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Large electroweak penguins in $B \to \pi\pi$ and $B \to \pi K$: implication for new physics

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We show that the anomalies in the $B \to \pi\pi$ decay channels, *viz.*, the large branching ratio of $B \to \pi^0\pi^0$ and large direct CP asymmetry of $B \to \pi^+\pi^-$, can both be explained, even in the framework of existing theoretical models which predict the relative strengths of the tree and the strong penguin amplitudes, if there is an abnormally large electroweak penguin contribution. We also critically examine the expectation of a similarly large electroweak penguin in the $B \to \pi K$ sector, and show that it can be accomodated, and may even be motivated from an SU(3) flavor symmetry between the $B \to \pi\pi$ and the $B \to \pi K$ amplitudes, but is not necessary to explain the data. We emphasize that if the experimental numbers on these charmless nonleptonic channels survive, they may turn out to be an indirect signal of new physics. Current data is insufficient to discriminate betwen flavor-specific and flavor-blind new physics scenarios.

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I. INTRODUCTION

Nonleptonic B meson decays pose a serious theoretical challenge to us. The main difficulty is to handle the uncertainties of low-energy QCD; to be very precise, to calculate the long-distance contribution $\langle M_1 M_2|\mathcal{H}_{eff}|B\rangle$. There are at least three models with varied degrees of sophistication to tackle the said uncertainty: the conventional factorization (CF) model [1, 2], the QCD-improved factorization (QCDF) model [3, 4] and the perturbative QCD (PQCD) model [5, 6]. These models provide definite theoretical predictions for branching ratios (BR) and CP asymmetries (A_{CP}) for nonleptonic charmless B decay modes. We still do not have comparable numbers from other approaches, *e.g.*, the soft-collinear effective theory (SCET) model [7], though they have predictions for some of the $B \to \pi\pi$ amplitudes.

One of the major obstacles is to calculate the strong phase, and hence the CP asymmetry, theoretically. (The direct CP asymmetry depends both on the strong phase difference and the weak phase difference of the two interfering amplitudes, and is zero if any one of them vanishes.) The color-transparency argument of Bjorken [8] is not expected to hold beyond the color-allowed tree decays. In the CF model the strong phases are computed from the imaginary parts of the respective Wilson coefficients [2], and no soft final-state interactions are taken into account. Among the more sophisticated versions, QCDF predicts a rather small strong phase difference between the dominant amplitudes. PQCD predictions are different as annihilation and exchange topologies are given more weightage than in QCDF (and the penguin amplitude is also enhanced), and they can generate a sizable strong phase. (A comparative study of these two models can be found in [4]). This is why the predictions for CP asymmetries in QCDF and PQCD are sometimes even opposite in sign. Needless to say, the relative importance of different amplitudes varies from model to model. In particular, strong penguins are much larger in the PQCD model due to a dynamical enhancement. There are, however, some common trends, which can be justified even intuitively. For example, the tree-amplitude in $B \to \pi^+\pi^-$ dominates over the strong penguin, whereas the situation is opposite for $B \to \pi^- K^+$. This is entirely due to the relative importance of the CKM matrix elements.

All these models also agree on the fact that the electroweak penguins (EWP) are much smaller than the strong penguins. That can be seen very easily, by comparing the relevant Wilson coefficients. Intuitively, it follows from the smallness of α compared to the strong coupling constant α_s. In fact, one can safely neglect all the electroweak penguins except that mediated by a top quark in the loop. However, one must remember that the strong phases coming from gluonic penguins (which is necessarily $I = 0$) need not be the same as those coming from γ, Z penguins (they do not have a definite isospin). Again, this complication does not matter since the EWPs are supposed to be very small. There is no known mechanism which can enhance the EWP amplitude to the level of, say, the corresponding gluonic penguins.

The predictions from the theoretical models are supposed to be robust when both the daughter mesons are light. For example, BRs and A_{CP}s have been computed for all the $B \to \pi\pi$ and $B \to \pi K$ modes. Such numbers mostly agree with each other, if we take into account the theoretical uncertainties. This leads us to believe that the leading predictions of these models may be taken seriously. These include the relative importance of the tree and strong penguin amplitudes, the prediction for strong phase differences, et cetera.

However, as is well known and will be substantiated in the next section, some of the experimental numbers are at variance with the theoretical predictions, though most of them agree. In the $B \to \pi\pi$ sector, there are three such discrepancies: (i) $Br(B \to \pi^0\pi^0)$ is abnormally large (the largest prediction is from PQCD, which is about 5 times less); (ii) $Br(B \to \pi^+\pi^-)$ is slightly on the lower side (the lowest prediction is again from PQCD, which is more than 2σ above the data; QCDF has a large theoretical uncertainty [4], and taking everything into account, may just fit the data); and (iii) the direct CP asymmetry in $B \to \pi^+\pi^-$ is larger than theoretical prediction, irrespective of the model chosen. One must be cautious: the error bars, particularly for the CP asymmetry data, are still large, and it may be too early to draw any definite conclusions based on them. Still, the trend is worth investigating.

In contrast, the individual BRs and A_{CP}s in the $B \to \pi K$ sector are more or less in agreement with the theory. But when one eliminates the theoretical uncertainties by taking suitable ratios of the BRs, it is claimed [9] that the EWP contribution is abnormally large—by about an order of magnitude. It is more puzzling since it appears that we understand the radiative decay $b \to s\gamma$ fairly well. We will critically investigate this issue also. It will be shown that solutions with vanishingly small EWP amplitudes can be obtained even with a rough SU(3) flavor symmetry between the tree and the strong penguin amplitudes of $B \to \pi\pi$ and $B \to \pi K$ channels. The strong phase difference between these two amplitudes for $B \to \pi K$ is small (modulo π), but is different from the analogous quantity for $B \to \pi\pi$ decays, which is exactly what is predicted in PQCD. However, a substantially large EWP can be easily accomodated in the data. But there is no apparent conflict between the $B \to \pi K$ channels and the radiative decay $b \to s\gamma$.

