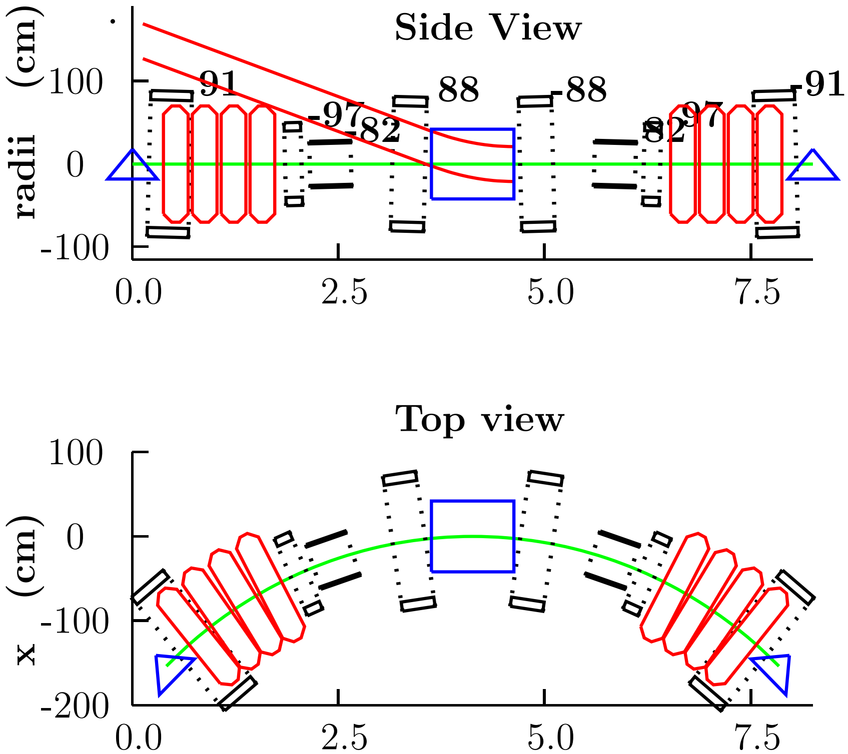
From equation 27 we expect

$$\frac{dp}{p}(\min) \; \approx 3\%$$

The observed value is 4%, but it is still falling.

An equilibrium of 3 % appears reasonably correct

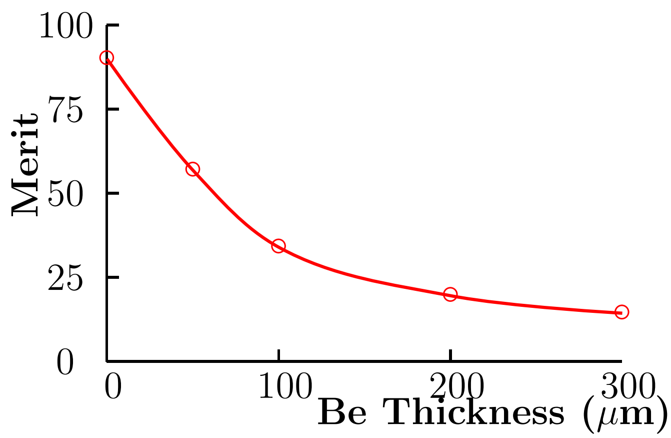
4.6.7 Insertion for Inject/Extract



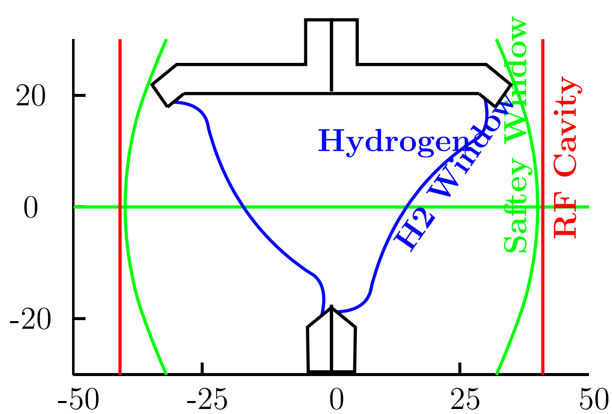
- First Simulation gave Merit = 10
  Synchrotron tune = 2.0: Integer
- Increase energy, wedge angle, and add matching.
- Merit 160 $\rightarrow$ 110

