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Short communication

Generalizing the definition of the bi-directional reflectance distribution function

Larry Di Girolamo*

Department of Atmospheric Sciences, University of Illinois at Urbana-Champaign, 105 S. Gregory Street, Urbana, IL 61801-3070, USA

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Abstract

The fundamental quantity that characterizes the reflectance property of a surface is the bi-directional reflectance distribution function (BRDF), defined as the ratio of the radiance scattered by a surface into a specified direction to the unidirectional irradiance incident on a surface. Its standard definition was derived under very restrictive conditions and it has only the angles of illumination and measurement as dependent variables. Several recent papers have attempted to generalize the BRDF to include the spatial attributes of illumination and measurement in order to make it applicable to heterogeneous media. The various BRDF definitions proposed by these papers are shown to be special cases of a generalized form of the BRDF derived herein. The spatial attributes of illumination and measurement are included as part of the nomenclature of the generalized BRDF. It is also shown that the generalized BRDF obeys reciprocity when properly weighted by the areas of illumination and measurement.

Keywords: Bi-directional reflectance; BRDF; Reciprocity; Heterogeneous media

1. Introduction

In the 1960s and 1970s, numerous papers were published recommending that the scientific community standardize the definition and nomenclature of a wide variety of reflectance quantities. This reached its apex in 1977 when the National Bureau of Standards published its recommended definition and nomenclature for these reflectance quantities (Nicodemus, Richmond, Hsia, Ginsberg, & Limperis, 1977). The most basic of these reflectance quantities is the bi-directional reflectance distribution function (BRDF), from which all other standardized reflectance quantities can be derived. The BRDF is defined as the ratio of the radiance scattered by a surface into a specified direction to the unidirectional (collimated) irradiance incident on a surface. More formally, the BRDF, f_r, is typically written as

$$f_r(-\mathbf{\Omega}_2;\mathbf{\Omega}_1) = \frac{I(-\mathbf{\Omega}_2;\mathbf{\Omega}_1)}{\mathbf{\Omega}_1 \cdot \boldsymbol{n}F(\mathbf{\Omega}_1)} \ [\mathrm{sr}^{-1}] \tag{1}$$

where $\mathbf{\Omega}$ represents the directional unit vector with outward direction from the surface as negative, $\boldsymbol{n}$ is a unit vector that is outward normal to the surface (i.e., $\mathbf{\Omega}\cdot\boldsymbol{n}<0$ represents a direction incident at the surface), and $I(-\mathbf{\Omega}_2;\mathbf{\Omega}_1)$ is the radiance in direction $-\mathbf{\Omega}_2$ caused by an irradiance $F(\mathbf{\Omega}_1)$ from direction $\mathbf{\Omega}_1$. Nicodemus et al. (1977) recommended f_r as the standard symbol for the BRDF, where the subscript r indicates a quantity associated with reflected irradiance. However, many other symbols are often used in published journals and textbooks. In addition, as discussed in Nicodemus et al. (1977), the definition of the BRDF in Eq. (1) may be written in terms of the ratio of infinitesimal quantities, which in practice cannot be directly measured. A real measurement requires integration of these quantities over finite intervals, leading to an estimated value of the BRDF. It is in this context that Eq. (1) and what follows should be interpreted.

There are three basic conditions described by Nicodemus et al. (1977) that must be met for the BRDF to be the fundamental quantity that characterizes the reflecting properties of a surface. These conditions are as follows:

Condition I: The surface must be horizontally homogeneous;
Condition II: Uniform irradiance from a single direction exists over a large enough area such that the radiance

* Tel.: +1-217-333-3080.
E-mail address: larry@atmos.uiuc.edu (L. Di Girolamo).

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leaving the top of the surface does not vary with horizontal position; and
Condition III: The BRDF is defined at a point.

With these conditions met, the BRDF obeys the following widely known reciprocal relationship (e.g., see Hapke, 1993, or most other textbooks dealing with radiative transfer or remote sensing):

$$f_r(-\boldsymbol{\Omega}_2;\boldsymbol{\Omega}_1) = f_r(-\boldsymbol{\Omega}_1;\boldsymbol{\Omega}_2) \tag{2}$$

However, there have been many observational experiments that question whether the measured BRDF should obey Eq. (2) (e.g., recent ones include Davies, 1994; Kriebel, 1996; Li & Wan, 1998; Loeb & Davies, 1997). This question has also been debated during the 1998 International Forum on BRDF, with a final recommendation that the validity of Eq. (2) be further investigated (Liang & Strahler, 1998). A complete discussion as to why the relationship shown in Eq. (2) fails to be observed in some common situations, including other practical problems (e.g., nonuniform detector response function), is given in Di Girolamo, Varnai, and Davies (1998) and Snyder (2002). At the heart of their discussion is the fact that *Conditions I–III* are difficult to meet in many common situations. For these situations, simply taking the ratio of measured radiance to irradiance, as in Eq. (1), leads to a BRDF that is not a fundamental quantity that characterizes the reflecting properties of a surface, because a measured BRDF fundamentally depends on the spatial characteristics of the illumination and measurement areas. Thus, there is a need to generalize the definition of the BRDF to include these spatial characteristics.