There are two ways of analyzing the data. First, one can take a particular model of his or her choice, and fit the data to get an idea of the relatively poorly known parameters. The trouble is that equal justice cannot be done to all the experiments. (A well-known example is the pre-2003 CKMfitter fit of the $B \to \pi^+\pi^-$ asymmetry using the BaBar data only, within the context of QCDF; the Belle data was so far away that it could not be fitted.) The second approach [9, 10] is to analyze the data without paying any heed to the models, except, maybe, for some basic symmetry ideas like flavor SU(3). As expected, the fitted amplitudes and strong phases, all a priori unknown, come out to be different from the model predictions.

The question is whether these discrepancies are due to the fact that we do not understand the low-energy QCD well, or whether there is new physics (NP) beyond the Standard Model (SM). If there is NP, that will generate some more effective four-Fermi operators, leading to new contributions in B decays. If these operators are flavor-blind, they should mimic the strong penguins, whereas the flavor-specific operators should appear more like a modification to the tree or the EWP amplitudes. The exact structure, of course, will depend on the specific model. Unfortunately, due to lesser number of $B \to \pi\pi$ channels, it is not possible to extract an observable which will clearly point to some abnormality in the EWP sector, and we have to look for an indirect answer.

The large BR of the $\pi^0\pi^0$ channel tells that if unknown EWP dynamics is the cause of the enhancement, the EWP amplitude should be quite large (we will later see that it is almost as large as the color-allowed tree amplitude). It is difficult to have such a large amplitude from intuitive arguments. The same may be said for the $B \to \pi K$ channels. (That is why we prefer a NP solution for the puzzle.)

This brings us to the justification of our approach. We assume that the model predictions are sensible, as far as the tree and strong penguins (and annihilation and exchange diagrams) go, and as a typical example we take the PQCD model. Qualitatively same results are found for other models too, but there is a reason why we choose PQCD. As we will show, a fit to the $B \to \pi\pi$ data needs large penguin contribution, and only PQCD can provide such a large enhancement, so that this is a more conservative approach to look for NP effects. Also, a large $|P/T|$ helps the angle γ to be fitted in the first quadrant, in accordance with the standard CKM fit [11]. We treat the parameters of the EWP sector as unknowns, and fit them from experimental data. This will show us that it is the $\pi\pi$ sector, rather than πK, that contains the puzzling EWP behaviour; and one should expect some deviation from the SM prediction for $B \to \rho\gamma$, instead of $B \to K^*\gamma$. We will also discuss how a large EWP amplitude can be correlated between $B \to \pi\pi$ and $B \to \pi K$ modes.

However, the main emphasis lies elsewhere. Such an analysis is the first step to look for effects coming from NP, since a number of NP models mimic EWP dynamics, by having unequal strength of $b \to u\overline{u}d$ and $b \to d\overline{d}d$ amplitudes. There are numerous examples, starting from models with an extra Z' [12] and/or exotic quarks (*e.g.*, vector singlets) to R-parity conserving and violating supersymmetry [13]. We do not go into any of these specific models; this is a model-independent study to show that such models may have an interesting future.

We wish to point out that this study implicitly assumes the unitarity of a 3×3 CKM matrix (thus, allowed ranges for the amplitudes for a model with a fourth chiral generation should be different). A weaker assumption is no NP effect in $B^0-\overline{B^0}$ mixing. Such an effect modifies the phase of the $B^0-\overline{B^0}$ box, and β as measured from $B \to J/\psi K_S$ need not be the true β. This also weakens the limits on V_{td}. Inclusion of such NP effects in $B^0-\overline{B^0}$ mixing does not invalidate our conclusions; it only changes the allowed range of the angle γ of the Unitarity Triangle (UT).

The paper is arranged as follows. In Section II, we tabulate the experimental and theoretical inputs for the analysis. Section III and IV deal with the $B \to \pi\pi$ and $B \to \pi K$ data respectively. In Section V, we summarize and conclude.

II. THEORETICAL AND EXPERIMENTAL INPUTS

Let us first set our notations and conventions. We work in the (α, β, γ) convention of the UT. We use B to indicate a flavor-untagged B^0 or $\overline{B^0}$, or even a charged B meson when there is no chance for confusion.

The CP asymmetry for $B^+ \to f$ is defined as

$$A_{CP} = \frac{\Gamma(B^+ \to f) - \Gamma(B^- \to \overline{f})}{\Gamma(B^+ \to f) + \Gamma(B^- \to \overline{f})}, \tag{1}$$

which agrees with the convention of [2, 3, 4] but is opposite to that used by [5, 14]. The same convention (according to the quark content, b or $\overline{b}$) is used for neutral B decays to flavor-specific final states, like $\pi^\pm K^\mp$. For a flavor-nonspecific final state f (*e.g.*, $\pi^+\pi^-$), we define $l = \exp(-i2\beta)\langle f|\mathcal{H}|\overline{B^0}\rangle / \langle f|\mathcal{H}|B^0\rangle$, and $a^d_{CP} = (1-|l|^2)/(1+|l|^2)$, $a^m_{CP} = 2\mathrm{Im}l/(1+|l|^2)$. For the $\pi^+\pi^-$ system, they are related to the $S_{\pi\pi}$ and $C(A)_{\pi\pi}$ of BaBar and Belle by

$$S_{\pi\pi} = -a^m_{\pi\pi}, \; C_{\pi\pi} = -A_{\pi\pi} = a^d_{\pi\pi}. \tag{2}$$

We also use the following valence quark convention:

$$\begin{aligned} &B^0 \equiv \overline{b}d, B^+ \equiv \overline{b}u, K^0 \equiv \overline{s}d, \\ &K^+ \equiv \overline{s}u, \pi^+ \equiv u\overline{d}, \pi^0 \equiv (u\overline{u} - d\overline{d})/\sqrt{2}, \end{aligned} \tag{3}$$

and for the antiparticles, the quark contents are just reversed, without any additional $-$ sign. This is different from those used by the Gronau-Rosner group [10], but is consistent with, say, [2]. In the convention of [10], the following mesons have an extra minus sign in their wavefunctions: B^-, K^-, π^0, π^-. This obviously does not change the BRs or A_{CP}s. However, they change the amplitude sum rules. This will be discussed later in this section.