4.6.8 Unanswered Questions

- RF windows must be very thin
- RF at 70 deg will help



- Design of wedge absorber



- But best with apex inside aperture

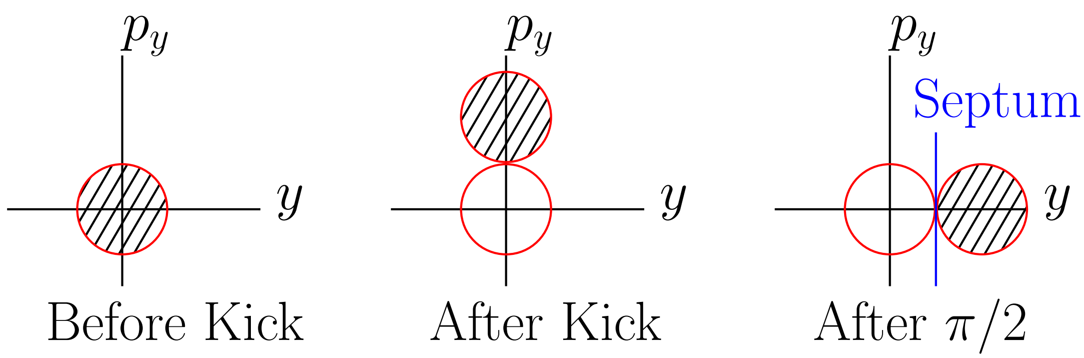
4.6.9 Conclusion for RFOFO Ring

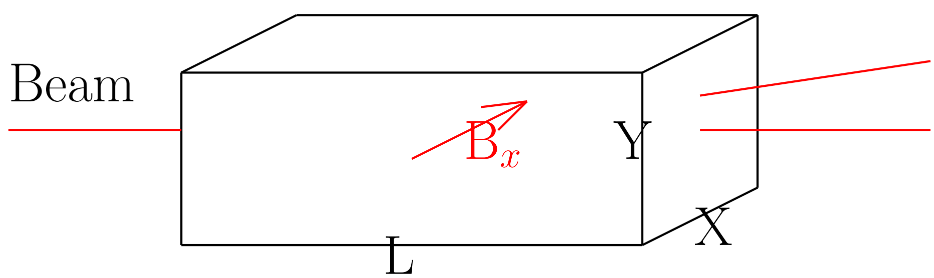
- RFOFO Ring Cools better than linear a channel
  Merit 160 for ring vs 15 for Study 2
- Uses fewer components
  33 m ring vs. 108 m Study 2
- Simulation done with Maxwellian Fields
  But exact coil positions need determining
- Simulation with GEANT Needed
- Injection insertion details not designed
- Kicker still problematical

5 Injection/Extraction

5.1 Kickers

5.1.1 Minimum Required kick





$$f_\sigma = \frac{\mathrm{Ap}}{\sigma} \qquad \mu \;=\; \inf \qquad F \;=\; \frac{Y}{X}$$

$$I \;=\; \left(\frac{4\ f_\sigma^2\ m_\mu}{\mu_o\ c}\right) \quad \frac{\epsilon_n}{L}$$

$$V \;=\; \left(\frac{4\ f_\sigma^2\ m_\mu\ R}{c}\right) \quad \frac{\epsilon_n}{\tau}$$

$$U \;=\; \left(\frac{m_\mu^2\ 8\ f_\sigma^4\ R}{\mu_o\ c^2}\right) \quad \frac{\epsilon_n^2}{L}$$

- muon ϵ_n $\gg$ other ϵ_n's
- So muon kicker Joules $\gg$ other kickers
- Nearest are $\bar{p}$ kickers

Compare with others

For $\epsilon_{\perp} = 10\ \pi$mm, $\beta_{\perp} = 1$m, & τ=50 nsec:

After correction for finite μ and leakage flux:

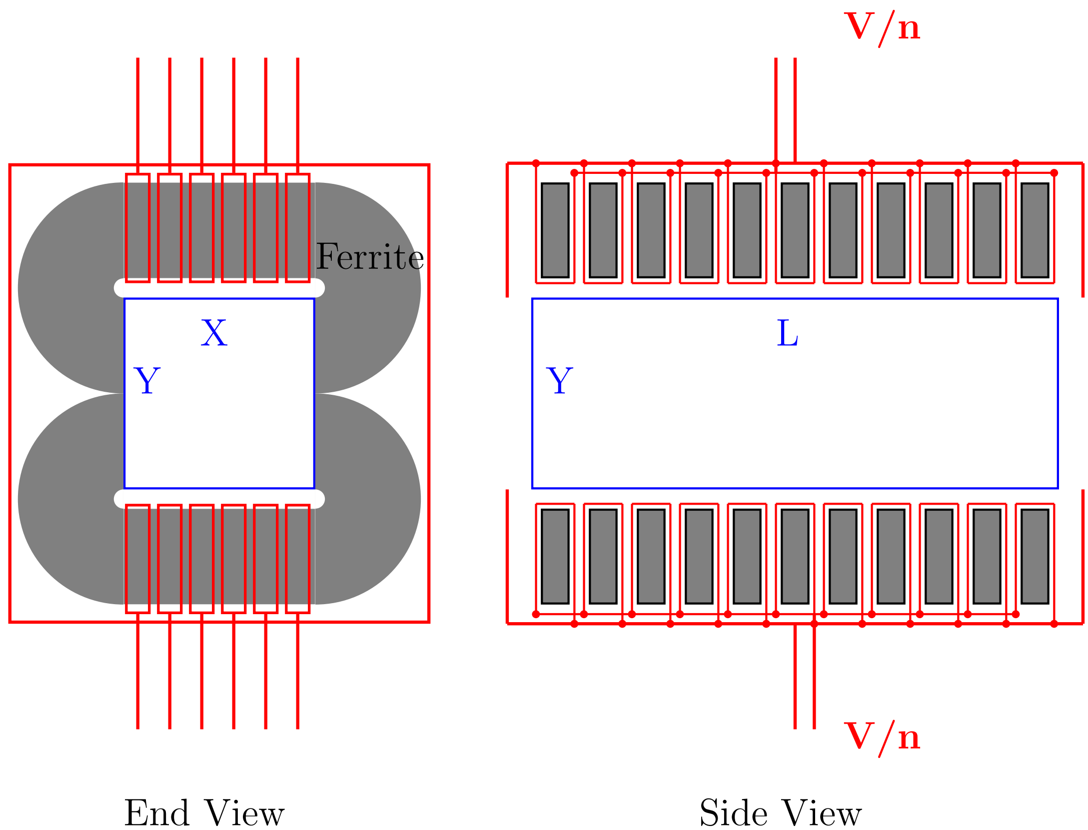
		μ Cooling	CERN $\bar{p}$	Ind Linac
$\int Bd\ell$	Tm	.30	.088	
L	m	1.0	≈5	5.0
$t_{\rm rise}$	ns	**50**	**90**	**40**
B	T	**.30**	**≈0.018**	**0.6**
X	m	.42	.08	
Y	m	.63	.25	
$V_{\rm 1turn}$	kV	**3,970**	**800**	**5,000**
$U_{\rm magnetic}$	J	**10,450**	**≈13**	**8000**

Note

- U is 3 orders above $\bar{p}$
- Same order as Induction
- And t same order
- But V is too High

