Pressure of the standard model

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Why do we want to know the pressure?

- To understand the hydrodynamics of heavy-ion collisions (only the QCD degrees of freedom are thermalized)

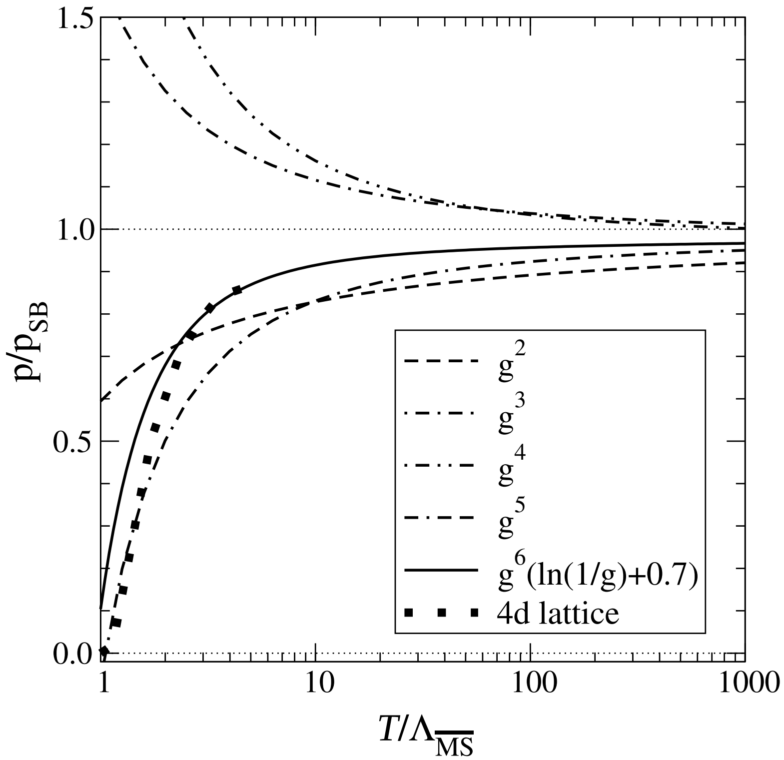
- Cosmology: the relic densities of particles left over from the Big Bang (WIMPs)

  - $\sim 10\%$ deviation from the ideal gas pressure leads to $\sim 1\%$ change in the relic density of WIMPs (Hindmarsh & Philipsen, Phys. Rev. D71, 087302 (2005))

- Theoretically important

$$\mathcal{Z} \;=\; \int \mathcal{D}\varphi \, \mathrm{e}^{-S_E[\varphi]}, \quad p = \frac{T}{V} \ln \mathcal{Z}.$$

QCD pressure



Kajantie *et al.*, Phys. Rev. D67, 105008 (2003).

- Pressure of QCD (with massless quarks) is studied extensively, expansion stretched to the last perturbatively computable term

- The expansion does not converge well unless the temperature is asymptotically high.

The electroweak sector

- Studies of electroweak thermodynamics centered on the electroweak phase transition (electroweak baryogenesis).

- Differences to the QCD computation
  - Presence of fundamental scalars driving the phase transition (crossover)
  - Another mass scale at tree level (Higgs mass), $p(T) \sim T^4 + \nu^2 T^2 + \nu^4$
  - Many independent couplings, formal powercounting:
    $g^2 \sim g'^2 \sim \lambda \; (\sim g^2_{Y,\mathrm{top}} \sim g_s^2)$, $\nu^2 \lesssim g^2 T^2$
  - Weakly coupled theory, expect the expansion to converge better.
    - Weakly coupled even close to the crossover transition!

Calculation of the pressure

- Straightforward perturbative expansions in the coupling constants inhibited by infrared divergences, $p(T)/T^4 \sim 1 + g^2 + g^4(1/\epsilon + 1) + \ldots$ ($d = 4 - 2\epsilon$)

- Can be cured by resummation

- Convenient framework: effective field theories

  – separate different mass scales into successive effective theories by integrating out the large scales one by one.