The experimental data (updated for Winter 2004), taken from the HFAG website [14] and quoted at 68% confidence level (CL), is as follows.

The BRs, multiplied by 10^6, are

$$\begin{aligned} Br(\pi^+\pi^-) &= 4.6 \pm 0.4; \\ Br(\pi^0\pi^0) &= 1.9 \pm 0.5; \\ Br(\pi^\pm\pi^0) &= 5.2 \pm 0.8; \\ Br(K^0\pi^+) &= 21.8 \pm 1.4; \\ Br(K^+\pi^0) &= 12.8^{+1.1}_{-1.0}; \\ Br(K^+\pi^-) &= 18.2 \pm 0.8; \\ Br(K^0\pi^0) &= 11.9^{+1.5}_{-1.4}. \end{aligned} \tag{4}$$

Mode	CF	QCDF	PQCD
$\pi^+\pi^-$	9.0 - 12.0	3.5 - 15.0	5.5 - 9.0
	19.7 - 21.6	-4 - 21	-30 - -16
$\pi^0\pi^0$	0.35 - 0.63	0 - 0.07	0.2 - 0.4
	-42.2 - 45.9	-100 - 0	
$\pi^+\pi^0$	3.0 - 6.8	4.9 - 6.1	2.6 - 5.0
	0.0 - 0.1	0.0	0.0
$K^+\pi^-$	14.0 - 18.0	3.0 - 24.5	13.0 - 18.6
	-10.6 - -3.7	-13 - -6	12.9 - 21.9
$K^0\pi^0$ ‡	5.0 - 7.4	1.5 - 10.0	8.3 - 8.9
	24.4 - 36.5	0 - 7	
$K^+\pi^0$	9.4 - 12.0	2.0 - 16.0	7.6 - 11.0
	-9.2 - -2.6	-10.6 - 3.0	10.0 - 17.3
$K^0\pi^+$ ‡	14.0 - 22.0	5.5 - 26.0	13.7 - 19.7
	-1.5 - -1.3	-1.75 - 0	0.6 - 1.5

TABLE I: Predictions for BRs (first row, multiplied by 10^6) and A_{CP}s (multiplied by 100, and in our convention) of different nonleptonic modes. The numbers are taken from [2, 4, 5, 6]. The uncertainties are not treated equally; in particular, QCDF pedictions take into account more sources of uncertainty than the other two. The UT angle γ is fixed to be in the first quadrant. For the modes indicated by ‡, the CP asymmetry is measured with a K_S in the final state.

The CP asymmetries are

$$\begin{aligned} a^d_{\pi\pi} = C_{\pi\pi} &= -0.46 \pm 0.13; \\ a^m_{\pi\pi} = -S_{\pi\pi} &= 0.74 \pm 0.16; \\ A_{CP}(\pi^+\pi^0) &= -0.07 \pm 0.14; \\ A_{CP}(K^0\pi^+) &= -0.02 \pm 0.06; \\ A_{CP}(K^+\pi^0) &= 0.00 \pm 0.07; \\ A_{CP}(K^+\pi^-) &= 0.095 \pm 0.028; \\ A_{CP}(K^0\pi^0) &= -0.03 \pm 0.37; \\ a^d_{K_S\pi^0} = C_{K_S\pi^0} &= 0.40^{+0.27}_{-0.28} \pm 0.09; \\ a^m_{K_S\pi^0} = -S_{K_S\pi^0} &= 0.48^{+0.38}_{-0.47} \pm 0.28. \end{aligned} \tag{5}$$

The theoretical predictions based on different models are given in Table 1. The 'large' BRs (*i.e.*, except that of $B \to \pi^0\pi^0$) more or less agree with each other; there is a nontrivial overlap region. Also to be noted that only the BR of $B \to \pi^0\pi^0$ is *significantly* off the mark. The predictions for A_{CP}s differ, mainly because an enhancement of penguins and annihilation topologies in the PQCD model. However, even taking into account all the theoretical uncertainties, some of the experimental resuls do not tally with the predictions. The error bars are large, and ultimately the difference might go away, but this is high time to be prepared for any indirect signal of NP.

Let us first discuss the theory of $B \to \pi\pi$. We follow the standard convention of writing the amplitudes [9] (*i.e.*, putting the top-mediated penguins together with up- and charm-mediated penguins by using the unitarity relationship $V_{tb}V^*_{td/s} = -V_{ub}V^*_{ud/s} - V_{cb}V^*_{cd/s}$) except that the EWPs, both color-allowed and color-suppressed, are written separately. This will help us to understand the

