Boundary Control Method for the 1-d Schrodinger Equation

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Abstract. We consider the inverse problem of determining the potential in the one-dimensional Schrodinger equation from dynamical boundary observations, which are the range values of the Neumann to Dirichlet map. Our approach and results are new in several respects: (i) Dynamical boundary data have not been used in the inverse problem for the Schrodinger equation, since the traditional Gelfand-Levitan-Marchenko (GLM) approach reconstructs the potential from spectral or scattering data. Here we show that one can completely recover the spectral data from the dynamical boundary data; (ii) The construction of the spectral data uses exact and spectral controllability for the Schrodinger equation, for which we give a new proof, based on the properties of exponential Riesz bases (nonharmonic Fourier series); (iii) From the spectral data one could recover the potential by using the GLM method. Instead, we choose to solve the inverse problem by the Boundary Control (BC) method, which is based on several deep connections between inverse problems and control theory of distributed parameter systems and on the propagation of singularities for the wave equation. This approach is more general and, in principle, could be more extended to higher dimensions.

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