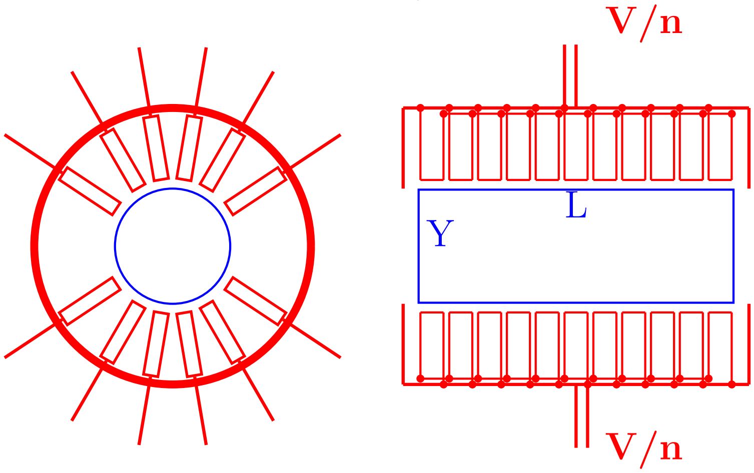
5.1.2 Induction Kicker

- Drive Flux Return
- Subdivide Flux Return Loops
  Solves Voltage Problem
- Conducting Box Removes
  Stray Field Return



Works with no Ferrite

- V = the same
- U≈ 2.25×
- I≈ 2.25×
- No rise time limit
- Not effected by solenoid fields