EWP dynamics of the $\pi\pi$ sector. The amplitudes are

$$
\begin{aligned}
A(\overline{B} \to \pi^+\pi^-) &= |T_c|e^{-i\gamma} + |P_c|e^{i(\delta_P - \delta_T)} \\
&\quad + |P_{EW}^C|e^{i(\delta_{EC}-\delta_T)}\mathcal{G}, \\
\sqrt{2}A(\overline{B} \to \pi^0\pi^0) &= -|C|e^{-i\gamma} + |P_c|e^{i(\delta_P-\delta_C)} \\
&\quad -\frac{1}{2}|P_{EW}^C|e^{i(\delta_{EC}-\delta_C)}\mathcal{G} \\
&\quad +\frac{3}{2}|P_{EW}|e^{i(\delta_E-\delta_C)}\mathcal{G}, \\
\sqrt{2}A(B^- \to \pi^-\pi^0) &= |T_c|e^{-i\gamma} + |C|e^{-i\gamma}e^{i(\delta_C-\delta_T)} \\
&\quad +\frac{3}{2}|P_{EW}^C|e^{i(\delta_{EC}-\delta_T)}\mathcal{G} \\
&\quad -\frac{3}{2}|P_{EW}|e^{i(\delta_E-\delta_T)}\mathcal{G},
\end{aligned}
\tag{6}
$$

where $\mathcal{G} = 1 + R_b \exp(-i\gamma)$, and $R_b = |\lambda_u/\lambda_c| = 0.37 \pm 0.04$, with $\lambda_i = V_{ib}V_{id}^*$. Conventionally, T_c and C contain, apart from the tree-level amplitudes, those parts of the strong penguin that are proportional to $e^{-i\gamma}$. For simplicity (and also guided by intuitive arguments) we take only the top-mediated EWP amplitudes; they are proportional to λ_t, and can be written in terms of λ_u and λ_c using the unitarity argument. Note that we have extracted the strong phase associated with the leading term, since only the difference of strong phases is a measurable quantity. The annihilation contributions have been neglected, but can be suitably dumped with T_c or P_c. To calculate the respective BRs, one has to remember that B^+ lives longer than B^0 by a factor of 1.086 ± 0.017 [15].

Due to our slightly different valence quark convention, the amplitude sum rule looks like

$$
A(\pi^+\pi^-) - \sqrt{2}A(\pi^0\pi^0) = \sqrt{2}A(\pi^+\pi^0). \tag{7}
$$

This convention, of course, yields identical results to that of, say, [10].

Thus, there are ten free parameters; five amplitudes, four independent strong phase differences and the weak phase γ. The observables are far less in number: only six, namely, three BRs and three CP asymmetries. One therefore cannot say in a model-independent way that the EWPs are going to be large. However, if one neglects the EWPs, the number of free parameters gets reduced by four (two amplitudes and two strong phases), so one can extract a unique solution [9]. We, on the other hand, will take the model predictions seriously. For example, we use the PQCD prediction $R_c = |P_c/T_c| = 0.23^{+0.07}_{-0.05}$ and $-41^\circ < \delta \equiv \delta_P - \delta_T < -32^\circ$. We also use the CKM fit value for γ: $50^\circ < \gamma < 72^\circ$ at 68% CL. This gives us three more theoretical inputs.

Next we come to the $B \to \pi K$ modes. One can invoke the flavor SU(3) and relate the $B \to \pi\pi$ amplitudes to the $B \to \pi K$ ones. The symmetry is supposed to be broken only by the corresponding CKM factors and the ratio of the meson decay constants f_K/f_π. This in turn implies that one takes the chiral enhancement factors coming from $(V-A)\otimes(V+A)$ type operators (O_5-O_8 in the standard literature) to be the same in both cases. Let us follow a different approach. We start with the assumption that the amplitudes are independent of each other; later we will see whether there is any trace of the flavor symmetry. If one can relate the amplitudes by SU(3), that will act only as a further boost to our analysis. In fact, the PQCD relationship between the amplitudes T_c and P_c, and the analogous one for $b \to s$ transitions, $|T_c'/P_c'| = 0.201 \pm 0.037$, is consistent with the SU(3) expectation

$$
\begin{aligned}
T_c' &= T_c \frac{f_K}{f_\pi}\frac{V_{us}}{V_{ud}} = 0.28 T_c; \\
P_c' &= P_c \frac{f_K}{f_\pi}\frac{V_{cs}}{V_{cd}} = -5.3 P_c,
\end{aligned}
\tag{8}
$$

where primes indicate $b \to s$ transitions. Note that PQCD also predicts the strong phase difference between P_c' and T_c' to be about 156° [6]. We, however, will not use that as an input; rather, we will show that the allowed solutions, obtained just from fitting the data, are entirely consistent with the PQCD prediction.

We use the Wolfenstein representation of the CKM matrix, and define $\overline{\lambda} = l/(1 - l^2/2) \approx 0.230$, and

$$
R_b{}' = R_b(d \to s) = R_b \frac{V_{us}}{V_{ud}}\frac{V_{cd}}{V_{cs}} = -\overline{\lambda}^2 R_b. \tag{9}
$$

Remember that V_{cd} is negative, hence the minus sign. We assume that there is no effect of NP or unknown SM dynamics in $B^0-\overline{B^0}$ mixing, and the phase from the $B^0-\overline{B^0}$ box is $\sin(2\beta)$, which has been directly measured as well as fitted. We use the CKMfitter fit result $\sin(2\beta) = 0.739 \pm 0.048$. It was discussed earlier what happens if one relaxes this assumption and allows NP effects in the mixing.