There have been two key papers that have attempted to further generalize the original definition of the BRDF. The first is by Greffet and Nieto-Vesperinas (1998). They argued that the original BRDF definition was for surfaces that exhibited local reflectivity, whereby the light entering a point also exits from that point, which makes *Condition II* easily satisfied for such surfaces. However, they argued that a perfectly viable definition of the BRDF can be obtained by relaxing this condition to include non-local reflectivity, whereby light entering a surface at a point can affect the observed reflectivity at a distant point on the same surface by way of multiple scattering through the medium. Based on coherence theory applied to a semi-infinite quasi-homogeneous medium, they proved that this point representation of emerging radiance and incident unidirectional irradiance obeys the following reciprocal relationship:

$$f_r(\boldsymbol{r}_2,-\boldsymbol{\Omega}_2;\boldsymbol{r}_1,\boldsymbol{\Omega}_1) = f_r(\boldsymbol{r}_1,-\boldsymbol{\Omega}_1;\boldsymbol{r}_2,\boldsymbol{\Omega}_2) \tag{3}$$

where $\boldsymbol{r}$ is a position vector on the top boundary of the medium; thus, $f_r(\boldsymbol{r}_2,-\boldsymbol{\Omega}_2;\boldsymbol{r}_1,\boldsymbol{\Omega}_1)$ is the BRDF at position $\boldsymbol{r}_2$ in direction $-\boldsymbol{\Omega}_2$ caused by illuminating the point at position $\boldsymbol{r}_1$ from direction $\boldsymbol{\Omega}_1$. Although this point-based definition of BRDF is useful in some situations, it does not address the practical aspect of the finite field-of-view of measuring instruments.

Snyder (2002) attempts to generalize the definition of the BRDF by directly addressing the finite field-of-view of measuring instruments. He argued that by simply averaging the point BRDF over the finite region A, the BRDF obeys reciprocity even for heterogeneous media within the measurement area, that is

$$f_r(A,-\boldsymbol{\Omega}_2;\boldsymbol{\Omega}_1) = \frac{\frac{1}{a}\int_A I(\boldsymbol{r},-\boldsymbol{\Omega}_2;\boldsymbol{\Omega}_1)\mathrm{d}\boldsymbol{r}}{\boldsymbol{\Omega}_1\cdot\boldsymbol{n}F(\boldsymbol{\Omega}_1)}\ [\mathrm{sr}^{-1}] \tag{4}$$

where a is the area of surface A, and

$$f_r(A,-\boldsymbol{\Omega}_2;\boldsymbol{\Omega}_1) = f_r(A,-\boldsymbol{\Omega}_1;\boldsymbol{\Omega}_2) \tag{5}$$

The nomenclature used here differs from that given by Snyder (2002). Most notable is the inclusion of surface A as a dependent variable explicitly in the nomenclature of the left-hand side of Eq. (4) (cf. Eq. (7) of Snyder, 2002).

Strictly, the definition of the BRDF in Eq. (4) obeys the reciprocity relationship of Eq. (5) if (i) only the measurement area is illuminated (Di Girolamo et al., 1998), (ii) there is no horizontal transport of light from outside the measurement area into the measurement area and toward the instrument (Di Girolamo et al., 1998), or (iii) the distribution of media within the measurement area is periodic outside the measurement area with illumination everywhere at the top boundary (Di Girolamo, 2002). Although Snyder's proposed definition of the BRDF will remain reciprocal over some class of surfaces and illumination conditions, it is not fully general.

2. Generalizing the BRDF

If a generalized definition of the BRDF is to obey reciprocity, then this definition can be derived from the appropriate reciprocity principle. In this case, the appropriate reciprocity principle is given by Di Girolamo (1999), which states that for external unidirectional illumination,

$$\boldsymbol{\Omega}_2\cdot\boldsymbol{n}\int_{A_2} F(\boldsymbol{r},\boldsymbol{\Omega}_2)I(\boldsymbol{r},-\boldsymbol{\Omega}_2;A_1,\boldsymbol{\Omega}_1)\mathrm{d}\boldsymbol{r} = \boldsymbol{\Omega}_1\cdot\boldsymbol{n}\int_{A_1} F(\boldsymbol{r},\boldsymbol{\Omega}_1)I(\boldsymbol{r},-\boldsymbol{\Omega}_1;A_2,\boldsymbol{\Omega}_2)\mathrm{d}\boldsymbol{r} \tag{6}$$