$$\begin{aligned}
\mathcal{L}_{\mathrm{E}} &= \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{2}\left(D_i A^a_0\right)^2 + \frac{1}{2}m_D^2 A^a_0 A^a_0 + \left|D_i\Phi\right|^2 + m_3^2\Phi^\dagger\Phi \\
&\quad + \mathcal{L}_{\mathrm{E,int}}(A^a_0, \Phi) \\
m_D^2 &\sim g^2T^2, \quad m_3^2 \sim -\nu^2 + g^2T^2, \quad g_3^2 = g^2T \quad \text{etc.}
\end{aligned}$$

- Recipe for calculating the pressure:
  - Compute the strict perturbative expansion of pressure in the full 4-dimensional theory, schematically

$$p_{\mathrm{E}}(T) \quad \sim \quad T^4 \left[1 + g^2 + g^4 \left(\frac{1}{\epsilon} + 1 + \ln \frac{\Lambda}{T} \right) + \mathcal{O}(g^6) \right].$$

  - Construct the 3-dimensional effective theory and compute its pressure, schematically

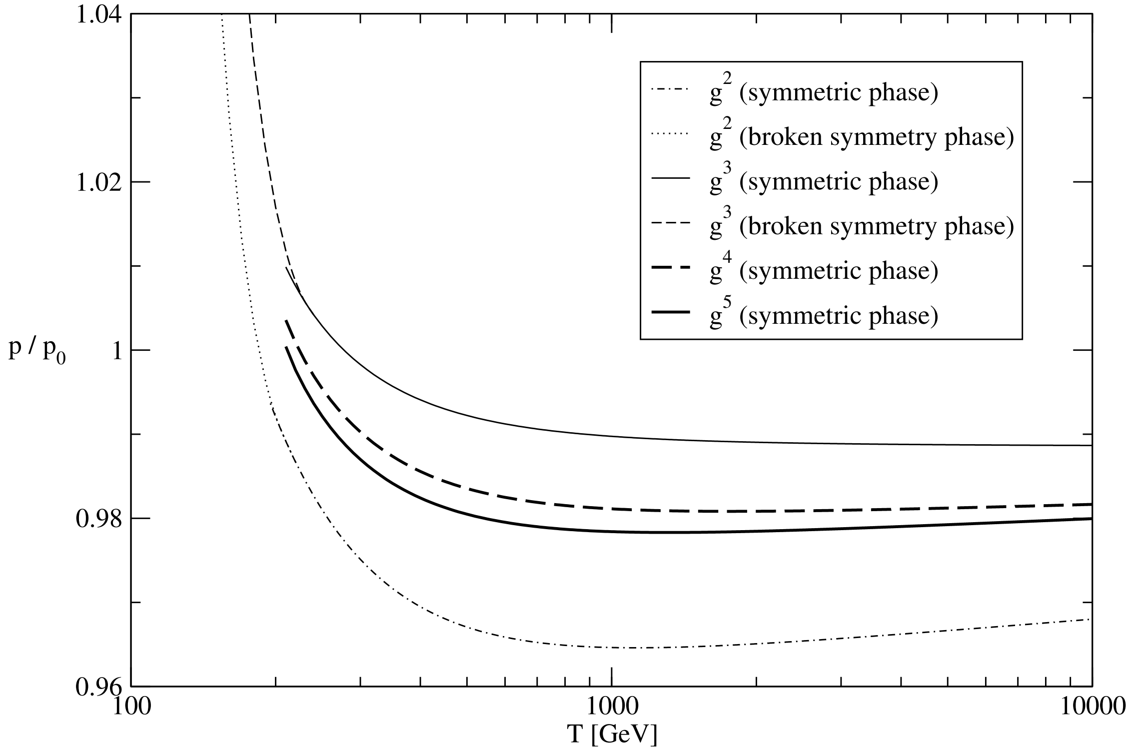
$$\frac{p_{\mathrm{M}}(T)}{T} \quad \sim \quad m^3 + g_3^2 m^2 \left(\frac{1}{\epsilon} + 1 + \ln \frac{\Lambda}{m} \right) + g_3^4 m (1 + \ln \frac{\Lambda}{m}) + \mathcal{O}(g_3^6).$$

$\Longrightarrow$ Pressure given by $p(T) = p_{\mathrm{E}}(T) + p_{\mathrm{M}}(T) + \mathcal{O}(g^6)$

$$\frac{p(T)}{T^4} \quad \sim \quad 1 + g^2 + g^3 + g^4(1 + \ln g) + g^5(1 + \ln g) + \mathcal{O}(g^6).$$