The amplitudes can be written in an analogous way to that of the $B \to \pi\pi$ ones. There is, howver, one important difference. The modes $B^+ \to K^0\pi^+$ and $B \to K^0\pi^0$ have no tree-level contributions. The strong penguin, which after rearrangement was absorbed into T_c, now gives a different amplitude $P_{ct}' \equiv P_c' - P_t'$ with a different strong phase δ_{CT}' (we use primes to indicate $b \to s$ transitions).

$$\begin{aligned}
A(\overline{B} \to K^-\pi^+) &= |T_c'|e^{-i\gamma} + |P_c'|e^{i(\delta_P'-\delta_T')} + |P_{EW}^{C}{}'|e^{i(\delta_{EC}'-\delta_T')}\mathcal{G}', \\
A(B^- \to \overline{K^0}\pi^-) &= -|P_c'| - |P_{ct}'|e^{-i\gamma}e^{i(\delta_{CT}'-\delta_P')} + \frac{1}{2}|P_{EW}^{C}{}'|e^{i(\delta_{EC}'-\delta_P')}\mathcal{G}', \\
\sqrt{2}A(B^- \to K^-\pi^0) &= |T_c'|e^{-i\gamma} + |P_c'|e^{i(\delta_P'-\delta_T')} + |P_{EW}^{C}{}'|e^{i(\delta_{EC}'-\delta_T')}\mathcal{G}' + \\
&\quad \left[\overline{\lambda}|\tilde{C}|e^{-i\gamma}e^{i(\delta_{\tilde{C}}-\delta_T)} + \frac{1}{\overline{\lambda}}\frac{3}{2}|P_{EW}|e^{i(\delta_E-\delta_T)}\mathcal{G}'\right] \\
\sqrt{2}A(\overline{B} \to \overline{K^0}\pi^0) &= |P_c'| + |P_{ct}'|e^{-i\gamma}e^{i(\delta_{CT}'-\delta_P')} - \frac{1}{2}|P_{EW}^{C}{}'|e^{i(\delta_{EC}'-\delta_P')}\mathcal{G}' + \\
&\quad \left[\overline{\lambda}|\tilde{C}|e^{-i\gamma}e^{i(\delta_{\tilde{C}}-\delta_P)} + \frac{1}{\overline{\lambda}}\frac{3}{2}|P_{EW}|e^{i(\delta_E-\delta_P)}\mathcal{G}'\right]. \qquad (10)
\end{aligned}$$

In the above expressions, $\mathcal{G}' = 1 + R_b' e^{-i\gamma}$, and $\tilde{C}$ is the same as C (in $B \to \pi\pi$) except that there is no strong penguin (this difference is non-negligible). In the last two amplitudes, there is a part where the π is emitted from the B meson; this, expectedly, is very similar in structure to that of $B \to \pi\pi$. In these terms, the relative sign between $\overline{\lambda}$ and $\overline{\lambda}^{-1}$ terms is positive whereas in the analogous expression for $B \to \pi\pi$, that is negative; this is because V_{cd} is negative. P_{EW} is, of course, the same as that in $B \to \pi\pi$. Thus, there are five new amplitudes and four strong phases.

III. ANALYSIS: $B \to \pi\pi$

The main emphasis of the $B \to \pi\pi$ analysis is to extract, unambiguously, a signature for large EWP amplitudes. In the SM, we expect T_c to be the largest amplitude, while P_c and C are smaller than T_c. In fact, models like PQCD predict $|P_c/T_c| \sim 0.25$, while the predictions from other models are generally below 10%. There is no such prediction available for C, but one can expect $|C/T_c| < 1$. The EWP amplitudes are expected to be highly suppressed compared to these. The color suppression further diminishes P_{EW}^C compared to P_{EW}.

For those readers who do not want to go through the nitty-gritties of the analysis, let us state our conclusions right here. We will see that the amplitudes T_c and C turn out to be as expected (P_c is related to T_c), and though the color-allowed EWP P_{EW} may happen to be large, there are a number of solutions (we have more unknowns than equations, so there is a large number of possible solutions) where this turns out to be small, almost as small as expected from the theory. Later, we will see that only those small values are allowed by the $B \to \pi K$ data. The surprise comes from the color-suppressed EWP amplitude P_{EW}^C; it turns out to be large, larger than P_c, and almost as large as T_c! There are other interesting features of the analysis; we discuss them at the end of this section.

We start by treating all the amplitudes as free parameters, with the exception of P_c. The latter is varied over its entire range allowed by the theoretical uncertainties of $|P_c/T_c|$. The other four amplitudes are varied over the range 0-6 $\times 10^{-8}$. We will see later that this is a rather conservative range. All the strong phase differences are varied over the range 0-2π, except $\delta_P - \delta_T$, which is again constrained by the theory. The weak angle γ is varied between 50° and 72°.

From the solutions that satisfy the experimental constraints, we get the following ranges for the amplitudes:

$$\begin{aligned}
2.5 < &\quad T_c \times 10^8 &< 4.1; \\
0.4 < &\quad P_c \times 10^8 &< 1.2; \\
0 < &\quad C \times 10^8 &< 3.8; \\
0 < &\quad P_{EW} \times 10^8 &< 2.5; \\
1.0 < &\quad P_{EW}^C \times 10^8 &< 3.7. \qquad (11)
\end{aligned}$$

Note that apart from the last line, none of these are unexpected. One may object that C is supposed to be smaller than T_c; if we take that constraint into account nothing changes except the upper limit of C; it becomes 3.5×10^{-8}. However, there is a concenration of solution points in the $C - P_{EW}$ plane near the origin (see Fig. 1). This vindicates the conjecture that both these amplitudes may be small. Later we will see from the $B \to \pi K$ channels that the upper limit of P_{EW} is smaller than the range predicted here by about 40%.

Why P_{EW}^C is so large? The main culprit is the $B \to \pi^0\pi^0$ BR. This, along with the facts that the other two BRs are more or less in the expected ballpark, while the direct CP asymmetry in $B \to \pi^+\pi^-$ is large (*i.e.*, there must be a significant interfering amplitude) generates a large P_{EW}^C. See Fig. 2 to have an idea of the solution points for P_{EW}^C; in fact, if we claim that P_{EW} is small, the range for P_{EW}^C becomes 2-3$\times 10^{-8}$. This is a large amplitude by any standard, and cannot be accomodated in any theoretical model.