where $I(\boldsymbol{r},-\boldsymbol{\Omega}_2;A_1,\boldsymbol{\Omega}_1)$ is the radiance at position $\boldsymbol{r}$ in direction $-\boldsymbol{\Omega}_2$ caused by illuminating the surface A_1 with an irradiance $F(\boldsymbol{r},\boldsymbol{\Omega}_1)$ from direction $\boldsymbol{\Omega}_1$, $I(\boldsymbol{r},-\boldsymbol{\Omega}_1;A_2,\boldsymbol{\Omega}_2)$ is the radiance at position $\boldsymbol{r}$ in direction $-\boldsymbol{\Omega}_1$ caused by illuminating the surface A_2 with an irradiance $F(\boldsymbol{r},\boldsymbol{\Omega}_1)$ from direction $\boldsymbol{\Omega}_2$, and surface integration is over surfaces A_1 and

A_2. Eq. (6) is general and applies to any absorbing and scattering media, regardless of heterogeneity. The only assumption used in its derivation is that the scattering phase function has time-reversal symmetry, which is also needed for Eq. (2) to be valid. Fig. 1 illustrates the generality in the position and orientation of A_1 and A_2.

In the special case where the incident irradiance is independent of position (i.e., uniform over the illuminated area), Eq. (6) can be written as

$$\frac{\int_{A_2} I(\boldsymbol{r}, -\boldsymbol{\Omega}_2; A_1, \boldsymbol{\Omega}_1)\mathrm{d}\boldsymbol{r}}{\boldsymbol{\Omega}_1 \cdot \boldsymbol{n}F(A_1, \boldsymbol{\Omega}_1)} = \frac{\int_{A_1} I(\boldsymbol{r}, -\boldsymbol{\Omega}_1; A_2, \boldsymbol{\Omega}_2)\mathrm{d}\boldsymbol{r}}{\boldsymbol{\Omega}_2 \cdot \boldsymbol{n}F(A_2, \boldsymbol{\Omega}_2)} \tag{7}$$

where $F(A,\boldsymbol{\Omega})$ is the uniform irradiance over area A and from direction $\boldsymbol{\Omega}$, as depicted in Fig. 1. Each side of Eq. (7) has units of $\mathrm{m}^2\ \mathrm{sr}^{-1}$.

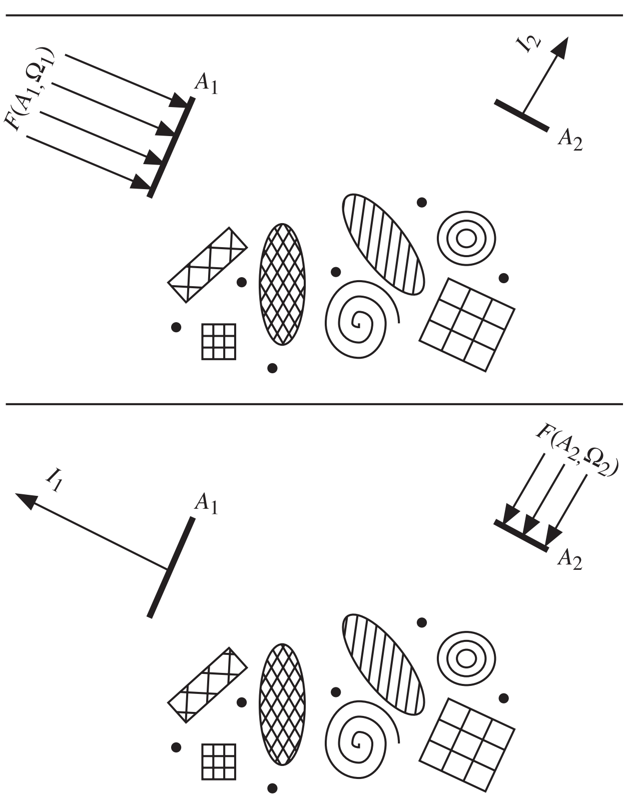

Fig. 1. The top and bottom panel represent an illustration of a reciprocal experiment in the context of Eqs. (6)–(9). The scattering and absorbing heterogeneous medium is represented by the odd geometric symbols. The area of illumination and measurement are represented by area A_1 and A_2. $I_2 = \frac{1}{a_2}\int_{A_2} I(\boldsymbol{r}, -\boldsymbol{\Omega}_2; A_1, \boldsymbol{\Omega}_1)\mathrm{d}\boldsymbol{r}$ and $I_1 = \frac{1}{a_1}\int_{A_1} I(\boldsymbol{r}, -\boldsymbol{\Omega}_1; A_2, \boldsymbol{\Omega}_2)\mathrm{d}\boldsymbol{r}$ in the context of Eq. (8). This illustration is meant to be completely general. Therefore, spatial scales do not matter (e.g., A_1 and A_2 can be larger or smaller than the photon mean free path within the medium). A_1 and A_2 may be in any orientation and position is space, as long as they are not embedded in the medium (see Di Girolamo, 1999). This includes A_2 being a small subset of A_1, which is a common (but not a general) arrangement when A_1 is the area being illuminated.