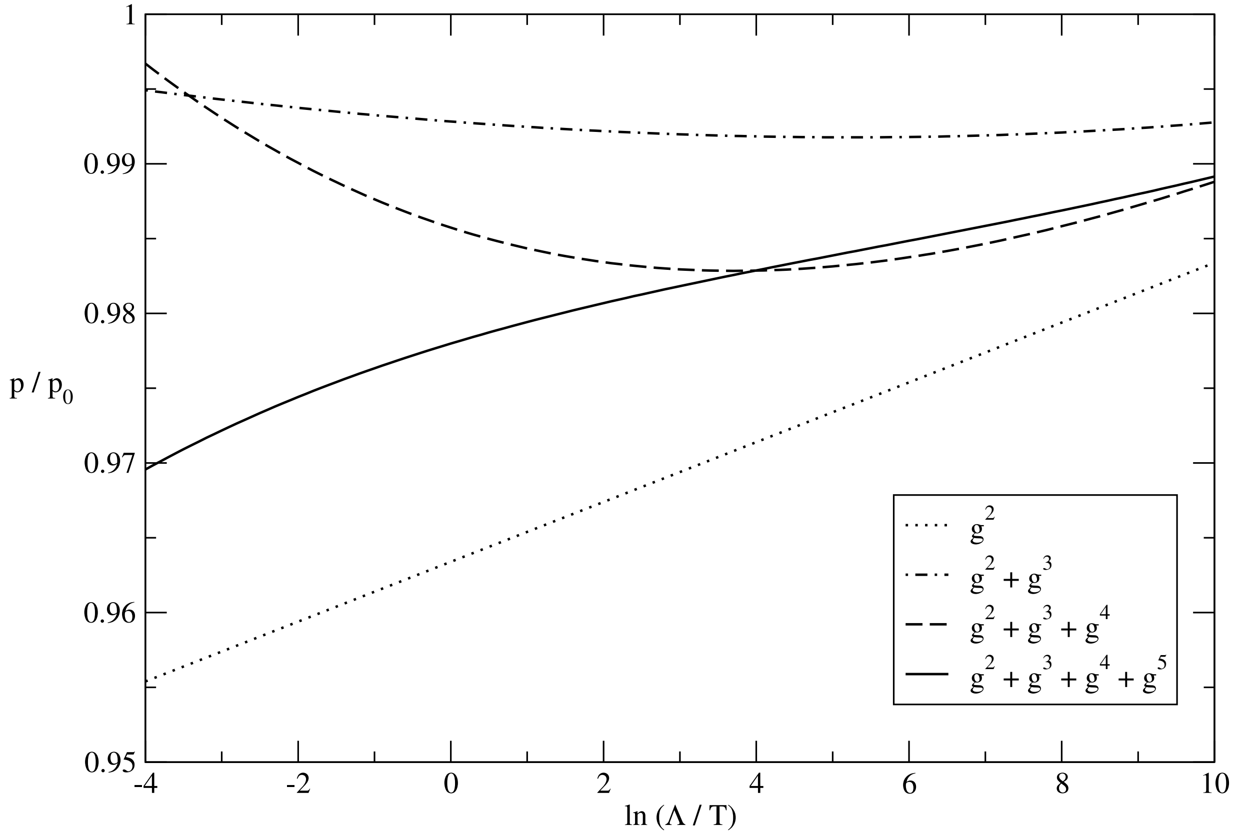
Numerical results

SU(2) + Higgs theory:



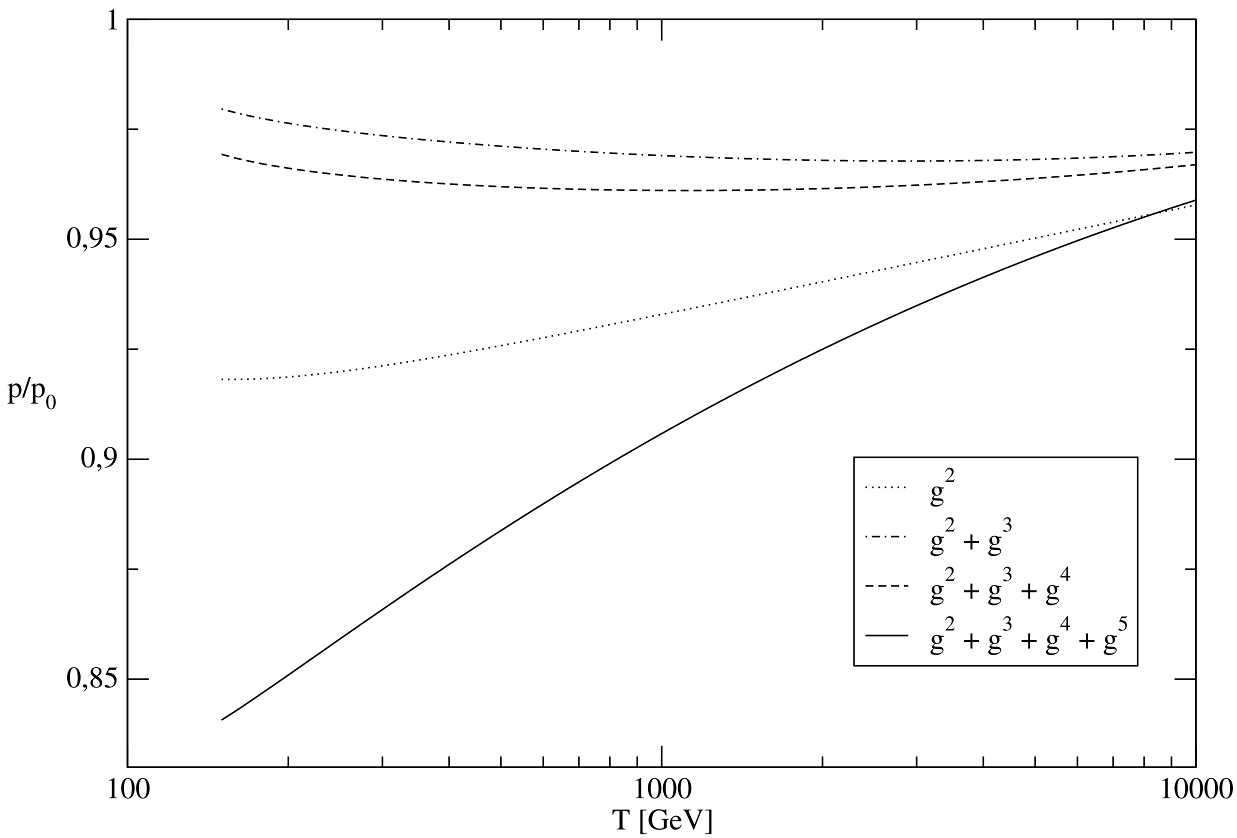
No large deviation from the ideal gas result, perturbative expansion appears to be converging well.

Scale dependence:



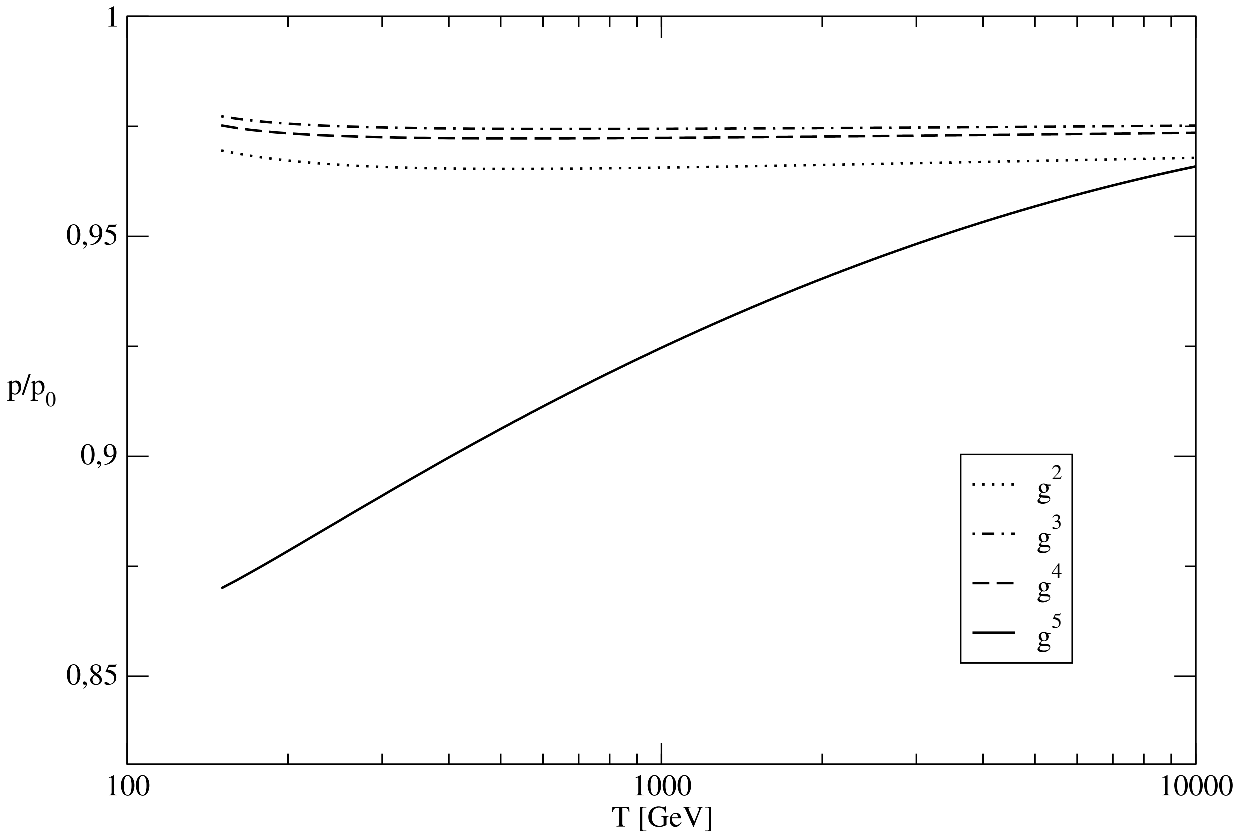
Only weak scale dependence, no significant reduction in the scale dependence as the expansion is stretched from $\sim g^2$ to $\sim g^5$.

Full standard model:



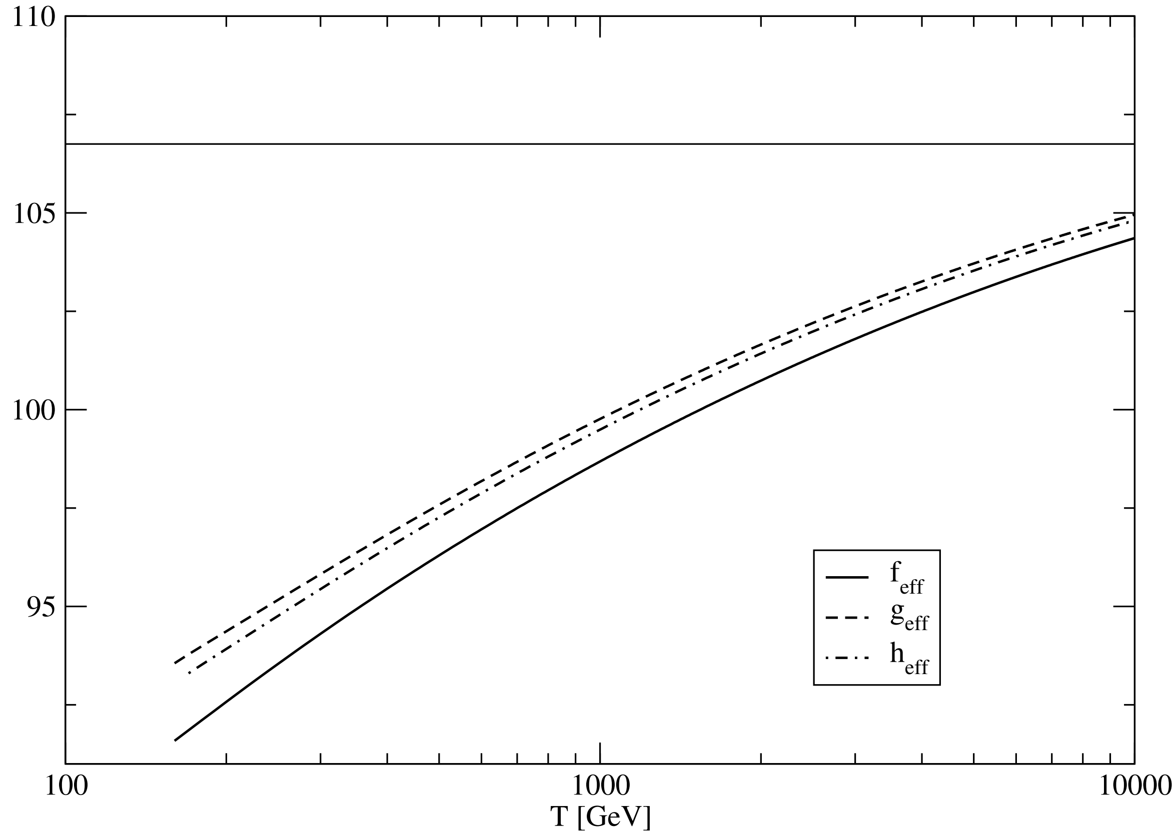
QCD contributions dominate (# of QCD degrees of freedom $= 79$ of the total of 106.75)