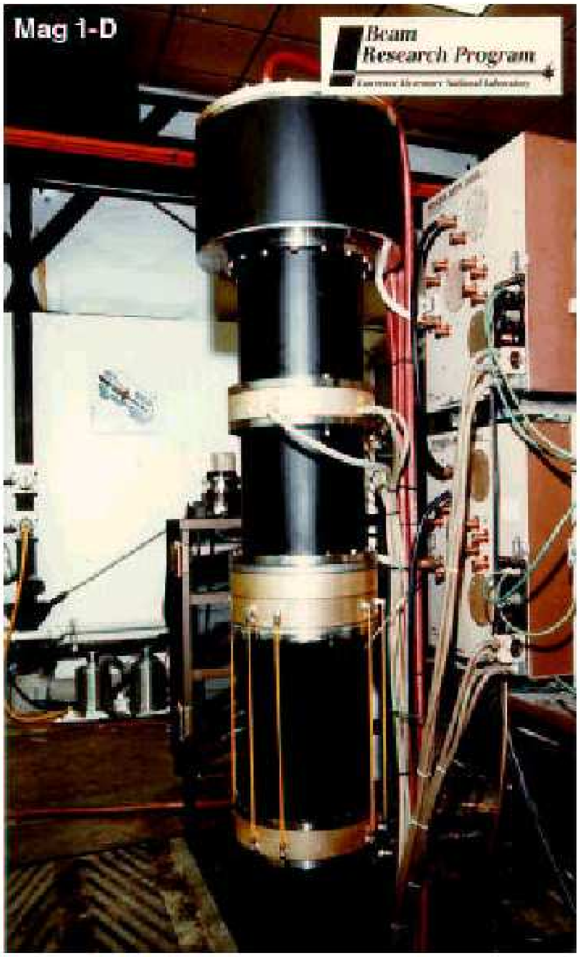


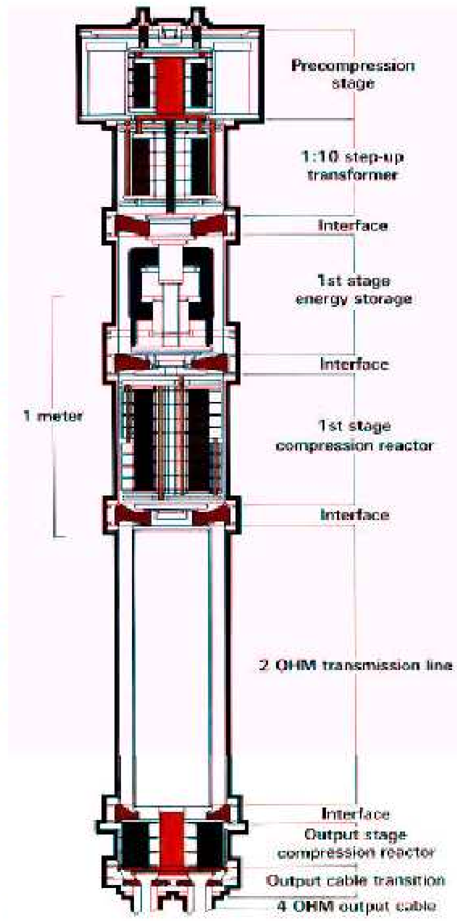
End View Side View

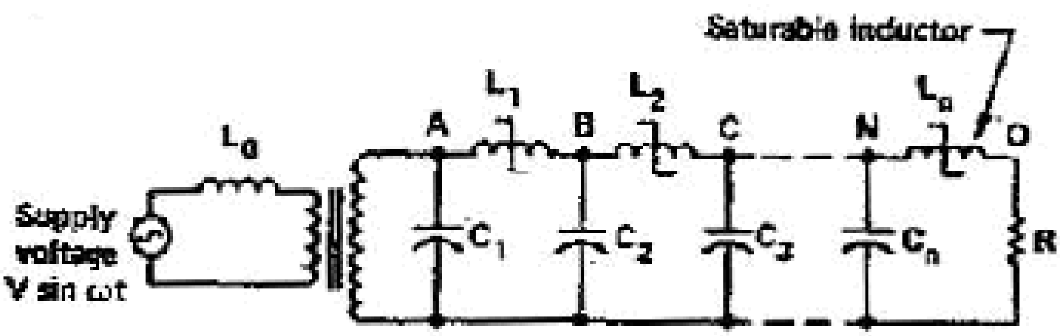
- If non Resonant: 2 Drivers
  for inj. & extract.
  Need 24 ×2 Magamps (≈ 20 M$)
- If Resonant: 1 Driver, 2×efficient
  Need 12 Magamps (≈ 5 M$)

5.2 Magnetic Amplifiers

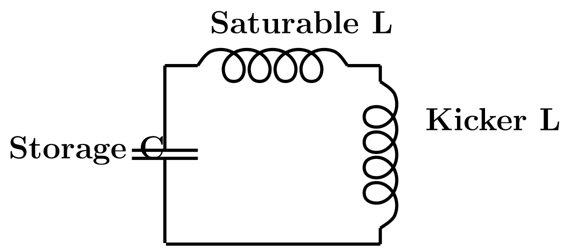
Used to drive Induction Linacs
similar to ATA or DARHT







Magamp principle



Initially Unsaturated, $L = L_1$ is large:

$$\tau_L = \sqrt{(L + L_1)C} \quad \text{is slow}$$

The current I rises slowly:

$$I = I_o \sin\left(\frac{t}{\tau_L}\right)$$

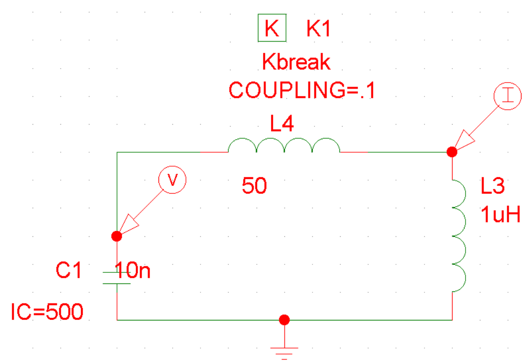
When the inductor saturates
$L = L_2$ is small:

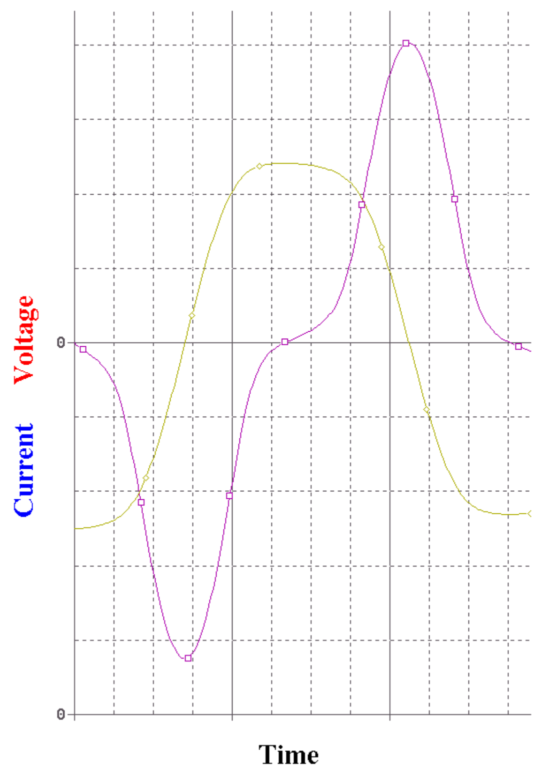
$$\tau_S = \sqrt{(L + L_2)C} \qquad \text{is fast}$$

After approx π phase
Inductor regains its high inductance
The oscillation slows before reversing.