The strong phase differences do not show any interesting pattern, except, again, for $\delta_{EC} - \delta_T$. Varied between 0 and 2π, solutions come out only for the range $174° < \delta_{EC} - \delta_T < 190°$. This shows a large destructive interference between P_{EW}^C and T_c, which is necessary to

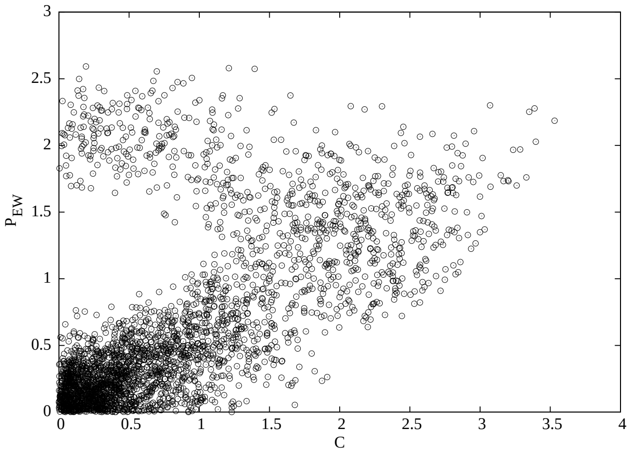

FIG. 1: C versus P_{EW} for $B \to \pi\pi$. Note the concentration of solutions near the origin, and the excluded region for P_{EW} for low C. The amplitudes have been multiplied by 10^8.

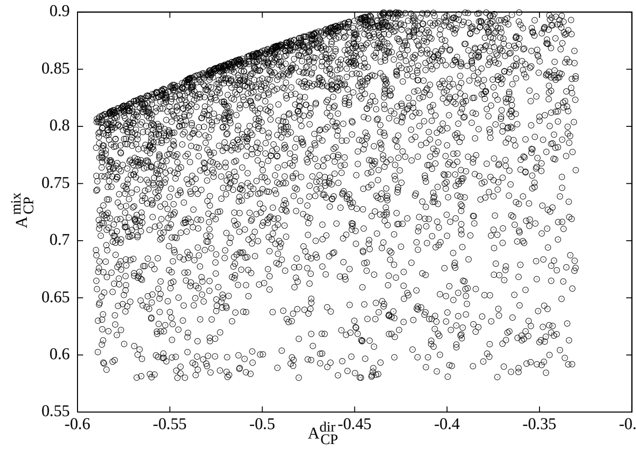

FIG. 3: Direct and mixing-induced CP asymmeries in $B \to \pi^+\pi^-$. The circular arc that bounds the parameter space at the upper left hand corner corresponds to $|a^d_{\pi\pi}|^2 + |a^m_{\pi\pi}|^2 = 1$.

reproduce the BRs of $B \to \pi^+\pi^-$ and $B^+ \to \pi^+\pi^0$ in the right ballpark.

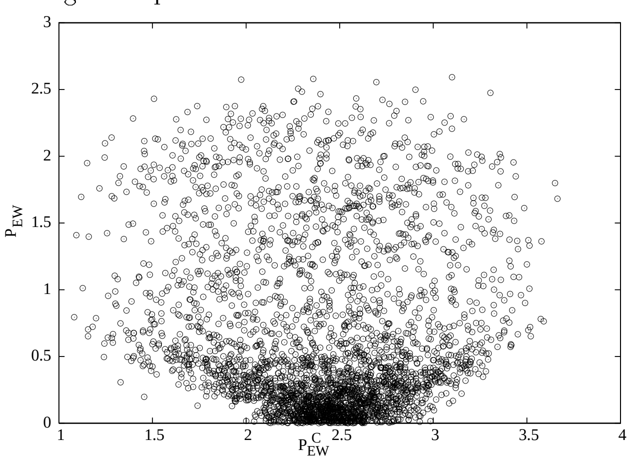

FIG. 2: P_{EW} versus P^C_{EW} for $B \to \pi\pi$. Note the large nonzero allowed values for the latter, and the concentration of solutions for small P_{EW}. The amplitudes have been multiplied by 10^8.

Fig. 3 shows the allowed parameter space of $a^d_{\pi\pi}$ versus $a^m_{\pi\pi}$; this shows a concentration of solutions near the physical boundary $|a^d_{\pi\pi}|^2 + |a^m_{\pi\pi}|^2 \leq 1$.

Let us emphasize again at this point that this treatment is valid if and only if the new physics, if any, contributes to the decay but not to the $B^0-\overline{B^0}$ mixing. Most of the new physics models are of this type; however, there are exceptions. For example, four-generation fermion models contribute mostly in mixing. Supersymmetry with R-parity violation may contribute to only mixing, only decay, or both, depending upon the nonzero couplings chosen. Generically, if NP contributes in $B^0 - \overline{B^0}$ mixing, the following modifications have to be taken into account (an example with R-parity violation has been exhaustively discussed in [16]):

- ΔM_d is affected by NP. Consequently, V_{td}, which is determined from the measurement of ΔM_d, becomes a free parameter, except for constraints coming from the unitarity of the CKM matrix. Obviously, for models with more than three quark generations, this is a very loose constraint. Fortunately, the precise value of V_{td} is not that important in our analysis.

- What is more important is the value of $\sin(2\beta)$. There is a SM value of β, which let us call β_{SM}. We have explicitly assumed that the CP asymmetry in, say, $B \to J/\psi K_S$, measures β_{SM}. If there is NP in mixing, it will measure β which should be different from β_{SM}. Note that this happens even if the NP mixing amplitude is real. The value of β is important in the analysis of CP asymmetries in the $B \to \pi^+\pi^-$ channel. To get a feeling of β_{SM}, one should perform the usual CKM fit for the vertex (ρ, η) of the UT without the ΔM_d constraint. Again, this is not a foolproof prescription; one implicitly assumes that NP does not contribute to, say, ε'/ε.