Full standard model – corrections due to massless QCD:

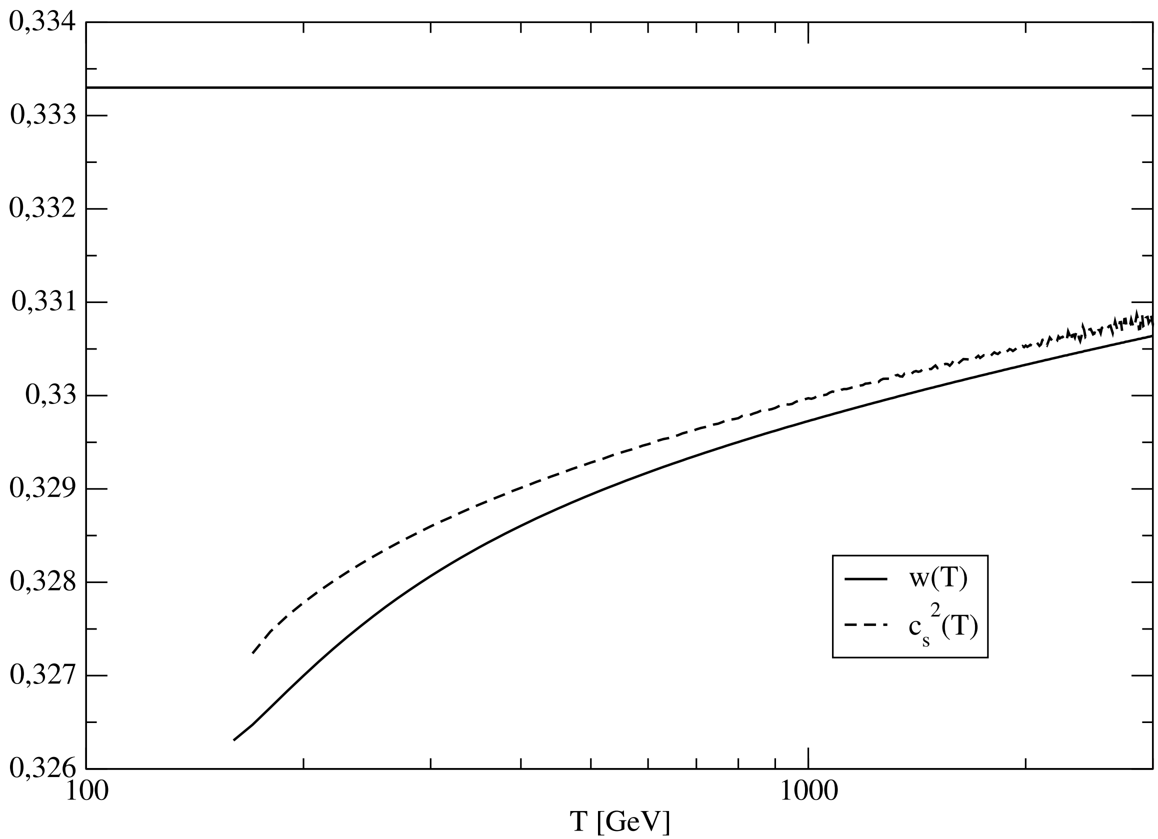


Large QCD contributions remain due to heavy top!