Let a_1 and a_2 be the surface areas of A_1 and A_2, respectively. Define the generalized BRDF as

$$f_r(A_2, -\boldsymbol{\Omega}_2; A_1, \boldsymbol{\Omega}_1) = \frac{\frac{1}{a_2}\int_{A_2} I(\boldsymbol{r}, -\boldsymbol{\Omega}_2; A_1, \boldsymbol{\Omega}_1)\mathrm{d}\boldsymbol{r}}{\boldsymbol{\Omega}_1 \cdot \boldsymbol{n}F(A_1, \boldsymbol{\Omega}_1)} [\mathrm{sr}^{-1}] \tag{8}$$

Substituting Eq. (8) into Eq. (7) yields the following reciprocal relationship:

$$a_1 f_r(A_2, -\boldsymbol{\Omega}_2; A_1, \boldsymbol{\Omega}_1) = a_2 f_r(A_1, -\boldsymbol{\Omega}_1; A_2, \boldsymbol{\Omega}_2) \tag{9}$$

Thus, the generalized BRDF measured over area A_2 in direction $-\boldsymbol{\Omega}_2$ when area A_1 is illuminated from direction $\boldsymbol{\Omega}_1$ will have the same magnitude as the generalized BRDF measured over area A_1 in direction $-\boldsymbol{\Omega}_1$ when area A_2 is illuminated from direction $\boldsymbol{\Omega}_2$ when properly weighted by a_1 and a_2. Note that the generalized BRDF and its reciprocal relationship encompass the definitions proposed by Greffet and Nieto-Vesperinas (1998) when A_1 and A_2 are reduced to points, and Snyder (2002) when $A_1 = A_2$.

3. Discussion

The generalized definition of the BRDF given in Eq. (8) has several advantages:

(1) The generalized definition of the BRDF has less restrictive conditions than the original definition given in Eq. (1), making it more amenable to real world situations. The only condition that remains is that the unidirectional irradiance over the illuminated area is uniform, which was necessary to go from Eq. (6) to Eq. (7). Without this condition, no reflectance quantity can be defined that obeys reciprocity.

(2) The generalized definition of the BRDF has the area of illumination and measurement as dependent variables, which reminds us that there are spatial scales to contend with when quantifying reflectance. This may help to interpret differences in measured BRDF of the same sample using different experimental apparatuses (e.g., Kim, 1988) and to avoid false claims of reciprocity failure (e.g., Venable, 1985).

(3) The generalized definition of the BRDF maintains a simple reciprocity principle given by Eq. (9).

From a practical standpoint, the generalized definition of the BRDF does not change how we estimate it from observations. The estimated generalized BRDF is still just the ratio of radiance to unidirectional irradiance, just as we have been measuring for years. The main difference is that the generalized BRDF stresses the importance of specifying the areas of illumination and measurement.

As an example, consider the case when the BRDF is estimated from a satellite measuring sunlight scattered from a heterogeneous forest canopy or clouds. In this case, the entire Earth's disc, E, is illuminated, and measurements are normally made over a much smaller area, M. If the solar illumination is from $\mathbf{\Omega}_1$ and measurement from $-\mathbf{\Omega}_2$, then it is straightforward to calculate $f_r(M,-\mathbf{\Omega}_2;E,\mathbf{\Omega}_1)$. If M is the area of a single instantaneous field-of-view referenced to some altitude at the top of the atmosphere, then the numerator in Eq. (6) is simply the measured radiance. The real difficulty arises when one tries to measure the reciprocal counterpart to $f_r(M,-\mathbf{\Omega}_2;E,\mathbf{\Omega}_1)$, namely $f_r(E,-\mathbf{\Omega}_1;M,\mathbf{\Omega}_2)$, because it is impossible to have the Sun illuminate only area M and nowhere else on the Earth's disk. But this does not make the proposed generalized definition of the BRDF impractical. In fact, one very practical aspect jumps out: $f_r(M,-\mathbf{\Omega}_2;E,\mathbf{\Omega}_1) \neq f_r(M,-\mathbf{\Omega}_1;E,\mathbf{\Omega}_2)$. Thus, we should not expect *directional* reciprocity (i.e., Eq. (2)) to be obeyed by *directionally* reciprocal sets of satellite measurements of reflected sunlight under global illumination. This is discussed in detail in Di Girolamo et al. (1998).

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