Even considering all this, it is hard to see how a NP that contributes only to mixing but not to decay can explain the $B \to \pi\pi$ results. It can affect the fit for γ, but cannot generate a large BR for $B \to \pi^0\pi^0$ or a large direct CP asymmetry for $B \to \pi^+\pi^-$.

IV. ANALYSIS: $B \to \pi K$

The analysis for $B \to \pi K$ follows that of $B \to \pi\pi$. There are five new amplitudes, apart from P_{EW}, and hence four new strong phase differences. We scan this ten-dimensional parameter space for possible solutions. Note that R'_b is a small number and hence for all practical purpose $\mathcal{G}'$ can be approximated by unity (but for our analysis we keep it as it is). Also note that these ten parameters are not really all free; the allowed range for P_{EW}, as well as the weak phase γ, are determined from the $B \to \pi\pi$ data, and we use the PQCD result $|T'_c/P'_c| = 0.201 \pm 0.037$. We also equate the strong phases of P'_c and P'_{ct}.

We obtain the following ranges for the amplitudes:

$$\begin{aligned} 0.2 < \quad & T'_c \times 10^8 & < 1.15; \\ 0.8 < \quad & P'_c \times 10^8 & < 4.8; \\ 0 < \quad & P'_{ct} \times 10^8 & < 3.2; \\ 0 < \quad & \tilde{C} \times 10^8 & < 3.8; \\ 0 < \quad & P_{EW} \times 10^8 & < 1.55; \\ 0 < \quad & {P^C_{EW}}' \times 10^8 & < 6.3. \end{aligned} \tag{12}$$

There are two points to note right here. First, $\tilde{C}$ could in principle have a bigger range. We constrained it to be smaller than C of $B \to \pi\pi$ modes. Second, both the electroweak penguin amplitudes can be close to zero; we refer to Figure 4 for a feeling of the solutions. Thus, *there is no apparent EWP anomaly in the data.* We have explicitly checked that solutions are possible even if one puts by hand both the EWP amplitudes equal to zero.

All the strong phase differences have been varied between 0 and 2π. Solutions are allowed for the entire range, unlike $B \to \pi\pi$, where we found a rather constrained fit $174^\circ < \delta_{EC} - \delta_T < 190^\circ$. However, from Figures 5 and 6, there appears to be an interesting correlation between the amplitudes and the strong phases.

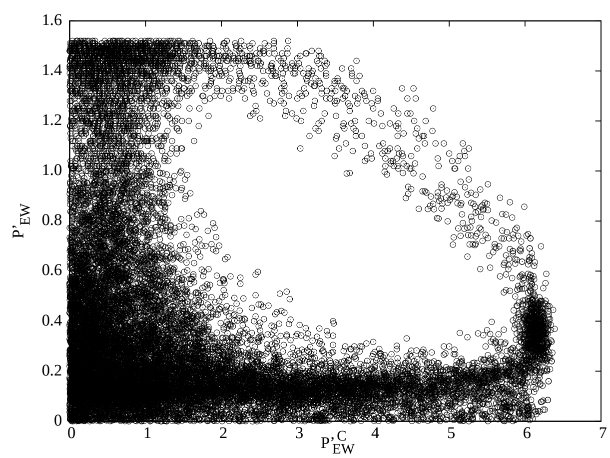

FIG. 4: Allowed solutions for color-allowed and color-suppressed EWP amplitudes, showing that there is no apparent EWP anomaly in $B \to \pi K$ data. The amplitudes have been multiplied by 10^8.

From Figure 5, it appears that small ${P^C_{EW}}'$ solutions are admitted only if T'_c is large: $|T'_c| > 0.55 \times 10^{-8}$. For smaller values of T'_c, ${P^C_{EW}}'$ must be large. This is intuitively clear: after all, one of them should be large enough to generate the required BRs. One could have done this taking T'_c and P'_c to be free parameters; but we wish to follow the theoretical prediction, which is also consistent with an SU(3) flavor symmetry, as far as possible.

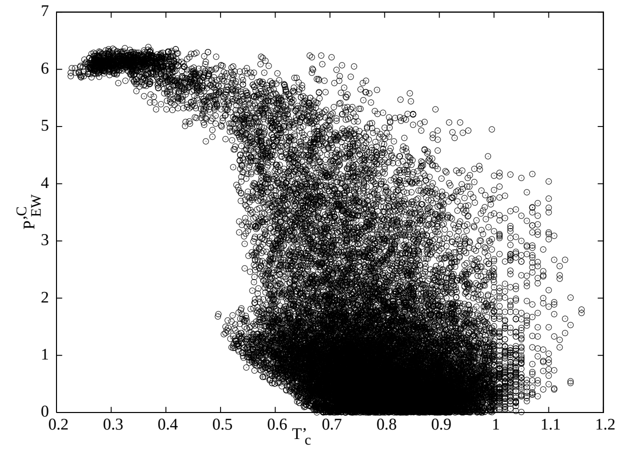

FIG. 5: T'_c versus ${P^C_{EW}}'$ for $B \to \pi K$. Note how small ${P^C_{EW}}'$ solutions necessarily corespond to large T'_c which is consistent with SU(3) relationship. The amplitudes have been multiplied by 10^8.

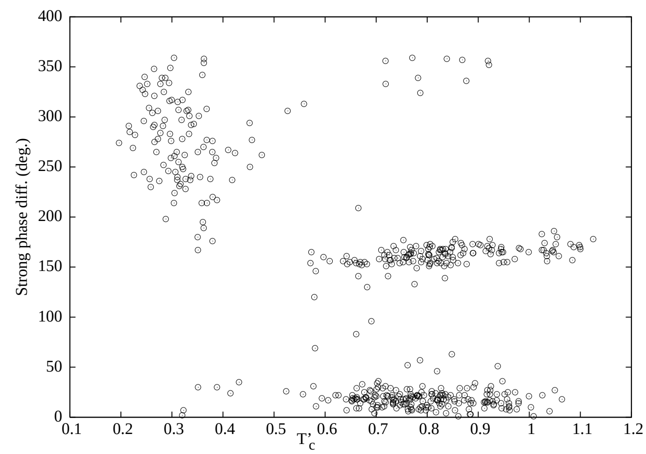

FIG. 6: Allowed solutions for the strong phase difference $\delta'_P - \delta'_T$ as a function of T'_c, multiplied by 10^8.