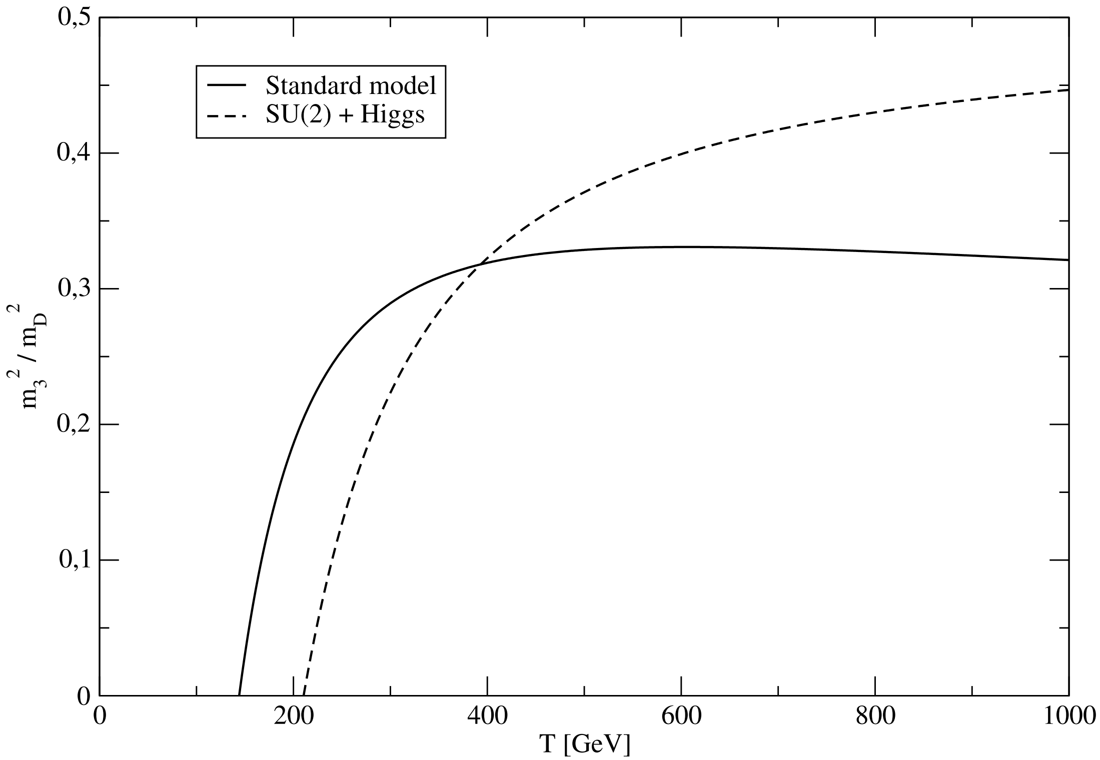
Number of bosonic degrees of freedom:



EoS variable $w(T) = p(T)/\epsilon(T)$ and the speed of sound $c_s^2(T) = p'(T)/\epsilon'(T)$:



What happens close to the electroweak crossover?



- Electroweak symmetry gets spontanously broken in a crossover transition as the temperature is lowered, driven by the Higgs field.