Pspice Simulation

a) Single stage





Circuit Model (Reginato)

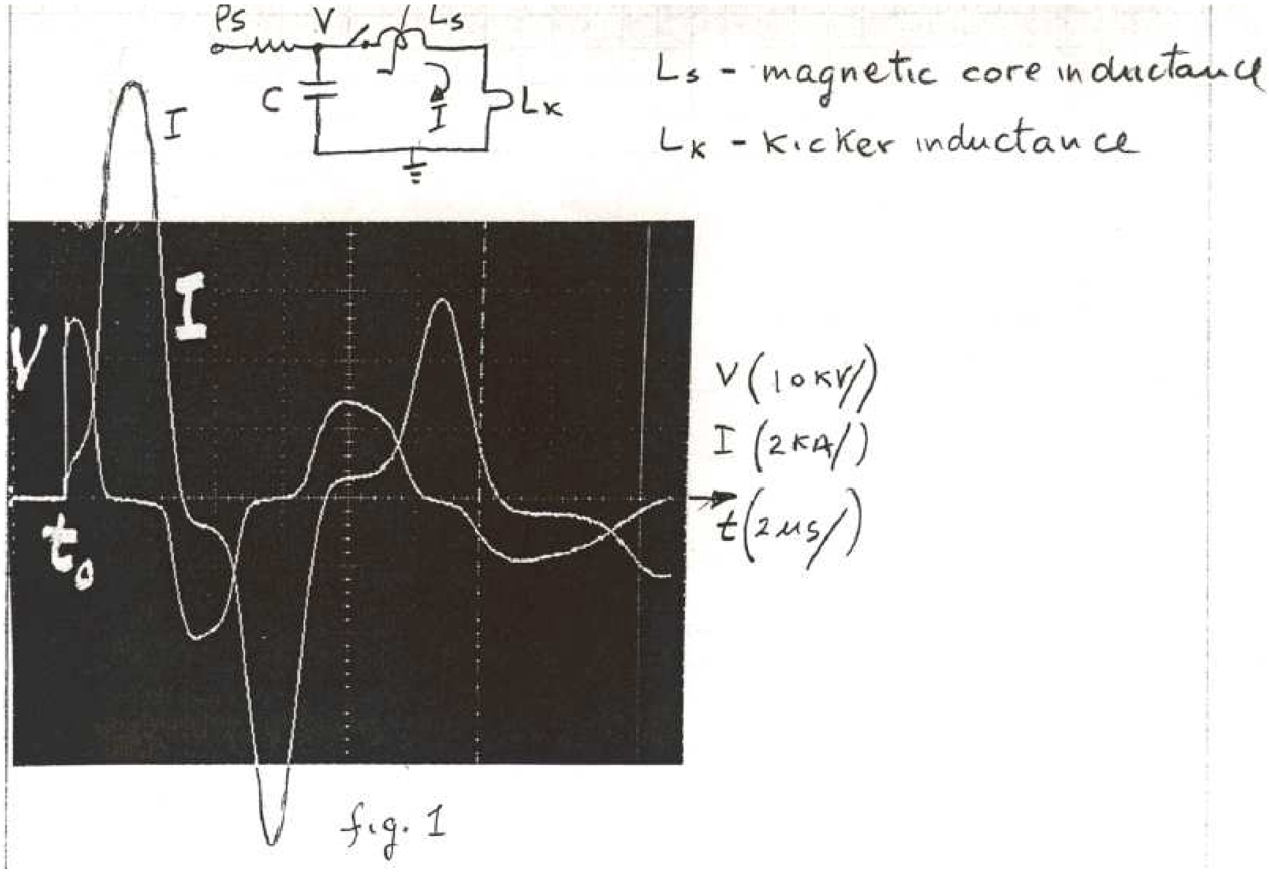
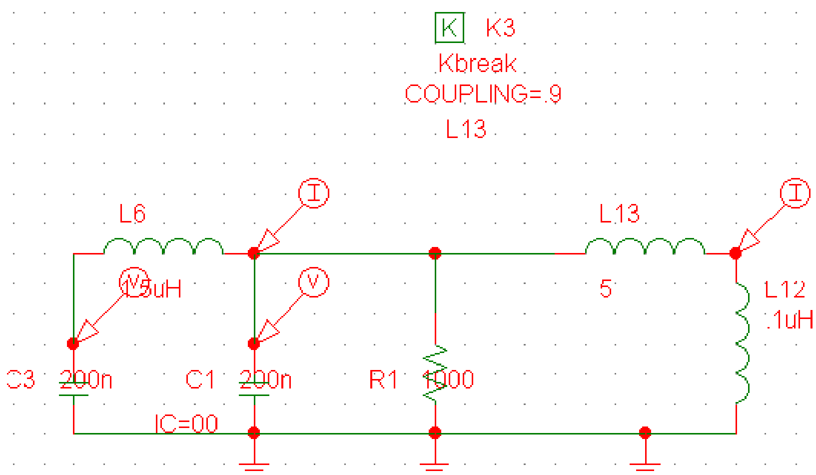
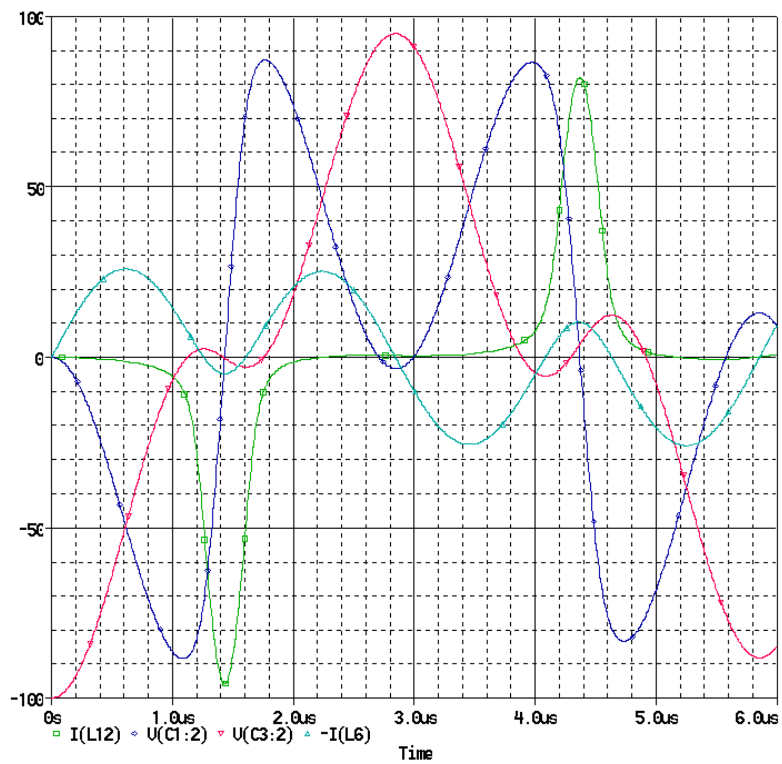


fig. 1

a) 2 Stage


K K3
Kbreak
COUPLING=.9
L13
L6
L13
1.5uH
5
L12
.1uH
C3 200n
C1 200n
IC=00
R1 1000


100
50
0
-50
-100
0s
1.0us
2.0us
3.0us
4.0us
5.0us
6.0us
I(L12) V(C1:2) V(C3:2) -I(L6)
Time

6 Ring Cooler Conclusion

- Rapid Progress has been made.
- Need for very thin windows is greater than for linear coolers
- Work needed on Hydrogen wedge design
- Much Work needed on Insertion
  but probably doable
- The Kicker is the least certain
- Need pre-cooler or other ideas to match phase space into short bunch train

BUT

- Performance better than linear coolers
- Might lower acceleration cost
- Real hope that Collider requirements may be met