Figure 6 shows the solutions for the strong phase difference $\delta'_P - \delta'_T$ as a function of T'_c. Note the band around the theoretical prediction of 156° from the PQCD model, which appears only for large T'_c and hence small EWPs. This shows that there is nothing to worry about the EWP sector; but one can accomodate large values, which may come from NP models.

Thus, the SU(3) flavor symmetry is there, as far as the magnitudes of $T_c^{(')}$, $P_c^{(')}$ and $P_{EW}^{(')}$ are concerned. However, the strong phase differences are not necessarily small. If one demands a correlation between both the amplitudes and the strong phases for $B \to \pi\pi$ and $B \to \pi K$, one must invoke large EWP contribution. But such a contribution is there for $B \to \pi\pi$; there is no a priori reason why it should not appear for $B \to \pi K$, and thus help to salvage the flavor symmetry.

V. SUMMARY AND CONCLUSIONS

We have juggled with a number of amplitudes and strong phases. The equations being horrendously coupled, we do not even envisage to attempt an analytic solution. Those solutions necessarily involve some simplification and run the risk of missing the point. Rather, we solved these equations numerically, and the equations being more in number than constraints (and remember that these constraints often have large error bars), found a range of allowed values for the free parameters. It is indeed interesting that one can form some idea about the nature of the solutions.

We assume that the theoretical models can catch the essence of these nonleptonic transitions successfully. There are points where they differ; for example, the way how one takes care of end-point suppresions, higher-twist corrections, or annihilation topologies. Fortunately, such subtleties are not really crucial for our analysis. What is important is the relative strength of the strong penguin amplitudes with respect to the tree amplitude. PQCD, by virtue of a dynamical enhancement mechanism, predicts a larger $|P_c/T_c|$ than other models like CF or QCDF. Since our strategy is to extract the minimal set that does not fit in these model predictions, we take PQCD as our working model. If one takes, for example, QCDF, one needs a larger color-suppressed EWP to explain the $B \to \pi\pi$ data. PQCD also helps to fit γ in the first quadrant, which is compatible with the CKM fit.

We found that the explanation of the $B \to \pi\pi$ data needs a large color-suppressed EWP contribution (the color-allowed EWP may be large, but need not be). This is entirely due to the large BR of $B \to \pi^0\pi^0$. If this data would have been in the expected ballpark, the large direct CP asymmetry in $B \to \pi^+\pi^-$ could be explained by a much smaller EWP contribution. Such a large EWP contribution also means that there should be something worth watching in $B \to \rho\gamma$.

The situation is not so drastic for $B \to \pi K$. Even if one takes the SU(3) predictions of T'_c and P'_c seriously, there are solutions with vanishingly small EWP amplitudes. Of course, there is a price to pay: the other two amplitudes $\tilde{C}$ and P'_{ct} become essentially free, except for the constraints that $|\tilde{C}| < |C|$ and $\delta'_P = \delta'_{CT}$. We have tried to see what happens to the fit if all amplitudes and strong phases are taken from some model (we used CF, because that predicts all the amplitudes and also the strong phases coming from the imaginary parts of the respective Wilson coefficients), and there one indeed requires a sizable EWP contribution!

So, what is the lesson from the $B \to \pi K$ analysis? Probably that it is yet early to say that there is any definite evidence for large EWP amplitudes, but they can be easily accomodated, as is evident from our figures. All the penguins are expected to be enhanced by about a factor of 5 when going from $B \to \pi\pi$ to $B \to \pi K$. This trend can be accomodated easily for the color-allowed EWPs, but appears to be very hard for the color-suppressed ones. Thus, even if the color-allowed EWPs can be explained within the present models, the color-suppressed one poses a serious challenge.

Is the situation so bizarre because we do not understand the dynamics of penguins, or to be more general, that of low-energy QCD? No one knows, but from intuitive arguments it is very hard to establish such a large EWP amplitude compared to the strong penguins. One should not also forget the possibility that the experimental data may change over the years, though such an assumption does not help the analysis.

Is the solution to be found in physics beyond the SM? Again, it is early days, but one may be hopeful, more so because this may provide an indirect evidence for such NP before the Large Hadron Collider (LHC) comes into operation. NP models generate new operators in the effective Hamiltonian, or modify the Wilson coefficients for the existing operators (an example is the Universal Extra Dimension model of Appelquist *et al* [17]). If the interactions are flavor-specific, they will appear as a modification to the tree or the EWP amplitudes. These models include exotic quarks, new Z' gauge bosons, R-parity conserving and violating supersymmetry and so on.

A couple of cautions here. First, one needs the exact structure of a model to be more specific than the present analysis. What one can do is to eliminate models: for example, supersymmetry with flavor alignment will not generate such a large FCNC amplitude. The exact weightage of new operators will also depend on the particular model chosen. Second, this analysis has to be slightly modified if there is new physics in $B^0 - \overline{B^0}$ mixing. This has been discussed in Section 3.

Where should one get confirmatory signals for such NP effects? Again, that depends on the specific model. We have already mentioned $B \to \rho\gamma$; another place would be the semileptonic $b \to d$ decays. For nonleptonic decays, one should carefully analyse the $B \to \rho\pi$, $B \to \rho\rho$, $B_s \to \pi K$, and $B_s \to KK$ channels. An analysis of angular correlations in $B \to \rho\rho$ [18] should be useful.

Acknowledgments

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