- Dimensional reduction as performed breaks down

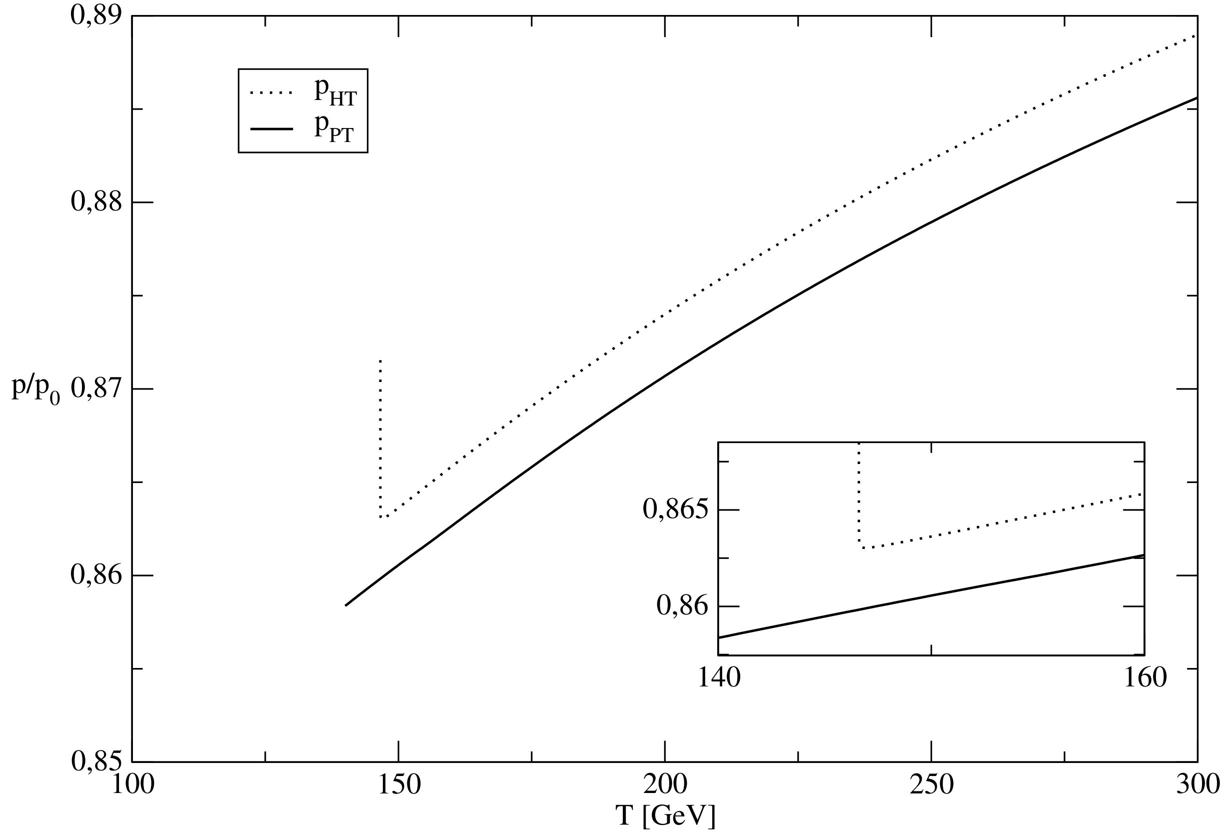
- Theory remains weakly coupled at the transition, can hope to use similar methods to calculate the pressure near the transition.

- $\Longrightarrow$ construct another effective theory by integrating out the adjoint scalars A_0^a

$$
\begin{aligned}
\mathcal{L}_{\mathrm{E2}} &= \frac{1}{4} F_{ij}^a F_{ij}^a + \left|D_i \Phi\right|^2 + \widetilde{m}_3^2 \Phi^\dagger \Phi + \widetilde{\lambda}_3 \left(\Phi^\dagger \Phi\right)^2, \\
\widetilde{m}_3^2 &\sim \underbrace{m_3^2}_{\sim g^3 T^2} - g_3^2 m_D \sim g^3 T^2.
\end{aligned}
$$

Then the contribution from the effective theories divides into two parts:

$$
\begin{aligned}
p_{\mathrm{M}}(T) &= p_{\mathrm{M1}}(T) + p_{\mathrm{M2}}(T), \\
\frac{p_{\mathrm{M1}}(T)}{T} &\sim m_D^3 + g_3^2 m_D^2 (1/\epsilon + 1 + \ln \frac{\Lambda}{m_D}) \\
&\quad + g_3^4 m_D (1/\epsilon + 1 + \ln \frac{\Lambda}{m_D}) + \ldots, \\
\frac{p_{\mathrm{M2}}(T)}{T} &\sim \widetilde{m}_3^3 + \widetilde{g}_3^2 \widetilde{m}_3^2 (1/\epsilon + 1 + \ln \frac{\Lambda}{\widetilde{m}_3}) + \ldots
\end{aligned}
$$



- Singular behavior cured.
- A new class of diagrams resummed to the expansion.

Conclusions

- Pressure of the full standard model is calculated to three loop order.
- Deviation from the ideal gas is large, the expansion of the electroweak sector converges better.
- New type of terms in the expansion:
  - Another mass scale leading to qualitatively different type of terms in the expansion, $\nu^2 T^2$, ν^4.
  - Terms of order $g^5 \ln g$ from the fundamental scalars.
- Potential concequences to WIMP relic density.